Birzeit University
Faculty of Engineering and Technology

Department of Electrical and Computer Engineering

Communication Systems ENEE 339

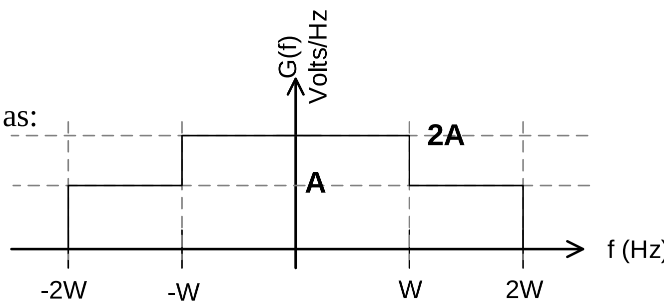
Midterm Exam

Instructors: Dr. Wael Hashlamoun, Dr. Mohammad Jubran Date: April 23, 2017

Problem 1: 25 Points

The Fourier transform G(f) of a signal g(t) is given as:

$$G(f) = \begin{cases} 2A & -W \le f \le W \\ A & W \le |f| \le 2W \\ 0 & |f| > 2W \end{cases}$$



a. Find the absolute bandwidth of g(t)
b. Find the energy in g(t).
c. If g(t) is passed through an ideal low pass filter with bandwidth 3W/2, find the energy in the signal at the filter output.
d. Use the table of Fourier transform pairs at the end of the exam to find g(t).

Problem 2: 25 Points

The message sign $m(t) = 2\cos(2\pi 40t) + 4\cos(2\pi 80t)$alalong with the carrier signal$c(t) = 4\cos(2\pi 1000t)$ are applied to a modulator that generates the double sideband suppressed carrier signals(t)

a. Find the average power of m(t).
b. Find the time-domain expression of the modulated signal s(t).
c. Find the bandwidth of the transmitted signal in Hz.
d. Draw the block diagram of the demodulator used to recover m(t) from s(t) without distortion specifying the details of each block

Problem 3: 25 Points

The message $m(t) = 0.3\cos(2\pi 500t)$is applied to a normal amplitude modulatorwith a sensitivity k_a= 0.2/V and a carrier$c(t) = 10\cos(2\pi 10000t)$to produce the signal$s(t) = A_c \cos(2\pi f_c t)(1 + k_a m(t))$

a. Find the modulation index.
b. Find the average power in the carrier and in each of the sidebands.
c. Find the power efficiency

Problem 4: 25 Points

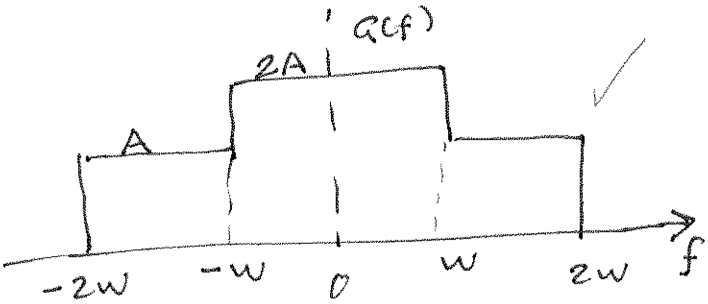
Consider the FM signal $s(t) = 10\cos[2\pi(10000)t + 1.2 sin 2\pi(200)t]$

a. Find the instantaneous frequency of $s(t)$
b. Find the peak frequency deviation of $s(t)$.
c. Find the 90% power bandwidth of $s(t)$.

Good Luck

ENEE 339

Solution to Midterm April 23, 2017

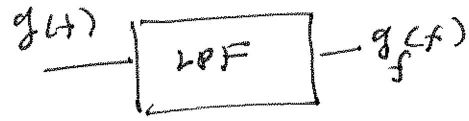


Problem 1

a. $B.W = 2W$

b. $E_g = 2\int_0^W (2A)^2 df + 2\int_W^{2W} (A)^2 df$

$E_g = 10A^2 W$



c. $E' = 2\int_0^W (2A)^2 df + 2\int_W^{5W/2} (A)^2 df$

$E' = 9A^2 W$

d. $G(f) = A\,\text{rect}\left(\frac{f}{4W}\right) + A\,\text{rect}\left(\frac{f}{2W}\right)$ (1)

From Table $\text{rect}\left(\frac{t}{T}\right) \rightarrow T\,\text{sinc}\, fT$

$\text{sinc}\, 2Wt \rightarrow \frac{1}{2W}\,\text{rect}\left(\frac{f}{2W}\right)$

$\Rightarrow 2W\,\text{sinc}\, 2Wt \longrightarrow \text{rect}\left(\frac{f}{2W}\right)$ (2)

using (2), (1) becomes in the time domain

$$\boxed{g(t) = A(4W)\,\text{sinc}\, 4Wt + A(2W)\,\text{sinc}\, 2Wt}$$

Problem 2

$m(t) = 2\cos 2\pi(40)t + 4\cos 2\pi(80)t$

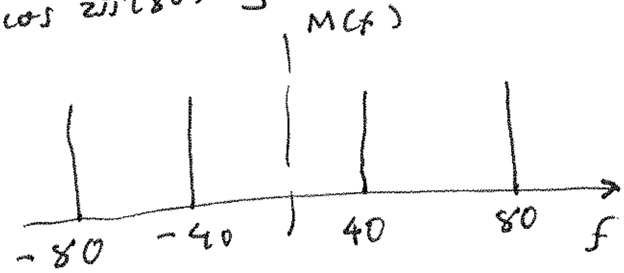
$c(t) = 4\cos 2\pi(1000)t$

a. $\langle m(t)^2 \rangle = \frac{(2)^2}{2} + \frac{(4)^2}{2}$; terms are orthogonal

$= 2 + 8 = 10\ W$

b. $s(t) = A_c\, m(t) \cos 2\pi f_c t$

$s(t) = 5\,[2\cos 2\pi(40)t + 4\cos 2\pi(80)t]\cos 2\pi(1000)t$

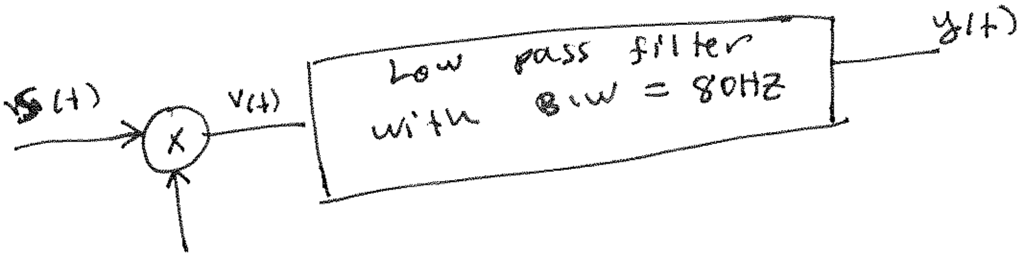


b. B.W $= 2W$

$= 2(80)$

$= 160\ HZ$

c.



$A_c' \cos 2\pi f_c t$

$A_c' \cos 2\pi(1000)t$

Analysis : $v(t) = A_c' \cos 2\pi f_c t\ s(t)$

$= A_c A_c' \cos 2\pi f_c t \cos 2\pi f_c t\ m(t)$

$= A_c A_c'\, m(t) \cos^2 2\pi f_c t$

$= \frac{A_c A_c'}{2}\, m(t)\,[1 + \cos 4\pi f_c t]$

$\Rightarrow \quad y(t) = \frac{A_c A_c'}{2}\, m(t).$

Problem 3

$$s(t) = A_c \cos 2\pi f_c t \,(1 + k_a m(t)) \quad ; \quad A_c = 10,\ f_c = 10000,\ k_a = 0.2$$

$$s(t) = A_c [1 + 0.2 \times 0.3 \cos 2\pi (500) t] \cos 2\pi f_c t$$

$$s(t) = A_c [1 + 0.6 \cos 2\pi (500) t] \cos 2\pi f_c t$$

a. $M.I. = 0.6$

b. $s(t) = 10 \cos 2\pi f_c t + 6 \cos 2\pi f_c t \cos 2\pi f_m t$

$$s(t) = \underbrace{10 \cos 2\pi f_c t}_{\text{carrier}} + \underbrace{3 \cos 2\pi (f_c + 500) t + 3 \cos 2\pi (f_c - 500) t}_{\text{sidebands}}$$

$$P_{av}(\text{carrier}) = \frac{A_c^2}{2} = \frac{10^2}{2} = 50$$

$$P_{av}(2 \text{ sidebands}) = \left(\frac{3^2}{2}\right) \times 2 = 9 \ ; \ \text{each with } 4.5 \text{ Watt}$$

c. $$\text{power efficiency} = \frac{\text{power in sidebands}}{\text{total transmitted power}}$$

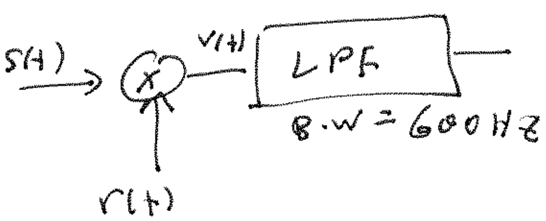
$$= \frac{9}{50 + 9} = \frac{9}{59}$$

$$= 0.152$$

Also, $$\text{power efficiency} = \frac{\mu^2}{2 + \mu^2} = \frac{(0.36)^2}{2 + (0.36)^2}$$

$$= 0.152 \quad ; \text{(formula derived in class)}$$

d. $v(t) = A_c [1 + \mu \cos w_m t] \cos w_c t \cdot \cos w_c t$



$$= \frac{A_c [1 + \mu \cos w_m t]}{2} [1 + \cos 2 w_c t]$$

$$\Rightarrow y(t) = \frac{A_c}{2} [1 + \mu \cos w_m t]$$

Problem 4 :

$$s(t) = 10 \cos\left(2\pi(10000)t + 1.2 \sin 2\pi(200)t\right)$$

a. $$f_i(t) = \frac{1}{2\pi}\frac{d}{dt}\left(2\pi(10000)t + 1.2 \sin 2\pi(200)t\right)$$

$$= f_c + \frac{1}{2\pi} \times 1.2 \ (2\pi(200)) \cos 2\pi(200)t \qquad (1)$$

$$= f_c + 240 \cos 2\pi(200)t$$

b. peak frequency deviation = 240 from (1)

Also, $\beta = \frac{\Delta f}{f_m} \Rightarrow \Delta f_o = \beta f_m = (1.2)(200)$

$= 240 \text{ HZ}.$

c. $$s(t) = (10) J_0(1.2) \cos 2\pi f_c t \qquad \equiv 6.711$$
$$+ (10) J_1(1.2) \cos 2\pi(f_c + f_m)t \qquad \equiv 4.983$$
$$+ (10) J_{-1}(1.2) \cos 2\pi(f_c - f_m)t \qquad = 4.983$$
$$+ (10) J_2(1.2) \cos 2\pi(f_c + 2f_m)t \qquad = 0.1593$$
$$+ (10) J_{-2}(1.2) \cos 2\pi(f_c - 2f_m)t \qquad = 0.1593$$

Total average power $= \frac{(10)^2}{2} = 50$

carrier

$\frac{(6.711)^2}{2}$

22.5

$f_c + f_m$ $(f_c - f_m)$

$2 \times \frac{(4.983)^2}{2}$

24.83

$f_c + 2f_m$ / $f_c - 2f_m$)

$2 \times \frac{(1.593)^2}{2}$

2.537

94.6%

not enough

$\Rightarrow$ B.W $= 2 \times (2 f_m)$

$= 4 f_m$

$= 4(200)$

$= 800 \text{ HZ}$

BIRZEIT UNIVERSITY

Faculty of Engineering and Technology

Department of Electrical and Computer Engineering

Communication Systems ENEE 339

Instructor: Dr. Wael Hashlamoun

Midterm Exam

First Semester 2018-2019

Date: Sunday 18/11/2018 — Time: 75 minutes

Name: — Student #:

Opening Remarks:

- Calculators are allowed, but mobile phones, books, notes, formula sheets, and other aids are not allowed.
- You are required to show all your work and provide the necessary explanations everywhere to get full credit.

Problem 1: 25 Points

The Fourier transform, $G(f)$, of a signal g(t) is given by:

$$G(f) = \begin{cases} \sqrt{1-\left(\frac{f}{f_0}\right)^2} & -f_0 \le f \le f_0 \\ 0 & |f| > f_0 \end{cases} ; \ f_0 = 1000\ Hz$$

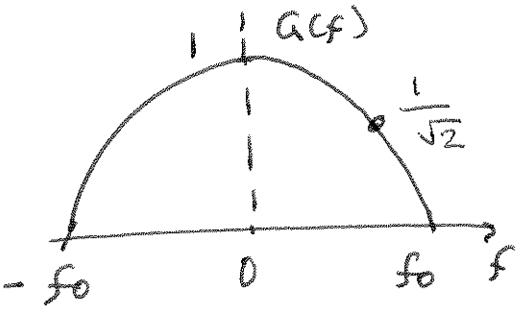
9 a. Find the 3-dB bandwidth of g(t).

9 b. If g(t) is passed through an ideal low pass filter with bandwidth $f_0/2$ and unity gain, find the energy of the signal at the filter output.

7 c. The signal $s(t) = g(t)cos2\pi(1000t)$ is passed through an ideal low pass filter with bandwidth 1000 Hz, sketch the spectrum of s(t).

a.

$G(0) = 1$



$$-3 = 20\log\frac{G(B)}{G(0)} = 20\log\sqrt{1-\left(\frac{B}{f_0}\right)^2}$$

$$-3 = 10\log\left(1-\left(\frac{B}{f_0}\right)^2\right)$$

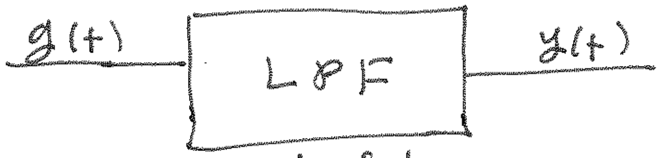
$$-0.3 = \log\left(1-\left(\frac{B}{f_0}\right)^2\right) \Rightarrow 10^{-0.3} = 1-\left(\frac{B}{f_0}\right)^2$$

$$\frac{B}{f_0} = \sqrt{1-10^{-0.3}} = \sqrt{0.498} = 0.706$$

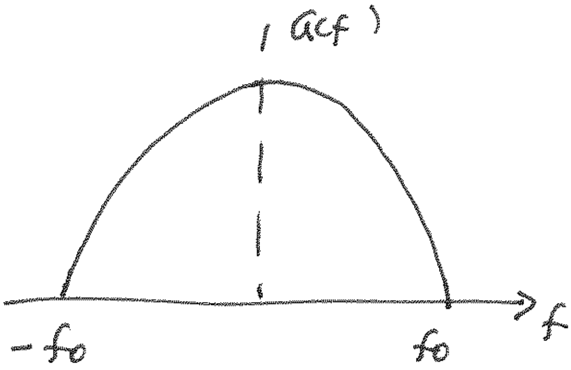
$$\Rightarrow \boxed{B = 0.706 f_0 = 706\ Hz}$$

Problem 1

b.

