A review of neuro-symbolic AI integrating reasoning and learning for advanced cognitive systems

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A B S T R A C T

Neuro-symbolic AI represents the convergence of two principal paradigms in artificial intelligence: neural networks, which are efficient in data-driven learning, and symbolic reasoning, which offers explainability and logical inference. This hybrid methodology combines the adaptability of neural networks with symbolic AI's interpretability and formal reasoning abilities, which provide a practical framework for advanced cognitive systems. This paper analyzes the present condition of neuro-symbolic AI, emphasizing essential techniques that combine reasoning and learning. We explore models such as Logic Tensor Networks, Differentiable Logic Programs, and Neural Theorem Provers. The study analyzes their impact on the advancement of cognitive systems in natural language processing, robotics, and decision-making. The paper examines the challenges faced by neuro-symbolic AI, such as scalability, integration with multimodal data, and maintaining interpretability without compromising efficiency. By evaluating the strengths and weaknesses of many methodologies, we comprehensively understand the field's development and its potential to revolutionize intelligent systems. In addition, we identify emerging research areas, including the incorporation of ethical frameworks and the development of adaptive dynamic neuro-symbolic systems that respond in real-time. This review aims to guide future research by providing insights into the potential of neuro-symbolic AI to influence the development of the next generation of intelligent, explainable, and adaptive systems.

1. Introduction

1.1. Overview of artificial intelligence paradigms

Symbolic AI was a major focus in the early days of AI research (Steels et al. (2007)), which relies on formal logic and rule-based reasoning (Garcez et al. (2023)). In symbolic systems, knowledge is represented through facts, rules, and relationships (Grimm et al. (2009)), which makes these systems easy to understand and perform deductive reasoning. However, symbolic systems have difficulty learning from data and dealing with ambiguity or uncertainty (Woods et al. (1986) and Zhang et al. (2021)). Neural networks are models that learn from data and can identify patterns by being trained on large datasets (Dongare et al. (2012)). These networks, especially deep learning models, have shown excellent image recognition and natural language processing results (Abiodun et al. (2018)). But still, they usually act like "black boxes" and are challenging to interpret. Combining these two paradigms results in neuro-symbolic AI (Sarker et al. (2021)), where neural networks handle pattern identification and learning, while symbolic reasoning ensures organized decision-making and explicability. The

hybrid technique enables AI systems to do complex tasks, such as commonsense reasoning, which would be challenging for neural networks independently. In tasks related to natural language comprehension, a neuro-symbolic system can utilize the learning ability of neural networks to analyze unprocessed text while employing symbolic logic to deduce deeper meanings and relationships based on established rules.

1.2. Emergence of neuro-symbolic AI

The development of neuro-symbolic AI can be attributed to the shortcomings observed in purely symbolic and purely neural methodologies. Initial symbolic AI systems demonstrated significant capabilities in logical reasoning (Bronkhorst et al. (2020); however, they needed to improve their ability to generalize from data, a critical requirement for practical applications in the real world. Conversely, neural networks demonstrated superior performance in data-driven tasks; however, they have been critiqued for their limited interpretability and dependence on large data sets for training purposes. This insight led to the exploration of hybrid models that aim to integrate the advantages of both paradigms. Neuro-symbolic AI advances the foundation by integrating

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symbolic knowledge representation (Diday et al. (1990)) alongside the learning capabilities inherent in neural networks. In the early 2000s (Panksepp et al. (2000)), there was a notable increase in interest surrounding neuro-symbolic models, which emerged as a potential solution to connect the domains of learning and reasoning. For example, symbolic knowledge can help a neural network throughout the training process and enable it to concentrate on relevant features or rules, which enhances its performance and explainability. Moreover, symbolic reasoning facilitates post-hoc explanations (Malandri et al. (2024)), which can be essential in sensitive domains such as healthcare and legal reasoning. The ongoing enhancements in computational capabilities, coupled with the growing accessibility of data, have significantly accelerated the evolution of neuro-symbolic systems, which makes them an increasingly feasible choice for advanced AI applications.

1.3. Relevance of neuro-symbolic AI in advanced cognitive systems

Advanced cognitive systems require AI models that perform exceptionally in data-driven tasks and incorporate high-level reasoning, contextual comprehension, and interpretability. Neuro-symbolic AI is essential for addressing these needs (Thomas et al. (2022)), as it facilitates the integration of deep learning's perceptual strengths with the organized reasoning inherent in symbolic logic. In practical scenarios like autonomous systems and robotics, neuro-symbolic AI offers the essential framework for real-time decision-making (Edwards et al. (2024)) characterized by precision and transparency. An autonomous vehicle must identify challenges managed by neural networks while following traffic regulations and ethical considerations addressed through symbolic reasoning. Another significant domain in which neuro-symbolic AI demonstrates its importance is healthcare (Roy et al. (2022)). In the context of medical decision support systems, physicians need to have confidence in and understand the reasoning underlying the recommendations provided by artificial intelligence. The combination of symbolic logic, capable of following medical guidelines or diagnostic pathways, with neural networks (Reategui et al. (1997)), which excel in analyzing patient data, enables neuro-symbolic systems to provide precise predictions and the underlying reasoning for those predictions. The hybrid approach ensures that advanced cognitive systems exhibit high performance and transparency.

2. Background and evaluation of neuro-symbolic AI

2.1. Historical and evolution of neuro-symbolic AI and neural networks

The origin of neuro-symbolic AI can be traced to the 1950s (Mijwel et al. (2015)) when it was characterized by initial systems grounded in formal logic, as demonstrated by the Logic Theorist and General Problem Solver. These systems used formal logic to do deductive reasoning using symbolic rules, which paved the way for knowledge representation and automated reasoning. However, their inability to learn from data limited its scalability and practical application. For example, while these systems performed well in organized, rule-based situations, they struggled with noisy, unstructured data, which prompted more investigation. In the 1980s, connectionist models and neural networks were introduced to solve this problem (Pollack et al. (1989)). The back-propagation technique revolutionized neural networks, which allows them to learn from data while minimizing errors via gradient descent (Cilimkovic et al. (2015)). This era saw substantial advances in pattern recognition, which includes image and audio processing (Traore et al. (2018) and Siniscalchi et al. (2014)). Despite their effectiveness, neural networks were criticized for their "black-box" character, which provided limited interpretability and required large data for training (Ding et al. (2022)).

In the early 2000s, researchers launched efforts to connect symbolic reasoning with neural learning (DeLoache et al. (2004)), which recognizes these paradigms' relative strengths and drawbacks. Hybrid models

combined symbolic knowledge representations, such as rules and ontologies, with neural networks' data-driven learning capabilities. For example, Logic Tensor Networks (LTNs) embed logical constraints as tensors within neural network designs, which allows for both reasoning and learning. Similarly, DeepProbLog combines probabilistic logic programming and neural networks to enable decision-making under uncertainty. These advances address the limits of early symbolic systems by including learning capabilities and the limitations of neural systems by introducing interpretability and reasoning.

In 2016, advancements like LTNs and DeepProbLog established neuro-symbolic AI as a viable paradigm. These models effectively merged symbolic reasoning with neural network learning capability, which results in higher performance in knowledge graph reasoning and probabilistic decision-making under uncertainty. For example, LTNs made it easier to encode logical principles directly into neural networks (Serafini et al. (2016)). In contrast, DeepProbLog enabled the management of noisy input and ambiguity in applications such as robotics and natural language processing.

By the 2020s, neuro-symbolic AI has gained popularity in real-world applications such as healthcare diagnostics, autonomous systems, and natural language processing. Models such as neural theorem provers (NTPs) and differentiable logic programs helped to bridge the gap between symbolic logic and deep learning. These developments enabled systems to achieve high accuracy while still being interpretable, which is crucial for sensitive applications such as medical decision assistance. As a result, modern neuro-symbolic AI has developed into a strong framework that smoothly combines learning and reasoning to produce scalable, interpretable solutions. The evolution of AI paradigms from symbolic AI to neural networks is shown in Fig. 1.

