Machine Learning – Bayesian learning exercises

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Bayesian learning exercises

X_1	X_2	X_3	Y
0	0	1	**0**
0	1	0	**0**
1	1	0	**0**
0	0	1	**1**
1	1	1	**1**
1	0	0	**1**
1	1	0	**1**

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Bayesian learning exercises

Y = 1 :

$$if => P(h|D) = \frac{P(D|h)P(h)}{\cancel{P(D)} => not\ used} =>$$

$$P\left(Y = 1 \,\middle|\, X_1 = 0 \,\&\, X_2 = 0 \,\&\, X_3 = 1\right) = ?$$

$$P\left(Y = 0 \,\middle|\, X_1 = 0 \,\&\, X_2 = 0 \,\&\, X_3 = 1\right) = ?$$

$$P\left(Y = 1 \,\middle|\, X_1 = 0 \,\&\, X_2 = 0 \,\&\, X_3 = 1\right)$$
$$= P(X_1 = 0 \,\& X_2 = 0 \,\&\, X_3 = 1) * P(Y = 1)$$
$$= P(X_1 = 0|Y = 1) * P(X_2 = 0|Y = 1) * P(X_3 = 0|Y = 1)$$

$$P(Y = 1) = \frac{4}{7} = 0.57$$

$$P(X_1 = 0|Y = 1) = \frac{1}{4} = 0.25$$

$$P(X_2 = 0|Y = 1) = \frac{2}{4} = 0.5$$

$$P(X_3 = 1|Y = 1) = \frac{2}{4} = 0.5$$

$$P(Y = 1 \,|\, X_1 = 0 \,\&\, X_2 = 0 \,\&\, X_3 = 1) = 0.25 * 0.5 * 0.5 * 0.57 = 0.035$$

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Y = 0 :

$$P\left(Y = 0 \,\middle|\, X_1 = 0 \,\&\, X_2 = 0 \,\&\, X_3 = 1\right)$$

$$= P(X_1 = 0 \,\& X_2 = 0 \,\&\, X_3 = 1) * P(Y = 0)$$

$$= P(X_1 = 0|Y = 0) * P(X_2 = 0|Y = 0) * P(X_3 = 0|Y = 0)$$

$$P(Y = 0) = \frac{3}{7} = 0.42$$

$$P(X_1 = 0|Y = 0) = \frac{2}{3} = 0.66$$

$$P(X\_2 = 0 \,|\, Y = 0) = 1/3 = 0.33)$$

$$P(Y = 0 \,|\, X\_1 = 0 \,\&\, X\_2 = 0 \,\&\, X\_3 = 1)$$

$$= 0.33 * 0.33 * 0.66 * 0.42 = 0.030$$

$$P(X_3 = 1|Y = 0) = \frac{1}{3} = 0.33$$

$$P\left(Y = 1 \,\middle|\, X_1 = 0 \,\&\, X_2 = 0 \,\&\, X_3 = 1\right) = \frac{0.035}{0.035 + 0.030}$$

$$= 0.538 => 53\%$$

$$P\left(Y = 0 \,\middle|\, X_1 = 0 \,\&\, X_2 = 0 \,\&\, X_3 = 1\right) = \frac{0.030}{0.035 + 0.030}$$

$$= 0.461 => 46\%$$

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Step 1 :

X1:

0 -> 1 incorrect, 3 correct

1 -> 1 incorrect, 2 correct

X2 :

0 -> 2 incorrect, 2 correct

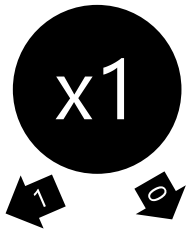
1 -> 2 incorrect, 1 correct

X3 :

0 -> 1 incorrect, 2 correct

1 -> 2 incorrect, 2 correct

•Step 1 Result : Root node is X1



Step 2 :

X2 :

1 -> 1 incorrect, 0 correct

0 -> 1 incorrect, 1 correct

X3 :

1 -> 0 incorrect, 1 correct

0 -> 0 incorrect, 1 correct

Step 3 : در این مسئله نمی توان درخت تصمیم رسم کرد.

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X_1	X_2	X_3	Y
1	0	1	**1**
1	1	1	**1**
0	1	1	**0**
1	1	0	**0**
1	0	1	**0**
0	0	0	**1**
0	0	0	**1**
0	0	1	**0**

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$$P(x1 = 1|Y = 1)\text{^}P(x2 = 1|Y = 1)\ \text{^}\ P(x3 = 0|Y = 1)\text{^}P(Y = 1) = 0.5 + 0.25 + 0.5 + 0.5 = 0.03125$$

$$P(x1 = 1|Y = 0)\text{^}P(x2 = 1|Y = 0)\ \text{^}\ P(x3 = 0|Y = 0)\text{^}P(Y = 0) = 0.5 + 0.25 + 0.5 + 0.5 = 0.03125$$

$$\frac{0.03125}{0.031125 + 0.031125} = 0.5 => 50\%$$

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$P(x1 = 1|Y = 1)\text{^}P(x2 = 1|Y = 1)\text{^}P(Y = 1)$
$= 0.5\text{^}0.25\text{^}.0.5 = 0.0625$

$P(x1 = 1|Y = 0)\text{^}P(x2 = 1|Y = 0)\text{^}P(Y = 1)$
$= 0.5\text{^}0.5\text{^}.0.5 = 0.125$

$$\frac{0.0625}{0.0625 + 0.125} = 0.33 = 33\%$$

$$\frac{0.125}{0.0625 + 0.125} = 0.125 = 12\%$$

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