Institut Teknologi Telkom Purwokerto

TEKNIK PENGINTEGRALAN

Dian Kartika Sari, S.Si., M.Pd

2. Integral Fungsi Trigonometri

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Bentuk : $\int \cos^n x\, dx \;\; \& \;\; \int \sin^n x\, dx$

* Untuk *n* ganjil, Tuliskan :

$$\sin^n x = \sin x \sin^{n-1} x \;\; \text{dan} \;\; \cos^n x = \cos x \cos^{n-1} x$$

dan gunakan identitas $\sin^2 x + \cos^2 x = 1$

* Untuk *n* genap, Tuliskan :

$$\sin^n x = \sin^2 x \sin^{n-2} x \;\; \text{dan} \;\; \cos^n x = \cos^2 x \cos^{n-2} x$$

dan gunakan identitas $sin^2 x = \frac{1-\cos 2x}{2}$ dan $cos^2 x = \frac{1+\cos 2x}{2}$

Contoh

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$\sin x\, dx = -d(\cos x)$

$\cos x\, dx = d(\sin x)$

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Hitung

1. $\int \sin^3 x\, dx$
2. $\int \sin^4 x\, dx$

$\int \cos^3 x\, dx = \int \cos^2 x \cos x\, dx$

$= \int (1-\sin^2 x)\, d(\sin x)$

$= \sin x - \frac{1}{3}\sin^3 x + C$

$(1-\cos^2 x)(1-\cos^2 x)$

$1 - \cos^2 x - \cos^2 x + \cos^4 x$

$1 - 2\cos^2 x + \cos^4 x$

Jawab

1. $\int \sin^3 x dx = \int \sin^2 x \sin x dx = -\int (1-\cos^2 x)\, d(\cos x) = -\cos x + \frac{1}{3}\cos^3 x + C$

$\int \sin^5 x\, dx = \int \sin^4 x \cdot \sin x\, dx$

$= \int (\sin^2 x)^2 \cdot -d(\cos x)$

$= \int -(1-\cos^2 x)^2\, d(\cos x)$

$= \int -(1 - 2\cos^2 x + \cos^4 x)\, d(\cos x)$

$= \int -1 + 2\cos^2 x - \cos^4 x\, d(\cos x)$

$= -\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C$

Contoh

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Hitung

1. $\int \sin^3 x\,dx$
2. $\int \sin^4 x\,dx$

Jawab

1. $\int \sin^3 x dx = \int \sin^2 x \sin x dx = -\int (1-\cos^2 x)\, d(\cos x) = -\cos x + \frac{1}{3}\cos^3 x + C$

2. $\int \sin^4 x\,dx = \int \sin^2 x \sin^2 x\,dx = \int (\frac{1-\cos 2x}{2})(\frac{1-\cos 2x}{2})dx$

$= \frac{1}{4}\int (1-2\cos 2x+\cos^2 2x)dx = \frac{1}{4}(\int dx - 2\int \cos 2x\,dx + \int \frac{1+\cos 4x}{2}dx)$

$= \frac{1}{4}x - \frac{1}{4}\sin 2x + \frac{1}{8}x + \frac{1}{32}\sin 4x + C$

$= \frac{3}{8}x - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$

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- Bentuk $\int \sin^m x \cos^n x \, dx$

a). Untuk *n* atau *m* ganjil, keluarkan sin *x* atau cos *x* dan gunakan identitas $\sin^2 x + \cos^2 x = 1$

b). Untuk *m* dan *n* genap, tuliskan $\sin^m x$ dan $\cos^n x$ menjadi jumlah suku-suku dalam cosinus, gunakan identitas $\cos 2x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$\sin^2 x = \frac{1 - \cos 2x}{2}$ $\cos^2 x = \frac{1 + \cos 2x}{2}$

Contoh

$\sin x\,dx = -d(\cos x)$

$\cos x\,dx = d(\sin x)$

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$$\int \sin^3 x \cos^2 x\,dx = \int \sin^2 x \cos^2 x \sin x\,dx = -\int (1-\cos^2 x)\cos^2 x\,d(\cos x)$$

$$= -\int (\cos^2 x - \cos^4 x)\,d(\cos x) = -\left(\frac{1}{3}\cos^3 x - \frac{1}{5}\cos^5 x\right) + C$$