2.2. Early attempt at Hybrid AI models

Early efforts to develop hybrid AI models stemmed from the realization that neither symbolic AI nor neural networks alone could achieve true artificial general intelligence (AGI). In the 1980s and 1990s, systems integrated rule-based reasoning with machine learning methodologies (Mefford et al. (2019)). For example, symbolic connectionism represented an initial effort to bridge the divide by integrating symbolic rules into neural networks. However, these systems exhibited constraints in scalability and came across challenges when processing real-world, noisy data. One significant initial attempt was SOAR, an architecture that employed symbolic reasoning for decision-making and learning (Wray et al. (2005)). The objective was to develop a comprehensive cognitive architecture to address diverse problems. Another example is the CYC project (Witbrock et al. (2005)), which concentrated on constructing an extensive knowledge base through symbolic reasoning to deduce relationships among facts. Despite these efforts, initial hybrid models faced challenges with integration, frequently approaching symbolic and neural components as distinct entities rather than as profoundly interconnected systems. Contemporary neuro-symbolic AI aims to address these limitations by combining the two paradigms in a manner that not only improves learning outcomes but also maintains interpretability.

2.3. Key advances in neuro-symbolic AI

A significant advancement in neuro-symbolic AI is the creation of models that facilitate a smooth integration of symbolic reasoning with neural networks, as shown in Fig. 2. Initial efforts faced limitations due to the insufficient integration of these two paradigms, frequently regarding symbolic logic merely as a post-hoc explanatory framework. Recent developments in differentiable programming and logic embedding have facilitated the incorporation of logical rules directly within neural network architectures, which enable an effective relationship between these components throughout the learning process. The Logic Tensor Network (LTN) model facilitates encoding logical constraints as
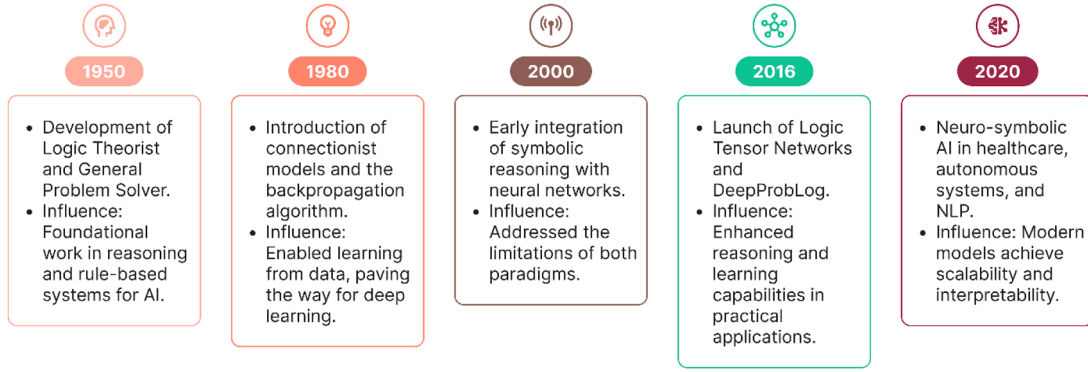
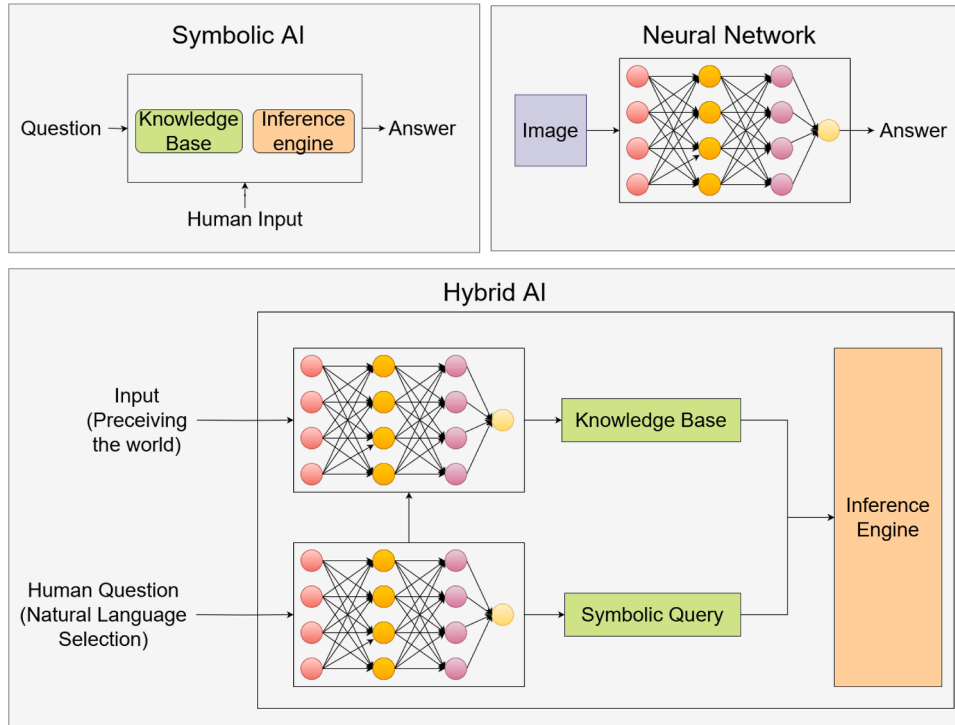**Fig. 1.** Evolution of neuro-symbolic AI and neural networks.

Fig. 2. The evolution of AI paradigms from symbolic AI to neural networks.

tensors within neural networks, which enhances reasoning capabilities while maintaining performance levels (Serafini et al. (2016)). LTNs signify a significant advancement in the field of neuro-symbolic AI. Conventional symbolic AI faced substantial challenges in managing uncertainty and dealing with incomplete information. The issue has been explored through models like DeepProbLog (Lu et al. (2024)), which integrates neural networks with probabilistic logic programming. DeepProbLog combines the probabilistic reasoning inherent in symbolic systems with the learning capabilities of neural networks, thereby enabling the management of complex and uncertain environments. These advancements hold significant value in domains such as robotics and natural language understanding, where systems must deduce missing information while navigating ambiguous data. The advancements in this field have significantly improved the practical use of neuro-symbolic AI in various tasks while laying the groundwork for future investigations to develop more robust, more interpretable, and scalable AI systems.

3. Related work

Recent advancements in neuro-symbolic AI have sparked considerable research interest, with numerous studies exploring the integration of neural networks and symbolic reasoning to enhance learning, reasoning, and interoperability across various domains. Table 1 summarizes the summary of related work. Oltramari et al. (2020) proposed a hybrid AI technique as a general framework for using the capabilities of both methodologies. Specifically, the authors adopted the concept of neuro-symbolism, which involves employing knowledge bases to control the learning process of deep neural networks. The authors further anchored the discussion in two applications of neuro-symbolism and, in both cases, showed that the systems maintain interpretability while obtaining similar performance with the state-of-the-art.

Additionally, Hitzler et al. (2022) explored the combination of neural and symbolic techniques in neuro-symbolic AI. The authors emphasized the benefits of combining deep learning and symbolic reasoning, which highlights that neural networks manage noisy input while symbolic systems provide interpretability and logical rigor. The study looked at various applications, including planning, logical

Table 1
Summary of related work.

Author	Methods	Applications	Key Results	Limitations
Hitzler et al. (2022)	Neural-symbolic integration, metadata, knowledge graphs	AI explainability, metadata usage, complex problem-solving	Highlighted potential of neuro-symbolic AI	Scalability issues in large datasets
Colelough and Regli (2024)	Systematic literature review, PRISM methodology	AI evolution, Neuro-symbolic AI applications	Identified gaps in explainability and meta-cognition	Historical focus without detailed technical innovations
Agrawal et al. (2024)	Neuro-symbolic AI for decision making	Healthcare, finance, and law	Highlighted social benefits of explainable AI	Limited to domain-specific applications
Oltramari et al. (2023)	Cognitive architecture, ACT-R framework	Commonsense reasoning, novel situation handling	Theoretical insights on commonsense reasoning	Focuses primarily on commonsense reasoning
Garcez et al. (2023)	Symbolic representations for robustness	AI systems, explainable AI, commonsense reasoning	Proposed future challenges	Challenges with scalability in real-world applications
Hamilton et al. (2022)	Neuro-symbolic NLP models	NLP reasoning, out-of-distribution generalization	Logic-augmented models achieved neuro-symbolic goals	Limited generalization in complex real-world NLP tasks
Yu et al. (2023)	Neuro-symbolic learning systems	Perception and cognition in neuro-symbolic systems	A comprehensive overview lacked domain-specific insights	Scalability challenges in large systems
Himabindu et al. (2023)	Neuro-symbolic framework	Knowledge transfer reasoning tasks	Improved reasoning, generalization, and interpretability	Framework complexity and implementation
Oltramari et al. (2020)	Hybrid AI methodology	AI context understanding neural network guidance	Combined knowledge bases with neural networks	Performance is limited in specific AI tasks.

deduction, and explainability, all supported by metadata and structured knowledge graphs. However, the authors mentioned important research areas, such as dealing with symbolic challenges with deep learning and linking neural-symbolic components for complicated problem-solving. Based on these findings, Hamilton et al. (2022) systematically assessed neuro-symbolic AI applications for natural language processing (NLP). The authors analyzed studies that integrated symbolic reasoning with neural models to increase reasoning, generalization, and interpretability. The review discovered that logic-augmented neural networks achieved various neuro-symbolic objectives, including reasoning and out-of-distribution generalization.

