Supplementary material to "3-D radial gravity gradient inversion"

Vanderlei C. Oliveira Jr. [1], *and Valeria C. F. Barbosa* [2]

Observatório Nacional, Rio de Janeiro, Brazil. Contact e-mail: 1 vandscoelho@gmail.com, 2 valcris@on.br,

July 24, 2013

Abstract

We have provided supplementary material to our article titled "*3-D radial gravity gradient inversion*" and whose authors are *Vanderlei C. Oliveira Jr, and Valeria C. F. Barbosa*. In this material, we presented extra results obtained by inverting the gravity-gradient data from a survey over the Vinton salt dome (Louisiana, USA). In these extra results, we investigate the effect on the results caused by the assumption of uncertainties in the density contrast of the cap rock with the host rocks.

Introduction

Oliveira Jr & Barbosa (2013) used the 3-D radial inversion for inverting a gravity-gradient data to estimate the shape of an isolated 3-D geologic body, given its depth to the top and density contrast. This method follows the lines developed by Oliveira Jr *et al.* (2011), in which the 3-D body is approximated by a set of vertically stacked right prisms with known thicknesses and density contrasts. Each prism has a polygonal horizontal cross section whose vertices are described by polar coordinates referred to an origin within the polygon. By jointly inverting all gravity-gradient tensor components, the radial inversion retrieves the shape of depth slices of the 3-D body by estimating the horizontal Cartesian coordinates of the origins and the radii of the vertices describing the horizontal cross sections of all prisms. This estimate is formulated as a non-linear constrained inverse problem.

Oliveira Jr & Barbosa (2013) inverted the *xx*-, *xy*, *xz*, *yy*, *yz*, and *zz*-components of the gravity-gradient tensor (grey scale maps in Fig. 1) from a survey over the Vinton salt dome (Louisiana, USA). According to Coker *et al.* (2007), the Vinton salt dome is characterized by a massive cap rock extending above the salt rock. This cap rock is formed by gypsum and anhydrite which is embedded in sediments characterized by intercalated layers of sandstone and shale. Based on the Ennen & Hall's (2011) interpretation, Oliveira Jr & Barbosa (2013) assumed that the cap rock has a depth of the top at 160 m and that the surrounding sediments (shale and sandstone) and the salt dome have the same density 2.2 g/cm³. This implies that the observed gravity-gradient data are predominantly caused by the cap rock. Based on the density ranges of the lithologies of the study area and Ennen & Hall's (2011) interpretation, Oliveira Jr & Barbosa (2013) assumed that the cap rock has a density contrast of 0.55 g/cm³ with the host rocks.

Here, we apply the 3-D radial gravity gradient inversion to investigate the effect on the estimated 3-D source's shape caused by the assumption of uncertainties in the density contrast of the cap rock with the host rocks.

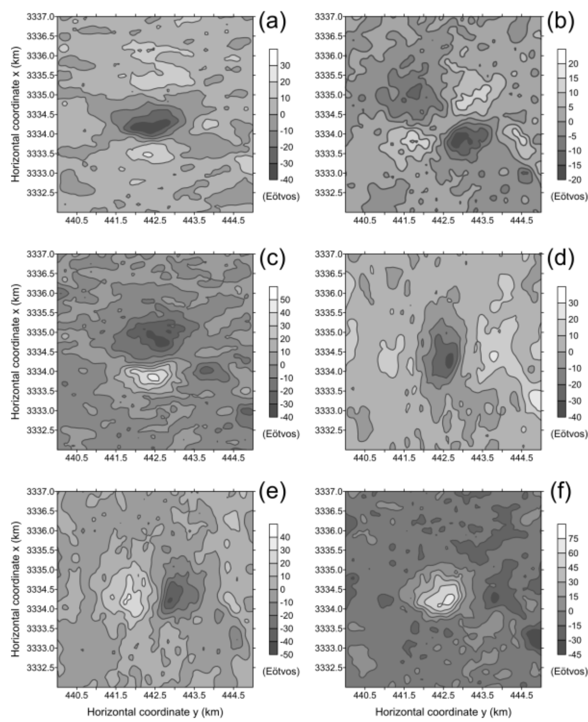

Figure 1. Observed (grey scale maps) (a) *xx*-, (b) *xy*-, (c) *xz*-, (d) *yy*-, (e) *yz*- and (f) *zz*-components of the gravity-gradient tensor from a survey over the Vinton salt dome (Louisiana, USA). The observed data are terrain corrected by using a density of 2.20 g/cm³.