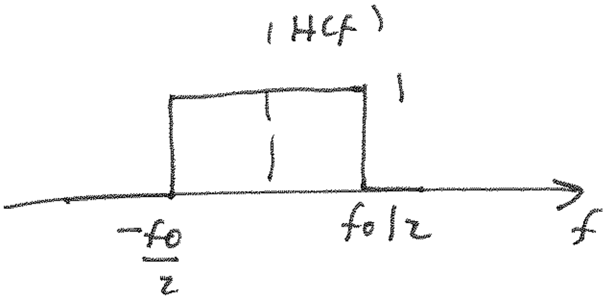


B.W = $f_0/2$

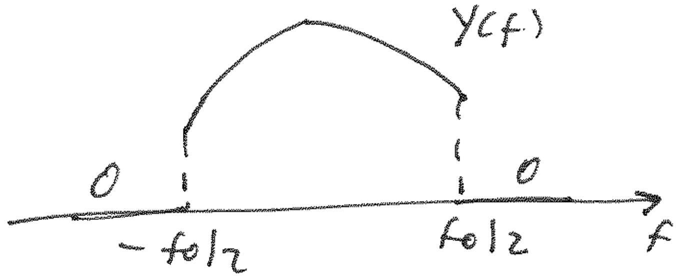


$$Y(f) = G(f)\,H(f)$$

$$|Y(f)|^2 = |G(f)|^2 |H(f)|^2$$



$$E_y = \int_{-f_0/2}^{f_0/2} |G(f)|^2\,df$$



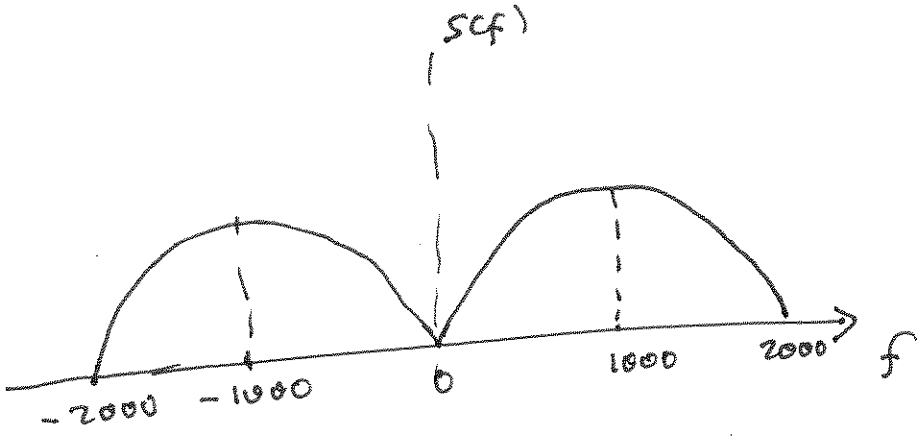
$$= 2\int_0^{f_0/2} \left(1-\left(\frac{f}{f_0}\right)^2\right) df$$

$$= 2\left[f - \frac{f^3}{3f_0^2}\right]_0^{f_0/2}$$

$$= 2\left[\frac{f_0}{2} - \frac{f_0^3}{24 f_0^2}\right] = 2\left[\frac{f_0}{2} - \frac{f_0}{24}\right]$$

$$= f_0 - \frac{f_0}{12} = \boxed{\frac{11}{12} f_0 = \frac{11}{12} * 1000 = 916.6}$$

c.



Problem 2: 25 Points

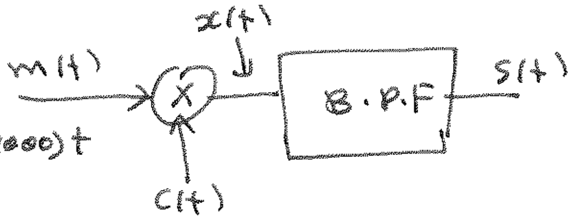
The message $m(t) = 2\cos(2\pi 100t)$ along with the carrier $c(t) = 10\cos(2\pi 1000t)$ are applied to an upper single sideband modulator, which uses the frequency discrimination method, to generate the modulated signal s(t).

6 a. Find the average power in m(t).

6 b. If $c(t)$ is applied to an ideal envelope detector, sketch the signal observed at its output.

6 c. Find the time-domain expression for the modulated signal s(t).

7 d. If s(t) is applied to a coherent demodulator consisting of a multiplier, which uses the signal $c'(t) = \cos(2\pi 1000t + \varphi)$, followed by a low pass filter with bandwidth 120 Hz and unity gain, find the filter output.



$x(t) = 20 \cos 2\pi(100)t \cos 2\pi(1000)t$

$x(t) = 10 \cos 2\pi(1100)t + 10 \cos 2\pi(900)t$

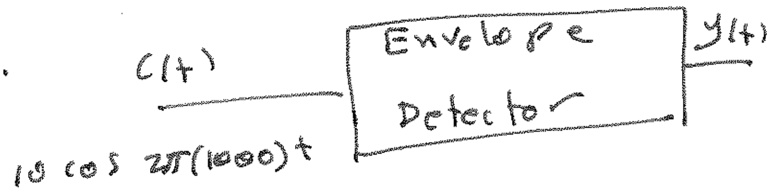
The B.P.F removes lower band

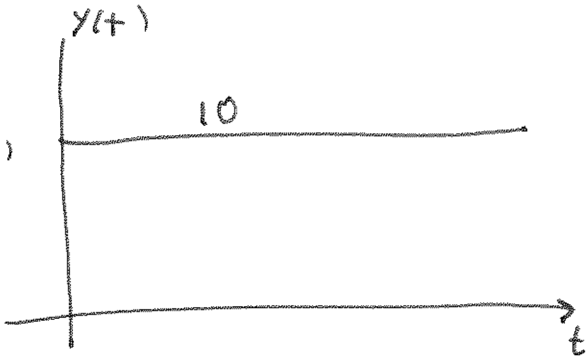
$\Rightarrow$

c. $\boxed{s(t) = 10 \cos 2\pi(1100)t}$

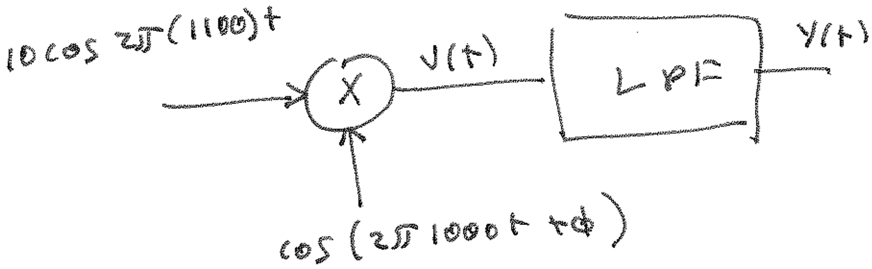
a. $\langle m(t)^2 \rangle = \frac{(2)^2}{2} = 2$

b.





d.



$V(t) = 10 \cos 2\pi(1100)t \cdot \cos(2\pi 1000t + \phi)$

$= 5\cos(2\pi 2100t + \phi)$

$+ 5\cos(2\pi 100t \; \phi)$

$\boxed{y(t) = 5\cos(2\pi 100t + \phi)}$

$\boxed{y(t) = 5\cos\phi \cos 2\pi(100)t - 5\sin\phi \sin 2\pi(100)t}$

Problem 3: 25 Points

The Fourier transform of a message m(t) is given as:

$$M(f) = \begin{cases} 5|f| & -W \le f \le W \\ 0 & |f| > W \end{cases} ;\; W = 1000$$

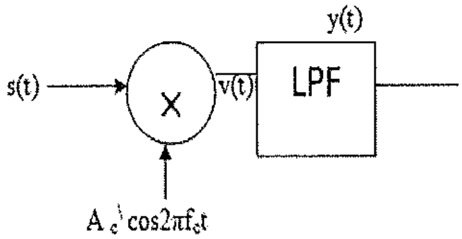
This message is applied to a double sideband suppressed carrier modulator along with the carrier $c(t) = 10\cos(2\pi(f_c)t)$ to produce the modulated signal $s(t)$

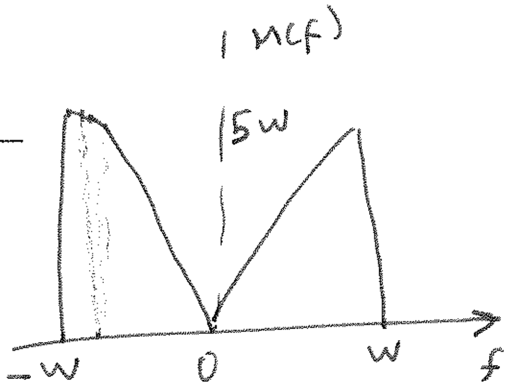
6 a. Find and sketch $S(f)$, the Fourier transform of $s(t)$, for an arbitrary value of f_c.

6 b. Find the transmission bandwidth of s(t).

6 c. Find the minimum required value of f_c in terms of W.

7 d. Show that the receiver in the figure below can demodulate m(t) from s(t) without distortion (you do not have to find what m(t) is).

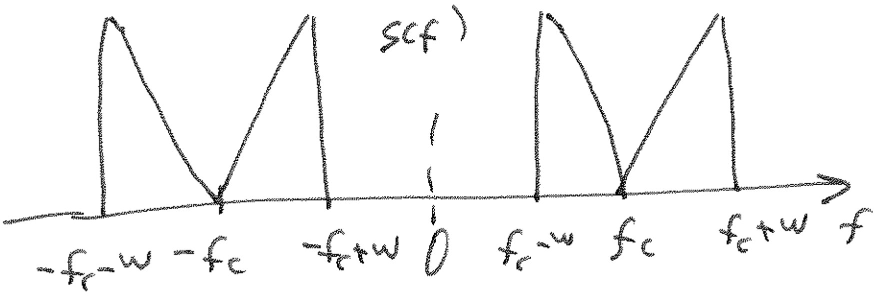




$s(t) = 10 \cos 2\pi f_c t \; m(t)$

$S(f) = 5\, M(f - f_c)$
$\quad + 5\, M(f + f_c)$

a. $\longrightarrow$



b.

$B.W = 2W$

$\boxed{B.W = 2(1000) = 2000 \text{ Hz}}$

c. $f_c - W \ge 0 \Rightarrow \boxed{f_c \ge W = 1000 \text{ Hz}}$

d. $v(t) = 10(\cos 2\pi f_c t)\, m(t) \cdot A_c' \cos 2\pi f_c t$

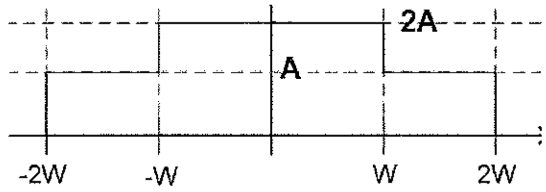
$= 5A_c' m(t) [\cos 4\pi f_c t + 1]$

$\Rightarrow \boxed{y(t) = 5A_c' m(t)}$

Problem 4: 25 Points

Consider the message signal g(t), shown in the figure below

$$g(t) = \begin{cases} 2A & -W \le t \le W \\ A & W \le |t| \le 2W \\ 0 & |f| > 2W \end{cases}; A = 1, W = 1$$



g(t) is applied to an FM modulator with sensitivity $k_f = 20\ Hz/V$ to produce an FM signal s(t). The unmodulated carrier frequency is 1000 Hz.

8.5 a. Use the time-bandwidth relationship to find the equivalent rectangular bandwidth of g(t). ✓

5 b. Find and plot the instantaneous frequency of s(t) versus time.

5 c. Find the peak frequency deviation of s(t).

