

# Machine Learning

Empirical and Structural Risk. Error Decomposition. Model Selection. Underfitting and overfitting

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ML Research



## ① Structural Risk and its Minimization

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- 1 Structural Risk and its Minimization
- 2 Overfittning and underfitting

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- 3 Model Selection overview

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- 5 Recent results: Double Descent

# Instance-based learning

- $X$  – set of objects descriptions,  $Y$  – set of objects labels
- Unknown target dependency: mapping  $y : X \rightarrow Y$
- Finite training set:  $X^m = \{(x_1, y_1), \dots, (x_m, y_m)\}$ , so as  $y_i = y(x_i)$

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- **Empirical Risk** – average error of  $a$  on  $X^m$
- **Empirical Risk Minimization (ERM)** – the common approach to solve the broad range of tasks of inductive learning (e.g., classification / regression tasks)

# Empirical risk: definitions

Loss function  $L(\hat{y}, y)$

Characteristics of difference between the prediction  $\hat{y} = a(x)$  and the *ground truth* label  $y = y(x)$  for object  $x \in X$

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Empirical Risk (ER)

Performance metric reflecting the average error made by an algorithm  $a$  upon the set  $X^m$ :

$$R(a, X^m) = \frac{1}{m} \sum_{i=1}^m L(a(x_i), y(x_i))$$

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### ERM cons

Overfitting on the training set  $X^m$ . Happens almost always when using ERM, because the performance criteria is the error **on the very same set** (solution: to measure the performance it makes sense to change the set)



# Loss functions examples

## Classification task

- Classification error:  $L(a, x) = L(\hat{y}, y) = [\hat{y} \neq y] = 1 - \delta_y(\hat{y})$
- The function is discontinuous  $\Rightarrow$  ERM is a task of combinatorial optimization  $\Rightarrow$  in many practical applications can be reduced to the search of maximal consistent subsystem of inequality system (number of inequalities is equal to the number of training examples  $m$ )  $\Rightarrow$  NP-hard

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## Regression task

- Squared error:  $L(\hat{y}, y) = (\hat{y} - y)^2$

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- **Structural Risk Minimization** (SRM):  $S(a, X^m) = R(a, X^m) + \lambda C(a) \rightarrow \min$ , where  $\lambda > 0$  is some weight of the regularization term, and  $C(a) \geq 0$  is the regularization cost associated with the function  $a : X \rightarrow Y$

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## SRM pros

A simple add-on to ERM allowing to avoid the overfitting



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## Definition

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## Handling methods

Regularization + decreasing model complexity

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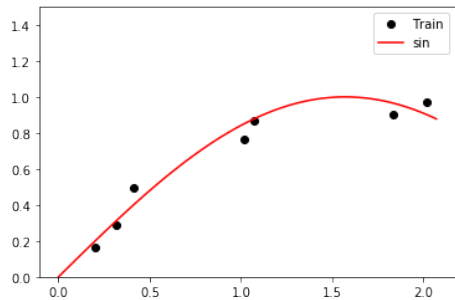
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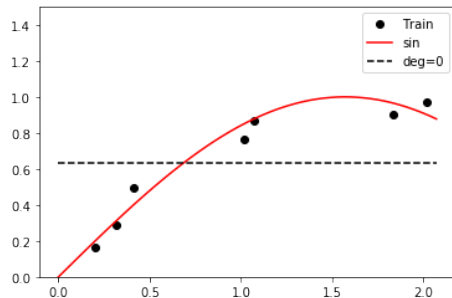
Better / bigger model