Solution sensitivity to uncertainties in the density contrast

We conducted a numerical analysis to investigate the effect on the solution obtained via 3-D radial gravity gradient inversion caused by uncertainties in the density contrast of the targeted geologic body. This analysis is performed by inverting the gravity-gradient data from the Vinton salt dome (grey scale maps in Fig. 1).

Based on Ennen & Hall's (2011) interpretation and Coker *et al.* (2007), we assumed that the density contrast of the cap rock with the host rocks is $\rho = 0.55$ g/cm³, with an uncertainty of 0.10 g/cm³. To analyze the effect of this uncertainty on the estimated source, we applied the Oliveira Jr & Barbosa's (2013) method by using three interpretation models with density contrasts equal to: (a) 0.45 g/cm³; (b) 0.55 g/cm³; and (c) 0.65 g/cm³. All the interpretation models are formed by an ensemble of $L = 10$ prisms, each one with the same number of polygon vertices $M^k = 16$ ($k = 1, \ldots, 16$) describing the horizontal cross sections of the polygons. Based on the prior information, we also assumed the knowledge about the actual depth to the top of the cap rock; hence we set the depth to the top of all interpretation models as $z_0 = 160$ m. The ten prisms which make up the used initial approximations have the same horizontal Cartesian coordinates of $x_0^k = 3{,}334{,}150$ m and $y_0^k = 442{,}606$ m, $k = 1, \ldots, 10$. In these three initial approximations, the radii forming the shallowest prism (r_j^1, $j = 1, \ldots, 16$) are equal to 100 m and the radii forming the second prism (r_j^2, $j = 1, \ldots, 16$) are equal to 200 m. The sizes of radii of the third prism up to the tenth prism are increased successively by adding 100 m, until the radii of the deeper prism (r_j^{10}, $j = 1, \ldots, 16$) attain 1,000 m.

Based on the Oliveira Jr & Barbosa's (2013) criterion for analyzing the data resolution and determining the estimate having both the optimum depth to the bottom and the optimum volume, we construct three estimated $s \times v_p$ curves (Fig. 2), each one with a different density contrast.

The 14 pairs of s and v_p (colored dots) forming each curve in Fig. 2 are produced by a set of estimated 3-D sources with different maximum depths $z_{\max}$. In all curves, the value of $z_{\max}$ varies from 320 m to 580 m, in steps of 20 m, leading to an uncertainty of ± 10 m in the estimated depth to the bottom. Following the Oliveira Jr & Barbosa's (2013) criterion to choose the estimate having the *optimum* depth to the bottom, we choose the three estimated cap rocks (not shown) producing the minimum ℓ_2-norm s and volume v_p over its respective $s \times v_p$ curves. The estimated $s \times v_p$ curves in red ($\rho = 0.65$ g/cm³) and black ($\rho = 0.55$ g/cm³) present well-defined minima of s associated with estimated 3-D cap rocks (not shown) whose maximum depths $z_{\max}$ are, respectively, 380 and 440 m. In contrast, the estimated $s \times v_p$ curve in blue ($\rho = 0.45$ g/cm³) exhibits multiple minima of s, which are associated with $z_{\max}$ greater than or equal to 480 m.

These three estimated $s \times v_p$ curves in Fig. 2 are consistent with the ambiguity of potential-field data interpretation. The potential-field data interpretation is subject to a fundamental ambiguity involving the product of the physical property by the volume. Hence, the larger the density contrast assumed for the source, the smaller the predicted volume of the estimated source; hence the shallower will be the depth-to-bottom estimate of the source. These aspects are observed in Fig. 2.