7.5 d. Find the time domain representation for the modulated signal s(t) for all time t.

a. $$T_{eq} = \frac{\left(\int_{-\infty}^{\infty} |g(t)|\,dt\right)^2}{\int_{-\infty}^{\infty} |g(t)|^2\,dt}$$

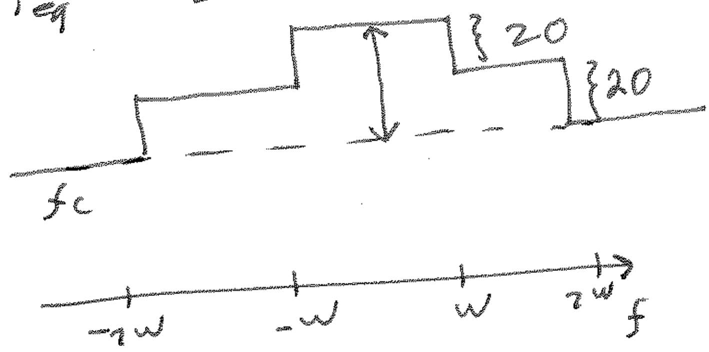
$$\int_{-2W}^{2W} |g(t)|\,dt = 2AW + 2W(2A) = 6AW = 6$$

$$\int_{-2W}^{2W} |g(t)|^2\,dt = (A^2)(2W) + 4A^2(2W) = 10A^2W = 10$$

$$\Rightarrow T_{eq} = \frac{(6)^2}{10} = \frac{36}{10} = 3.6\frac{A^2W^2}{A^2W} = 3.6W$$

$$B_{eq}T_{eq} = \frac{1}{2} \Rightarrow B_{eq} = \frac{1}{2T_{eq}} = \frac{1}{2 * 3.6} = 0.138$$

b. ⟶



c. $\Delta f_{max} = 40\ Hz$

d. $$s(t) = \begin{cases} A_c \cos 2\pi f_c t & t < -2W,\quad t \geq 2W \\ A_c \cos 2\pi(f_c + 20)t & -2W \leq t < -W,\quad W < t \leq 2W \\ A_c \cos 2\pi(f_c + 40)t & -W \leq t \leq W \end{cases}$$

ABET : a

BIRZEIT UNIVERSITY

Faculty of Engineering and Technology

Department of Electrical and Computer Engineering

Communication Systems ENEE 339

Instructor: Dr. Wael Hashlamoun

Midterm Exam

First Semester 2019-2020

Date: Monday 2/12/2019 Time: 75 minutes

Name: Student #:

Problem 1: 25 Points

The Fourier transform, $G(f)$, of a signal g(t) is given by:

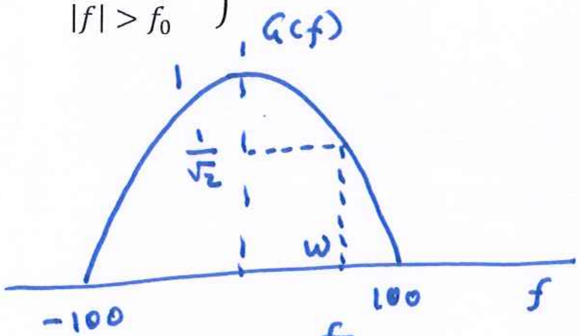
a

$$G(f) = \begin{cases} \sqrt{1-\left(\frac{f}{f_0}\right)^2} & -f_0 \le f \le f_0 \\ 0 & |f| > f_0 \end{cases}; \; f_0 = 100\ Hz$$

7 a. Find the absolute bandwidth of g(t)

9 b. Find the total energy in g(t)

9 c. Find the 3-dB bandwidth of g(t).



a. $B_{abs} = 100\ HZ.$

b. $E = 2\int_0^{f_0} |G(f)|^2 df$

$= 2\int_0^{f_0} \left[1-\left(\frac{f}{f_0}\right)^2\right] df = 2\left(f - \frac{f^3}{3f_0^2}\right)\Big|_0^{f_0} = \frac{4f_0}{3} = \frac{400}{3}$

$E = 133.33$ 5

c. $\frac{1}{\sqrt{2}} = \sqrt{1-\left(\frac{w}{f_0}\right)^2}$; $\frac{G(w)_{max}}{\sqrt{2}} = G(w) \Rightarrow \frac{1}{\sqrt{2}} = G(w)$

$\frac{1}{2} = \left(1-\left(\frac{w}{f_0}\right)^2\right) \Rightarrow \left(\frac{w}{f_0}\right)^2 = \frac{1}{2}$

$w^2 = \frac{f_0^2}{2}$

or $w = \frac{f_0}{\sqrt{2}} = 70.71\ HZ$

Problem 2: 25 Points

The message $m(t) = 3\cos(2\pi 100t)$ along with the carrier $c(t) = 10\cos(2\pi 1000t)$ are applied to a double sideband modulator, to generate the modulated signal s(t).

6 a. Find the expression for s(t)
6 b. Find the average power in m(t).
6 c. Find the average power in s(t).
7 d. If s(t) is applied to an ideal envelope detector to produce an output y(t), sketch y(t).

a. $s(t) = m(t)\ c(t)$

$s(t) = 30 \cos 2\pi(100t) \cos 2\pi(1000t)$

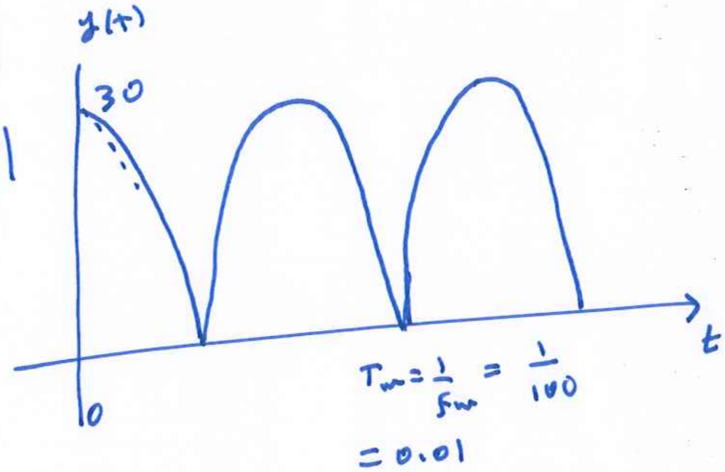
b. $\langle m(t)^2 \rangle = \frac{A_m^2}{2} = \frac{(3)^2}{2} = \frac{9}{2}$

c. $s(t) = 15 \cos 2\pi(1100t) + 15 \cos 2\pi(900)t$

$\langle s(t)^2 \rangle = \frac{(15)^2}{2} \times 2 = (15)^2 = 225\ W$

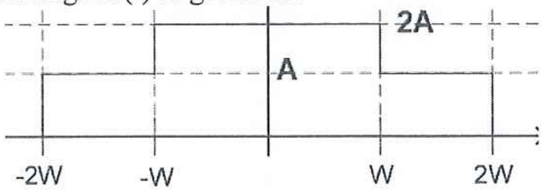
d. $y(t) = |30 \cos 2\pi(100)t|$

$= 30 |\cos 2\pi(100)t|$



Problem 3: 25 Points

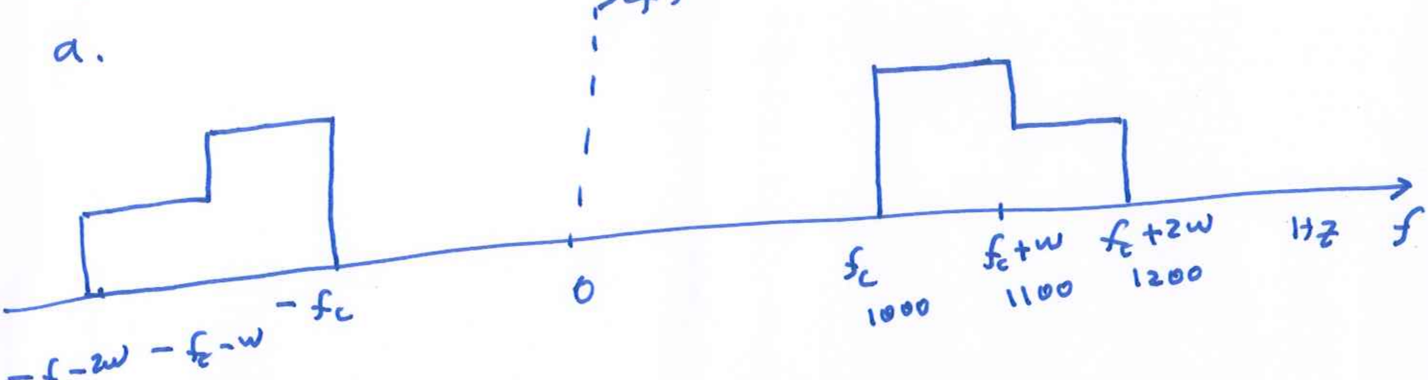
The Fourier transform of a message m(t) is given as:



where $W = 100\ Hz$. This message is applied to an upper single sideband modulator along with the carrier $c(t) = 10\cos(2\pi(f_c)t)$ to produce the modulated signal $s(t)$

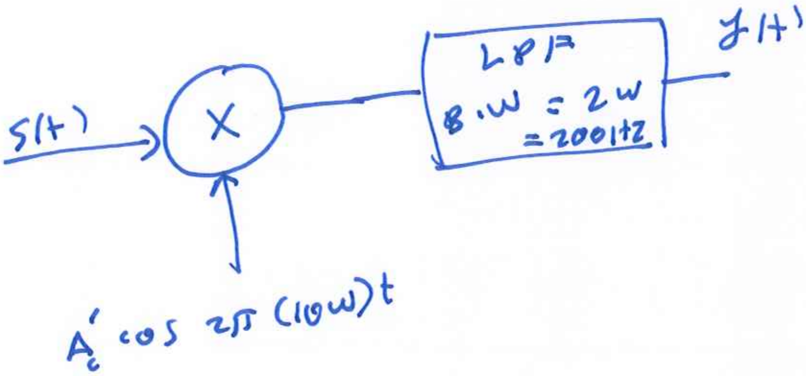
a. Find and sketch $S(f)$, the Fourier transform of $s(t)$, assuming $f_c = 10W$.
b. Find the transmission bandwidth of s(t).
c. Draw the block diagram of the receiver that would recover m(t) from s(t) identifying the details of each block.

a.



b. B.W = 2W = 200 HZ

c.



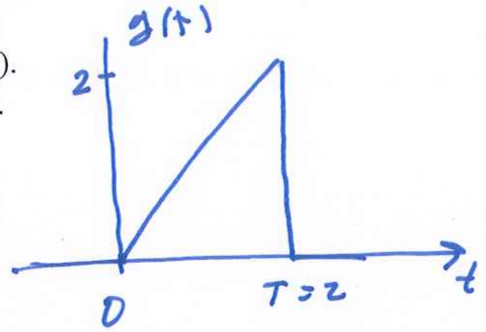
Problem 4: 25 Points

Consider the message signal g(t),

$$g(t) = \begin{cases} t & 0 \le t \le T \\ 0 & otherwise \end{cases} ; T = 2\ sec$$

g(t) is applied to an FM modulator with sensitivity $k_f = 2\ Hz/V$ to produce an FM signal s(t). The unmodulated carrier frequency is 100 Hz.

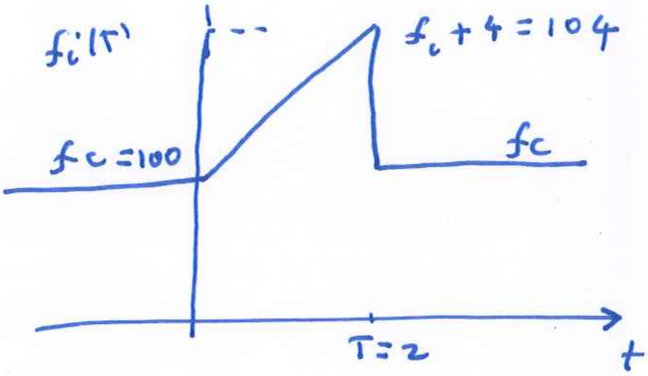
10 a. Find and plot the instantaneous frequency of s(t) versus time.
5 b. Find the peak frequency deviation of s(t).
10 c. Suggest a method via which g(t) can be recovered from s(t).
d. **BONUS**: Find the time-domain expression for s(t) for all t.



Good Luck

a. $f_i = f_c + k_f\ m(t)$

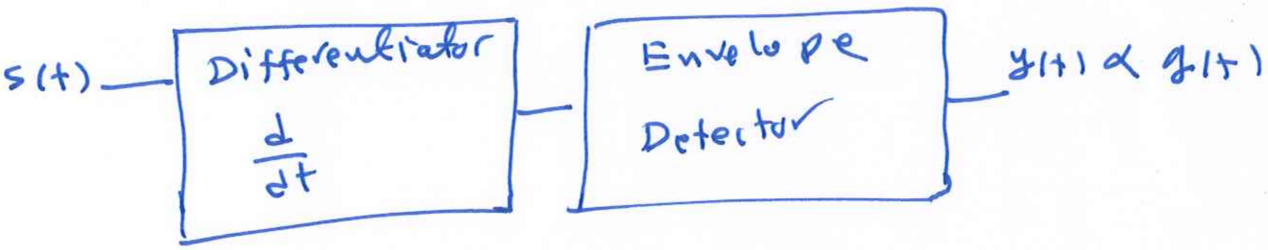
$f_i = 100 + 2\ m(t)$



b. $(\Delta f)_{max} = f_i(t)\Big|_{max} - f_c$

$= 4HZ$

c.



d. For $t < 0$, $f_i(t) = f_c \Rightarrow s(t) = A_c \cos 2\pi f_c t$

For $0 < t < T$, $s(t) = A_c \cos\left(2\pi f_c t + \int_0^t 2\pi k_f\ m(\alpha)\, d\alpha\right)$

$= A_c \cos\left(2\pi f_c t + 2\pi k_f \frac{t^2}{2}\right)$

For $t > T$, $s(t) = A_c \cos\left(2\pi f_c t + \int_0^T 2\pi k_f\ m(\alpha)\, d\alpha + \int_T^t m(\alpha)\, d\alpha\right)$

$= A_c \cos\left(2\pi f_c t + 2\pi k_f \frac{T^2}{2}\right)$

$= A_c \cos\left(2\pi f_c t + 2\pi k_f (2)\right)$

BIRZEIT UNIVERSITY

Faculty of Engineering and Technology
Department of Electrical and Computer Engineering
Second Semester, 2018/2019
COMMUNICATION SYSTEMS, ENEE339
First Exam, March 26, 2019
Time Allowed: 80 Minutes.

Name: ....

ID: ....

Section: ....

Question #	SOC	Max Grade	Achieved
1		20	
2		20	
3		10	
Total		50	

Opening Remarks:

- This is a 80-minutes exam. Calculators are allowed. Books, notes, formula sheets, and other aids are not allowed.
- You are required to show all your work and provide the necessary explanations everywhere to get full credit.