# Overfitting and Underfitting: examples



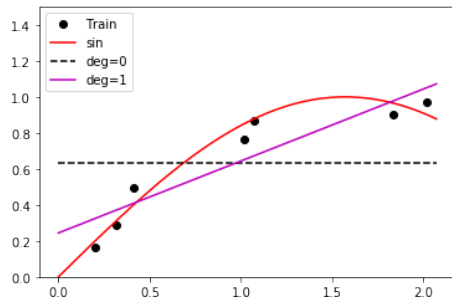


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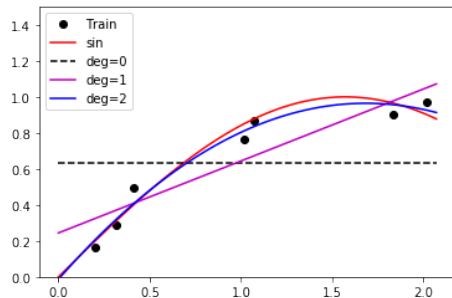
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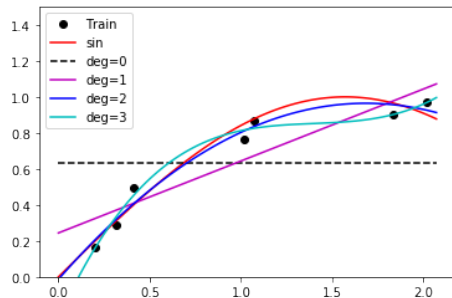
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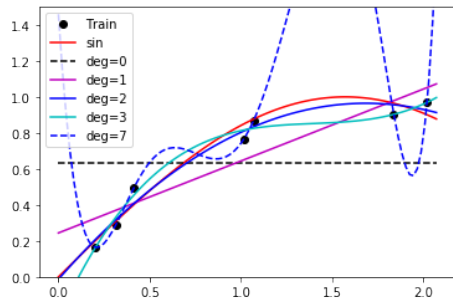
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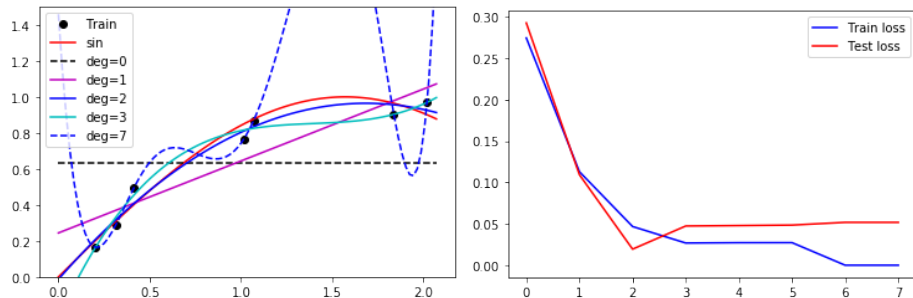
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- **Parameters:** coefficients  $a_n, a_{n-1}, \dots, a_1, a_0$ , and they are adjusted during model training
- **Hyperparameters:** the degree of the polynomial  $n$ , which is chosen before training starts; then chosen from the set of hyperparameters tested on the validation set

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- Explainability (tradeoff between good and interpretable model)

# Derivation of mean squared error expression

## Definitions

Let  $y = y(x) = f(x) + \varepsilon$  be the target dependence, where  $f(x)$  is the deterministic function,  $\varepsilon \sim N(0, \sigma^2)$  and  $a(x)$  is the machine learning algorithm.

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**Remark.**  $x$ : test point; expectation  $E$  is done over all possible training datasets

# Additional definitions

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The mean squared error decomposition in the example above is called the **bias-variance tradeoff**

# Model of Optimal Complexity: Classic View

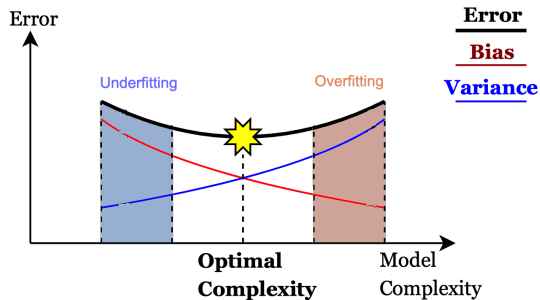
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- Complex models tend to overfit
- The optimal complexity of the model is somewhere between



# Model of Optimal Complexity: Recent Empirical Evidence

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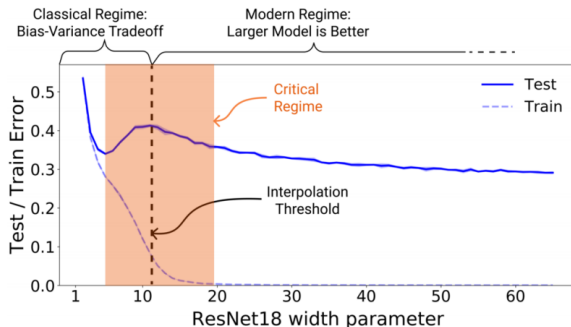
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- Tipping point — the point at which the complexity of the model is comparable to the cardinality of the training set (interpolation threshold)
- This behavior is called **double descent**

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# Model of Optimal Complexity: Double Descent

- Example of double descent in practice<sup>2</sup>:



<sup>2</sup>Image source: <https://arxiv.org/pdf/1912.02292.pdf>

## Mandatory external links to read

- 1 Read the sections 5.2 and 5.4 from "**The Hundred-Page Machine Learning Book**"(see "*References*" course page)
- 2 Re-read the section about How Supervised Learning algorithms work paying more attention to the *Empirical* and *Structural Risk* subsections
- 3 Read the sections "*Introduction*" "*Motivation*" and "*Bias-variance decomposition of mean squared error*" of the Bias-Variance Tradeoff page
- 4 Read the material about Overfitting and Underfitting

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- 5 In case of a huge amount of data and parameters ( $\approx$ billions), classical estimates stop working

# Thank you!