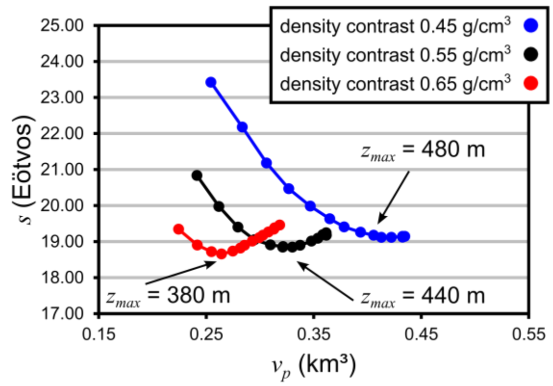

Figure 2. Estimated $s \times v_p$ curves obtained by inverting the real data shown in Fig. 1. These curves are obtained by using interpretation models with density contrasts equal to: 0.45 g/cm³ (blue curve); 0.55 g/cm³ (black curve); and 0.65 g/cm³ (red curve). Each curve is constructed by varying the depth to the bottom $z_{\max}$ of the corresponding interpretation model from 320 to 580 m, in steps of 20 m. The well-defined minima of s on the red curve (depth to the bottom of 380 m and predicted volume of 0.263 km³) and black curve (depth to the bottom of 440 m and predicted volume of 0.327 km³) are pinpointed. The blue curve shows multiple minima of s, which are associated with $z_{\max}$ greater than or equal to 480 m.

These estimated cap rocks (not shown) give rise to the question of whether they yield acceptable data fittings. The histograms (Figs. 3-5) of the residuals between the predicted and the observed gravity-gradient data confirm the unacceptable data fittings, because the sample means μ (Fig. 3-5) are different from zero. These non-zero means indicate the presence of systematic errors in the observed data (grey scale maps in Fig. 1).

Supplementary material to "*3-D radial gravity gradient inversion*"

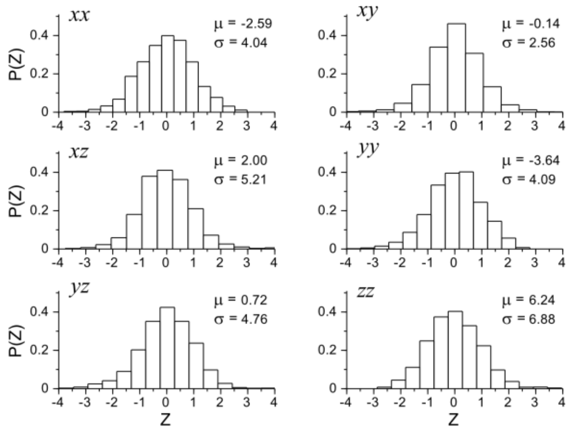

Figure 3. Histograms of the residuals between predicted (not shown) and real (grey scale maps in Fig. 1) (a) *xx*-, (b) *xy*-, (c) *xz*-, (d) *yy*-, (e) *yz*- and (f) *zz*-components of the gravity-gradient tensor from a survey over the Vinton salt dome, USA. The predicted data were obtained using the estimated cap rock (not shown) with optimum bottom depth of 440 m provided by the minimum of *s* on the black curve in Fig. 2. The sample mean μ and the sample standard deviation σ are shown in each histogram. The residuals are transformed in a dimensionless variable Z by subtracting the residual value from the sample mean μ and then dividing the difference by the sample standard deviation σ. P(Z) is the frequency curve of the variable Z.

To remove the systematic errors detected in the histograms shown in Figs. 3-5, we added the sample non-zero means μ to the corresponding observed component of the gravity-gradient, yielding three sets of corrected gravity-gradient data (not shown). Next, we repeat the procedure for calculating the three new estimated $s \times v_p$ curves shown in Fig. 6. Each curve in Fig. 6 has 14 pairs of s and v_p (colored dots), each one produced by an estimated 3-D source with a different depth to the bottom z_{max}. The value of z_{max} varies from 320 to 580 m, in steps of 20 m, leading to an uncertainty of ± 10 m in the estimated depth to the bottom. Following the Oliveira Jr & Barbosa's (2013) criterion to choose the estimate having the *optimum* depth to the bottom, we choose the three estimated cap rocks (not shown) producing the minimum ℓ_2-norm s and volume v_p over its respective $s \times v_p$ curves. The estimated $s \times v_p$ curves in red and black (Fig. 6) present well-defined minima of s associated with estimated 3-D cap rocks (not shown) whose maximum depths to the bottom z_{max} are, respectively, 400 and 460 m. In contrast, the estimated $s \times v_p$ curve in blue (Fig. 6) exhibits multiple minima of s, which are associated with z_{max} greater than or equal to 500 m. These estimated cap rocks (not shown) yield acceptable data fittings. The histograms (Figs. 7-9) of the residuals between the predicted and the corrected gravity-gradient data confirm the acceptable data fittings, because of the sample means μ (Fig. 7-9) are very close to zero.