Problem#1 [20 Points]

Consider the signal $x(t) = cos(2000\pi t) + 2cos(4000\pi t) + 0.5cos(8000\pi t)$. If this signal passes through a channel with amplitude spectrum and phase spectrum as shown in figure 1. Assuming $y(t)$ is signal at the output of the channel, answer the following questions:

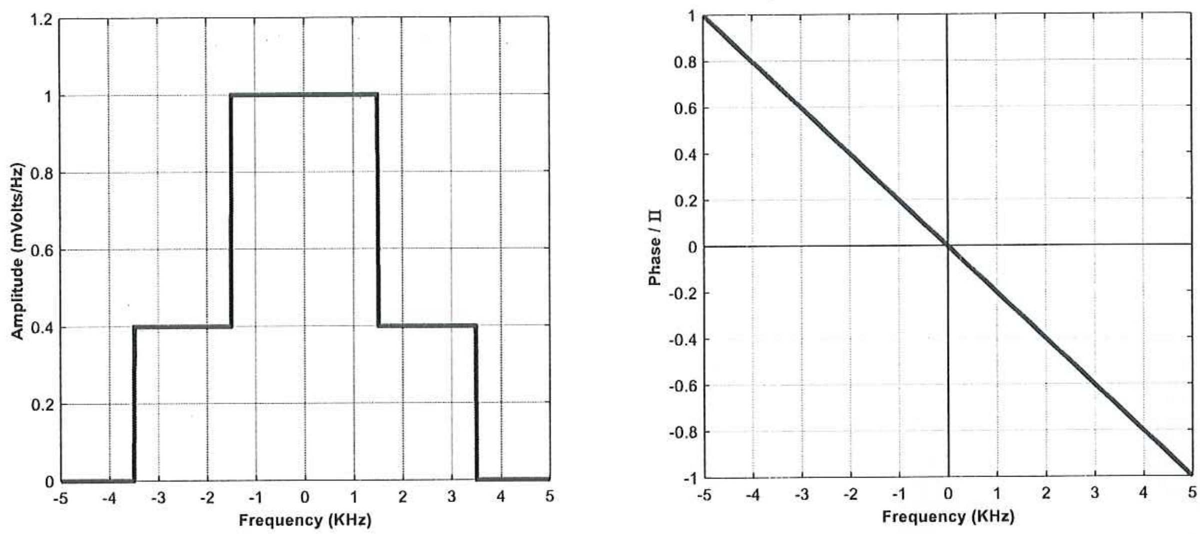
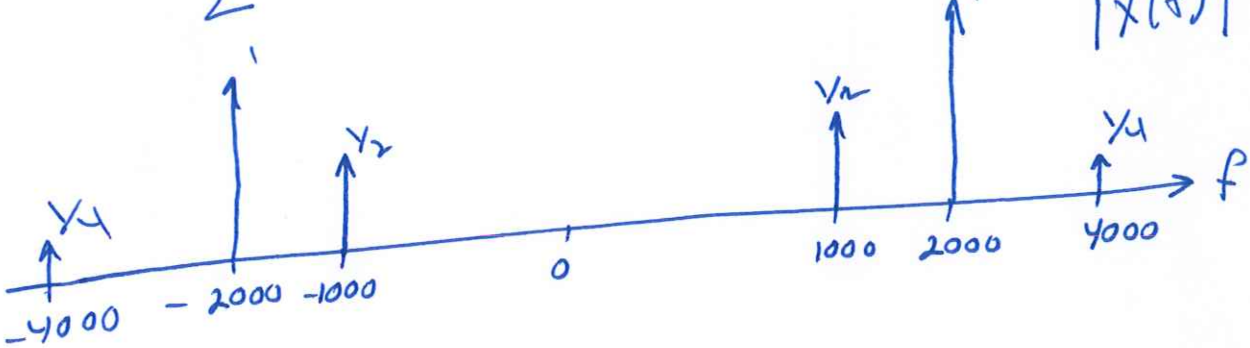


Figure 1: amplitude spectrum and phase spectrum of problem 1

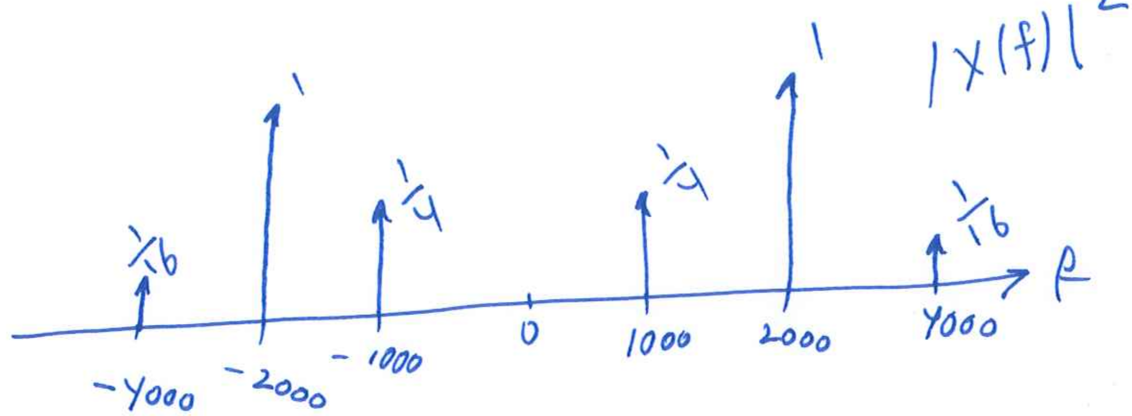
a. Determine and plot the Amplitude Spectrum of $x(t)$.

$$X(f) = \frac{\delta(f-1000) + \delta(f+1000)}{2} + 2\left[\frac{\delta(f-2000) + \delta(f+2000)}{2}\right]$$

$$+ 0.5\left[\frac{\delta(f-4000) + \delta(f+4000)}{2}\right]$$



b. Plot the Power Spectral Density of $x(t)$.



c. Determine the average power of $x(t)$.

$$P_{av} = 2 \times \frac{1}{4} + 2 \times 1 + 2 \times \frac{1}{16} = 2.625 \text{ watts.}$$

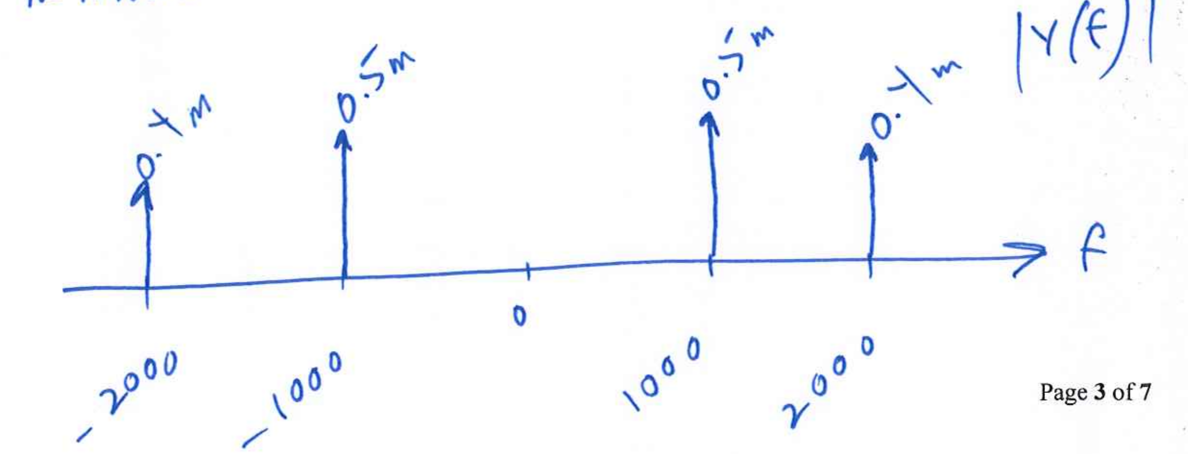
d. Determine the absolute bandwidth of $x(t)$.

$$BW_{abs} = 4000 \text{ Hz}.$$

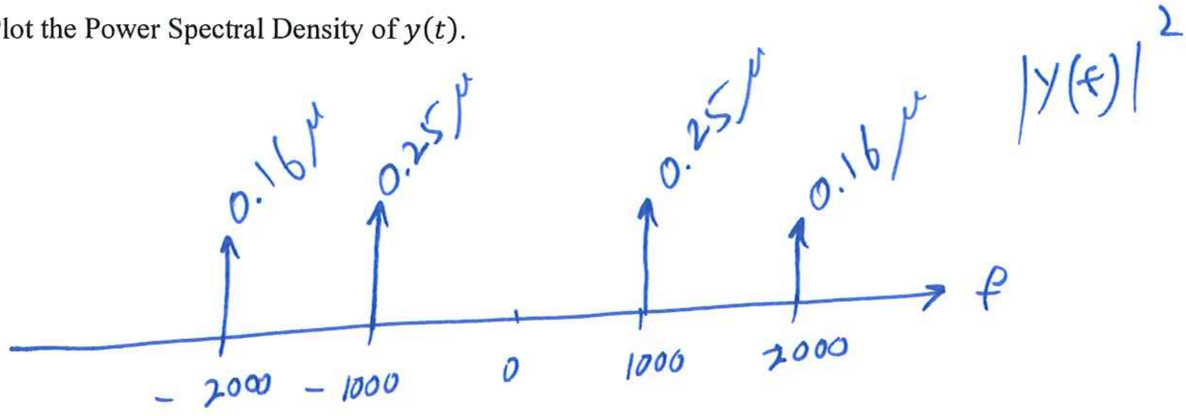
e. Determine and plot the Amplitude Spectrum of $y(t)$.

$$|Y(f)| = \frac{\delta(f-1000) + \delta(f+1000)}{2} + 0.4\left[\frac{\delta(f-2000) + \delta(f+2000)}{2}\right]$$

in mVolts.



f. Plot the Power Spectral Density of $y(t)$.



g. Determine the average power of $y(t)$.

$P_{av} = 2 \times 0.25 + 2 \times 0.16 = 0.82\mu$ Watts.

h. Determine the absolute bandwidth of $y(t)$.

$BW_{ABS} = 2000\ Hz$

i. Is this a distortionless transmission? Explain briefly.

No,
- Different harmonics are multiplied by different coefficients.
- The harmonic with $f = 4000\ Hz$ is totally distorted (disappeared).

Problem#2 [20 Points]

For the Amplitude Spectrum of the signal $g(t)$ shown in figure 2, answer the following:

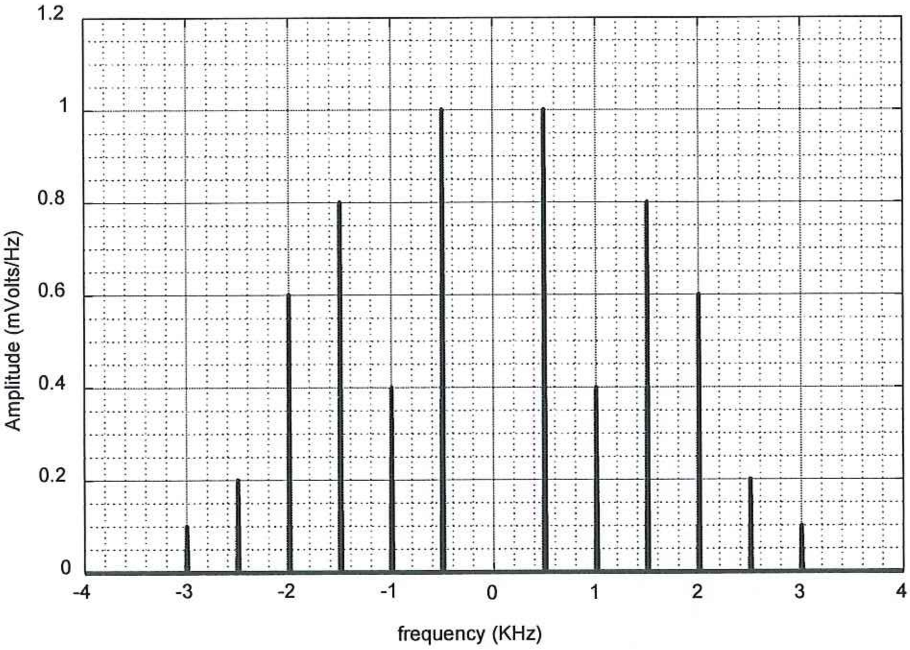


Figure 2: Amplitude Spectrum of $g(t)$

a. Determine the 90% power bandwidth of $g(t)$.

$f = \pm 0.5 KHz \rightarrow P_{0.5K} = 2 \times 1^2 = 2 \mu watts.$

$f = \pm 1 KHz \rightarrow P_{1K} = 2 \times 0.4^2 = 0.32 \mu watts$

$f = \pm 1.5 KHz \rightarrow P_{1.5K} = 2 \times 0.8^2 = 1.28 \mu watts$

$f = \pm 2 KHz \rightarrow P_{2K} = 2 \times 0.6^2 = 0.72 \mu watts.$

$f = \pm 2.5 KHz \rightarrow P_{2.K} = 2 \times 0.2^2 = 0.08 \mu watts.$

$f = \pm 3 KHz \rightarrow P_{3K} = 2 \times 0.1^2 = 0.02 \mu watts.$

$P_{total} = P_{0.5K} + P_{1K} + P_{1.5K} + P_{2K} + P_{2.5K} + P_{3K} = 4.42 \mu watts.$

① $\Rightarrow \dfrac{P_{0.5K}}{P_{total}} = \dfrac{2\ \mu}{4.42\ \mu} = 45.248\% < 90\%$

② $\Rightarrow \dfrac{P_{0.5K} + P_{1K}}{P_{total}} = \dfrac{2.32\ \mu}{4.42\ \mu} = 52.49\% < 90\%$

③ $\Rightarrow \dfrac{P_{0.5} + P_{1K} + P_{1.5K}}{P_{total}} = \dfrac{3.6\ \mu}{4.42\ \mu} = 81.45\% < 90\%$

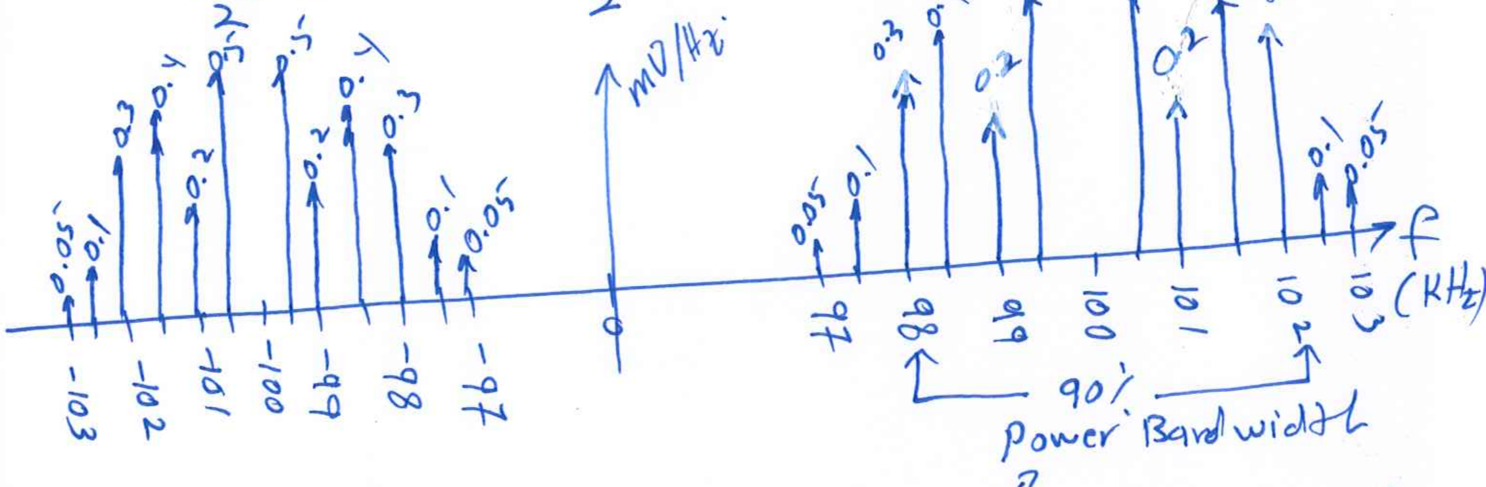
④ $\dfrac{P_{0.5} + P_{1K} + P_{1.5K} + P_{2K}}{P_{total}} = \dfrac{4.37\ \mu}{4.42\ \mu} = 97.77\% > 90\%$

so 90% Power Bandwidth is 2KHz.

b. Plot the Amplitude Spectrum of $s(t) = g(t) \times cos(2\pi 10^5 t)$.

