report

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1 M6e: Gyroscope with three Axes

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2 Introduction

A gyroscope is a rotating body that exhibits unique dynamical behavior due to the conservation of angular momentum. When subjected to external torques, its axis of rotation undergoes characteristic motions known as precession and nutation. In this experiment, these rotational properties of a gyroscope are investigated in detail.

The moment of inertia of the gyro disk is first determined from measurements of angular acceleration under a known applied torque, applied by a falling mass and from the corresponding rotation speed. Then, the behavior of the gyroscope under no external torques is measured, in order to determine a damping coefficient which will be used in the study of precession and nutation.

Subsequently, the precession and nutation frequencies of the gyroscope are studied as functions of the disk's rotation speed, and the correlations of the same with theoretical relations are examined.

3 Experimental Setup

For the experiment, we used a gyroscope by 3B Scientific with two counterweights, with the following specifications:

Parameter	Symbol	Value	Unit
Diameter of disk	$2R$	250	mm
Mass of disk	m	1500	g
Distance disk–vertical axis	z	165	mm
Counterweight masses	$m_{(}G1_{)}, m_{(}G2_{)}$	1400, 50	g
Additional mass pieces	m_z	47	g
Distance additional mass–vertical axis	z_z	275	mm
Diameter of bobbin	$2r$	65	mm

For measurements of the periods of the gyroscope, the stopwatch on a smartphone was used, and for the measurements of the rotation speed, a Laser RPM meter was used, which used a set of 8 reflective stripes on the gyroscope disk.

3.1 Methodology

For the first task to measure the moment of inertia of the gyroscope, firstly a string was wound on the bobbin behind the disk and then attached to a weight of m_Z. The weight was then held at a height $h = 73\,cm$ from the ground, and then released in order to accelerate the disk to a certain measured rotational speed ω. The time t for the weight to reach the ground was also measured. The experiment was repeated ten times for higher accuracy.

For the second task, the gyro was set into rotation at a certain rotational speed on the order of 500 RPM using the spring. Then the decrease of the rotation speed was measured every 10 s over a period of 120 s.

For the third task, initially, the tripod stand was removed, and the gyroscope was brought into equilibrium by shifting the counterweights. Then, the gyroscope was brought to a certain rotation speed, before a mass was hooked onto the end of the gyroscope. Then at the start and at the end of each half precession period, the rotation speed of the gyro disk was measured, and the time for each half period was measured as well. The experiment was repeated for ten different rotation frequencies and for both one and two masses.

Finally for the fourth task, the gyroscope was brought to a certain rotation speed and then a nutation was generated by a brief vertical impulse on the gyro axis. The time for three nutation periods was measured with the stopwatch and the rotation speed was recorded at the start and end. The experiment was repeated for ten different rotation frequencies.

4 Theoretical Background

4.1 Moment of Inertia

From the lab manual, we see that there are in total three ways to calculate the moment of inertia, I_3. Firstly, we can go from a basic definition of the geometry of the disk:

$$I_3 = \frac{1}{2} \cdot m \cdot R^2 \tag{1}$$

Secondly, we can derive an equation using the basic equations of motion:

$$I_3 = m_Z \cdot r \cdot \left(\frac{g}{\alpha} - r \right) \tag{2}$$

Finally, we can derive I_3 using the conservation of energy:

$$I_3 = \frac{2 \cdot m_Z \cdot g \cdot h}{\omega^2} - m_Z \cdot r^2 \tag{3}$$

In our analysis, we shall derive I_3 through these three methods and compare the accuracy of the last 2 methods with the initial geometrical definition

4.2 Frictional Dampening

We can model the damping of the rotational frequency through the following equation:

$$\omega(t) = \omega_0 e^{-\beta t} \tag{4}$$

We can fit this equation by taking the logarithm of both sides:

$$ln\, \omega(t) = ln\, \omega_0 - \beta t \tag{5}$$

As can be clearly seen, this gives us a linear relation with slope $-\beta$ and intercept \$ln: _0 \$.

4.3 Precession

When a mass is affixed onto the end of the gyroscope, an external torque τ acts on the system, and changes the direction of the angular momentum according to

$\tau = \frac{d\mathbf{L}}{dt}$.

The torque is given by:

$\tau = mgr$,

where m is the attached mass, g the acceleration due to gravity, and \$r \$ the perpendicular distance from the pivot to the center of mass of the gyro disk or additional weight. This torque acts perpendicular to the angular momentum vector, causing the axis to move at right angles to both torque and angular momentum. As a result, instead of toppling, the gyroscope's axis slowly rotates around the vertical direction — a motion known as **precession**.

The angular velocity of precession, Ω_p , is obtained by equating the rate of change of angular momentum to the applied torque:

$\tau = \frac{dL}{dt} = L\Omega_p \sin\theta$,

where θ is the inclination angle of the gyro axis. For small angles (or when $\sin\theta \approx 1$), this gives

$\Omega_p = \frac{mgr}{I_3 \omega}$.

In terms of measurable rotation and precession frequencies $f = \omega_3/2\pi$ and $f_p = \Omega_p/2\pi$, this becomes $f_p = \frac{mgr}{4\pi^2 I_3} \cdot \frac{1}{f}$.

This expression shows that the precession frequency f_p is inversely proportional to the spin frequency f . A faster-spinning gyroscope experiences a slower precession, illustrating the stabilizing effect of angular momentum.

Experimentally, the precession period is measured for different rotation speeds of the gyro disk. By plotting the precession frequency f_p as a function of the reciprocal of the rotation frequency $1/f$, a linear relationship is obtained with slope

slope $= \frac{mgr}{4\pi^2 I_3}$.

4.4 Nutation

When the gyroscope is slightly disturbed from its equilibrium orientation whilst rotating with angular speed ω, the axis does not simply return smoothly to its original position. Instead, due