

PILSTM

April 13, 2025

```
[1]: import numpy as np
import os
import sys
import gym
import zipfile
import autograd
import matplotlib.gridspec as gridspec
# Use tf.random.set_seed for TensorFlow 2.0 and above
#from scipy.signal.waveforms import square
import matplotlib.pyplot as plt
from scipy.integrate import solve_ivp
from sklearn.model_selection import train_test_split
import random
import tensorflow as tf
from tensorflow import keras
from tensorflow.keras.models import Sequential, model_from_json
from keras.layers import Dense
from keras.layers import Input
from tensorflow.keras import layers
```

```
2025-04-13 00:03:15.314761: I external/local_xla/xla/tsl/cuda/cudart_stub.cc:32]
Could not find cuda drivers on your machine, GPU will not be used.
2025-04-13 00:03:15.513475: I external/local_xla/xla/tsl/cuda/cudart_stub.cc:32]
Could not find cuda drivers on your machine, GPU will not be used.
2025-04-13 00:03:15.590707: E
external/local_xla/xla/stream_executor/cuda/cuda_fft.cc:485] Unable to register
cuFFT factory: Attempting to register factory for plugin cuFFT when one has
already been registered
2025-04-13 00:03:15.774702: E
external/local_xla/xla/stream_executor/cuda/cuda_dnn.cc:8454] Unable to register
cuDNN factory: Attempting to register factory for plugin cuDNN when one has
already been registered
2025-04-13 00:03:15.813402: E
external/local_xla/xla/stream_executor/cuda/cuda_blas.cc:1452] Unable to
register cuBLAS factory: Attempting to register factory for plugin cuBLAS when
one has already been registered
2025-04-13 00:03:16.077384: I tensorflow/core/platform/cpu_feature_guard.cc:210]
This TensorFlow binary is optimized to use available CPU instructions in
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performance-critical operations.

To enable the following instructions: AVX2 FMA, in other operations, rebuild TensorFlow with the appropriate compiler flags.

2025-04-13 00:03:17.664630: W

tensorflow/compiler/tf2tensorrt/utils/py_utils.cc:38] TF-TRT Warning: Could not find TensorRT

```
[15]: # @title Hp meristor's state variable:
from IPython.display import display, Math

latex_equation = r"""
\text{State variable:}\quad \frac{dw}{dt} = \mu_v \text{\textit{v}} \cdot \left( \frac{R_{\text{on}}}{D^2} \right) \cdot i(t) \cdot f(w) \\
\text{Window function:}\quad f(w) = w(1 - w) \\
\text{state variable in this code is w:}\quad w = \frac{X}{D}
"""
display(Math(latex_equation))
```

State variable: $\frac{dw}{dt} = \mu_v \cdot \left(\frac{R_{\text{on}}}{D^2} \right) \cdot i(t) \cdot f(w)$

Window function: $f(w) = w(1 - w)$

state variable in this code is w: $w = \frac{X}{D}$

```
[2]: import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import solve_ivp
from sklearn.preprocessing import StandardScaler, MinMaxScaler

# ----- Physical Parameters -----
frequency = 1
A_train = 1.5
W_train = 2 * np.pi * frequency

mu_v = 10**4
D = 60
r_on = 0.1
r_off = 16
r0 = 4
w0 = (r0 - r_off) / (r_on - r_off)

points_per_period = 600

total_points = 10 * points_per_period

# ----- Solving ODE -----
def f(t, w, A, W, mu_v, D, r_on, r_off):
    k = mu_v * (r_on / D**2)
```

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    f_w = w * (1 - w)
    r = r_on * w + r_off * (1 - w)
    I = A * np.sin(W * t) / r
    return I * f_w * k

t_all = np.linspace(0, 6, total_points)
sol_all=solve_ivp(f, (0, 6), [w0], t_eval=t_all, args=(A_train, W_train, mu_v, u
    ↪D, r_on, r_off),
    method='RK45', max_step=0.001)

w_all = sol_all.y[0]
v_all = A_train * np.sin(W_train * t_all)
r_all = r_on * w_all + r_off * (1 - w_all)
I_all = v_all / r_all

X_all = np.column_stack([t_all[:-1], w_all[:-1], I_all[:-1]])
y_all = w_all[1:]

# ----- Split Data -----
test_ratio = 0.2
test_size = int(test_ratio * len(X_all))

test_index = np.arange(len(X_all) - test_size, len(X_all))
train_index = np.arange(0, len(X_all) - test_size)

X_train, X_test = X_all[train_index], X_all[test_index]
y_train, y_test = y_all[train_index], y_all[test_index]

print(X_train.shape)
print(X_test.shape)
print(y_train.shape)
print(y_test.shape)
plt.figure(figsize=(10, 4))
plt.plot(t_all[:-1], w_all[:-1], label="Original Data", alpha=0.3)
plt.plot(X_train[:, 0], y_train, color='blue', alpha=0.5, label='Train')
plt.plot(X_test[:, 0], y_test, color='red', alpha=0.5, label='Test')
plt.xlabel("Time")
plt.ylabel("w (State Variable)")
plt.title("Train/Test Split for Time-Series Memristor Data")
#plt.grid()
plt.legend()
plt.savefig("rungekutta_train_test.pdf")
plt.show()