$S(f) = F\{g(t) \times \cos(2\pi \times 10^5 t)\} = G(f) \underset{\text{conv}}{*} \left[\dfrac{\delta(f - 100K) + \delta(f + 100K)}{2}\right]$

$= \frac{1}{2} G(f - 100K) + \frac{1}{2} G(f + 100K)$



c. Determine the 90% power bandwidth of $s(t)$.

$\Rightarrow$ 90% Power Bandwidth $= 2 \times \underset{\substack{\uparrow \\ \text{answer of part a.}}}{2\text{ Hz}} = 4\text{ KHz}$

Problem#3 [10 Points]

Determine and plot the amplitude spectrum of the signal $y(t)$ shown in figure 3.

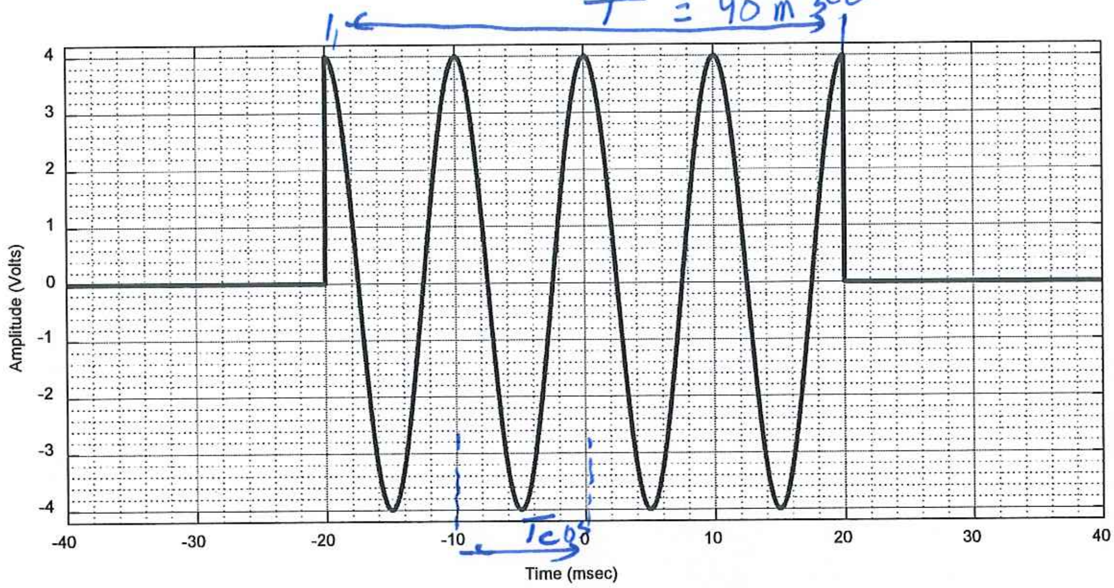


Figure 3: time domain plot of $y(t)$

$$y(t) = 4\cos\left(2\pi * \frac{1}{T_c} * t\right) rect\left[\frac{t}{40m}\right]$$

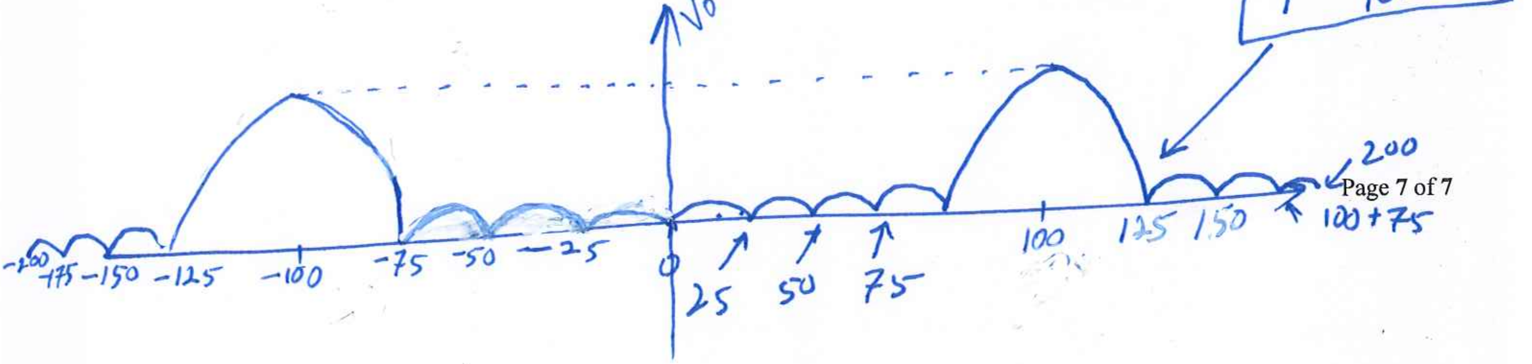
$$'' = 4\cos\left(2\pi * \frac{1}{10m} t\right) rect\left[\frac{t}{40m}\right]$$

$$'' = 4\cos(2\pi\, 0.1k\, t)\, rect\left(\frac{t}{40m}\right)$$

$$Y(f) = 4\left[\frac{\delta(f-0.1k)+\delta(f+0.1k)}{2}\right] \underset{\text{conv}}{*} 40m\, sinc(40m f)$$

$$= 0.16\left[\delta(f-0.1k)+\delta(f+0.1k)\right] \underset{\text{conv}}{*} sinc(\overset{Tf}{40m f})$$

$$\boxed{\frac{1}{T} = \frac{1}{40m} = 25}$$



BIRZEIT UNIVERSITY

Faculty of Engineering and Technology
Department of Electrical and Computer Engineering
Communication Systems ENEE 339

Instructor: Dr. Wael Hashlamoun

Midterm Exam
First Semester 2017-2018

Date: Wednesday 29/11/2017 Time: 75 minutes
Name: Student #:

Opening Remarks:

- Calculators are allowed, but mobile phones, books, notes, formula sheets, and other aids are not allowed.
- You are required to show all your work and provide the necessary explanations everywhere to get full credit.

Problem 1: 25 Points
The Fourier transform $G(f)$ of a signal g(t) is given by:

$$G(f) = \begin{cases} A\cos(\frac{\pi f}{2W}) & -W \le f \le W \\ 0 & |f| > W \end{cases}$$

7 a. Find the absolute bandwidth of g(t)
8 b. Find the equivalent rectangular bandwidth of g(t).
5 c. Use the time-bandwidth relationship to find the equivalent effective time duration of g(t)
5 d. The signal $g(t)cos2\pi(3Wt)$ is passed through an ideal low pass filter with bandwidth W, find the filter output.

Problem 2: 25 Points
The message signal $m(t) = 2\cos(2\pi 50t) + 4\cos(2\pi 100t)$ along with the carrier signal $c(t) = 4\cos(2\pi 1000t)$ are applied to an upper single sideband modulator to generate the modulated signal s(t):

5 a. Find the average power of m(t).
5 b. Find the bandwidth of m(t)
10 c. Find the time-domain expression of the modulated signal s(t).
5 d. Explain how m(t) can be recovered from s(t) without distortion. Use a block diagram to illustrate your method.

Problem 3: 25 Points

The Fourier transform of a message m(t) is given as:

$$M(f) = \begin{cases} M_0 & -W \le f \le W \\ 0 & |f| > W \end{cases}$$

This message is applied to a double sideband modulator along with the carrier $c(t) = 10\cos(2\pi(10000)t)$ to produce the modulated signal $s(t)$

5 a. Find the message m(t).
5 b. Find the time-domain representation of s(t)
5 c. Find and sketch $S(f)$, the Fourier transform of $s(t)$.
5 d. Find the transmission bandwidth
5 e. If $s(t)$ is applied to an ideal envelope detector, find its output.

Problem 4: 25 Points

Consider the FM signal $s(t) = 10\cos[2\pi(10000)t + 1.6 sin 2\pi(100)t]$. The FM modulator sensitivity is $k_f = 10\ Hz/V$. The modulated signal $s(t)$ is passed through an ideal bandpass filter with bandwidth 500 Hz centered at the carrier frequency $f_c = 10000$ Hz to produce the signal $g(t)$

6 ← a. Find the instantaneous frequency of $s(t)$
6 ← b. Find the message m(t)
3 ← c. Find the peak frequency deviation of $s(t)$.
10 { 6 ← d. Find the filter output $g(t)$
4 e. Find the fraction of the power contained in $g(t)$ to that in $s(t)$.

Good Luck

TABLE A6.4 *Trigonometric Identities*

$\exp(\pm j\theta) = \cos\theta \pm j\sin\theta$

$\cos\theta = \frac{1}{2}[\exp(j\theta) + \exp(-j\theta)]$

$\sin\theta = \frac{1}{2j}[\exp(j\theta) - \exp(-j\theta)]$

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta - \sin^2\theta = \cos(2\theta)$

$\cos^2\theta = \frac{1}{2}[1 + \cos(2\theta)]$

$\sin^2\theta = \frac{1}{2}[1 - \cos(2\theta)]$

$2\sin\theta\cos\theta = \sin(2\theta)$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$

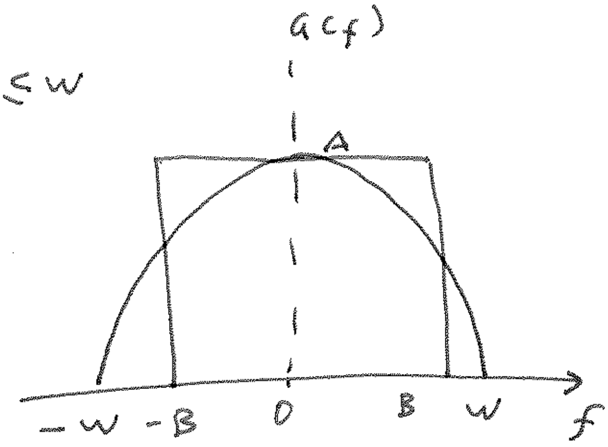
$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$

$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha - \beta) + \sin(\alpha + \beta)]$

Midterm Exam Communication Systems ENEE 339 29/11/2017

Problem 1

$$G(f) = A\cos\left(\frac{\pi f}{2W}\right) \quad -W \le f \le W$$



a. B.W (absolute) = W

b. $2B_{eq}A^2 = \int_{-W}^{W} |G(f)|^2 df$

$$\int_{-W}^{W} |G(f)|^2 df = \int_{-W}^{W} A^2 \cos^2\left(\frac{\pi f}{2W}\right) df$$

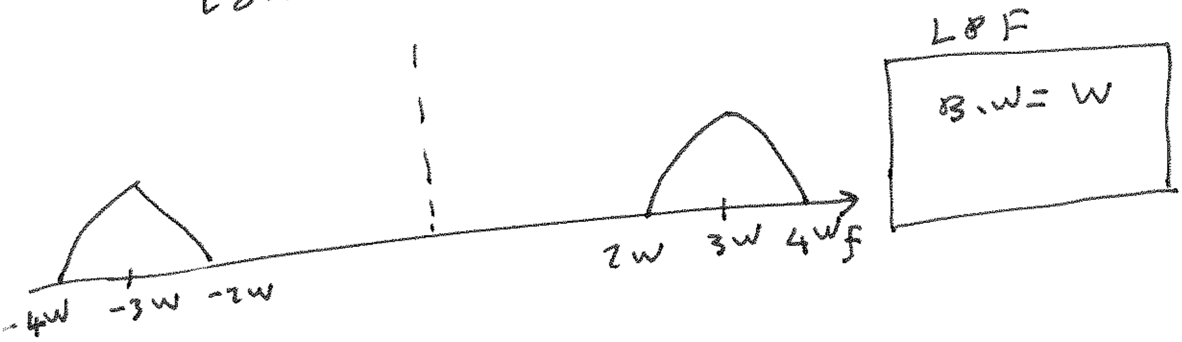
$$= \int_{-W}^{W} \frac{A^2}{2}\left[1 + \cos\frac{2\pi f}{2W}\right] df = \frac{A^2}{2}(2W) + \cancel{\frac{A^2}{2}\int_{-W}^{W} \cos\frac{2\pi f}{2W}\, df}^{\,0}$$

$$2BA^2 = \frac{A^2}{2}\, 2W \Rightarrow \boxed{B_{eq} = \frac{W}{2}}$$

c. $B_{eq}T_{eq} = \frac{1}{2} \Rightarrow T_{eq} = \frac{1}{2B_{eq}} = \frac{1}{2 \cdot \frac{W}{2}} = \frac{1}{W}$

$$\boxed{T_{eq} = \frac{1}{W}}$$

d. $\mathfrak{F}\{g(t)\cos 2\pi(3W)t\} = \frac{1}{2}G(f-3W) + \frac{1}{2}G(f+3W)$

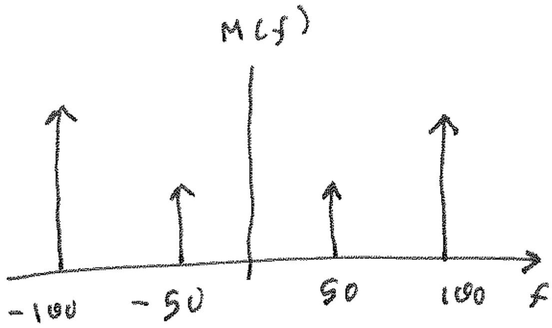


$\Rightarrow$ output $= 0$

Problem 2 :

$m(t) = 2\cos 2\pi(50)t + 4\cos 2\pi(100)t$

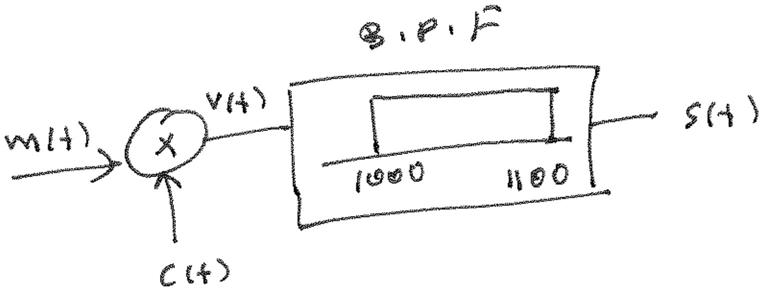
$c(t) = 4\cos 2\pi(1000)t$



a. $\langle m(t)^2 \rangle = \frac{(2)^2}{2} + \frac{(4)^2}{2} = 10 \text{ W}$

b. B.W = 100 Hz.

c.



