

SETS

$J \in \{1, \dots, 56\}$ customers ; k and $l \rightarrow$ alias for J

DECISION VARIABLES

$$I_J = \begin{cases} 1 & \text{if indirect delivery is used for customer } J \\ 0 & \text{o/w} \end{cases}$$

$$D_J = \begin{cases} 1 & \text{if direct delivery is used for customer } J \\ 0 & \text{o/w} \end{cases}$$

$$B_J = \begin{cases} 1 & \text{if large truck is used for customer } J \\ 0 & \text{o/w} \end{cases}$$

$$S_J = \begin{cases} 1 & \text{if small truck is used for customer } J \\ 0 & \text{o/w} \end{cases}$$

$$B1_J = \begin{cases} 1 & \text{if large truck is used for only customer } J \\ 0 & \text{o/w} \end{cases}$$

$$B2_{JK} = \begin{cases} 1 & \text{if large truck is used for customers } J \text{ and } k \\ 0 & \text{o/w} \end{cases}$$

$$B3_{JKL} = \begin{cases} 1 & \text{if large truck is used for customers } J, k \text{ and } l \\ 0 & \text{o/w} \end{cases}$$

$$S1_J = \begin{cases} 1 & \text{if small truck is used for only customer } J \\ 0 & \text{o/w} \end{cases}$$

$$S2_{JK} = \begin{cases} 1 & \text{if small truck is used for customers } J \text{ and } k \\ 0 & \text{o/w} \end{cases}$$

$$S3_{JKL} = \begin{cases} 1 & \text{if small truck is used for customers } J, k \text{ and } l \\ 0 & \text{o/w} \end{cases}$$

$$CB2_{JK} = \{ \text{Cost of carrying customers } J \text{ and } k \text{ by a large truck} \}$$

$$CB3_{JKL} = \{ \text{Cost of carrying customers } J, k \text{ and } l \text{ by a large truck} \}$$

$$CS2_{JK} = \{ \text{Cost of carrying customers } J \text{ and } k \text{ by a small truck} \}$$

$$CS3_{JKL} = \{ \text{Cost of carrying customers } J, k \text{ and } l \text{ by a small truck} \}$$

PROVIDED VALUES

$d_j^w = \{$ Demand weight of customer j

$d_j^v = \{$ Demand volume of customer j

$u_j = \{$ Indirect shipment cost of customer j

$a_{jk} = \{$ Clusterability of customers j and k

$c_j^{\text{small}} = \{$ Cost of small truck for customer j

$c_j^{\text{large}} = \{$ cost of large truck for customer j

MODEL

$$\begin{aligned} \min z = & \sum_j I_j \times d_j^w \times u_j \\ & + \sum_j B1_j \times c_j^{\text{large}} \\ & + \sum_j S1_j \times c_j^{\text{small}} \\ & + \sum_j \sum_k CB2_{jk} + 250 \times B2_{jk} \\ & + \sum_j \sum_k CS2_{jk} + 125 \times S2_{jk} \\ & + \sum_j \sum_k \sum_l CB3_{jkl} + 500 \times B3_{jkl} \\ & + \sum_j \sum_k \sum_l CS3_{jkl} + 250 \times S3_{jkl} \end{aligned}$$

Subject to

$$I_j + D_j = 1 \quad // \text{ choose either direct or indirect shipment for } j=1,2,\dots,56$$

$$B_j + S_j = D_j \quad // \text{ choose either large or small truck for } j=1,2,\dots,56$$

$$\begin{aligned} \sum_k \sum_l (B3_{klj} + B3_{ljk} + B3_{jkl}) + \sum_k (B2_{jk} + B2_{kj}) + B1_j &= B_j \\ \sum_k \sum_l (S3_{klj} + S3_{ljk} + S3_{jkl}) + \sum_k (S2_{jk} + S2_{kj}) + S1_j &= S_j \end{aligned} \quad \left. \begin{array}{l} \text{Carry} \\ \text{customer} \\ j\text{'s order} \\ \text{only once} \end{array} \right\}$$

for $j=1,2,\dots,56$

$$B_{2jk} \leq a_{jk} \quad j=1,2,\dots,56; \quad k=1,2,\dots,56$$

$$B_{3jkl} \leq a_{jk} \times a_{kl} \times a_{lj} \quad j=1,2,\dots,56; \quad k=1,2,\dots,56; \quad l=1,2,\dots,56$$

$$S_{2jk} \leq a_{jk} \quad j=1,2,\dots,56; \quad k=1,2,\dots,56$$

$$S_{3jkl} \leq a_{jk} \times a_{kl} \times a_{lj} \quad j=1,2,\dots,56; \quad k=1,2,\dots,56; \quad l=1,2,\dots,56$$

→ Can the customers shipped together?

$$B_{1j} \times d_j^v \leq 33 \quad j=1,2,\dots,56$$

$$B_{2jk} \times (d_j^v + d_k^v) \leq 33 \quad j=1,2,\dots,56; \quad k=1,2,\dots,56$$

$$B_{3jkl} \times (d_j^v + d_k^v + d_l^v) \leq 33 \quad j=1,2,\dots,56; \quad k=1,2,\dots,56; \quad l=1,2,\dots,56$$

$$S_{1j} \times d_j^v \leq 18 \quad j=1,2,\dots,56$$

$$S_{2jk} \times (d_j^v + d_k^v) \leq 18 \quad j=1,2,\dots,56; \quad k=1,2,\dots,56$$

$$S_{3jkl} \times (d_j^v + d_k^v + d_l^v) \leq 18 \quad j=1,2,\dots,56; \quad k=1,2,\dots,56; \quad l=1,2,\dots,56$$

→ Capacity constraints

$$CB_{2jk} \geq C_j^{\text{large}} \times B_{2jk}$$

$$CB_{2jk} \geq C_k^{\text{large}} \times B_{2jk}$$

$j=1,2,\dots,56; \quad k=1,2,\dots,56$ } Maximum cost of customers j or k for large truck

$$CS_{2jk} \geq C_j^{\text{small}} \times S_{2jk}$$

$$CS_{2jk} \geq C_k^{\text{small}} \times S_{2jk}$$

$j=1,2,\dots,56; \quad k=1,2,\dots,56$ } Maximum cost of customers j or k for small truck

$$CB_{3jkl} \geq C_j^{\text{large}} \times B_{3jkl}$$

$$CB_{3jkl} \geq C_k^{\text{large}} \times B_{3jkl}$$

$$CB_{3jkl} \geq C_l^{\text{large}} \times B_{3jkl}$$

$j=1,2,\dots,56$
 $k=1,2,\dots,56$
 $l=1,2,\dots,56$ } Maximum cost of customers j, k or l for large truck

$$CS_{3jkl} \geq C_j^{\text{small}} \times S_{3jkl}$$

$$CS_{3jkl} \geq C_k^{\text{small}} \times S_{3jkl}$$

$$CS_{3jkl} \geq C_l^{\text{small}} \times S_{3jkl}$$

$j=1,2,\dots,56$
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