الف :

Y = 1:

$$if => P(h|D) = \frac{P(D|h)P(h)}{P(D)} => not used =>$$

$$P(Y = 1 | X_1 = 0 \& X_2 = 0 \& X_3 = 1) = ?$$

$$P(Y = 0 | X_1 = 0 \& X_2 = 0 \& X_3 = 1) = ?$$

$$P(Y = 1 | X_1 = 0 \& X_2 = 0 \& X_3 = 1) = P(X_1 = 0 \& X_2 = 0 \& X_3 = 1) * P(Y = 1)$$

$$= P(X_1 = 0|Y = 1) * P(X_2 = 0|Y = 1) * P(X_3 = 0|Y = 1)$$

$$P(Y = 1) = \frac{4}{7} = 0.57$$

$$P(Y = 0|Y = 1) = \frac{1}{7} = 0.25$$

$$P(X_1 = 0|Y = 1) = \frac{1}{4} = 0.25$$

$$P(X_2 = 0|Y = 1) = \frac{2}{4} = 0.5$$

$$P(X_3 = 1|Y = 1) = \frac{2}{4} = 0.5$$

$$P(Y = 1 | X_1 = 0 \& X_2 = 0 \& X_3 = 1) = 0.25 * 0.5 * 0.5 * 0.57 = 0.035$$

Y = 0:

$$P(Y = 0 | X_1 = 0 \& X_2 = 0 \& X_3 = 1) = P(X_1 = 0 \& X_2 = 0 \& X_3 = 1) * P(Y = 0)$$
  
=  $P(X_1 = 0 | Y = 0) * P(X_2 = 0 | Y = 0) * P(X_3 = 0 | Y = 0)$ 

$$P(Y=0) = \frac{3}{7} = 0.42$$

$$P(X_1 = 0|Y = 0) = \frac{2}{3} = 0.66$$

$$P(X_2 = 0 \mid Y = 0) = 1/3 = 0.33)$$

$$P(Y = 0 \mid X_1 = 0 \& X_2 = 0 \& X_3 = 1) = 0.33 * 0.33 * 0.66 * 0.42 = 0.030$$

$$P(X_3 = 1|Y = 0) = \frac{1}{3} = 0.33$$

$$P(Y = 1 | X_1 = 0 \& X_2 = 0 \& X_3 = 1) = \frac{0.035}{0.035 + 0.030} = 0.538 = 53\%$$

$$P(Y = 0 \mid X_1 = 0 \& X_2 = 0 \& X_3 = 1) = \frac{0.030}{0.035 + 0.030} = 0.461 = > 46\%$$

ب :

Step 1:

X1:

0 -> 1 incorrect, 3 correct

1 -> 1 incorrect, 2 correct

X2:

0 -> 2 incorrect, 2 correct

1 -> 2 incorrect, 1 correct

X3:

0 -> 1 incorrect, 2 correct

1 -> 2 incorrect, 2 correct

⇒ Step 1 Result : Root node is X1



Step 2:

X2:

1 -> 1 incorrect, 0 correct

0 -> 1 incorrect, 1 correct

X3:

1 -> 0 incorrect, 1 correct

0 -> 0 incorrect, 1 correct

در این مسئله نمی توان درخ تصمیم رسم کرد. : Step 3

سوال ۲:

الف :

$$P(x1 = 1|Y = 1)^P(x2 = 1|Y = 1)^P(x3 = 0|Y = 1)^P(Y = 1) = 0.5 + 0.25 + 0.5 + 0.5 = 0.03125$$

$$P(x1 = 1|Y = 0)^P(x2 = 1|Y = 0)^P(x3 = 0|Y = 0)^P(Y = 0) = 0.5 + 0.25 + 0.5 + 0.5 = 0.03125$$

$$\frac{0.03125}{0.031125 + 0.031125} = 0.5 => 50\%$$

: ب

$$P(x1 = 1|Y = 1)^P(x2 = 1|Y = 1)^P(Y = 1) = 0.5^0.25^0.05 = 0.0625$$

$$P(x1 = 1|Y = 0)^P(x2 = 1|Y = 0)^P(Y = 1) = 0.5^0.5^0.5 = 0.125$$

$$\frac{0.0625}{0.0625 + 0.125} = 0.33 = 33\%$$

$$\frac{0.125}{0.0625 + 0.125} = 0.125 = 12\%$$