```

```

from sklearn.preprocessing import StandardScaler, MinMaxScaler
import numpy as np

# -----
scaler_time = StandardScaler()
X_train[:, 0] = scaler_time.fit_transform(X_train[:, 0].reshape(-1, 1)).
    ↪flatten()
X_test[:, 0] = scaler_time.transform(X_test[:, 0].reshape(-1, 1)).flatten()

scaler_features = MinMaxScaler(feature_range=(-1, 1))
X_train[:, 1:] = scaler_features.fit_transform(X_train[:, 1:])
X_test[:, 1:] = scaler_features.transform(X_test[:, 1:])

scaler_y = MinMaxScaler(feature_range=(-1, 1))
y_train_scaled = scaler_y.fit_transform(y_train.reshape(-1, 1))
y_test_scaled = scaler_y.transform(y_test.reshape(-1, 1))

X_train_scaled = X_train.copy()
X_test_scaled = X_test.copy()

# ----- Create Sequences -----
sequence_length = 10 #
def create_sequences(X, y, sequence_length):
    X_seq, y_seq = [], []
    for i in range(len(X) - sequence_length):
        X_seq.append(X[i:i+sequence_length])
        y_seq.append(y[i+sequence_length])
    return np.array(X_seq), np.array(y_seq).reshape(-1, 1)

X_train_seq, y_train_seq = create_sequences(X_train_scaled, y_train_scaled,
    ↪sequence_length)
X_test_seq, y_test_seq = create_sequences(X_test_scaled, y_test_scaled,
    ↪sequence_length)

print("Mean of X_train_scaled:", np.mean(X_train, axis=0))
print("Std of X_train_scaled:", np.std(X_train, axis=0))
print("Mean of X_test_scaled:", np.mean(X_test, axis=0))
print("Std of X_test_scaled:", np.std(X_test, axis=0))
print("Mean of y_train_scaled:", np.mean(y_train_scaled))
print("Std of y_train_scaled:", np.std(y_train_scaled))

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print("Mean of y_test_scaled:", np.mean(y_test_scaled))
print("Std of y_test_scaled:", np.std(y_test_scaled))

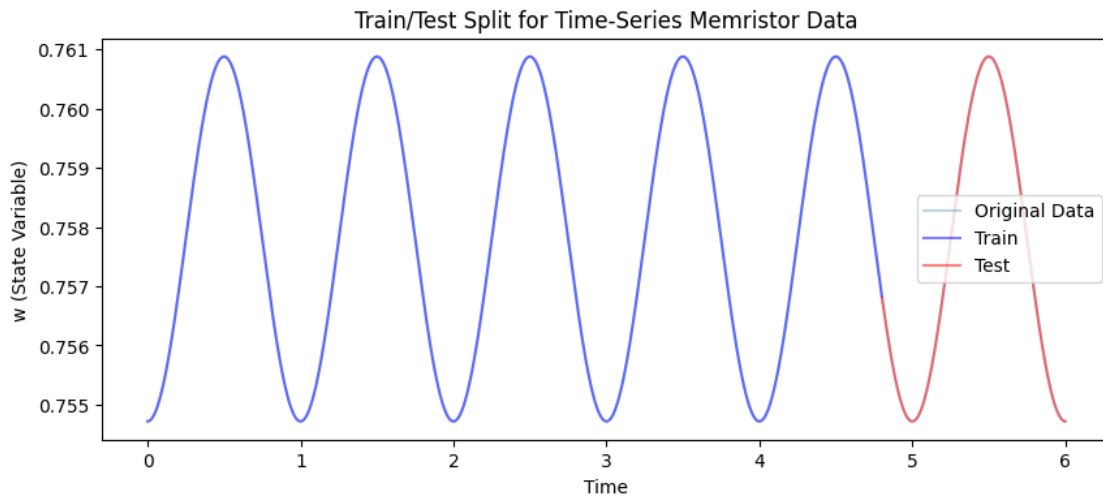
print(f'X_train_seq shape: {X_train_seq.shape}, y_train_seq shape: {y_train_seq.
↪shape}')
print(f'X_test_seq shape: {X_test_seq.shape}, y_test_seq shape: {y_test_seq.
↪shape}')

```

```

(4800, 3)
(1199, 3)
(4800,)
(1199,)

```



```

Mean of X_train_scaled: [-6.65671222e-17  3.04609012e-02  2.26622042e-02]
Std of X_train_scaled: [1.          0.70298873  0.71031816]
Mean of X_test_scaled: [ 2.16470271 -0.12662702 -0.09072466]
Std of X_test_scaled: [0.24979159  0.70958015  0.68660316]
Mean of y_train_scaled: 0.03060324437440435
Std of y_train_scaled: 0.7028492870931501
Mean of y_test_scaled: -0.12719823824974452
Std of y_test_scaled: 0.7100073018388313
X_train_seq shape: (4790, 10, 3), y_train_seq shape: (4790, 1)
X_test_seq shape: (1189, 10, 3), y_test_seq shape: (1189, 1)