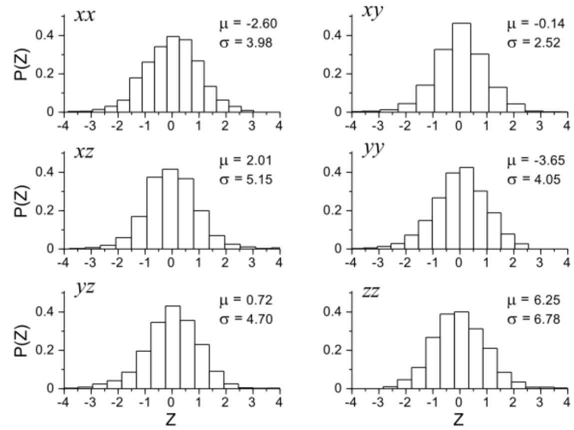

Figure 4. Histograms of the residuals between predicted (not shown) and real (grey scale maps in Fig. 1) (a) *xx*-, (b) *xy*-, (c) *xz*-, (d) *yy*-, (e) *yz*- and (f) *zz*-components of the gravity-gradient tensor from a survey over the Vinton salt dome, USA. The predicted data were obtained using the estimated cap rock (not shown) with optimum bottom depth of 380 m provided by the minimum of *s* on the red curve in Fig. 2. The sample mean μ and the sample standard deviation σ are shown in each histogram. The residuals are transformed in a dimensionless variable Z by subtracting the residual value from the sample mean μ and then dividing the difference by the sample standard deviation σ. P(Z) is the frequency curve of the variable Z.

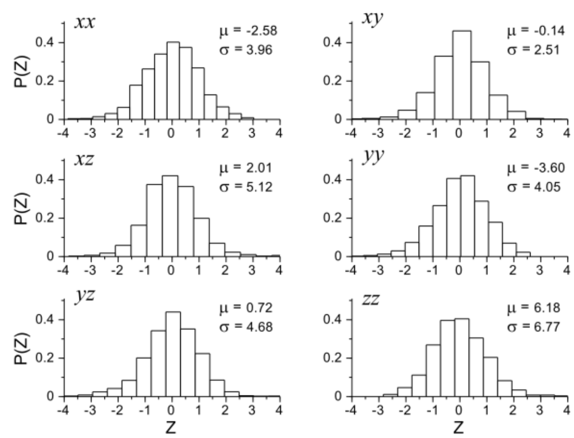

Figure 5. Histograms of the residuals between predicted (not shown) and real (grey scale maps in Fig. 1) (a) *xx*-, (b) *xy*-, (c) *xz*-, (d) *yy*-, (e) *yz*- and (f) *zz*-components of the gravity-gradient tensor from a survey over the Vinton salt dome, USA. The predicted data were obtained using the estimated cap rock (not shown) with optimum bottom depth of 480 m provided by the minimum of *s* on the blue curve in Fig. 2. The sample mean μ and the sample standard deviation σ are shown in each histogram. The residuals are transformed in a dimensionless variable Z by subtracting the residual value from the sample mean μ and then dividing the difference by the sample standard deviation σ. P(Z) is the frequency curve of the variable Z.