Shifting the focus to cognitive architectures, Oltramari et al. (2023) proposed integrating cognitive architectures with neuro-symbolic components to allow high-level reasoning and commonsense comprehension in AI systems. The authors proposed a hybrid framework based on the ACT-R cognitive architecture, which emphasizes the importance of generative models in boosting adaptation to novel contexts. Although the concept was intended to overcome limits in commonsense reasoning and generalization, the authors offered limited experimental evidence. Additionally, Garcez et al. (2023) examined improvements in neuro-symbolic computing, concentrating on how it might handle issues such as trust, safety, interpretability, and accountability in AI systems. The authors discussed how symbolic representations may help neural models be more resilient and provide better causal explanations. The report identified issues such as integrating commonsense reasoning and providing reasonable explainability.

Further expanding the scope, Yu et al. (2023) examined progress in neuro-symbolic learning systems, evaluating challenges, methodologies, applications, and prospective trajectories. The study aimed to deliver an extensive summary of the domain and its capacity to handle challenges related to perception and cognition. In terms of practical frameworks, Himabindu et al. (2023) proposed a paradigm for neuro-symbolic integration. The authors merged symbolic AI with deep learning. The framework was confirmed by tests that showed increased reasoning capacities, less data reliance, and enhanced interpretability in complex tasks necessitating knowledge transfer. Building on this foundation, Colelough and Regli (2024) conducted a PRISMA-based systematic review of neuro-symbolic AI initiatives from 2020 to 2024. The authors reviewed 167 publications and identified major study areas, including learning and inference (63%), logic and reasoning (35%), and knowledge representation (44%). The study also found gaps in explainability (28%) and metacognition (5%). The inclusion criteria assured the reproducibility of evaluated investigations by focusing on works using publicly available codebases despite a comprehensive field classification.

Similarly, Agrawal et al. (2024) explored the role of neuro-symbolic

AI in developing advanced reasoning skills for AI assistants in areas such as healthcare, finance, and law. The authors emphasized the need to combine deep learning models with symbolic reasoning formalisms to improve decision-making, interpretability, and problem-solving. The author addressed use cases in which AI assistants assess vast datasets, traverse regulatory systems, and provide reasonable recommendations.

4. Core components of neuro-symbolic AI

4.1. Symbolic reasoning mechanics in AI

Symbolic reasoning in artificial intelligence includes manipulating symbols to represent knowledge and execute logical inference. This framework provides formal logic and rule-based systems to facilitate decision-making. The fundamental mechanics are based on logical operators such as conjunction (AND), disjunction (OR), and negation (NOT), integrated with first-order logic (FOL) to depict complicated connections among entities. In artificial intelligence systems, symbolic representations model knowledge bases, which allows machines to perform logical inferences and address problems that necessitate a structured comprehension of the environment. For example, in 2005, Arocha et al. identified reasoning processes as essential in several medical tasks, including decision-making, clinical problem-solving, and the comprehension of medical literature. They emphasized that recognizing physicians' reasoning techniques is necessary for developing successful decision-support systems. the authors introduced a formal methodology for cognitive-semantic analysis to discover and characterize reasoning methods. Malandr et al. (2024) highlighted the significance of explaining the behavioral differences between two machine learning classification models, especially within eXplainable AI (XAI), due to the rising implementation of learning-based decision support systems. Human decision-makers must understand the distinctions across various machine learning models beyond predictive performance to enhance decision-making efficacy. The authors presented MERLIN, an innovative XAI methodology that offers model-contrastive explanations for two machine learning models. The authors presented an encoding that enables MERLIN to process textual and tabular data with continuous and discrete features.

4.2. Neural networks learning mechanism

Neural networks depend on data-driven mechanisms for their learning processes. A neural network is fundamentally organized into layers of interconnected nodes, commonly called neurons, with each connection linked to a specific weight. The adjustment of connection weights in these networks occurs through an optimization process

known as backpropagation, commonly implemented alongside gradient descent algorithms. The learning mechanism of neural networks involves the iterative minimization of an objective function, such as mean squared error or cross-entropy loss, which quantifies the difference between the predicted and actual output. Each neuron's output computes a weighted summation of its inputs, which is subsequently processed through an activation function (sigmoid or ReLU). This mechanism enables the network to acquire non-linear patterns effectively. Numerous authors explored various neural network learning mechanisms. Table 2 summarizes the neural network learning mechanism and their application domains. Radhakrishnan et al. (2024) proposed the Average Gradient Outer Product (AGOP) as a comprehensive mathematical framework for feature learning in neural networks. Their empirical results indicated that AGOP encompasses characteristics across several architectures, including transformers, CNNs, MLPs, and RNNs. Moreover, AGOP, as it does not rely on backpropagation, facilitated feature learning in machine learning models that were previously incapable of discerning task-specific characteristics. Yan et al. (2024) introduced an advanced deep convolutional neural network model to promote super-resolution reconstruction through multiple convolutional layers and a residual learning approach. Their model was engineered to capture more different image elements, which addresses the shortcomings of current algorithms in maintaining high-frequency details and textures, as seen by their higher performance on public datasets. Bao et al. (2024) extensively evaluated neural network technologies in stock forecasting, which examines a range of traditional and novel models, including RNNs, CNNs, Transformers, GANs, and GNNs. They emphasized the datasets and assessment metrics often utilized in the domain while delineating unsolved challenges and prospective research avenues for neural network applications in financial forecasting. Gong et al. (2024) proposed the Local Feature Masking (LFM) approach to enhance the generalization capacity and resilience of Convolutional Neural Networks (CNNs) against adversarial assaults. Incorporating random feature masking in the shallow layers during training resulted in substantial enhancements in generalization and resilience to adversarial assaults relative to previous methodologies. Lindsay et al. (2024) investigated the capacity of artificial neural network models to replicate alterations in brain circuits or activity to comprehend behavioral modifications. They highlighted the potential of these models in neuroscience for causal analysis. They examined how interpretability techniques in AI may offer novel approaches to discovering brain traits that influence performance and behavior. Sun et al. (2024) introduced a

multi-input operant conditioning neural network incorporating blocking and competing effects, which facilitates fast learning in complex situations. Their research also incorporated temporal discrepancies between signals, stochastic exploration, feedback learning, and adaptive learning, validated using PSPICE simulation for hardware implementation in artificial intelligence. Chen et al. (2024) devised an innovative graph convolutional network (TFM-GCAM), including attention techniques to enhance traffic flow prediction. By integrating traffic flow theory and introducing a Fusion Attention mechanism, their model captured the traffic node's spatial-temporal properties and dynamic attributes, which surpassed prior methodologies in experimental and ablation analyses.