```

```

[3]: import numpy as np
import random
import matplotlib.pyplot as plt
import tensorflow as tf

```

```

from tensorflow.keras.layers import Dense, Input, LSTM # Import LSTM here
from tensorflow.keras import Sequential, regularizers
from tensorflow.keras.layers import Dropout

class PhysicsInformedRNN(tf.keras.Model):
    def __init__(self, **kwargs):
        super().__init__(**kwargs)
        self.RNN = Sequential([
            LSTM(35, return_sequences=True, activation='tanh',
kernel_regularizer=tf.keras.regularizers.l2(1e-4)),
            Dropout(0.06),
            LSTM(39, return_sequences=True, activation='tanh',
kernel_regularizer=tf.keras.regularizers.l2(1e-4)), #
            Dropout(0.06),
            Dense(29, activation='tanh', kernel_regularizer=tf.keras.
kernel_regularizer=l2(1e-4)),
            Dense(1,)
        ])

    def call(self, inputs):
        return self.RNN(inputs)

    def build(self, input_shape):
        self.RNN.build(input_shape)
        super().build(input_shape)

pinn_rnn = PhysicsInformedRNN()
pinn_rnn.build((None, sequence_length, 3))
pinn_rnn.summary()
#####
from tensorflow.keras.optimizers import Adam
from tensorflow.keras.callbacks import ModelCheckpoint
import tensorflow as tf
import matplotlib.pyplot as plt

# -----

```

```

WARNING: All log messages before absl::InitializeLog() is called are written to
STDERR
I0000 00:00:1744490008.960806    6129 cuda_executor.cc:1015] successful NUMA
node read from SysFS had negative value (-1), but there must be at least one
NUMA node, so returning NUMA node zero. See more at
https://github.com/torvalds/linux/blob/v6.0/Documentation/ABI/testing/sysfs-bus-
pci#L344-L355
2025-04-13 00:03:28.961320: W

```

tensorflow/core/common_runtime/gpu/gpu_device.cc:2343] Cannot dlopen some GPU libraries. Please make sure the missing libraries mentioned above are installed properly if you would like to use GPU. Follow the guide at <https://www.tensorflow.org/install/gpu> for how to download and setup the required libraries for your platform.
 Skipping registering GPU devices...

Model: "physics_informed_rnn"

Layer (type)	Output Shape	Param #
sequential (Sequential)	?	18,350

Total params: 18,350 (71.68 KB)

Trainable params: 18,350 (71.68 KB)

Non-trainable params: 0 (0.00 B)

```
[4]: NO = 1
Nf = X_train_seq.shape[0]
Nd = y_train_seq.shape[0]

#col_weights = tf.Variable(1.0) # for ode loss
#u_weights = tf.Variable(1.0) # for ic loss
#data_weights = tf.Variable(1.5) # for data loss
#-----
col_weights = tf.Variable(tf.ones(Nf), dtype=tf.float32) #weight of ODE
data_weights = tf.Variable(tf.ones(Nd), dtype=tf.float32) #weight of data
u_weights = tf.Variable(tf.ones(NO), dtype=tf.float32)

optimizer_col_weights = tf.keras.optimizers.Adam(learning_rate=1e-2)
optimizer_data_weights = tf.keras.optimizers.Adam(learning_rate=1e-4)

print("Shape of col_weights:", col_weights.shape)
print("Shape of ode_res:", data_weights .shape)

print("done")
```

Shape of col_weights: (4790,)
 Shape of ode_res: (4790,)
 done

```

[5]: def compute_loss(X, y_true, model, col_weights, u_weights, data_weights):
    X = tf.convert_to_tensor(X, dtype=tf.float32)
    y_true = tf.convert_to_tensor(y_true, dtype=tf.float32)

    with tf.GradientTape(persistent=True) as tape:

        w_pred_sequence = model(X)
        w_pred = w_pred_sequence[:, -1, :]

        I_t = X[:, -1, 0:1]
        w_prev = X[:, -1, 1:2]
        T = X[:, -1, 2:3]

        f_w = w_pred * (1 - w_pred)

        with tf.GradientTape() as g:
            g.watch(T)

            inputs = tf.concat([I_t, w_prev, T], axis=1)
            w_pred_g = model(tf.expand_dims(inputs, axis=1)) # (batch_size, 1,
↪3)

            dw_dt = g.gradient(w_pred_g, T)

            ode_res = dw_dt - mu_v * (r_on / D**2) * I_t * f_w

            ode_loss = tf.reduce_mean(tf.square(col_weights[:, tf.newaxis] *
↪ode_res))

            data_loss = tf.reduce_mean(tf.square(data_weights * (w_pred - y_true)))

            #ic_input = tf.convert_to_tensor([[0.0, y_train[0], X_train[0, 2]]],
↪dtype=tf.float32)
            ic_input = X[:, 0:1, :]
            ic_pred = model(ic_input)[:, -1, :]

            ic_true = tf.convert_to_tensor(y_train[0], dtype=tf.float32)
            ic_loss = tf.reduce_mean(tf.square(u_weights * (ic_pred - ic_true)))

            total_loss = data_loss + ode_loss + ic_loss

        return total_loss, ode_loss, data_loss, ic_loss

##### training loop

