1 Single-Agent Markovian Persuasion Models

There is a hidden Markov state $S_t \in \mathcal{S}$ whose transition follows the time-homogeneous kernel $S_{t+1} \sim P(\cdot \mid S_t = s, A_t = a)$. The principal (sender) fully observes S_t and commits to a signaling or recommendation policy π that maps the state to a recommendation $R_t = \pi(S_t)$. The agent (receiver) does not observe S_t.

At each time t, the agent hold a belief $\mu_t \in \Delta(\mathcal{S})$ about S_t, observe the recommendation R_t, and then choose an action A_t. The agent's one-period payoff is given by a function $u(a, s)$ for $a \in \mathcal{A}$, $s \in \mathcal{S}$, and the principal's one-period payoff is $v(A_t, S_t)$. Let H_t denote the history observed by the agent up to time t (e.g., past recommendations and realized outcomes), which induces the belief $\mu_t(\cdot) = \mathbb{P}\left[S_t \in \cdot \,\middle|\, H_t\right]$ under the announced policy. Consider the following three models that differ in how the agent uses information over time:

- The agent at time t is a new, short-lived receiver who cares only about the current period. Their prior is a fixed distribution μ that does not depend on t or on H_t (so effectively $\mu_t \equiv \mu$), and they choose

$$A_t \in \arg\max_{a \in \mathcal{A}} \mathbb{E}_\mu \left[v(a, S_t) \,\middle|\, R_t\right] \tag{1}$$

  that maximizes only their current expected payoff. They do not track information over time and do not care about future rewards. This corresponds to the standard "short-lived receiver" dynamic persuasion setup [Wu et al., 2022].

- The same agent is present over all periods and updates their belief μ_t from history: $\mu_t(\cdot) = \mathbb{P}\left[S_t \in \cdot \,\middle|\, H_t\right]$. However, at each time t they still choose A_t to maximize onluythe current expected payoff, i.e.,

$$A_t \in \arg\max_{a \in \mathcal{A}} \mathbb{E} \left[v(a, S_t) \,\middle|\, H_t, R_t\right], \tag{2}$$

  while how current action affects future information or future payoffs. In this sense the agent may track information over time (their belief μ_t changes with H_t), but their behavior is myopic: they never take actions purely to learn. This captures a long-lived agent who learns passively but does not engage in strategic exploration [Renault et al., 2017, Iyer et al., 2023, Lehrer and Shaiderman, 2021].

- The same agent is present over all periods, forms a belief $\mu_t(\cdot) = \mathbb{P}\left[S_t \in \cdot \,\middle|\, H_t\right]$, and cares about a discounted stream of payoffs

$$\mathbb{E}\left[\sum_{k=t}^{\infty} \beta^{k-t} v(A_k, S_t) \,\middle|\, H_t, R_t\right], \tag{3}$$

  for some $\beta \in (0, 1)$, where the expectation is taken under the probability law induced by the transition kernel P, the principal's signaling policy π, and the agent's strategy. In this case it can be optimal for the agent to take actions that are suboptimal in the current period in order to improve future information and explore deliberately.

2 Networked Markovian Persuasion with Myopic Agents

We now start with the first, simplest model and extend it to a population of agents on a network. The Markov state is now a vector $S_t = (S_{1,t}, \dots, S_{n,t}) \in \mathcal{S}^n$, where $S_{i,t}$ describes the local state of region $i = 1, \dots, n$. The regions are connected by a graph $G = (V, E)$; we write $N(i)$ for the neighborhood of i (e.g., i together with its one-hop neighbors on the graph). The state still evolves according to a time-homogeneous transition kernel $S_{t+1} \sim P(\cdot \mid S_t = s, A_t = a)$, where $A_t = (A_{1,t}, \dots, A_{n,t})$ is the profile of actions taken by the agents in period t. We focus on the case with one representative agent per region and identify agent i with region i. The agent's one-period conditional expected utility in region i is given by a function $v_i : \mathcal{A}_i \times \mathcal{S}^{N(i)} \to \mathbb{R}$, while the principal's one-period conditional expected utility is the average utility across regions,

$$U(A_t, S_t) = \frac{1}{n} \sum_{i=1}^{n} v_i \left(A_{i,t}, S_{N(i),t} \right). \tag{4}$$

We impose the following locality assumption on rewards and dynamics. This assumption says that, conditional on the current local environment around a region i, neither the instantaneous payoff nor the law of $S_{i,t+1}$ depends on what happens in distant parts of the network. In particular, externalities and information propagate across the system only through the edges of G. This will potentially allow us to work with local posteriors over $S_{N(i),t}$ and to formulate the persuasion and incentive-compatibility constraints in terms of neighborhoods $N(i)$ rather than the full state S_t. Such locality assumptions are reasonable in many applications where interactions are predominantly spatial or geographic, for example, ride-sharing platforms where waiting times and prices in an area depend mainly on nearby demand and supply, or epidemic and diffusion models where the state of a location next period depends on its own and its neighbors' states (add some references here).