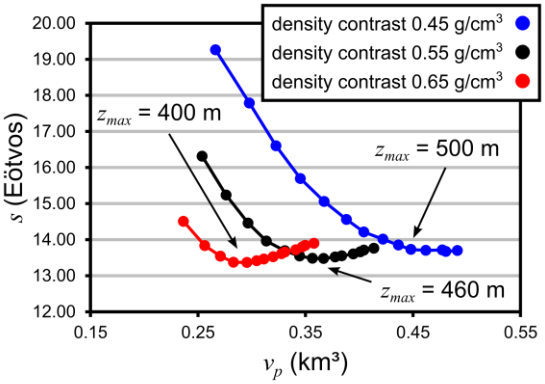

Figure 6. Estimated s x v_p curves obtained by inverting the corrected gravity-gradient data (not shown). These curves are obtained by using interpretation models with density contrasts equal to: 0.45 g/cm³ (blue curve); 0.55 g/cm³ (black curve); and 0.65 g/cm³ (red curve). Each curve is constructed by varying the depth to the bottom z_{max} of the corresponding interpretation model from 320 to 580 m, in steps of 20 m. The well-defined minima of s on the red curve (depth to the bottom of 400 m and predicted volume of 0.294 km³) and black curve (depth to the bottom of 460 m and predicted volume of 0.366 km³) are pinpointed. The blue curve shows multiple minima of s, which are associated with z_{max} greater than or equal to 500 m.

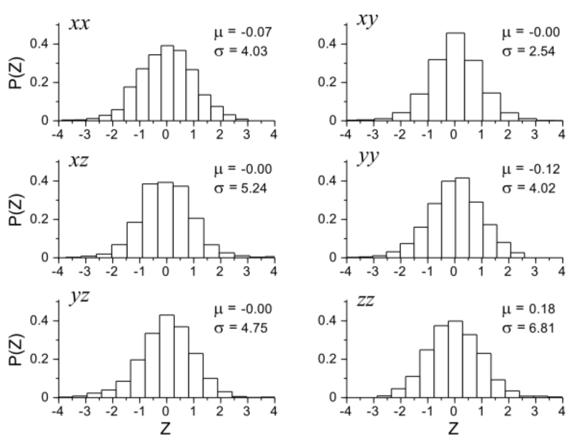

Figure 7. Histograms of the residuals between predicted (not shown) and corrected gravity-gradient data (a) *xx*-, (b) *xy*-, (c) *xz*-, (d) *yy*-, (e) *yz*- and (f) *zz*-components of the gravity-gradient tensor from a survey over the Vinton salt dome, USA. The predicted data were obtained using the estimated cap rock (not shown) with optimum bottom depth of 460 m provided by the minimum of s on the black curve in Fig. 6. The sample mean μ and the sample standard deviation σ are shown in each histogram. The residuals are transformed in a dimensionless variable Z by subtracting the residual value from the sample mean μ and then dividing the difference by the sample standard deviation σ. P(Z) is the frequency curve of the variable Z.

Fig. 10 shows two vertical cross-sections of the estimated cap rocks obtained by inverting the corrected gravity-gradient data (not shown) and assuming interpretation models with depth to the bottom given by the minima of the s x v_p curves (Fig. 6). These estimates (Fig. 10) are obtained by using density contrasts equal to 0.45 g/cm³ (blue contour), 0.55 g/cm³ (black contour), and 0.65 g/cm³ (red contour).

Discussion of the results

The 3-D radial gravity-gradient inversion proposed by Oliveira Jr & Barbosa (2013) depends on the prior knowledge about the density contrast between the targeted source rock and the host rocks. If the density contrast is wrongly assigned, the volume of the estimated source will be either greater or smaller than the true one. In the applications to real data, the interpreter does not know whether the density contrast was chosen correctly.

Here, we analyze how the estimated shape of the cap rock over the Vinton salt dome (USA) is changed by assuming different density contrasts. Fig. 10 confirms the well-known ambiguity involving the density contrast and the volume. The larger the density contrast assumed for the source, the smaller the predicted volume of the estimated source; hence the shallower will be the depth-to-bottom estimate of the source.