```



```

from tensorflow.keras.optimizers.schedules import ExponentialDecay
#earning_rate = tf.keras.optimizers.schedules.ExponentialDecay(
    #initial_learning_rate=1e-3
    #decay_steps=10000 ,
    #decay_rate=0.75
#
learning_rate=1e-3

optimizer=tf.keras.optimizers.Adam(learning_rate, clipnorm=1.0)

epochs=700

train_loss_record,ode_loss_record,data_loss_record,ic_loss_record=[],[],[],[]
for epoch in range(epochs):
    with tf.GradientTape(persistent=True) as tape:
        total_loss, ode_loss, data_loss, ic_loss = compute_loss(X_train_seq,
↪y_train_seq
                                                , pinn_rnn, col_weights,
↪u_weights, data_weights)

        grads = tape.gradient(total_loss, pinn_rnn.trainable_variables)

        grads_data = tape.gradient(data_loss, pinn_rnn.trainable_variables)
        grads_ode = tape.gradient(ode_loss, pinn_rnn.trainable_variables)

        grads_col = tape.gradient(ode_loss, col_weights)
        grads_data_weights = tape.gradient(data_loss, data_weights)

        optimizer.apply_gradients(zip(grads, pinn_rnn.trainable_variables))

        if grads_col is not None:
            optimizer_col_weights.apply_gradients(zip([grads_col], [col_weights]))
        if grads_data_weights is not None:
            optimizer_data_weights.apply_gradients(zip([grads_data_weights],
↪[data_weights]))

        train_loss_record.append(total_loss.numpy())
        ode_loss_record.append(ode_loss.numpy())
        data_loss_record.append(data_loss.numpy())
        ic_loss_record.append(ic_loss.numpy())

```

```

if (epoch + 1) % 100 == 0:
    print(f"Epoch {epoch+1}/{epochs} | "
          f"Total Loss: {train_loss_record[-1]:.6f} | "
          f"ODE Loss: {ode_loss_record[-1]:.6f} | "
          f"Data Loss: {data_loss_record[-1]:.6f} | "
          f"IC Loss: {ic_loss_record[-1]:.6f} | ")

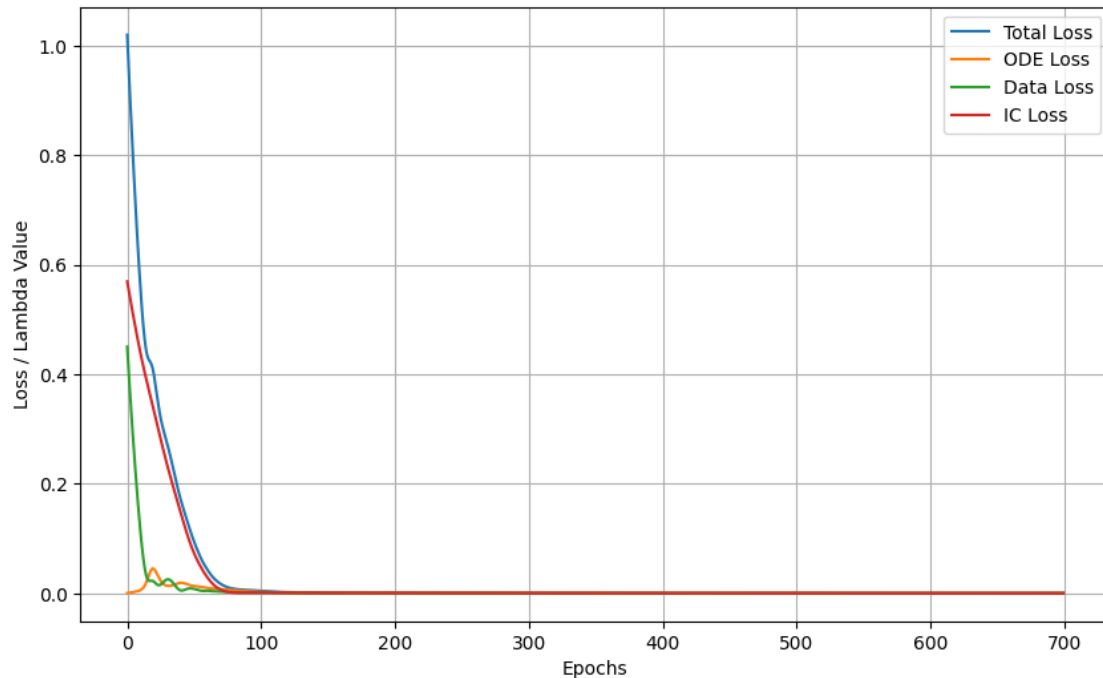
# ----- Plot Adaptive Lambda Values -----
plt.figure(figsize=(10, 6))
plt.plot(train_loss_record, label='Total Loss')
plt.plot(ode_loss_record, label='ODE Loss')
plt.plot(data_loss_record, label='Data Loss')
plt.plot(ic_loss_record, label='IC Loss')
plt.xlabel("Epochs")
plt.ylabel("Loss / Lambda Value")
plt.legend()
plt.grid(True)
plt.show()