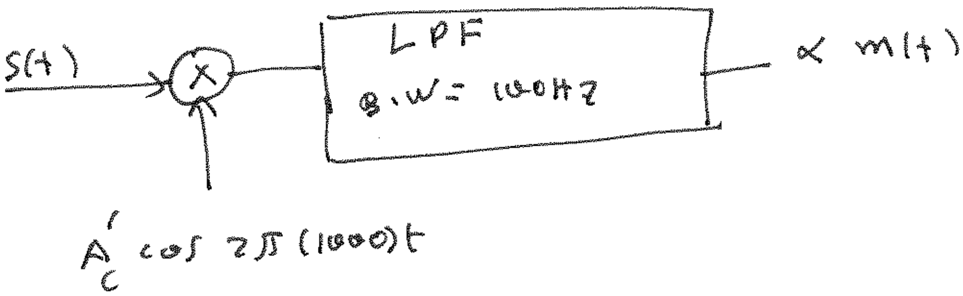
$v(t) = 4\cos 2\pi(1000)t\left[2\cos 2\pi(50)t + 4\cos 2\pi(100)t\right]$

$= 4\cos 2\pi(1050)t + 4\cos 2\pi(950)t$

$+ 8\cos 2\pi\, 1100\, t + 8\cos 2\pi(900)t$

$\Rightarrow \boxed{s(t) = 4\cos 2\pi(1050)t + 8\cos 2\pi(1100)t}$

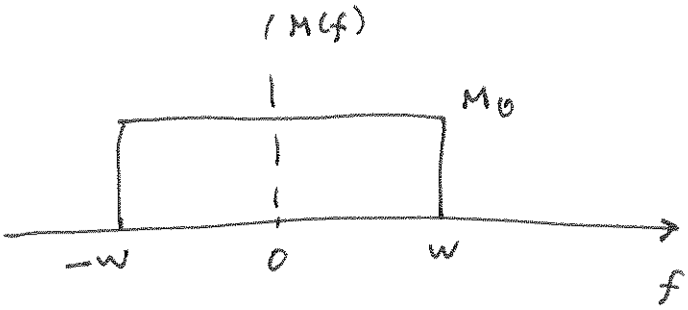
d.



Problem 3

$c(t) = 10\cos 2\pi(10{,}000)t$

$m(t) = M_0(2W)\,\mathrm{sinc}(2Wt)$



a. From Tables

$A\,\mathrm{rect}\left(\frac{t}{T}\right) \rightarrow AT\,\mathrm{sinc}(fT)$

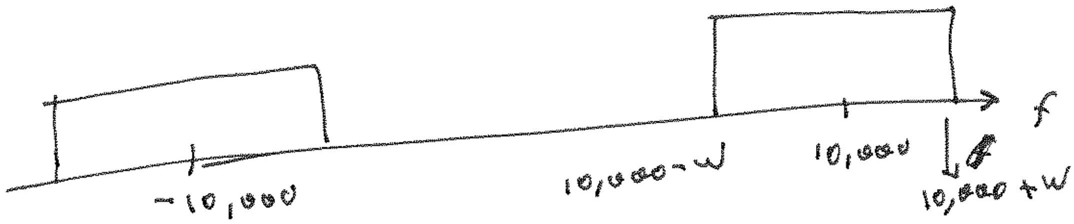
$A\,\mathrm{rect}\left(\frac{f}{T}\right) \leftarrow AT\,\mathrm{sinc}(tT)$

$M_0\,\mathrm{rect}\left(\frac{f}{2W}\right) \leftarrow M_0(2W)\,\mathrm{sinc}(t\,\underset{2W}{T})$

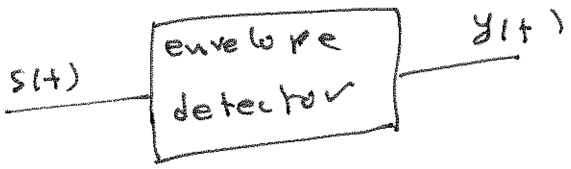
$$\boxed{m(t) = M_0(2W)\,\mathrm{sinc}(2Wt)}$$

b. $s(t) = m(t)\;10\cos 2\pi(10{,}000)t$

c. $S(f) = 5\left[M(f-10{,}000) + M(f+10{,}000)\right]$



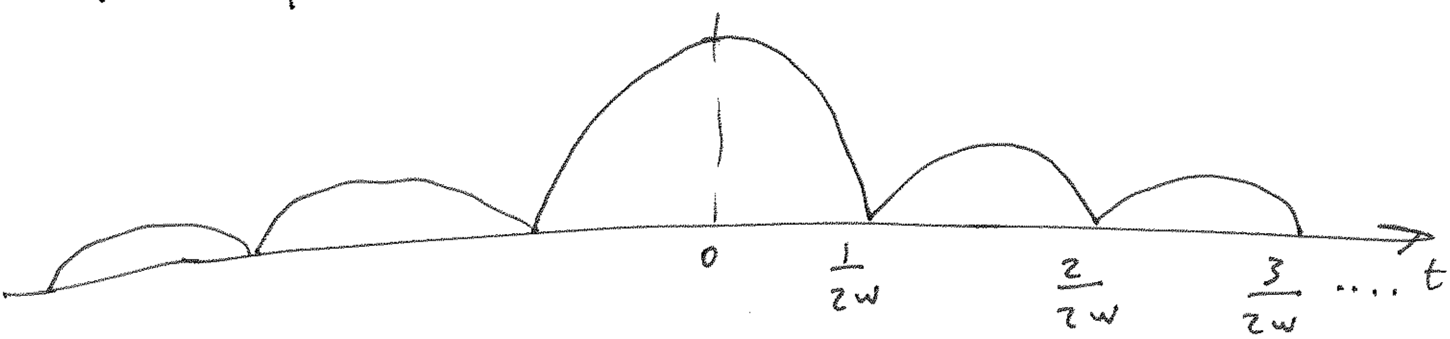
d. $B.W = 2W$.



e. $y(t) = |s(t)|$

$LP\left|10\cos 2\pi(10{,}000)t\;(2WM_0)\,\mathrm{sinc}(2Wt)\right|$

$y(t) = |m(t)| = 2WM_0\left|\mathrm{sinc}\,2Wt\right|$



Problem 4

$$s(t) = 10 \cos\left[2\pi(10{,}000)t + 1.6 \sin 2\pi(100)t\right]$$

$$K_f = 10 \text{ Hz/V}.$$

a. $$f_i(t) = \frac{1}{2\pi}\frac{d}{dt}\left[2\pi(10{,}000)t + 1.6 \sin 2\pi(100)t\right]$$

$$= 10{,}000 + \frac{1}{2\pi}\cdot(1.6)(2\pi\,100)\cos 2\pi(100)t$$

$$\boxed{f_i(t) = 10{,}000 + (1.6)(100)\cos 2\pi(100)t}$$

b. $$f_i(t) = f_c + K_f\, m(t)$$

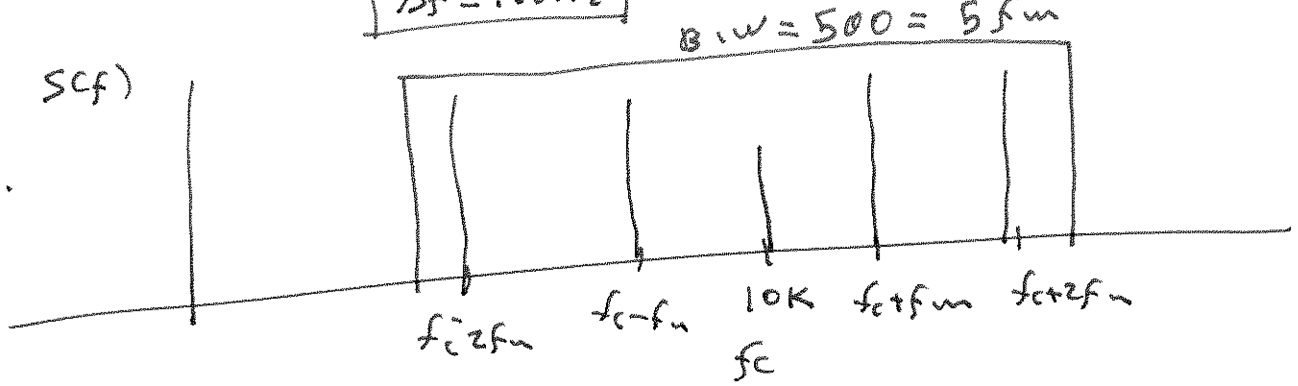
$$10\, m(t) = (1.6)(100)\cos 2\pi(100)t \Rightarrow$$

$$\boxed{m(t) = 16 \cos 2\pi(100)t}$$

c. $\Delta f = (1.6)(100) = 160\text{ Hz}$; Also, $\beta = \frac{\Delta f}{f_m} \Rightarrow f_m = \beta f_m$

$$\boxed{\Delta f = 160\text{ Hz}}$$

d.



filter output consists of 5 terms $\quad f_m = 100$

$$g(t) = A_c\Big[\underset{0.4554}{J_0(1.6)}\cos 2\pi f_c t + \underset{0.5699}{J_1(1.6)}\cos 2\pi(f_c+f_m)t$$
$$+ \underset{-0.5699}{J_{-1}(1.6)}\cos 2\pi(f_c-f_m)t$$
$$+ \underset{0.2570}{J_2(1.6)}\cos 2\pi(f_c+2f_m)t$$
$$+ \underset{0.2570}{J_2(1.6)}\cos 2\pi(f_c-2f_m)t\Big]$$

$$\frac{\langle g(t)^2\rangle}{\langle s(t)^2\rangle} = \frac{\frac{A_c^2}{2}\left[(0.4554)^2 + 2(0.5699)^2 + 2(0.2570)^2\right]}{(A_c^2/2)} = 0.9890$$

Birzeit University
Faculty of Engineering and Technology
Department of Electrical and Computer Engineering
Information and Coding Theory ENEE 3306
Midterm Exam

Instructors: Dr. Wael Hashlamoun   Date: June 7, 2022

Problem 1: 18 Points

The signal $x(t) = 4cos(2\pi f_0 t)$ is applied to a uniform quantizer with L quantization levels and a dynamic range $(-4, 4)$ V. Find the minimum value of L that will achieve a signal to quantization noise ratio $SQNR \geq 1000$.

$$SQNR = \frac{\langle x(t)^2 \rangle}{\Delta^2/12} = \frac{12 * \langle x(t)^2 \rangle}{\Delta^2} \geq 1000$$

$$x(t) = 4\cos 2\pi f_0 t \Rightarrow \langle x(t)^2 \rangle = \frac{(4)^2}{2} = \frac{16}{2} = 8$$

$$\Rightarrow SQNR = \frac{12 * 8}{\Delta^2} \geq 1000$$

$$\Delta^2 \leq \frac{12 * 8}{1000} = 0.096 \Rightarrow \Delta \leq 0.3098$$

$$\Delta = \frac{2 * 4}{L} = \frac{8}{L}$$

-4 L levels 4

$$\frac{8}{L} \leq 0.3098$$

$$\frac{8}{0.3098} \leq L \Rightarrow L \geq 25.8$$

$$\Rightarrow \boxed{L \geq 26} \quad L = 26 \text{ levels}$$

$$\Rightarrow SQNR = 1014$$

Problem 2: 22 Points

A digital communication signaling scheme employs the two signals s(t) and 0 to transmit binary digits 1 and 0, respectively, over a channel corrupted by AWGN with zero mean and power spectral density $N_0/2$. Let P(1) = P(0) = ½ and let s(t) be defined as:

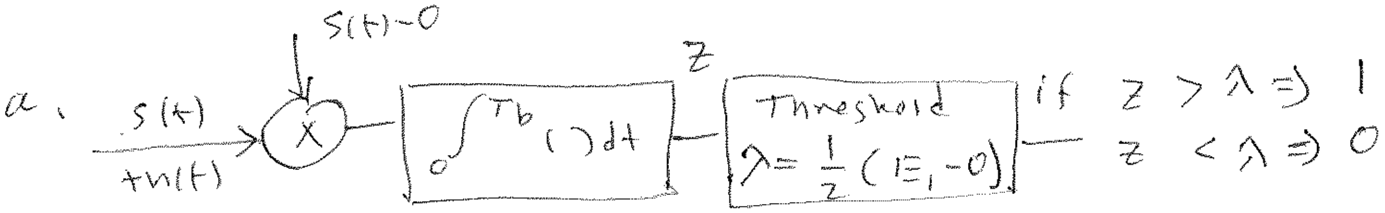
$$s(t) = \begin{cases} A\sin\left(\frac{\pi t}{T_b}\right) & ,0 \le t \le T_b \\ 0 & ,otherwise \end{cases}$$

6 a. Draw the block diagram of the optimum receiver implemented in terms of correlators.

8 b. Find the average probability of error of the optimum receiver.