Although the three estimated cap rocks shown in Fig. 10 present different volumes, their upper portions present approximately the same shape. The main differences between these three estimated cap rocks are their bottom depths. Notice that the three estimated cap rocks (Fig. 10b) disclose the same steep vertical border on the south. On the other hand, the three estimated cap rocks (10b) display approximately the same gradual declivity (about 21°) on the northern border. In agreement with the prior knowledge about the Vinton salt dome reported by Coker *et al*. (2007), the three estimated cap rocks have a northeast-southwest elongated form, being consistent with the strike of the main fault in the study area.

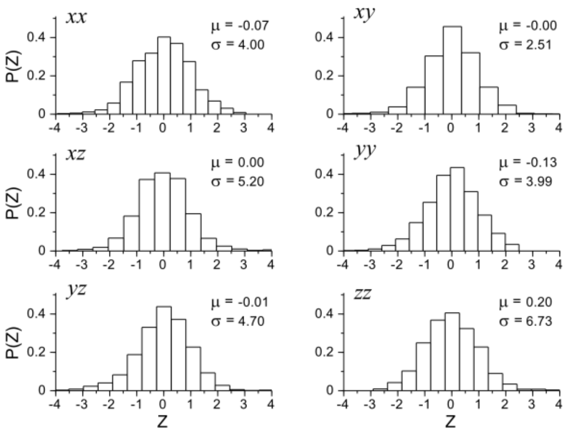

Figure 8. Histograms of the residuals between predicted (not shown) and corrected gravity-gradient data (a) *xx*-, (b) *xy*-, (c) *xz*-, (d) *yy*-, (e) *yz*- and (f) *zz*-components of the gravity-gradient tensor from a survey over the Vinton salt dome, USA. The predicted data were obtained using the estimated cap rock (not shown) with optimum bottom depth of 400 m provided by the minimum of s on the red curve in Fig. 6. The sample mean μ and the sample standard deviation σ are shown in each histogram. The residuals are transformed in a dimensionless variable Z by subtracting the residual value from the sample mean μ and then dividing the difference by the sample standard deviation σ. P(Z) is the frequency curve of the variable Z.

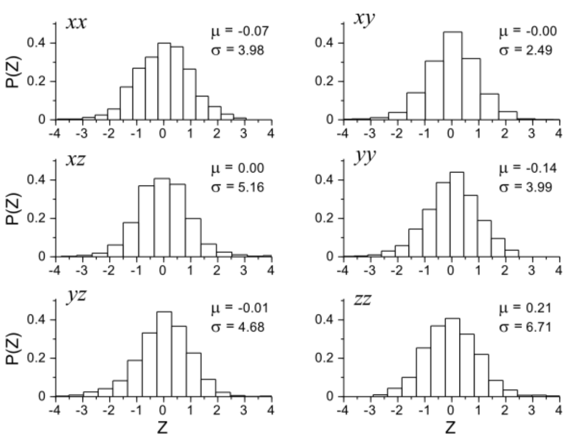

Figure 9. Histograms of the residuals between predicted (not shown) and corrected gravity-gradient data (a) *xx*-, (b) *xy*-, (c) *xz*-, (d) *yy*-, (e) *yz*- and (f) *zz*-components of the gravity-gradient tensor from a survey over the Vinton salt dome, USA. The predicted data were obtained using the estimated cap rock (not shown) with optimum bottom depth of 500 m provided by the minimum of s on the blue curve in Fig. 6. The sample mean μ and the sample standard deviation σ are shown in each histogram. The residuals are transformed in a dimensionless variable Z by subtracting the residual value from the sample mean μ and then dividing the difference by the sample standard deviation σ. P(Z) is the frequency curve of the variable Z.

Notice that the estimated $s \times v_p$ curve (blue curve in Fig. 6) obtained by inverting the corrected gravity-gradient data with an interpretation model with density contrast of 0.45 g/cm³ reveals multiple minima of *s*. This ambiguous smallest value of *s* in the $s \times v_p$ curve (blue curve in Fig. 6) indicates that the gravity-gradient data do not have enough in-depth resolution for completely recover the geometry of the source if its density contrast is lower than or equal to 0.45 g/cm³. In this case, the $s \times v_p$ curve (blue curve in Fig. 6) provides, at most, a lower bound ($z_{\max} = 500 \pm 10$ m) for the maximum depth of the cap rock. If the interpreter has confidence that density contrast is 0.45 g/cm³, the estimated cap rock (blue line in Fig 10) should be taken as a conservative estimate of the true cap rock, i.e., the true depth to the bottom of the cap rock will be greater than or equal to 500 m.