```

```

Epoch 100/700 | Total Loss: 0.004325 | ODE Loss: 0.002554 | Data Loss: 0.000665
| IC Loss: 0.001106 |
Epoch 200/700 | Total Loss: 0.000526 | ODE Loss: 0.000045 | Data Loss: 0.000055
| IC Loss: 0.000426 |
Epoch 300/700 | Total Loss: 0.000265 | ODE Loss: 0.000013 | Data Loss: 0.000020
| IC Loss: 0.000232 |
Epoch 400/700 | Total Loss: 0.000132 | ODE Loss: 0.000007 | Data Loss: 0.000009
| IC Loss: 0.000116 |
Epoch 500/700 | Total Loss: 0.000061 | ODE Loss: 0.000004 | Data Loss: 0.000005
| IC Loss: 0.000052 |
Epoch 600/700 | Total Loss: 0.000027 | ODE Loss: 0.000003 | Data Loss: 0.000003
| IC Loss: 0.000021 |
Epoch 700/700 | Total Loss: 0.000012 | ODE Loss: 0.000002 | Data Loss: 0.000002
| IC Loss: 0.000008 |

```



```
[6]: w_pred_seq = pinn_rnn.predict(X_test_seq)
w_pred = w_pred_seq[:, -1, :]

w_pred_original = scaler_y.inverse_transform(w_pred)

y_test_original = scaler_y.inverse_transform(y_test_seq)

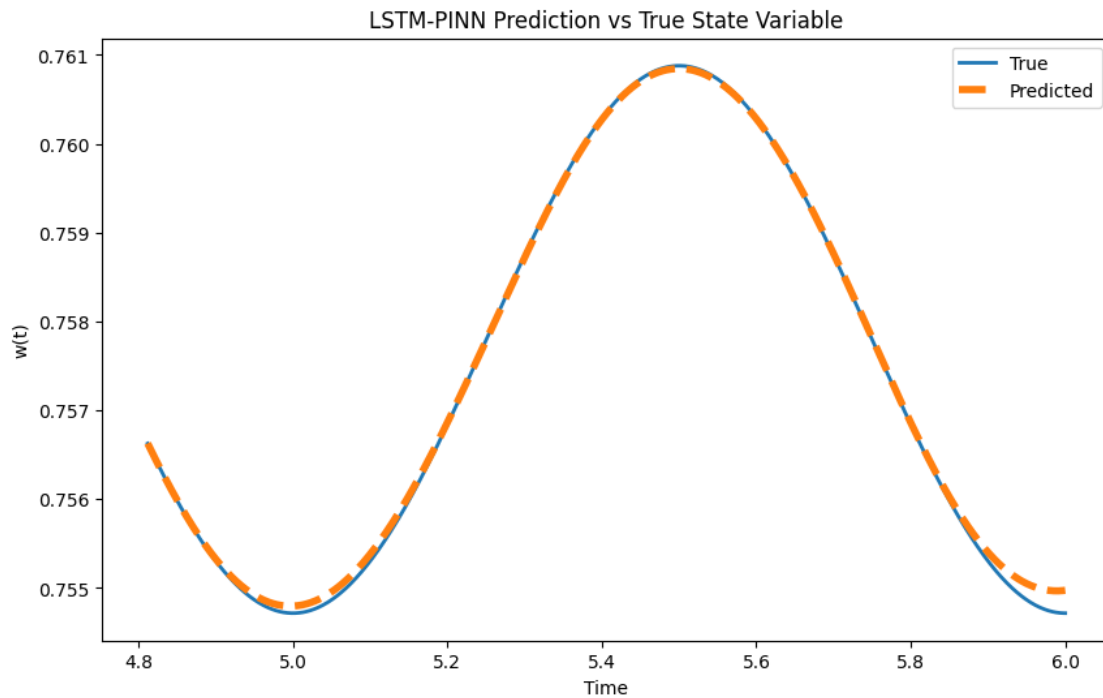
t_test_plot = t_all[len(t_all) - len(w_pred_original):]

# ----- Plotting -----
plt.figure(figsize=(10, 6))
plt.plot(t_test_plot, y_test_original, label='True', linewidth=2)
plt.plot(t_test_plot, w_pred_original, label='Predicted', linestyle='--',
        linewidth=4)
plt.xlabel('Time')
plt.ylabel('w(t)')
plt.title('LSTM-PINN Prediction vs True State Variable')
plt.savefig("testlstm.pdf")
plt.legend()

plt.show()
```