4.3. Bridging the gap: hybridization techniques

The combination of symbolic reasoning and neural networks in neuro-symbolic AI has been accomplished using various techniques that capitalize on the advantages of both approaches, as shown in Fig. 3. A technique involves the integration of differentiable logic, in which symbolic rules are embedded within the neural network as constraints. In Logic Tensor Networks (LTNs) (Badreddine et al. (2022) and Carraro et al. (2024)), logical rules are represented as tensors, which enables neural networks to learn while maintaining logical consistency. The primary challenge in hybridization involves integrating symbolic knowledge without compromising the learning flexibility of neural networks. Hybrid systems allow logic in the optimization process to enforce limitations like consistency and validity while simultaneously adapting to data through neural learning. A popular hybridization method involves the integration of probabilistic reasoning with neural learning. Khashei et al. (2012) proposed a hybrid model that integrates ARIMA with a Probabilistic Neural Network (PNN) to enhance forecasting precision by adjusting ARIMA's estimated values according to residual trends. Empirical results showed that this method outperformed conventional ARIMA models across multiple datasets. Models such as DeepProbLog utilizes neural networks to derive representations from data while employing probabilistic logic to manage uncertainty and facilitate inferences based on these representations (Manhaeve et al. (2021) and Vilamala et al. (2021)). These models demonstrate superior performance in tasks characterized by ambiguous or incomplete data, where symbolic reasoning comes across difficulties. Neural-Symbolic Integration (NSI) approaches have appeared, such as neural networks managing perception tasks, such as image classification, while symbolic components address reasoning tasks, including decision-making and planning. These techniques integrate the adaptive learning capabilities of neural networks with the structured reasoning of symbolic systems.

5. Prominent models and techniques in neuro-symbolic AI

5.1. Logic Tensor Networks (LTNs)

Logic Tensor Networks (LTNs) are a hybrid neuro-symbolic framework that combines the representational capabilities of neural networks with the logical framework of symbolic reasoning. LTNs enable encoding first-order logic (FOL) rules into tensor-based representations, whereby logical predicates and variables are mapped to a continuous vector space, which enables neural network processing (Roychowdhury et al. (2021)). LTNs aim to acquire knowledge from data while preserving logical coherence. The essence of LTNs lies in fuzzy logic, where the truth values of logical statements are denoted as real numbers ranging from 0 to 1 (Jalaian et al. (2023)). This allows LTNs to convey varying degrees of truth and robust reasoning to uncertainty and data noise (Garcez et al. (2023)). Mathematically, LTNs represent logical expressions as tensors, which provides a degree of satisfaction to each formula to maximize the truth of these formulae while learning knowledge from data. For example, for a predicate *P(x)*, instead of assigning a binary truth value *(0 or 1)*, Logic Tensor Networks (LTNs) assign a continuous

Table 2
Summary of neural network learning mechanisms and their application domains.

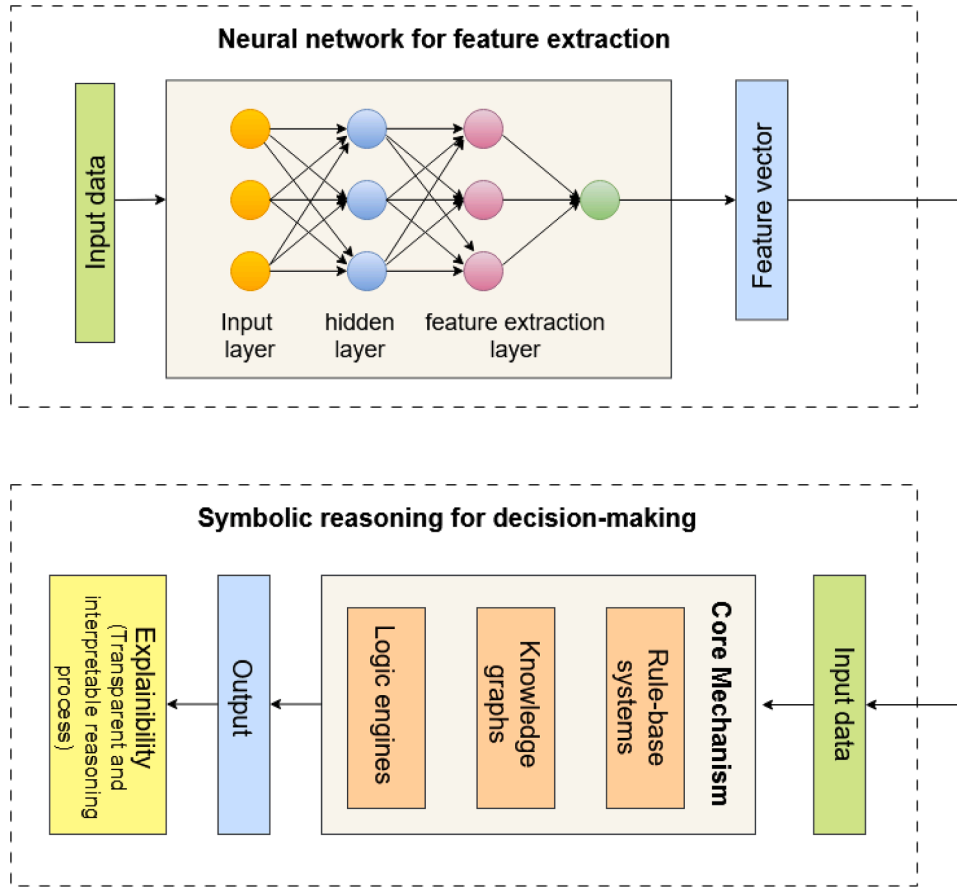| Author and publication year	Learning mechanism or approach	Application domain
Radhakrishnan et al. (2024)	Average Gradient Outer Product (AGOP), backpropagation-free feature learning	Feature learning in neural networks
Yan et al. (2024)	Deep CNN, Super-resolution reconstruction, Residual learning strategy	Super-resolution image reconstruction
Bao et al. (2024)	Neural networks in stock forecasting, RNNs, CNNs, Transformers, GANs, GNNs	Stock forecasting
Gong et al. (2024)	Local Feature Masking (LFM) strategy for generalization and adversarial robustness	Adversarial robustness in CNNs
Lindsay et al. (2024)	Artificial neural network models for causal testing in neuroscience	Neuroscience and behavior simulation
Sun et al. (2024)	Multi-input operant conditioning neural network, Adaptive learning, Random Exploration	AI hardware implementation
Chen et al. (2024)	Graph Convolution Network (TFM-GCAM), Attention mechanisms, Traffic flow forecasting	Traffic flow forecasting

Fig. 3. Hybrid model architecture integrating neural networks for feature extraction and symbolic reasoning for decision-making.

truth value $\mu(P(x)) \in [0,1]$, which shows the degree to which the predicate is satisfied. Differentiable processes compute the satisfaction level. Logical Tensor Networks (LTNs) are employed in domains such as knowledge graph completion, which helps infer absent relations within a graph using logical rules, and natural language processing (NLP), which helps impose logical constraints during the learning process. Furthermore, LTNs support multi-label classification by integrating logical reasoning into the learning process, which improves interpretability and decision-making (Martone et al. (2022)). However, scaling issues occur due to the high computational cost of encoding logical rules for large datasets (Bizzarri et al. (2024)).

Badreddine et al. (2022) emphasized the increasing interest in integrating symbolic logic with neural networks using neuro-symbolic frameworks such as LTN. The research presented Real Logic, a differentiable first-order logic language based on data, using neural computational graphs and fuzzy logic semantics. The study shows that LTN can effectively tackle various AI problems, which encompasses multi-label classification, relational learning, and query responding. By executing these tasks with TensorFlow 2, the authors demonstrated that LTN is a multifaceted and potent instrument for the progression of neuro-symbolic AI. Building on this, Carraro et al. (2024) presented Logic Tensor Networks (LTN) as a neuro-symbolic framework amalgamating deep learning with logical reasoning. The major contribution of LTN is its capacity to establish a logical knowledge base that functions as the aim of the neural model, which enables the model to learn by minimizing a loss function based on logical formulae. The training procedure employs gradient-descent optimization, guided by fuzzy logic, and involves formulating, assessing, and backpropagating gradients via the neural network. The study introduced LTNtorch, a PyTorch implementation of LTN, and showed its use in a binary classification scenario.