8 c. Find the optimum threshold of the receiver which minimizes the probability of error.

a.



$$P(E) = Q\left(\sqrt{\frac{\int (s_1 - s_2)^2 dt}{2N_0}}\right)$$

$$\int_0^{T_b} (s_1(t) - 0)^2 dt = \int_0^{T_b} A^2 \sin^2 \frac{\pi t}{T_b}\, dt = \frac{A^2}{2}\int_0^{T_b} \left[1 + \cos\frac{2\pi t}{T_b}\right] dt$$

$$= \frac{A^2 T_b}{2} + \frac{A^2}{2}\int_0^{T_b} \cos\frac{2\pi t}{T_b}\, dt \longrightarrow \left.\frac{\sin\frac{2\pi t}{T_b}}{2\pi/T_b}\right|_0^{T_b} = 0$$

$$\Rightarrow \frac{A^2 T_b}{2}$$

$$P(E) = Q\left(\sqrt{\frac{A^2 T_b/2}{2N_0}}\right) = Q\left(\sqrt{\frac{A^2 T_b}{4N_0}}\right)$$

$$\lambda = \frac{1}{2}(E_1 - E_0) = \frac{1}{2} * E_1 = \frac{1}{2} * \frac{A^2 T_b}{2}$$

$$\lambda = \frac{A^2 T_b}{4}$$

Problem 3: 20 Points

An analog base-band signal with 3 KHz bandwidth is to be transmitted over a PCM digital communication system. The signal is sampled at its Nyquist rate, uniformly quantized into one of 256 levels, and then encoded into binary digits.

12 4 a. Find the data rate in bits per second at the binary encoder output.

6 b. If the binary data is converted into a polar NRZ baseband signal m(t), find the 90% bandwidth of m(t).

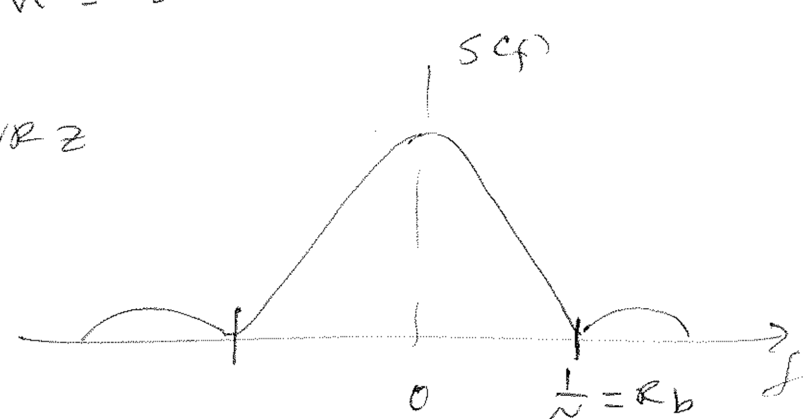
B.W = 3000 HZ

4 $f_s = 2W = 6000$ HZ

$L = 256 \Rightarrow L = 2^8 \Rightarrow$ 8-bit quantizer

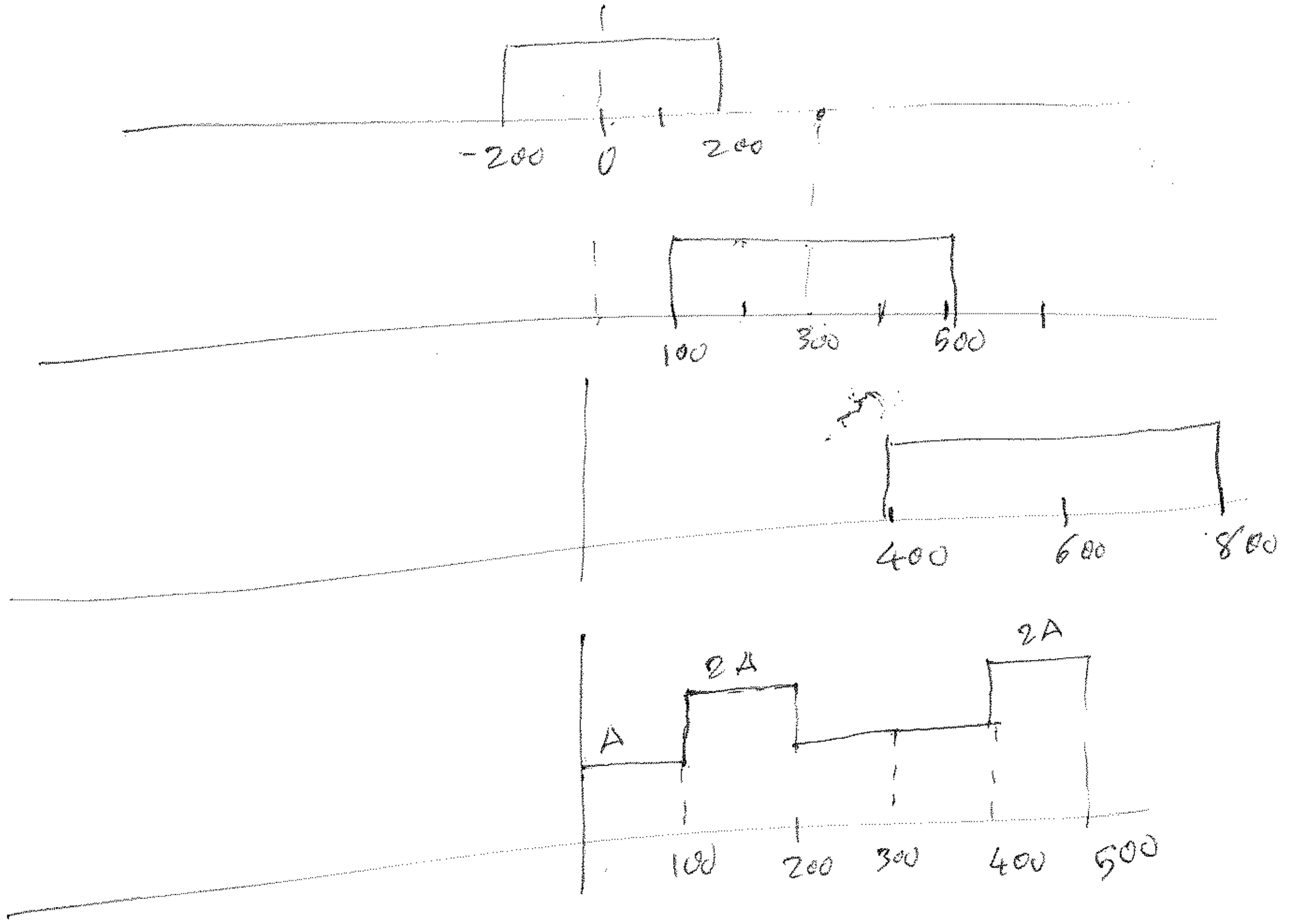
6 $R_b = 2Wn = 6000 * 8 = 48000$ bps

b. polar NRZ

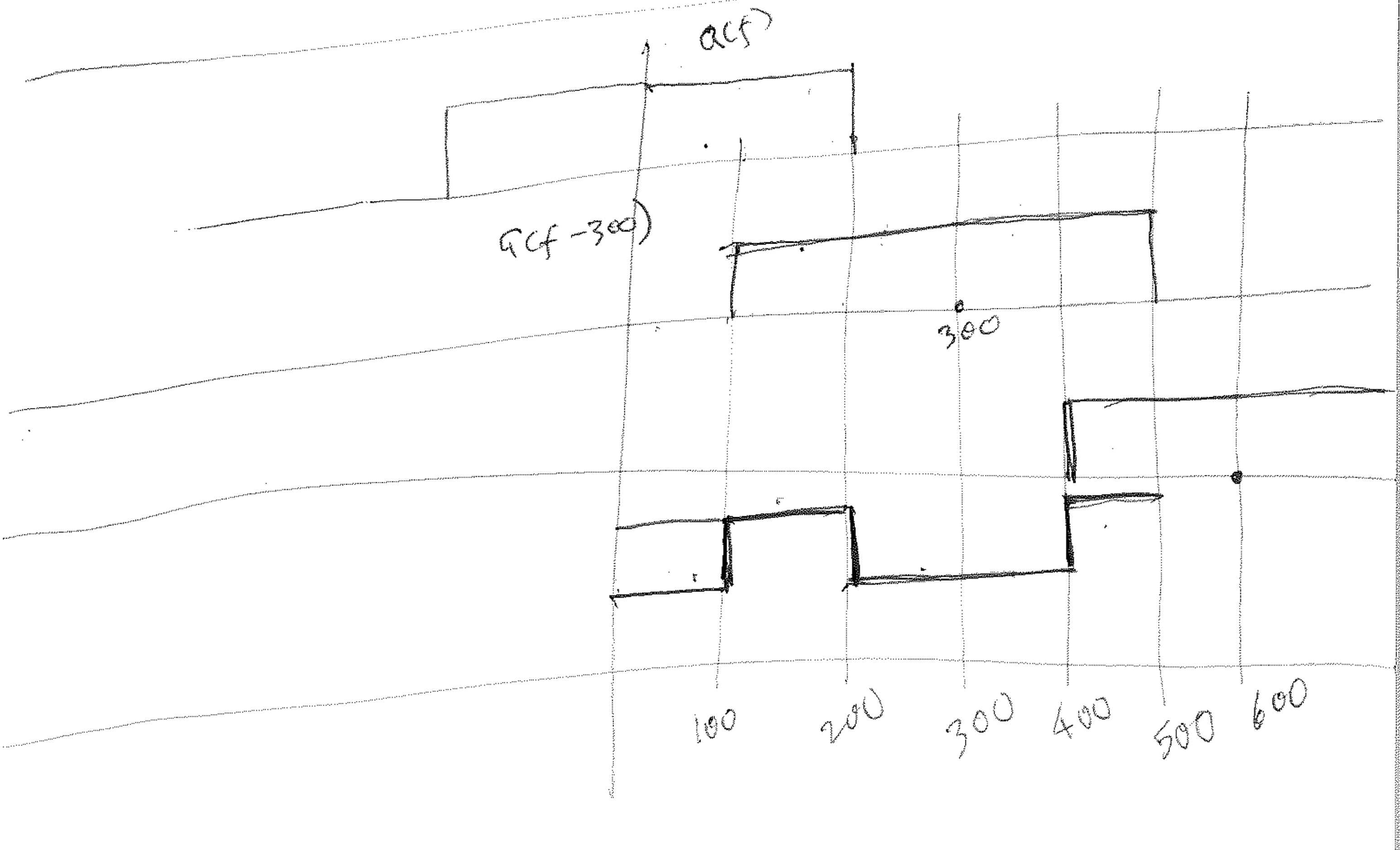


6 B.W = R_b =

= 48,000 HZ



-200
0
200
100
300
500
400
600
800
A
2A
2A
100
200
300
400
500



G(f)
G(f -300)
300
100
200
300
400
500
600

Problem 4: 20 Points

The Fourier transform, $G(f)$, of a signal $g(t)$ is given as:

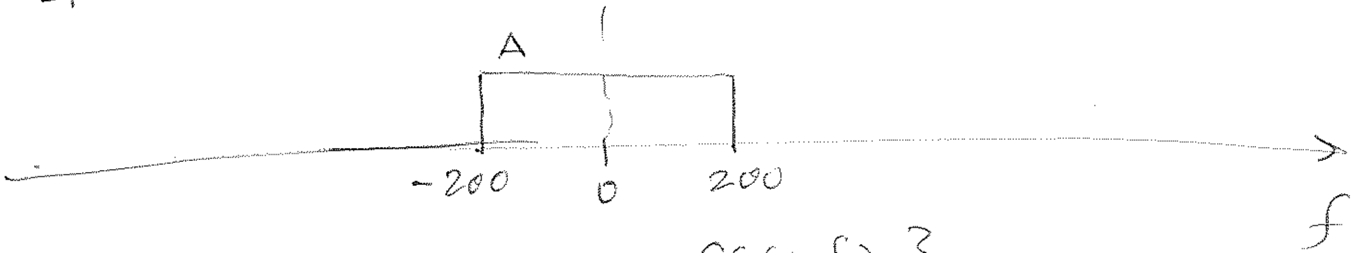
$$G(f) = \begin{cases} A, & -200 \leq f \leq 200 \\ 0, & |f| > 200 \end{cases}$$

The signal $g(t)$ is ideally sampled at a rate of 300 samples/sec to produce the sampled signal $g_s(t)$.

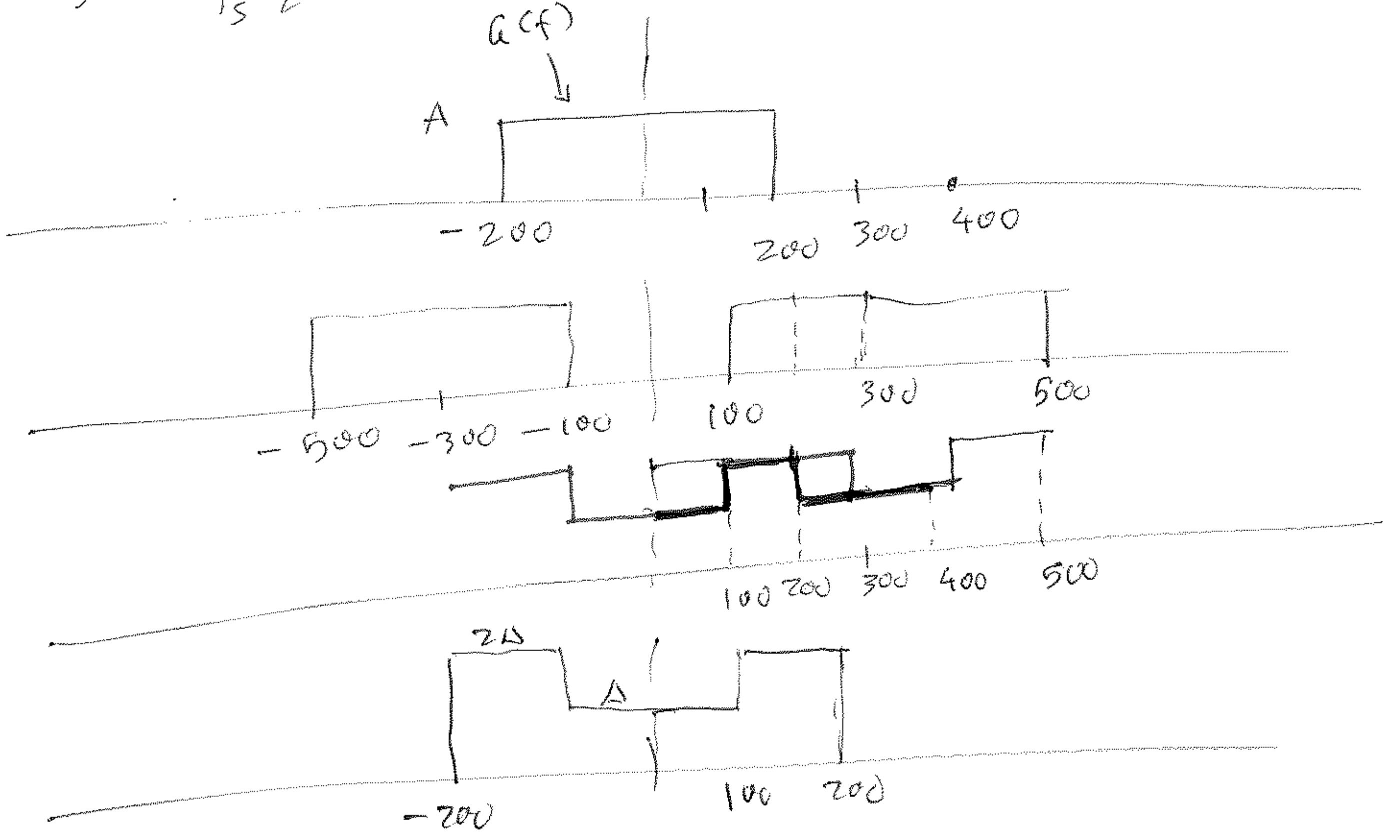
12 a. Find and sketch $G_s(f)$, the Fourier transform of $g_s(t)$ for $-500 \leq f \leq 500$ Hz

5 b. If $g_s(t)$ is applied to an ideal low pass filter with a bandwidth of 200 Hz, sketch the Fourier transform of the signal appearing at the output of the filter.