$$= \frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x + C$$

$$\int \sin^3 x \cos^3 x\,dx = \int \sin^2 x \cos^3 x \sin x\,dx$$

$$= \int (1-\cos^2 x)\cos^3 x \cdot -d(\cos x)$$

$$= \int -(\cos^3 x - \cos^5 x)\,d(\cos x)$$

$$= \int -\cos^3 x + \cos^5 x\,d\cos x$$

$$= -\frac{1}{4}\cos^4 x + \frac{1}{6}\cos^6 x + C$$

atau

$$= \int \sin^3 x \cos^2 x \cdot \cos x\,dx$$

$$= \int \sin^3 x (1-\sin^2 x)\cdot d(\sin x)$$

$$= \int \sin^3 x - \sin^5 x\,d(\sin x)$$

$$= \frac{1}{4}\sin^4 x - \frac{1}{6}\sin^6 x + C$$

Contoh

$$\int \sin^2 x \cos^2 x \, dx = \int \frac{1-\cos 2x}{2} \frac{1+\cos 2x}{2} \, dx$$

$$= \frac{1}{4}\int (1-\cos^2 2x)dx \quad = \frac{1}{4}\left(\int 1 - \frac{1+\cos 4x}{2} dx\right)$$

$\sin 2x = 2 \cdot \sin x \cdot \cos x$

$$= \frac{1}{8}\int dx - \frac{1}{8}\int \cos 4x \, dx$$

$$= \frac{1}{8}x - \frac{1}{32}\sin 4x + C$$

$\int \sin^2 x \cdot \cos^2 x \, dx = \int (\sin x \cdot \cos x)^2 \, dx$

$= \int \left(\frac{1}{2}\sin 2x\right)^2 dx$

$= \int \frac{1}{4}\sin^2 2x \, dx = \int \frac{1}{4}\left(\frac{1}{2} - \frac{1}{2}\cos 4x\right)dx = \int \frac{1}{8} - \frac{1}{8}\cos 4x + c$

$= \frac{1}{8}x - \frac{1}{8}\cdot\frac{1}{4}\sin 4x + c$

$(1-\cos 2x)(1+\cos 2x)$

$= 1 + \cos 2x - \cos 2x - \cos^2 2x$

$= \frac{1}{4}\int 1 - \left(\frac{1}{2} + \frac{1}{2}\cos 4x\right) dx$

$= \frac{1}{4}\int 1 - \frac{1}{2} - \frac{1}{2}\cos 4x \, dx$

$= \frac{1}{4}\int \frac{1}{2} - \frac{1}{2}\cos 4x \, dx$

$= \frac{1}{4}\left(\frac{1}{2}x - \frac{1}{2}\cdot\frac{1}{4}\sin 4x\right) + C$

$= \frac{1}{8}x - \frac{1}{32}\sin 4x + C$

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4. Substitusi Bentuk Akar

Integran memuat $\sqrt[n]{ax+b}$,misal $u=\sqrt[n]{ax+b}$

Contoh Hitung $\int \frac{dx}{2+2\sqrt{x}}$

Jawab :

$$\int \frac{dx}{2+2\sqrt{x}} = \int \frac{2u\,du}{2+2u} = \int \frac{u}{u+1}\,du$$

$$= \int \frac{u+1-1}{u+1}\,du = \int \left(1-\frac{1}{u+1}\right)du$$

$$= u - \ln(u+1) + C$$

$$= \sqrt{x} - \ln\left(1+\sqrt{x}\right) + C$$

Misal $u=\sqrt{x} \longrightarrow u^2 = x$

Dengan turunan implisit

$2u\frac{du}{dx}=1 \longrightarrow$ dx=2udu

Contoh

Hitunglah :

1. $\int x\sqrt[3]{x+4}\,dx$

Misal : $u = \sqrt[3]{x+4}$

$u = (x+4)^{1/3}$

$u^3 = x+4 \rightarrow x = u^3 - 4$

$3u^2 \frac{du}{dx} = 1$

$3u^2\,du = dx$

$\int x\sqrt[3]{x+4}\,dx$

$= \int (u^3-4)\cdot u \cdot 3u^2\,du$

$= \int (u^3-4)\,3u^3\,du$

$= \int 3u^6 - 12u^3\,du$

$= \frac{3}{7}u^7 - \frac{12}{4}u^4 + C$

$= \frac{3}{7}(x+4)^{7/3} - 3(x+4)^{4/3} + C$

$= \frac{3}{7}\sqrt[3]{(x+4)^7} - 3\sqrt[3]{(x+4)^4} + C$

TUGAS

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Hitunglah:

1. $\int 5xe^{2x}\,dx$
2. $\int sin^4x\,cos^5x\;\,dx$
3. $\int cos^5x\;\,dx$
4. $\int \frac{x^2+2x}{\sqrt{x+1}}\,dx$

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TERIMA KASIH