5.2. DeepProbLog

DeepProbLog is an extension of ProbLog, a probabilistic logic programming framework that combines the reasoning power of probabilistic logic with the perceptual capabilities of neural networks (De Smet et al. (2023)). This hybrid approach is designed to tackle problems involving both structured and unstructured data. DeepProbLog facilitates reasoning under uncertainty by combining the advantages of symbolic logic in organized decision-making with the adaptability of neural networks in learning representations from unstructured input (Jalaian et al. (2023)). DeepProbLog integrates neural networks as input processors that provide probabilistic outputs to symbolic logic programs. DeepProbLog calculates the probability of a query utilizing a collection of probabilistic information and logical principles. The total probability of a question q is determined by the aggregate of the probabilities of all proofs (valid logical derivations) that meet q. The expression is equation (1):

$$P(q) = \sum_{proofs} P(proof) \times P(evidence) \tag{1}$$

each proof corresponds to a particular instantiation of the logical variables, with the evidence probabilities provided by neural networks trained on data. DeepProbLog has been utilized in various domains, such as image classification, where it combines visual recognition through neural networks with symbolic reasoning to derive decisions based on the interrelations of objects within an image, and robotics, which facilitates decision-making that accounts for uncertainty (Jalaian et al. (2023)). Manhaeve et al. (2018) demonstrate how DeepProbLog handles ambiguous data by combining neural network outputs with logical principles. This enables the framework to produce probabilistic results

that incorporate both data-driven patterns and logical limitations. Building on this, Manhaeve et al. (2021) introduced DeepProbLog, a neural probabilistic logic programming language incorporating deep learning via neural predicates. The framework facilitates symbolic and subsymbolic inference, program induction, and deep learning through instances. This inaugural framework integrates general-purpose neural networks with probabilistic-logical reasoning, which facilitates end-to-end training. Expanding the application of DeepProbLog, Vilamala et al. (2021) introduced a complicated Event Processing (CEP) methodology with DeepProbLog, designed to manage subsymbolic data, establish complicated event rules with flexibility, and facilitate end-to-end training. Their neuro-symbolic design integrates neural networks for subsymbolic data processing with a probabilistic logic layer for rule formulation. The method shows resilience in identifying complex occurrences from audio streams despite noise. Despite its advantages, DeepProbLog has high processing requirements that make it difficult to scale to big applications (Vilamala et al. (2023)). More research is needed to improve its performance in real-world systems while maintaining its capacity to handle uncertainty.

5.3. Differentiable logic programs

Differentiable Logic Programs are a key innovation that translates symbolic logic into a continuous domain, which makes it compatible with neural network training methods (Bueff et al. (2024)). This approach allows logical principles into AI models while preserving differentiability, which is essential for gradient-based optimization techniques (Andelfinger et al. (2021)).

A differentiable logic program may be formally expressed by linking each predicate *P(x)* with a differentiable function $f_{P(x)}$, which measures the degree to which the predicate is fulfilled. For example, if *P(x)* denotes a rule asserting that "if *x* is a dog, then *x* barks," the validity of this rule may be shown as a soft constraint $f_{P(x)}$ that assigns a degree of truth dependent upon the degree of alignment between the input data and the expected outcome. This facilitates end-to-end training of the model. Differentiable logic programs are particularly beneficial for knowledge graph completion and reasoning with relational data, where it is essential to apply rational principles while learning from noisy data. Zimmer et al. (2021) presented the Differentiable Logic Machine (DLM). The proposed innovative neural-logic architecture addresses inductive logic programming (ILP) and reinforcement learning (RL) challenges through first-order logic programs. The inventions encompass the ongoing refinement of first-order logic program spaces, gradient-based training methodologies, a unique critic architecture for reinforcement learning, and an incremental training approach. DLM surpassed leading approaches in both ILP and RL challenges, which exhibited scalability for memory and computational efficiency. Building upon advancements in logic programming, Shindo et al. (2021) introduced a novel framework for learning logic programs from noisy and structured examples by creating an adaptive clause search approach and an enumeration algorithm for ground atoms. The proposed contributions encompass enabling the framework to manage structured data, including sequences and trees, and facilitating the composition of complex logic programs with numerous clauses. The studies demonstrated the framework's capacity to manage noisy, structured instances and its scalability for complex logic systems. Expanding the domain further, Gao et al. (2022) presented Differentiable Learning from Interpretation Transition (D-LFIT), an innovative framework for acquiring symbolic logic programming using neural networks, embeddings, and algebraic techniques. D-LFIT exhibited linear time complexity and resilience in managing mislabeled and incomplete datasets. The trials show that D-LFIT performs similarly to other models, such as RIPPER and NN-LFIT while surpassing them in handling incomplete and noisy data. Earlier foundational work by Yang et al. (2017) introduced Neural Logic Programming, which acquires probabilistic first-order logical rules for knowledge base reasoning through integrating parameter and structure

learning. The method, influenced by TensorLog, transforms inference problems into differentiable processes and utilizes a neural controller system for their composition. Empirically, their approach surpassed previous techniques on datasets such as Freebase and WikiMovies. These systems may effectively learn logical correlations from noisy or incomplete data, which overcomes a key shortcoming of classical symbolic AI (Yu et al., 2023). However, using continuous representations may obscure the interpretability of the underlying reasoning process. Furthermore, the computational difficulty of training these models can provide considerable challenges, particularly in large-scale applications (Shen et al. (2023)).

5.4. Neural theorem provers

Neural Theorem Provers (NTPs) provides an innovative method for combining neural networks with automated theorem proving, which conventionally depends on symbolic logic to generate proofs from a collection of axioms and rules (Riegel et al. (2020)). NTPs enhance this concept through the inclusion of neural networks into the proof process (Minervini et al. (2020)). The fundamental idea of NTPs is to represent logical proofs as differentiable computations, with each proof evaluated according to its compliance with logical principles and the evidence supplied by the neural network (Zhang et al. (2023)). In NTPs, each proof step is represented as a differentiable function. Given a query *q* and a set of axioms *A*, the NTP tries to generate a proof tree by applying inference rules. The probability of each step in the proof is assessed according to the neural network's acquired representations, and the cumulative proof score is calculated as the product of the scores of the individual stages. This can be expressed mathematically as in equation (2):

$$P(q|A) = \prod_i f_i(A) \tag{2}$$

where $f_i(A)$ represents the score of the *i*-th interference step, NTPs have been applied to tasks such as question answering, where the system learns to infer answers from a knowledge base by constructing logical proofs, and natural language understanding explains relationships between entities in a text. Rocktäschel et al. (2016) presented Neural Theorem Provers (NTPs), a comprehensive differentiable neural-symbolic methodology for executing first-order inference using vector representations of symbols and function-free rules. NTPs distinguish themselves from conventional neural-symbolic systems by facilitating the acquisition of predicate representations and complex logical relationships using differentiable backward chaining methods. Building this foundation, Lample et al. (2022) introduced a transformer-based automated theorem prover with a novel HyperTree Proof Search (HTPS) search algorithm. The proposed methodology, integrated online training, resulted in substantial enhancements in theorem proving accuracy, which surpassed the prior state of the art by successfully proving 82.6% of a reserved set of Metamath theorems. In contrast, de Jong et al. (2019) critically analyzed Neural Theorem Proving (NTP) and emphasized its shortcomings in extracting correlations from complex logical datasets. The authors revealed that modifying the NTP method to facilitate exploration might enhance performance, which advocates for using synthetic datasets containing ground-truth relationships to evaluate relation-learning algorithms better.

5.5. Probabilistic symbolic models

Probabilistic symbolic models combine symbolic logic with probabilistic reasoning to address uncertainty in real-world data. Conventional symbolic reasoning systems depend on deterministic principles, which makes them inappropriate for situations characterized by inadequate or chaotic information. Probabilistic models mitigate this limitation by attributing probabilities to logical propositions. These models

are generally constructed on frameworks like Markov Logic Networks (MLNs) (Richardson et al. (2006)), where logical formulae are linked to weights. In probabilistic symbolic models, the joint probability distribution of a collection of logical variables is determined by the weighted sum of the formulae. The likelihood of a question q, contingent upon a collection of formulae F, is expressed in equation (3):

$$P(q|F) = \frac{1}{Z}\exp\left(\sum_i w_i.F_i\right) \tag{3}$$

where w_i is the weight of the formula F_i, and Z is the normalization constant. These models allow for reasoning under uncertainty in complex domains such as medical diagnosis, where symptoms may only partially support a particular disease, or robotics, where sensor data may be noisy or incomplete. Probabilistic symbolic models provide a powerful tool for combining logical reasoning with uncertainty management. Kwiatkowska (2002) proposed PRISM, a tool designed to analyze probabilistic systems, which accommodates three models: discrete-time Markov chains, continuous-time Markov chains, and Markov decision processes. PRISM conducts analysis utilizing probabilistic temporal logics (PCTL and CSL) and provides three model-checking engines: symbolic approaches employing BDDs and MTBDDs, sparse matrices, and a hybrid methodology. The tool has been effectively utilized in diverse applications, including randomized distributed algorithms and industrial systems. In follow-up, Kwiatkowska (2004) introduced effective symbolic methods for probabilistic model checking, which were incorporated into PRISM. These strategies aim to address the performance limitations of MTBDD-based numerical computing, which has exhibited slower speeds compared to explicit methods utilizing sparse matrices. The research presented an innovative hybrid strategy that integrates symbolic and explicit methods, which leads to enhanced performance and facilitates the verification of significantly bigger systems. However, these models suffer scalability issues as the data becomes more complex (Das et al. (2010)). Furthermore, their inflexible logical frameworks may limit flexibility in dynamic situations. To solve these issues, future research should focus on developing flexible probabilistic-symbolic systems that can scale successfully while maintaining interpretability and accuracy (Hayakawa et al. (2021)).