38/38

1s 8ms/step



```
[7]: import os

save_dir = "./models"
if not os.path.exists(save_dir):
    os.makedirs(save_dir)

save_path = os.path.join(save_dir, "my_pinn_model.keras")

pinn_rnn.save(save_path)
print(f"Model saved at: {save_path}")
from tensorflow.keras.models import load_model

save_path = "./models/my_pinn_model.keras"

if os.path.exists(save_path):
    loaded_pinn_model = load_model(save_path,
    ↪ custom_objects={'PhysicsInformedRNN': PhysicsInformedRNN})
    print(" Model loaded successfully!")
else:
    print(" Error: Model file not found!")
```

Model saved at: ./models/my_pinn_model.keras

Model loaded successfully!

```

[9]: import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import solve_ivp
from sklearn.preprocessing import StandardScaler, MinMaxScaler
import tensorflow as tf

# ----- Physical Parameters -----
frequency = 5
A_train = 10
W_train = 2 * np.pi * frequency
mu_v = 10**4
D = 60
r_on = 0.1
r_off = 16
r0 = 4
w0 = (r0 - r_off) / (r_on - r_off)

points_per_period = 600
total_points = 10 * points_per_period

# ----- ODE Solver -----
def f(t, w, A, W, mu_v, D, r_on, r_off):
    k = mu_v * (r_on / D**2)
    f_w = w * (1 - w)
    r = r_on * w + r_off * (1 - w)
    I = A * np.sin(W * t) / r
    return I * f_w * k

t_all = np.linspace(0, 1, total_points)
sol_all = solve_ivp(
    f, (0, 1), [w0], t_eval=t_all,
    args=(A_train, W_train, mu_v, D, r_on, r_off),
    method='RK45',
    max_step=0.001
)

# ----- Data Preparation -----
w_all = sol_all.y[0]
v_all = A_train * np.sin(W_train * t_all)
r_all = r_on * w_all + r_off * (1 - w_all)
I_all = v_all / r_all

X_all = np.column_stack([t_all[:-1], w_all[:-1], I_all[:-1]])
y_all = w_all[1:]

# ----- Data Normalization -----
X_all_scaled = X_all.copy()

```

```

scaler_time = StandardScaler()
X_all_scaled[:, 0] = scaler_time.fit_transform(X_all_scaled[:, 0].reshape(-1, 1)).flatten()

scaler_features = MinMaxScaler(feature_range=(-1, 1))
X_all_scaled[:, 1:] = scaler_features.fit_transform(X_all_scaled[:, 1:])

scaler_y = MinMaxScaler(feature_range=(-1, 1))
y_all_scaled = scaler_y.fit_transform(y_all.reshape(-1, 1))

# ----- Create Sequences -----
sequence_length = 10
def create_sequences(X, y, sequence_length):
    X_seq, y_seq = [], []
    for i in range(len(X) - sequence_length):
        X_seq.append(X[i:i+sequence_length])
        y_seq.append(y[i+sequence_length])
    return np.array(X_seq), np.array(y_seq).reshape(-1, 1)

X_seq, y_seq = create_sequences(X_all_scaled, y_all_scaled, sequence_length)

# ----- PINN Prediction -----
w_pred_seq_new = pinn_rnn.predict(X_seq)
w_pred = w_pred_seq_new[:, -1, :]
w_pred_original_new = scaler_y.inverse_transform(w_pred)
y_all_rescaled = scaler_y.inverse_transform(y_seq)

t_test_plot = t_all[:len(w_pred_original_new)]

# ----- Compute Memristance and Current -----
r_M_pred = r_on * w_pred_original_new + r_off * (1.0 - w_pred_original_new)

print(f"Shape of v_all[:-1]: {v_all[:-1].shape}")
print(f"Shape of r_M_pred.flatten(): {r_M_pred.flatten().shape}")

min_length = min(len(v_all[:-1]), len(r_M_pred.flatten()))
v_all_adjusted = v_all[:min_length]
r_M_pred_adjusted = r_M_pred.flatten()[:min_length]

I_pred_new = v_all_adjusted / r_M_pred_adjusted

# ----- Data Trimming -----
t_trimmed = t_all[:min_length][sequence_length:]
v_trimmed = v_all[:min_length][sequence_length:]
I_trimmed = I_pred_new[sequence_length:]
w_trimmed = w_pred_original_new[:min_length][sequence_length:]
y_trimmed = y_all_rescaled[:min_length][sequence_length:]