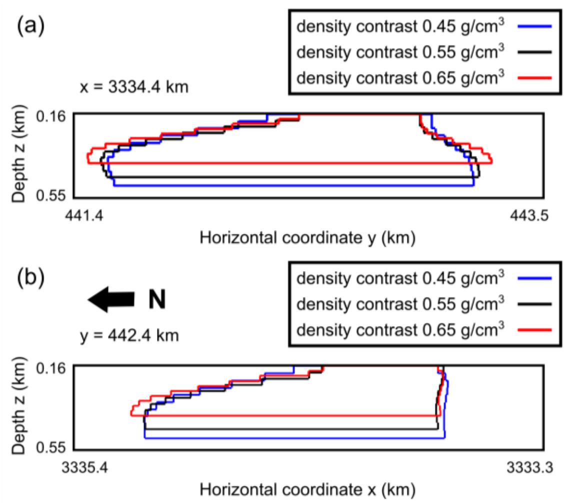

Figure 10. Profiles across three estimated cap rocks over the Vinton salt dome (Louisiana, USA) along (a) east-west and (b) north-south directions. The estimated cap rock (black) is obtained by using the optimum bottom depth of 460 m and density contrast of 0.55 g/cm³. The estimated cap rock (red) is obtained by using the optimum bottom depth of 400 m and density contrast of 0.65 g/cm³. The estimated cap rock (blue) is obtained by using the optimum bottom depth of 500 m and density contrast of 0.45 g/cm³.

Conclusions

We have presented an extra material to our article titled "*3-D radial gravity gradient inversion*" by *Vanderlei C. Oliveira Jr* and *Valeria C. F. Barbosa*. By using the gravity-gradient tensor from a survey over the Vinton salt dome (Louisiana, USA), we have investigated the effect on the solution caused by the assumption of uncertainties in the density contrast of the cap rock with the host rocks.

Supplementary material to "*3-D radial gravity gradient inversion*"

Based on the geologic knowledge and the density ranges of the lithologies of the study area, we assumed that the density contrast between the cap rock and the host rocks is 0.55 g/cm³, with an uncertainty of 0.10 g/cm³. In order to analyze the effect of this assumed uncertainty, we have compared three estimated cap rocks with density contrasts of 0.45 g/cm³, 0.55 g/cm³, and 0.65 g/cm³.

As expected, the estimated cap rock with large density contrast displays a smaller volume than the one having small density contrast in order to preserve the total mass. However, the six components of the gravity-gradient data are still fitted within the supposed measurement errors.

The most striking feature of the three estimated cap rocks above the Vinton salt dome is the very close resemblance between their upper shapes. The shapes of the three estimated cap rocks differ from each other mainly by their depths to the bottom. The smaller the density contrast, the deeper the estimated bottom depth. Because the upper parts of the three estimated cap rocks are very similar, we can conclude that the southern border of the cap rock is steeper than the northern border.

Our investigation shows that is certainly impossible to select the best estimated shape of the cap rock obtained via 3-D radial gravity gradient inversion among a set of estimates with density contrast varying from 0.45 g/cm³ to 0.65 g/cm³. However, we draw attention to a striking feature in our investigation. Regardless of the assumed density contrast within this closed interval, the upper portions of the estimated cap rocks have practically the same shapes. An uncertainty of 0.10 g/cm³ in the density contrast of the cap rock with the host rocks is mainly affecting the depth-to-bottom estimates.

Acknowledgments

The authors were supported in this research by fellowships (V. C. F. Barbosa) and scholarship (V. C. Oliveira Jr) from Conselho Nacional de Desenvolvimento e Tecnológico (CNPq), Brazil. Additional supports for authors were provided by the Brazilian agencies CNPq (grant 471693/2011-1) and FAPERJ (grant E-26/103.175/2011). The authors also thank the Bell Geospace Inc. for permission to use the real data from the Vinton salt dome.

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