5.6. Comparative analysis of existing models

Neuro-symbolic AI models have distinct strengths and limitations. Their performance varies greatly depending on parameters such as accuracy, scalability, interpretability, and computing efficiency, as shown in Fig. 4. Among the prominent models, Logic Tensor Networks (LTNs) stand out for their ability to integrate deep learning and logical reasoning (Roychowdhury et al. (2021)). LTNs use fuzzy logic to embed first-order logic rules directly into neural network designs, which allows them to retain high interpretability while learning from the input (Martone et al. (2022)). This interpretability significantly benefits models such as DeepProbLog and Neural Theorem Provers (NTPs), which have less visible reasoning procedures. However, the scalability of LTNs is frequently questioned, as their reliance on logical restrictions leads to greater computing complexity, especially when applied to large datasets or extensive knowledge bases (Bizzarri et al. (2024)).

DeepProbLog, on the other hand, excels at dealing with uncertainty and noisy data, which makes it the ideal choice for applications that need probabilistic reasoning, such as robotics and natural language comprehension. DeepProbLog's ability to combine neural networks and probabilistic logic enables reliable decision-making even with inadequate data (De Smet et al. (2023)). For example, it outperforms LTNs in multi-label classification tasks, especially when ambiguity exists. However, DeepProbLog is computationally costly, with inference times substantially longer than models such as Differentiable Logic Programs (Vilamala et al. (2023)). These long inference durations may restrict scalability in real-time systems, where speed is important.

Differentiable Logic Programs maintain a unique equilibrium by translating symbolic rules into a continuous domain, which allows them to act as soft limitations while learning. This flexibility improves their capacity to deal with noisy or unclear input, which makes them competitive in tasks requiring inductive logic programming (Andelfinger et al. (2021)). However, their reliance on continuous representations might lead to reduced interpretability compared to LTNs, as learned rules do not necessarily coincide with rigid logical frameworks (Shen et al. (2023)). Despite this disadvantage, their computational efficiency outperforms NTPs, which makes them more appropriate for large-scale applications. Neural theorem Provers are known for their organized reasoning ability, notably in theorem proving and question answering (Riegel et al. (2020)). These models achieve impressive precision by producing logical proofs using differentiable computations, which shows their use in fields that require accurate logical inference. However, their reliance on computationally complex differentiable proof production limits their usefulness in situations requiring scalability or real-time decision-making. NTPs may be more accurate in organized tasks than DeepProbLog but frequently fall short when dealing with probabilistic reasoning or ambiguous information (Ho et al. (2024)).

Finally, probabilistic symbolic models, such as PRISM, combine symbolic logic and probabilistic reasoning to address uncertainty effectively. These models have significant computing efficiency in small state spaces, which makes them appropriate for probabilistic system analysis (Das et al. (2010)). However, their performance deteriorates as system complexity rises, which shows a scalability barrier similar to that of LTNs (Hayakawa et al. (2021)). Unlike LTNs, PRISM's probabilistic approach provides a stronger basis for dealing with real-world uncertainty but with less flexibility when dealing with huge datasets.

Overall, model selection is significantly influenced by application context and metric prioritizing. LTNs are appropriate for tasks that require high interpretability, but DeepProbLog is better suited to probabilistic reasoning in unpredictable contexts. Differentiable Logic Programs maintain a compromise between efficiency and flexibility while sacrificing some interpretability, whereas NTPs excel at organized, logical tasks at the expense of computational efficiency. PRISM and other probabilistic symbolic models solve issues efficiently on a smaller scale but fail to scale in large systems. Evaluating these trade-offs reveals that no model dominates across all measures, and hybrid techniques may be required to obtain optimal performance in various applications.

To provide deeper insights, we introduce a SWOT (Strengths, Weaknesses, Opportunities, and Threats) analysis of existing neuro-symbolic AI models, as shown in Table 3. This research emphasizes significant elements of each model and gives actionable insights into

Models	Accuracy	Scalability	Interpretability	Computational Efficiency
LTNs	Moderate	Low	Hgh	Moderate
DeepProbLog	Hgh	Moderate	Moderate	Low
DLPs	Low	Hgh	Moderate	Hgh
NTPs	Hgh	Low	Moderate	Low
PRISM	Hgh	Moderate	Hgh	Hgh

Fig. 4. Comparative analysis of existing neuro-symbolic models.

Table 3

SWOT analysis of existing models.

Model	Strengths	Weaknesses	Opportunities	Threats
Logic Tensor Networks (LTNs) Serafini et al. (2016), Carraro et al. (2024), and Badreddine et al. (2022)	Efficiently combines logical reasoning with deep learning. High flexibility and fuzzy logic-based learning.	Scalability issues when handling large datasets. Requires careful tuning of logic rules.	Applications in knowledge graph completion and structured learning.	Risks of underperformance in noisy environments and computational inefficiency in large-scale tasks.
DeepProbLog Manhaeve et al. (2021), and Vilamala et al. (2021)	Supports probabilistic reasoning under uncertainty. Enables end-to-end training with symbolic logic.	Computationally expensive inference. Complex to implement for large problems.	Widely applicable in robotics, NLP, and multi-label classification.	Limited adaption due to high computational demands and implementation complexity.
Differentiable Logic Machine (DLM) Zimmer et al. (2021), Shindo et al. (2021), Gao et al. (2022), and Yang et al. (2017)	Robust handling of noisy and structured data. Efficient in inductive logic programming (ILP) tasks.	High complexity in training logic systems. Extensive resource requirements for large-scale data.	Ideal for knowledge graphs and real-time decision-making.	Risks of scalability bottlenecks in handling complex logical frameworks.
Neural Theorem Provers (NTPs) Rocktäschel et al. (2016), Lample et al. (2022), and de Jong et al. (2019)	Differentiable approach to logical inference. Excels in structured reasoning tasks like theorem proving.	High computational demands. Limited to function-free rules, restricting generalizability.	Promising for question answering and knowledge-based reasoning.	Dependency on differentiable methods limits efficiency in real-world tasks.
PRISM Kwiatkowska (2002) and Kwiatkowska (2004)	Highly efficient in probabilistic system analysis. Supports hybrid approaches for Markov models.	Limited scalability with large state spaces. Complexity in hybrid methods.	Applications in distributed algorithms and industrial systems.	Threat from more modern hybrid probabilistic-symbolic systems.

their practical uses, future potential, and inherent challenges.

6. Applications of neuro-symbolic AI

6.1. Natural Language Processing (NLP)

Neuro-symbolic AI in Natural Language Processing (NLP) provides substantial benefits by integrating the capabilities of neural networks for language comprehension with symbolic reasoning for managing structured information. Neural networks, especially transformer-based models like BERT and GPT, are proficient in deriving semantic meaning from extensive text corpora (Gillioz et al. (2020) and Salıcı et al. (2024)). However, individuals cannot frequently reason about language in a systematic, rule-governed fashion. This is essential for commonsense reasoning or question answering, which requires a thorough understanding of logical links. Neuro-symbolic AI enhances NLP systems by including symbolic reasoning, enabling models to utilize logical principles in text interpretation and improving performance in tasks such as machine translation, semantic parsing, and automated theorem proving (Bhuyan et al. (2024)).