```

```

# ----- Visualization -----
fig, axes = plt.subplots(2, 2, figsize=(12, 8))

# Voltage and Current vs. Time
axes[0, 0].plot(t_trimmed, v_trimmed, '#0072B2', label='Voltage $v(t)$')
axes[0, 0].plot(t_trimmed, I_trimmed, 'r', label='Predicted Current $I(t)$')
axes[0, 0].set_xlabel('Time (s)')
axes[0, 0].set_ylabel('Amplitude')
axes[0, 0].set_title(f'Voltage & Current for f={frequency} Hz, A={A_train}')
axes[0, 0].legend()

# I-V Characteristics
axes[0, 1].plot(v_trimmed, I_trimmed, 'g-', label='PI-lstm Prediction')
axes[0, 1].set_xlabel('Voltage (V)')
axes[0, 1].set_ylabel('Current (I)')
axes[0, 1].set_title(f'I-V Characteristics for f={frequency} Hz')

# Memristance vs. Voltage
axes[1, 0].plot(v_trimmed, r_M_pred_adjusted[sequence_length:], '#0072B2')
axes[1, 0].set_xlabel('Voltage (V)')
axes[1, 0].set_ylabel('Memristance ( $\Omega$ )')
axes[1, 0].set_title(f'Memristance vs. Voltage for f={frequency} Hz')

# State Variable  $w(t)$  Comparison
axes[1, 1].plot(t_trimmed, y_trimmed, label='True', color='blue')
axes[1, 1].plot(t_trimmed, w_trimmed, 'r', label='PI-lstm Prediction')
axes[1, 1].set_xlabel('Time (s)')
axes[1, 1].set_ylabel('State Variable $w(t)$')
axes[1, 1].set_title(f'State Variable Over Time for f={frequency} Hz')
axes[1, 1].legend()

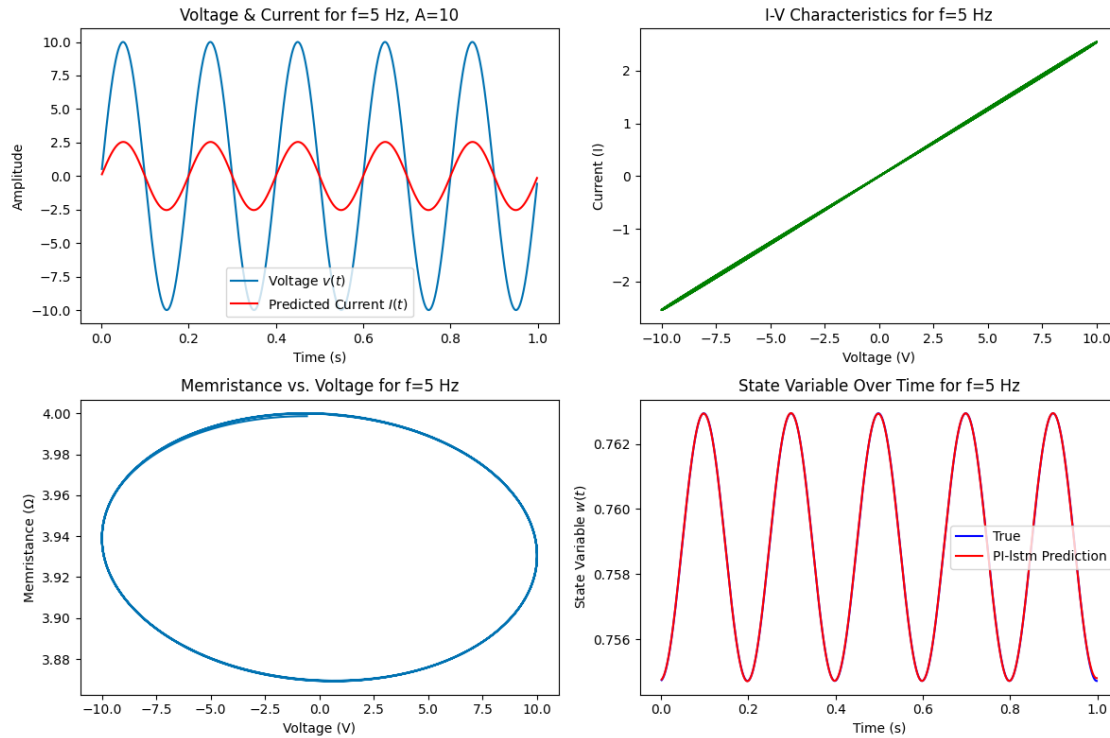
plt.tight_layout()
plt.savefig(f"piMemristor_Plots_square{frequency}Hz.pdf", format="pdf",
            bbox_inches="tight")
plt.show()

```

```

188/188          0s 2ms/step
Shape of v_all[:-1]: (5999,)
Shape of r_M_pred.flatten(): (5989,)