3 c. Based on the results of Part b, do you think that $g(t)$ can be recovered from $g_s(t)$ without distortion? Explain why.



$$G_s(f) = \frac{1}{T_s}\{G(f) + G(f - f_s) + G(f + f_s)\}$$



$\Rightarrow$ c

Problem 5: 20 Points

Consider two binary digital communication systems: The first employs the signals + g(t) and –g(t) to transmit the equally likely bits 1 and 0, respectively, over a channel corrupted by AWGN with zero mean and power spectral density $N_0/2$. The signal g(t) is given by:

$$g(t) = \begin{cases} A & ,0 \le t \le T_b \\ 0 & ,otherwise \end{cases}$$

The second system uses the signals + s(t) and –s(t) to represent the digits 1 and 0. The signal s(t) is given as:

$$s(t) = \begin{cases} \frac{2}{T_b}t & ,0 \le t \le T_b/2 \\ 0 & ,T_b/2 \le t \le T_b \end{cases}$$

If the two systems are to have the same probability of error, find the value of A.

$$E_g = \int_0^{T_b} A^2 dt = A^2 T_b$$

$$E_s = \int_0^{T_b/2} \left(\frac{2}{T_b}t\right)^2 dt = \frac{4}{T_b^2}\int_0^{T_b/2} t^2 dt$$

$$= \frac{4}{T_b^2}\frac{T^3}{3}\Bigg|_0^{T_b/2} = \frac{4}{T_b^2}\cdot\frac{T_b^3}{3(8)} = \frac{T_b}{6}$$

$$= \frac{4T_b}{3}$$

If $P(E_1) = P(E_2)$, then

$$E_g = E_s$$

$$= A^2 \cancel{T_b} = \frac{4\cancel{T_b}}{3(8)} = \frac{T_b}{6}$$

$$A = \sqrt{\frac{4}{24}}$$

$$= 0.408$$

Birzeit University
Faculty of Engineering and Technology
Department of Electrical and Computer Engineering
Communication Systems ENEE 3309
Midterm Exam

Instructors: Dr. Wael Hashlamoun, Dr. Ashraf Rimawi Date: Nov. 18, 2021

Problem 1: 25 Points

Consider the normal AM signal $s(t) = A_c\,[1 + \mu \cos(2\pi 150 t)]\, \cos 2\pi(1500)t$.
When $\mu = 0.42$, s(t) has a total average power of 47.3 W.

a. Find the power efficiency η
b. Find the bandwidth of s(t)
c. Calculate the average power in the carrier
d. Calculate the average power in the upper sideband.

a. $\eta = \dfrac{\mu^2}{2+\mu^2} = \dfrac{(0.42)^2}{2+(0.42)^2} = 0.081$

b. $B.W = 2 f_m = 2(150) = 300\ Hz$

c. $s(t) = A_c \cos 2\pi f_c t + A_c \mu \cos 2\pi f_c t \cos 2\pi f_m t$

$= A_c \cos 2\pi f_c t + \dfrac{A_c \mu}{2} \cos 2\pi (f_c + f_m) t$

$+ \dfrac{A_c \mu}{2} \cos 2\pi (f_c - f_m) t$

$P_{av} = \dfrac{A_c^2}{2} + \dfrac{A_c^2 \mu^2}{8} + \dfrac{A_c^2 \mu^2}{8}$

$P_{av} = \dfrac{A_c^2}{2}\left(1 + \dfrac{1}{2}\mu^2\right) \Rightarrow 47.3 = P_c\left(1 + \dfrac{1}{2}(0.42)^2\right)$

$\Rightarrow \boxed{} = \dfrac{A_c^2}{2}$

$\Rightarrow$

$47.3 = 1.088\ P_c \Rightarrow P_c = 43.46 = A^2/2$

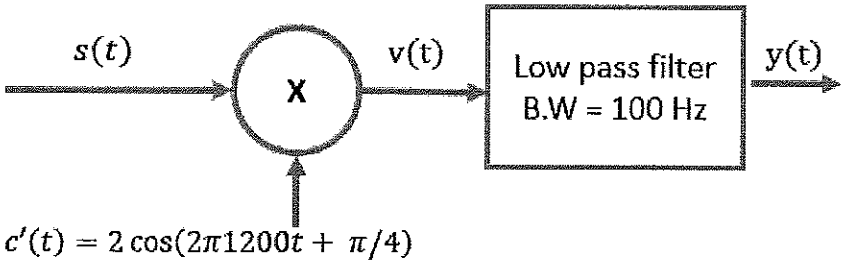
d. upper side band $= \dfrac{A_c \mu}{2} \cos 2\pi (f_c + f_m) t$

Power $= \dfrac{A_c^2 \mu^2}{8} = \dfrac{A_c^2}{2} \cdot \dfrac{\mu^2}{4}$

$= (43.46) * \dfrac{(0.42)^2}{4} = 1.9$[illegible]

Problem 2: 25 Points

The message signal $m(t) = 3\cos(2\pi 60t) + 6\cos(2\pi 120t)$ along with the carrier signal $c(t) = 4\cos(2\pi 1200t)$ are applied to a modulator that generates the double sideband suppressed carrier signal s(t). The demodulator is as shown in the figure below. It consists of a multiplier followed by a low pass filter, where the locally generated signal is $c'(t) = 2\cos(2\pi 1200t + \pi/4)$ and the bandwidth of the low pass filter is 100 Hz.



a. Find the bandwidth of m(t).
b. Find the time-domain expression of the modulated signal s(t).
c. Find the total average transmitted power.
d. Find the signal at the demodulator output.

a. B.W = 120 Hz

b. $s(t) = 4\left[\cos 2\pi(1200)t\right]\left[3\cos 2\pi(60)t + 6\cos 2\pi(120)t\right]$

$= 12\cos 2\pi(1200)t \cos 2\pi(60)t$

$+ 24\cos 2\pi(1200)t \cos 2\pi(120)t$

$= 6\cos 2\pi(1260)t + 6\cos 2\pi(1140)t$

$+12\cos 2\pi(1320)t + 12\cos 2\pi(1080)t$

c. Power in $s(t) = \frac{(6)^2}{2} + \frac{(6)^2}{2} + \frac{(12)^2}{2} + \frac{(12)^2}{2}$

$= 36 + 144 = 180\ W$

d. component at 120 HZ will not pass plus other high frequency terms

$y(t) = s(t) * 2\cos(2\pi 1200t + \pi/4)$

$= LP\{6\cos 2\pi(1260)t \times 2\cos(2\pi 1200t + \pi/4)\}$

$= 6\cos 2\pi(60)t \cos\frac{\pi}{4}$

$y(t) = \frac{6}{\sqrt{2}}\cos 2\pi(60)t = 4.24\cos 2\pi(60)t$

Problem 3: 25 Points

Consider the double sideband suppressed carrier signal

$$s(t) = 2\cos(2\pi 140t)\cos(2\pi 1750t)$$

An upper single sideband signal g(t) is to be generated from s(t) using the filtering method

7 a. Find g(t), assuming an ideal bandpass filter is used.
5 b. Find the best choice for the center frequency of the bandpass filter used to produce g(t)
8 c. Draw the block diagram of the receiver used to recover m(t) from g(t) without distortion identifying the details and properties of each block.
5 d. What will be the output of the diagram of part c?

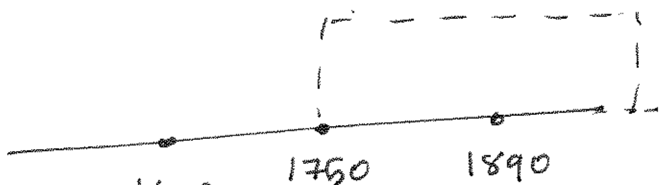
$s(t) = 2\cos 2\pi(140)t \cos 2\pi(1750)t$

$1750 - 140$

7 $= \cos 2\pi(140 + 1750)t + \cos 2\pi(1610)$

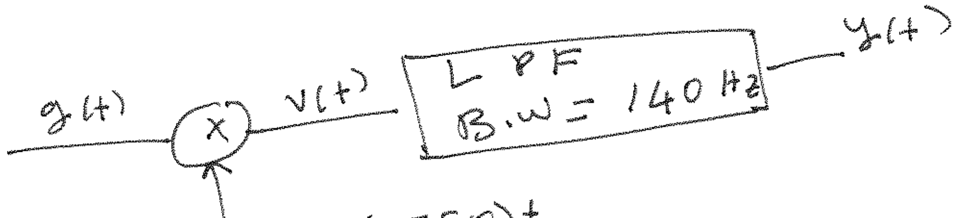
a. $s(t) = \cos 2\pi(1890)t + \cos 2\pi(1610)t$

$g(t) = \cos 2\pi(1890)t$



b. $f_o = 1890$

c.



$A_c' \cos 2\pi(1750)t$

$v(t) = A_c' \cos 2\pi(1890)t \cos 2\pi(1750)t$

$v(t) = \frac{A_c'}{2} \cos 2\pi(3640)t + \frac{A_c'}{2} \cos 2\pi(140)t$

$y(t) = \frac{A_c'}{2} \cos 2\pi(140)t$

Problem 4: 25 Points

The audio signal $m(t) = A_m\cos(2\pi(100)t)$ frequency modulates the carrier $c(t) = \cos2\pi(1000)t$. The resulting FM signal is

$$s(t) = \cos[2\pi(1000)t + \beta sin2\pi(100)t].$$

When $A_m = 1.8$, s(t) shows a peak frequency deviation of 320 Hz.

7 a. Find the FM modulation index
6 b. Use Casron's rule to estimate the bandwidth of s(t)
6 c. Find k_f , the sensitivity of the FM modulator in Hz/V
6 d. If A_m changes to 3.2 V, find the new frequency modulation index.

Good Luck

$s(t) = \cos(2\pi(1000)t + \beta \sin 2\pi(100)t)$

$A_m = 1.8$, $\Delta f = 320$ HZ

a. $\beta = \dfrac{\Delta f}{f_m} = \dfrac{320}{100} = 3.2$

b. $B.W = 2(\beta + 1) f_m = 2(3.2+1) * 100$

$= 840$ HZ

c. $\Delta f = K_f \, A_m$

$320 = K_f \, (1.8) \Rightarrow K_f = 177.77$ Hz/V

d. $\Delta f = K_f \, A_m$

$= 177.77 * 3.2$

$= 568.864$ Hz.

$\beta = \dfrac{568.4}{100} = 5.68$

TABLE A6.4 *Trigonometric Identities*

$\exp(\pm j\theta) = \cos\theta \pm j\sin\theta$

$\cos\theta = \frac{1}{2}[\exp(j\theta) + \exp(-j\theta)]$

$\sin\theta = \frac{1}{2j}[\exp(j\theta) - \exp(-j\theta)]$

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta - \sin^2\theta = \cos(2\theta)$

$\cos^2\theta = \frac{1}{2}[1 + \cos(2\theta)]$

$\sin^2\theta = \frac{1}{2}[1 - \cos(2\theta)]$

$2\sin\theta\cos\theta = \sin(2\theta)$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$

$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$

$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha - \beta) + \sin(\alpha + \beta)]$

TABLE A6.2 *Fourier-Transform Pairs*

Time Function	*Fourier Transform*
$\mathrm{rect}\left(\frac{t}{T}\right)$	$T\,\mathrm{sinc}(fT)$
$\mathrm{sinc}(2Wt)$	$\frac{1}{2W}\mathrm{rect}\left(\frac{f}{2W}\right)$
$\exp(-at)u(t), \quad a > 0$	$\frac{1}{a + j2\pi f}$
$\exp(-a\lvert t\rvert), \quad a > 0$	$\frac{2a}{a^2 + (2\pi f)^2}$
$\exp(-\pi t^2)$	$\exp(-\pi f^2)$
$\begin{cases} 1 - \frac{\lvert t\rvert}{T}, & \lvert t\rvert < T \\ 0, & \lvert t\rvert \geq T \end{cases}$	$T\,\mathrm{sinc}^2(fT)$
$\delta(t)$	1
1	$\delta(f)$
$\delta(t - t_0)$	$\exp(-j2\pi f t_0)$
$\exp(j2\pi f_c t)$	$\delta(f - f_c)$
$\cos(2\pi f_c t)$	$\frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)]$
$\sin(2\pi f_c t)$	$\frac{1}{2j}[\delta(f - f_c) - \delta(f + f_c)]$
$\mathrm{sgn}(t)$	$\frac{1}{j\pi f}$
$\frac{1}{\pi t}$	$-j\,\mathrm{sgn}(f)$
$u(t)$	$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$
$\sum_{i=-\infty}^{\infty} \delta(t - iT_0)$	$\frac{1}{T_0}\sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_0}\right)$