6.2. Robotics and autonomous systems

Neuro-symbolic AI is essential in robotics and autonomous systems to facilitate data-driven learning and advanced decision-making (Lu et al. (2024)). Robots functioning in dynamic situations must analyze sensory input (processed by neural networks) and make decisions based on logical regulations and constraints (managed by symbolic reasoning) (Bhuyan et al. (2024)). In autonomous cars, neural networks interpret visual inputs, such as pedestrian detection and road sign recognition (Chen et al. (2021) and Galvao et al. (2021)), while symbolic reasoning ensures adherence to traffic regulations and ethical standards. Neuro-symbolic models facilitate real-time reasoning and adaptive planning in robots by amalgamating acquired representations from sensory input with established behavioral norms.

6.3. Healthcare

Healthcare is a sector where neuro-symbolic AI has demonstrated significant potential, especially in decision-support systems for diagnoses and treatment recommendations. Neural networks excel at processing complex medical data, including imaging scans and genetic information, although they frequently fail to explain their predictions (Zafar et al. (2023), and Tran et al. (2021)). Neuro-symbolic systems can provide interpretable diagnoses and recommendations using symbolic

reasoning grounded in medical guidelines or clinical expertise (Hassan et al. (2022) and Vidal et al. (2024)). A neuro-symbolic system in healthcare may evaluate patient data using a neural network and implement symbolic rules based on medical protocols to propose therapies, which ensures that the recommendations are accurate and clinically explicable.

6.4. Knowledge representation and reasoning

Knowledge representation and reasoning are fundamental to neuro-symbolic AI's capacity to manage structured information and execute logical judgments. Symbolic reasoning offers a fundamental framework for expressing relationships among things, facts, and rules inside a domain, whereas neural networks are proficient at obtaining representations from unstructured input (Lu et al. (2024)). In neuro-symbolic systems, symbolic representations such as knowledge graphs encode links between ideas, while neural networks learn to infer new facts or recognize patterns in the data (Zhang et al. (2024)). This integration facilitates automated reasoning inside knowledge bases, which enables computers to extract new insights from existing data, address complex inquiries, and execute functions such as logical inference and rule-based decision-making (Zhu et al. (2024)). Neuro-symbolic AI augments conventional knowledge representation systems by incorporating data-driven learning.

6.5. Intelligent agents

Intelligent agents utilizing neuro-symbolic AI can demonstrate adaptive learning and advanced reasoning. In situations where agents engage with people or other systems, they must acquire knowledge from experience and reason about their behaviors coherently and understandably. Neuro-symbolic AI enables virtual assistants and autonomous decision-making systems to learn from human interactions using neural networks while following logical rules that dictate permissible behavior (Ciatto et al. (2024)). These agents can execute tasks such as automated planning, decision-making in unpredictable conditions, and conversational AI by combining acquired knowledge with symbolic reasoning (Lemaignan et al. (2017)).

7. Advancements in explainability and their impact on regulated industries

Explainability has emerged as a vital component for adopting neuro-symbolic AI, particularly in banking and health industries where trust and transparency are essential (Parveen et al. (2024)). The capacity of

these systems to provide interpretable and transparent decision-making processes is critical for maintaining regulatory compliance and ethical accountability. Explainable neuro-symbolic models, for example, can clarify the reasoning behind decisions such as credit risk assessment and fraud detection in the banking industry (Longo et al. (2024)). These models give organized explanations that meet strict legal and ethical norms by including symbolic reasoning components. Similarly, explainable models in healthcare empower clinicians by providing interpretable diagnostic advice and treatment strategies based on medical criteria (Parveen et al. (2024)). Transparency is vital for building confidence between professionals and patients.

Recent advances in neurosymbolic AI have greatly improved explainability. Models such as Logic Tensor Networks (LTNs) incorporate logical principles into neural networks, which provide unambiguous decision-making explanations. DeepProbLog uses probabilistic reasoning and neural learning to handle uncertainty while being interpretable. These advancements directly address the "black-box" critique leveled at standard AI systems (Chinu & Bansal, 2024), which makes neuro-symbolic models more appropriate for regulated applications that require responsibility and clarity. The practical implications of these breakthroughs are enormous. For example, neuro-symbolic AI could provide accurate predictions and traceable reasoning routes consistent with recognized medical norms in medical diagnostics (Lu et al. (2024)). These technologies can improve decision-making transparency in finance, which makes audits easier and minimizes regulatory scrutiny.

8. Challenges in neuro-symbolic AI

8.1. Scalability of hybrid systems

A primary problem in neuro-symbolic AI is the scalability of hybrid systems that integrate symbolic reasoning with neural networks (Bhuyan et al. (2024) and Lu et al. (2024)). Symbolic reasoning is conventionally resource-intensive, especially in complicated fields requiring the processing of extensive knowledge bases and elaborate rules (Zhou et al. (2021)). Conversely, deep learning models and vast neural networks need substantial computing resources, such as memory and processing capacity—the amalgamation of these two paradigms results in the system adopting the computing requirements (Capra et al. (2020)). Neuro-symbolic models, such as Logic Tensor Networks (LTNs), encode logical formulae as tensors and then learn from data, augmenting system complexity and complicating scalability for extensive datasets or knowledge bases.

8.2. Integration of multimodal data

Integrating multimodal data is essential for neuro-symbolic AI systems, particularly in applications using different inputs such as images, text, audio, and sensor measurements (Han et al., 2024). Neural networks proficiently manage unstructured data, such as images or sounds. In contrast, symbolic reasoning is conventionally more effective in processing organized data, such as knowledge graphs or relational databases (Chen et al. (2020)). The difficulty occurs when two categories of data must be handled concurrently. In an autonomous driving context, the system must analyze visual data from cameras, processed by neural networks, and integrate it with symbolic reasoning on traffic regulations and road signs, represented as structured knowledge (Gilpin et al., 2021). The amalgamation of multimodal input in these systems necessitates a fluid integration of brain representations and symbolic logic to guarantee coherent decision-making.

8.3. Balancing interpretability and performance

Finding a balance between interpretability and performance is a major challenge in neuro-symbolic AI. Neural networks, particularly deep learning models, perform exceptionally well in tasks such as image recognition, language translation, and gaming (Alam et al. (2020)). However, they usually operate as black-box models, which makes their decision-making processes difficult to understand (Chinu et al., 2024). In contrast, symbolic reasoning provides explicit, rule-based reasoning that is intrinsically interpretable, although typically lacking the flexibility and scalability of neural networks (Besold et al. (2021)). This contradiction creates tension between building AI systems with high performance and those capable of explaining their decision-making processes.

Explainability is more than a preference in highly regulated businesses like banking and health. For example, decision-making algorithms for credit scoring or fraud detection in banking must present explicit reasons for their recommendations to meet regulatory requirements and preserve consumer trust. Similarly, in healthcare, clinicians want interpretable models that provide accurate diagnoses and explanations consistent with medical standards (Hassan et al. (2022) and Vidal et al. (2024)). A lack of transparency in certain industries could hinder the adoption of neuro-symbolic AI since stakeholders may view these systems as untrustworthy or non-compliant. To address this difficulty, hybrid models must be developed that combine high performance with the capacity to deliver extensive, user-friendly explanations.

8.4. Computational complexity and optimization

The computational complexity of neuro-symbolic systems stems from the necessity to integrate symbolic reasoning, which typically encompasses NP-hard problems like theorem proving and rule-based inference, with the deep learning frameworks of neural networks. Hybrid models such as Neural Theorem Provers (NTPs) (Rocktäschel et al. (2016), Lample et al. (2022), and de Jong et al. (2019)) and Logic Tensor Networks (LTNs) strive to integrate these two paradigms (Serafini et al. (2016), Carraro et al. (2024), and Badreddine et al. (2022)); however, they frequently result in computational inefficiencies. Symbolic reasoning components may create bottlenecks when managing extensive knowledge bases or necessitating accurate, logical inference, yet neural networks are computationally intensive owing to the back-propagation training process.

8.5. Data scarcity and learning in resource-constrained environments

Data scarcity and the necessity to function in resource-limited contexts present considerable hurdles for neuro-symbolic AI systems. Neural networks often need substantial quantities of labeled data to attain optimal performance, but in several real-world situations, such data may be limited or costly to acquire (Song et al. (2022)). Conversely, symbolic reasoning systems do not need considerable training data, as they depend on established rules and logical correlations. However, these algorithms frequently exhibit an inability to learn and adapt from limited data sets. The problem lies in developing neuro-symbolic models that can learn efficiently from small inputs while preserving strong reasoning abilities (Hassan et al. (2022)). This is especially crucial in fields like healthcare or disaster response, where data may be partial or inaccessible, necessitating judgments with limited computational resources.