```



```
[14]: import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import solve_ivp
from sklearn.preprocessing import StandardScaler, MinMaxScaler
import tensorflow as tf

# ----- Physical Parameters -----
frequency = 5
A_train = 10
W_train = 2 * np.pi * frequency
mu_v = 10**4
D = 60
r_on = 0.1
r_off = 16
r0 = 4
w0 = (r0 - r_off) / (r_on - r_off)

points_per_period = 600
total_points = 10 * points_per_period

# Voltage function
def v(t):
```



```

    return A_train* signal.square(W_train * t)

# ----- ODE Solver -----
def f(t, w, A, W, mu_v, D, r_on, r_off):
    k = mu_v * (r_on / D**2)
    f_w = w * (1 - w)
    r = r_on * w + r_off * (1 - w)
    I = v(t) / r
    return I * f_w * k

t_all = np.linspace(0, 1, total_points)
sol_all = solve_ivp(
    f, (0, 1), [w0], t_eval=t_all,
    args=(A_train, W_train, mu_v, D, r_on, r_off),
    method='RK45',
    max_step=0.001
)

# ----- Data Preparation -----
w_all = sol_all.y[0]
v_all = v(t_all)
r_all = r_on * w_all + r_off * (1 - w_all)
I_all = v_all / r_all

X_all = np.column_stack([t_all[:-1], w_all[:-1], I_all[:-1]])
y_all = w_all[1:]

# ----- Data Normalization -----
X_all_scaled = X_all.copy()
scaler_time = StandardScaler()
X_all_scaled[:, 0] = scaler_time.fit_transform(X_all_scaled[:, 0].reshape(-1, 1))

scaler_features = MinMaxScaler(feature_range=(-1, 1))
X_all_scaled[:, 1:] = scaler_features.fit_transform(X_all_scaled[:, 1:])

scaler_y = MinMaxScaler(feature_range=(-1, 1))
y_all_scaled = scaler_y.fit_transform(y_all.reshape(-1, 1))

# ----- Create Sequences -----
sequence_length = 10
def create_sequences(X, y, sequence_length):
    X_seq, y_seq = [], []
    for i in range(len(X) - sequence_length):
        X_seq.append(X[i:i+sequence_length])
        y_seq.append(y[i+sequence_length])
    return np.array(X_seq), np.array(y_seq).reshape(-1, 1)

```

```

X_seq, y_seq = create_sequences(X_all_scaled, y_all_scaled, sequence_length)

# ----- PINN Prediction -----
w_pred_seq_new = pinn_rnn.predict(X_seq)
w_pred = w_pred_seq_new[:, -1, :]
w_pred_original_new = scaler_y.inverse_transform(w_pred)
y_all_rescaled = scaler_y.inverse_transform(y_seq)

t_test_plot = t_all[:len(w_pred_original_new)]

# ----- Compute Memristance and Current -----
r_M_pred = r_on * w_pred_original_new + r_off * (1.0 - w_pred_original_new)

print(f"Shape of v_all[:-1]: {v_all[:-1].shape}")
print(f"Shape of r_M_pred.flatten(): {r_M_pred.flatten().shape}")

min_length = min(len(v_all[:-1]), len(r_M_pred.flatten()))
v_all_adjusted = v_all[:min_length]
r_M_pred_adjusted = r_M_pred.flatten()[:min_length]

I_pred_new = v_all_adjusted / r_M_pred_adjusted

# ----- Data Trimming -----
t_trimmed = t_all[:min_length][sequence_length:]
v_trimmed = v_all[:min_length][sequence_length:]
I_trimmed = I_pred_new[sequence_length:]
w_trimmed = w_pred_original_new[:min_length][sequence_length:]
y_trimmed = y_all_rescaled[:min_length][sequence_length:]

# ----- Visualization -----
fig, axes = plt.subplots(2, 2, figsize=(12, 8))

# Voltage and Current vs. Time
axes[0, 0].plot(t_trimmed, v_trimmed, '#0072B2', label='Voltage $v(t)$')
axes[0, 0].plot(t_trimmed, I_trimmed, 'r', label='Predicted Current $I(t)$')
axes[0, 0].set_xlabel('Time (s)')
axes[0, 0].set_ylabel('Amplitude')
axes[0, 0].set_title(f'Voltage & Current for f={frequency} Hz, A={A_train}')
axes[0, 0].legend()

# I-V Characteristics
axes[0, 1].plot(v_trimmed, I_trimmed, 'g-', label='PI-lstm Prediction')
axes[0, 1].set_xlabel('Voltage (V)')
axes[0, 1].set_ylabel('Current (I)')
axes[0, 1].set_title(f'I-V Characteristics for f={frequency} Hz')

```

```

# Memristance vs. Voltage
axes[1, 0].plot(v_trimmed, r_M_pred_adjusted[sequence_length:], '#0072B2')
axes[1, 0].set_xlabel('Voltage (V)')
axes[1, 0].set_ylabel('Memristance ( $\Omega$ )')
axes[1, 0].set_title(f'Memristance vs. Voltage for f={frequency} Hz')

# State Variable w(t) Comparison
axes[1, 1].plot(t_trimmed, y_trimmed, label='True', color='blue')
axes[1, 1].plot(t_trimmed, w_trimmed, 'r', label='PI-lstm Prediction')
axes[1, 1].set_xlabel('Time (s)')
axes[1, 1].set_ylabel('State Variable $w(t)$')
axes[1, 1].set_title(f'State Variable Over Time for f={frequency} Hz')
axes[1, 1].legend()

plt.tight_layout()
plt.savefig(f"piMemristor_Plots_square{frequency}Hz.pdf", format="pdf",
           bbox_inches="tight")
plt.show()

```

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Shape of v_all[: -1]: (5999,)
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