Assumption 1. For each region i there exists a neighborhood $N(i) \subseteq V$ such that:

1. The reward of agents in region i depends only on local state in $N(i)$. In other words, for all $s \in \mathcal{S}^n$,
$$v_i(a_i, s) = v_i(a_i, s_{N(i)}). \tag{5}$$

2. he next-period state of region i depends on (S_t, A_t) only through the local configuration $(S_{N(i),t}, A_{N(i),t})$. Formally, for all measurable $\mathcal{S}' \subseteq \mathcal{S}$,
$$\mathbb{P}\left[S_{i,t+1} \in \mathcal{S}' \,\middle|\, S_t = s, A_t = a \right] = \mathbb{P}\left[S_{i,t+1} \in \mathcal{S}' \,\middle|\, S_t = s', A_t = a' \right] \tag{6}$$
whenever $(s_{N(i)}, a_{N(i)}) = (s'_{N(i)}, a'_{N(i)})$.

Throughout, the principal fully observes S_t and commits to a (possibly randomized) recommendation policy $\pi : \mathcal{S}^n \to \Delta(\mathcal{A}_1 \times \cdots \times \mathcal{A}_n)$ so that in period t a recommendation profile $R_t = (R_{1,t}, \dots, R_{n,t}) \sim \pi(\cdot \,|\, S_t)$ is drawn and privately communicated to the agents. We again interpret $R_{i,t}$ as a direct recommendation for the action of agent i. All agents share a common prior μ over S_t and know the policy π, but they do not observe S_t or the recommendations sent to other agents.

At each time t, agent i observes their own recommendation $R_{i,t}$ and then chooses an action $A_{i,t} \in \mathcal{A}_i$ to maximize their current expected payoff:

$$A_{i,t} \,\middle|\, R_{i,t} = r \in \arg\max_{a \in \mathcal{A}_i} \mathbb{E}_\mu \left[u_i(a, S_t) \,\middle|\, R_{i,t} = r \right], \tag{7}$$

where the expectation is taken over the posterior distribution of S_t given $R_{i,t} = r$. By Bayes' rule, this posterior is

$$\mathbb{P}_\mu \left[S_t = s \,\middle|\, R_{i,t} = r \right] = \frac{\mu(s)\mathbb{P}\left[R_{i,t} = r \,\middle|\, S_t = s \right]}{\sum_{s' \in \mathcal{S}^n} \mu(s')\mathbb{P}\left[R_{i,t} = r \,\middle|\, S_t = s' \right]}, \tag{8}$$

where

$$\mathbb{P}\left[R_{i,t} = r \,\middle|\, S_t = s \right] = \sum_{r_{-i} \in \mathcal{A}_{-i}} \pi\left((r, r_{-i}) \,\middle|\, s \right). \tag{9}$$

Following the classic work of Kamenica and Gentzkow [2011], we describe feasible outcomes in terms of the induced distributions over states and recommendations. For a given prior μ on S_t, and for each region i, we define the persuasion set $\mathcal{P}_i(\mu)$ as the set of probability measures $P \in \Delta(\mathcal{S}^n \times \mathcal{A}_i)$ such that the marginal of P on $\mathcal{S}^n$ is μ, and for every $a \in \mathcal{A}_i$ with $P(R_i = a) > 0$,

$$\mathbb{E}\left[u_i(a, S_t) \,\middle|\, R_i = a \right] \geq \mathbb{E}\left[u_i(a', S_t) \,\middle|\, R_i = a \right] \tag{10}$$

for all $a' \in \mathcal{A}_i$. It is well known that, with Bayesian rational agents, any signaling policy is outcome-equivalent to a direct recommendation policy characterized by some $P \in \mathcal{P}_i(\mu)$. Hence it is without loss of generality to work directly with the persuasion set.

2.1 Learning the Optimal Persuasion Policy

2.2 Local v.s. Global Policies

Remark 1. Questions to think about:

- How is this different from simply treating $R_{i,t}$ as the action and running a vanilla policy learning algorithm? Possible ans: once we change the recommendation rule, the agents may change how they react to signals, so the environment seen by the principal is no longer stable, and the usual guarantees for policy learning can break down.
- What happens if the prior μ is not observed? In that case, the principal can no longer learn a policy π as an explicit function of μ. What can the principal do instead, and what additional regret is induced by not observing μ?
- What if the prior μ_t varies over time (as in the second case in Section 1)? The persuasion set can still be defined in a similar way period by period, but finding the optimal policy becomes more complicated. It should be relatively straightforward to extend the analysis when μ_t is exogenous, but may require substantially more work when μ_t is endogenous and depends on past signals and actions.

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