9. Novel contributions and future directions

9.1. Dynamic neuro-symbolic systems

Dynamic neuro-symbolic systems signify the further advancement in hybrid AI models by facilitating real-time adaptation of AI to evolving settings (Bhuyan et al. (2024)). Conventional symbolic systems depend on fixed rules, but neural networks may evolve through data-driven learning. Dynamic neuro-symbolic systems integrate these advantages by enabling the symbolic component to modify or develop regulations in response to new information the neural networks achieve. This

flexibility is essential for systems functioning in non-stationary contexts, such as autonomous agents or industrial robots, where norms and patterns fluctuate constantly (Lu et al. (2024)). A dynamic neuro-symbolic system may modify its symbolic reasoning framework in response to new sensory inputs from the neural network. These systems are being investigated for continuous learning tasks and can mitigate the inflexibility of conventional symbolic methods.

9.2. Ethical consideration in neuro-symbolic AI

The integration of neuro-symbolic AI into sensitive sectors such as healthcare and finance demands rigorous ethical considerations to assure accountability, justice, and public confidence. (Albahri et al. (2023)) The combination of symbolic reasoning and neural networks allows us to insert ethical restrictions into decision-making processes directly (Lu et al. (2024)). To successfully solve real-world challenges, abstract discussions on ethics must be transformed into tangible frameworks and implementations. In the healthcare sector, neuro-symbolic systems are increasingly used in decision-support applications such as disease diagnosis and therapy recommendation predictions (Zafar et al. (2023), and Tran et al. (2021)). These systems must resolve ethical conflicts, such as balancing patient autonomy with clinical effectiveness. For example, symbolic thinking might incorporate clear ethical standards based on the "Four Principles of Biomedical Ethics"—autonomy, beneficence, nonmaleficence, and justice—to ensure that recommendations are consistent with recognized medical practices (Tsai et al. (1999)). Neural networks, on the other hand, may use complicated patient data, such as imaging or genetic profiles, to improve diagnosis accuracy. Combining these techniques allows a neuro-symbolic system to create interpretable suggestions while offering explicit reasons for its decisions.

Ethical issues are equally important in finance, particularly in credit scoring, fraud detection, and algorithmic trading applications. For example, credit scoring systems are frequently checked for biases that may penalize specific demographic groups. A neuro-symbolic method might reduce these biases by including fairness indicators, such as demographic parity or equalized chances, into its symbolic reasoning framework. While neural networks examine large datasets to determine creditworthiness, symbolic rules can assure adherence to regulatory requirements and ethical values, which reduces the possibility of discriminating outcomes (Morettin et al. (2024) and Xie et al. (2023)). Such a system can enhance financial decision-making while maintaining public trust by explaining its conclusions using ethical and regulatory frameworks. The development of ethical neuro-symbolic systems requires a multifaceted approach. First, symbolic reasoning components should be designed to encode domain-specific ethical principles that inform decision-making processes (Bhuyan et al. (2024)). Second, these components must evolve dynamically when neural networks analyze new input, which ensures the system is sensitive to changing ethical and cultural standards (Wallach et al. (2020)). Finally, explainability is crucial; symbolic reasoning must provide unambiguous post-hoc explanations of decisions, which allows stakeholders to assess the system's ethical reasoning transparently.

Future research should focus on testing these methodologies in real-world settings. In healthcare, this may imply incorporating neuro-symbolic systems into diagnostic instruments and tracking their adherence to ethical ideals over time (Pareveen et al. (2024)). Similarly, financial applications might improve fairness limitations in credit scoring or fraud detection to provide equal results while maintaining performance. Neuro-symbolic AI can establish novel standards for morally sound decision-making in critical domains through these initiatives.

9.3. Real-time decision making for IoT

In the context of the Internet of Things (IoT), neuro-symbolic AI facilitates real-time decision-making among interconnected devices (Lu et al. (2024)). IoT systems provide extensive data from sensors, cameras, and many sources. Neural networks analyze unstructured data proficiently, whereas symbolic reasoning facilitates decision-making based on logical norms and constraints, like safety laws and energy consumption thresholds (Zeng et al. (2023)). In smart cities, neuro-symbolic systems can evaluate real-time traffic data and implement symbolic rules on traffic management to make adaptive decisions, such as rerouting cars to alleviate congestion. The difficulty with IoT systems is the need for rapid, dependable decisions in settings with limited processing resources (Gubbi et al. (2013) and Lee et al. (2015)). In conjunction with lightweight neuro-symbolic architectures, edge computing is being investigated as a solution that facilitates decision-making near the data source and diminishes delays linked to cloud-based processing.

9.4. Advancements in explainability

Future research should focus on developing domain-specific frameworks to improve the explainability of neuro-symbolic AI in regulated businesses. For example, such frameworks might standardize how models explain credit risk or investment strategies in finance. In healthcare, they might directly incorporate ethical issues and clinical guidelines into model structures to guarantee that recommendations are both interpretable and practical (Hassan et al. (2022) and Vidal et al. (2024)). One interesting path is to develop hybrid models that dynamically tailor their explanations to the demands of various stakeholders. For example, in medicine, a neuro-symbolic system might deliver thorough, rule-based explanations to professionals while providing patients with simplified insights (Parveen et al. (2024)). Another avenue of investigation is post-hoc explainability approaches that use symbolic reasoning to illuminate neural network decisions after they have been made. This method can close the gap between performance and interpretability, which allows models to function effectively in high-stakes scenarios.

Furthermore, improvements in visualization tools may play an important role in making neuro-symbolic AI more approachable. Interactive dashboards or graphical interfaces that follow decision-making paths can improve user understanding and trust (Iftikhar et al. (2019)). As explainability drives the adoption of neuro-symbolic AI, these advancements will ensure that the technology satisfies the transparency, trust, and accountability needs in regulated businesses.

9.5. Towards fully differentiable neuro-symbolic AI

Completely differentiable neuro-symbolic AI aims to develop systems where neural and symbolic elements may be taught end-to-end using gradient-based optimization methods, including backpropagation (Ionescu et al. (2015) and Zucchet et al. (2022)). Recent improvements have concentrated on differentiating symbolic logic by integrating logical rules into neural network architectures or employing fuzzy logic to express symbolic knowledge as continuous values. Differentiable frameworks such as Logic Tensor Networks (LTNs) enable encoding logical rules as tensors, which may be optimized with neural network parameters (Badreddine et al. (2022)). Transitioning to entirely differentiable systems will facilitate uninterrupted training. Such breakthroughs might transform fields such as automated reasoning and decision-making systems, which need advanced reasoning and data-driven learning.

10. Conclusion

Neuro-symbolic AI proposes a paradigm change in artificial intelligence by integrating neural networks' data-driven flexibility with symbolic systems' structured logical inferences. This study has thoroughly reviewed key neuro-symbolic frameworks such as Logic Tensor

Networks (LTNs), DeepProbLog, and Neural Theorem Provers, demonstrating their ability in various fields, including natural language processing, robotics, and healthcare. These models exemplify the synergy between learning and reasoning, which allows for sound decision-making in complicated, real-world circumstances. For example, LTNs easily incorporate logical constraints into neural networks, improving interpretability, whereas DeepProbLog excels at addressing uncertainty and integrating probabilistic reasoning into symbolic logic. The paper also highlights key problems, including the computational complexity of hybrid systems, the scalability of reasoning methods, and the integration of several data sources. Furthermore, balancing interpretability and performance remains a major problem, particularly in sensitive healthcare and finance applications where transparency and trustworthiness are crucial. These limitations point to future approaches, including the development of completely differentiable structures, dynamic neuro-symbolic systems capable of real-time adaptation, and ethical frameworks integrated into decision-making processes.

Looking forward, neuro-symbolic AI has the potential to redefine the frontiers of intelligent systems. By overcoming current challenges, it has the potential to become a cornerstone for developing scalable, interpretable, and ethical AI systems. In high-stakes situations such as autonomous systems and regulated industries, the combination of explainability and robustness will be critical to encouraging confidence and wide adoption. The paper contributes to the growing understanding of neuro-symbolic AI's strengths and limits, providing insights that will pave the way for future advances in this transformative field.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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