Suppose a child psychologist claims that the average time working mothers spend talking to their children is at least 11 minutes per day. You conduct a random sample of 1000 working mothers and find they spend an average of 11.5 minutes per day talking with their children. Assume prior research suggests the population standard deviation is 2.3 minutes.Conduct a test with a level of significance of alpha = 0.05.

**Step 1: State the Hypotheses**

* **Null Hypothesis (H0):** The average time, μ, that working mothers spend talking to their children per day is at least 11 minutes. Mathematically, H0: μ ≥ 11.
* **Alternative Hypothesis (H1):** The average time, μ, that working mothers spend talking to their children per day is less than 11 minutes. Mathematically, H1: μ < 11.

**Step 2: Set the Level of Significance**

* α = 0.05

**Step 3: Calculate the Test Statistic**

Since the population standard deviation is known (σ = 2.3 minutes), we can use the z-test for the mean. The test statistic is calculated as follows:

𝑧=𝑥ˉ−𝜇0𝜎/𝑛*z*=*σ*/*n*​*x*ˉ−*μ*0​​

where:

* 𝑥ˉ*x*ˉ = sample mean = 11.5 minutes
* 𝜇0*μ*0​ = hypothesized population mean under the null hypothesis = 11 minutes
* 𝜎*σ* = population standard deviation = 2.3 minutes
* 𝑛*n* = sample size = 1000

𝑧=11.5−112.3/1000*z*=2.3/1000​11.5−11​ 𝑧=0.52.3/1000*z*=2.3/1000​0.5​ 𝑧=0.50.0727≈6.88*z*=0.07270.5​≈6.88

**Step 4: Determine the Critical Value or P-Value**

For a left-tailed test at α = 0.05, the critical z-value is approximately -1.645. This means if our calculated z is less than -1.645, we would reject the null hypothesis.

**Step 5: Make the Decision**

Since the calculated z-value is 6.88, which is far greater than -1.645, we do not reject the null hypothesis. In fact, our test statistic falls in the region that supports the null hypothesis.

**Step 6: Conclusion**

There is not sufficient evidence at the 0.05 level of significance to reject the null hypothesis. Therefore, we do not have enough statistical evidence to dispute the psychologist's claim that the average time working mothers spend talking to their children is at least 11 minutes per day. Indeed, our sample data suggests that working mothers might actually be spending more time (an average of 11.5 minutes) talking to their children each day.

The result of this test supports the psychologist's claim and suggests that the average talking time could even be higher than the claimed 11 minutes per day.

Top of Form

Q2

A coffee shop claims that their average wait time for customers is less than 5 minutes. To test this claim, a sample of 40 customers is taken, and their wait times are recorded. The sample mean wait time is found to be 4.6 minutes with a standard deviation of 0.8 minutes. Perform a hypothesis test at a significance level of 0.05 and determine whether there is enough evidence to support the coffee shop's claim.

**Step 1: State the Hypotheses**

* **Null Hypothesis (H0):** The average wait time, μ, is 5 minutes or more. Mathematically, H0: μ ≥ 5.
* **Alternative Hypothesis (H1):** The average wait time, μ, is less than 5 minutes. Mathematically, H1: μ < 5.

**Step 2: Set the Level of Significance**

* α = 0.05

**Step 3: Calculate the Test Statistic**

Since the sample size is relatively small (n = 40), and assuming the population distribution is approximately normal, we can use the t-test for the mean. The test statistic is calculated using:

𝑡=𝑥ˉ−𝜇0𝑠/𝑛*t*=*s*/*n*​*x*ˉ−*μ*0​​

where:

* 𝑥ˉ*x*ˉ = sample mean = 4.6 minutes
* 𝜇0*μ*0​ = hypothesized population mean under the null hypothesis = 5 minutes
* 𝑠*s* = sample standard deviation = 0.8 minutes
* 𝑛*n* = sample size = 40

𝑡=4.6−50.8/40*t*=0.8/40​4.6−5​ 𝑡=−0.40.8/6.3246*t*=0.8/6.3246−0.4​ 𝑡=−0.40.1266≈−3.16*t*=0.1266−0.4​≈−3.16

**Step 4: Determine the Critical Value or P-Value**

Since this is a left-tailed test, we find the critical t-value or the p-value for t = -3.16 with df = 40 - 1 = 39 degrees of freedom.

Using t-distribution tables or a calculator, we find that:

* The critical t-value for α = 0.05 and df = 39 is approximately -1.685 (you might need to check more precise values depending on the source).

**Step 5: Make the Decision**

Since the calculated t-value of -3.16 is less than the critical t-value of -1.685, we reject the null hypothesis.

**Step 6: Conclusion**

There is sufficient evidence at the 0.05 level of significance to support the coffee shop's claim that the average wait time for customers is less than 5 minutes. The calculated mean of 4.6 minutes along with the statistical test aligns with and supports the coffee shop's claim.

Top of Form