18. *(a)* Show that if two inductors L_1 and L_2 are in series in a circuit, as shown in Fig. 29-15*a*, the combination is equivalent to an inductance $L = L_1 + L_2$. Assume that no flux from either inductor links the other inductor. *(b)* What is the equivalent inductance if the two are in parallel, as shown in Fig. 29-15*b*?

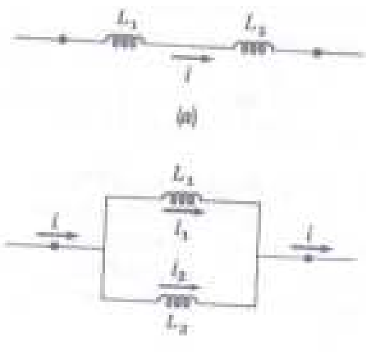


Figure 29-15. Exercise 18: *(a)* Inductors in series. *(b)* Inductors in parallel.

a|i

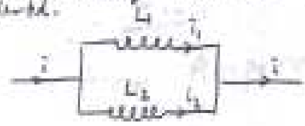
a) The current is the same for both inductors

$$|\mathcal{E}| = L_1 \frac{di}{dt} + L_2 \frac{di}{dt}$$

$$= L_{eq} \frac{di}{dt} \quad \Rightarrow \quad L_1 + L_2 = L_{eq}$$

In order that this is valid the inductances should be sufficiently separate otherwise mutual inductance should be considered.

b)



$$|\mathcal{E}| = L_1 \frac{di_1}{dt} = L_2 \frac{di_2}{dt} = L_{eq} \frac{di}{dt}$$

$$i = i_1 + i_2 \Rightarrow \frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$

$$\frac{|\mathcal{E}|}{L_{eq}} = \frac{|\mathcal{E}|}{L_1} + \frac{|\mathcal{E}|}{L_2} \Rightarrow \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$

17. A conducting rod of length $\ell = 120$ mm is pivoted at one end as the other end slides on a circular conductor perpendicular to a uniform magnetic field $B = 400$ mT, as shown in Fig. 28-23. The rod rotates counterclockwise with constant angular speed $\omega = 370$ rad/s. Assume that all of the $R = 1200\text{-}\Omega$ resistance of the circuit is contained in the resistance symbol in the figure. *(a)* Determine an expression for the induced current in the circuit in terms of ℓ, B, ω, and R. *(b)* Evaluate the current using the values above. *(c)* What is the sense of the induced current? *(d)* Evaluate the magnitude of the magnetic torque on the rotating rod about an axis parallel to **B** and through the pivot point. How can the rod rotate with constant angular speed?

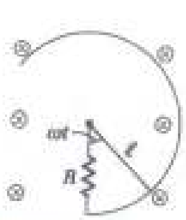


Figure 28-23. Exercise 17.

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a) $$\mathcal{E} = -N\frac{d\Phi_m}{dt} = -\frac{d}{dt}(\vec{B}\cdot\vec{A}) = -\frac{d}{dt}(BA\cos 180^\circ)$$

$$= \frac{d}{dt}(BA)$$

The area A shown for $\theta = \omega t$, $A = \frac{1}{2}(\omega t)\ell^2$

$$\mathcal{E} = \frac{d}{dt}\left[B\left(\frac{1}{2}\omega t \ell^2\right)\right] = \frac{1}{2}B\ell^2\omega$$

$$I = \frac{\mathcal{E}}{R} = \frac{\frac{1}{2}B\ell^2\omega}{R} = \frac{B\ell^2\omega}{2R}$$

b) $I = 888\ \mu A$

c) By Faraday's law, the induced current flows counterclockwise

d) For an element dr, the force $dF = (I\,dr)B$

Torque $= (I\,dr)Br$

$$\tau = \int_0^\ell (I\,dr)Br = IB\int_0^\ell r\,dr = IB\frac{\ell^2}{2} = 2.55\ \mu N{\cdot}m.$$

5. ***Emf in a sliding-wire circuit with a time-varying magnetic field.*** Suppose that the sliding wire in Fig. 28-6 and in the previous problem is initially at rest at $x = x_0 > 0$. Further, the magnetic field is not constant, but its magnitude increases from B_0 at $t = 0$ at a constant rate $dB/dt = C$ (with $C > 0$). An induced current will exist in the wire, and the wire will move in response to the magnetic force acting on it. *(a)* Show that the induced emf is given by $\mathcal{E} = \ell[B(t)v_x(t) + Cx(t)]$, where $v_x(t)$ is the x component of the velocity of the wire. *(b)* What is the sense of the induced current at $t = 0$? *(c)* In what direction does the wire begin to move at $t = 0$? *(d)* What are the answers to parts *(b)* and *(c)* if B decreases ($C < 0$)?

Page 718/Pr 5

a) $\phi_B = \vec{B}\cdot\vec{A} = BA$

$$\mathcal{E} = \frac{d\phi_B}{dt} = B\frac{dA}{dt} + A\frac{dB}{dt} \qquad A = \ell x$$

$$\mathcal{E} = B\frac{d}{dt}(\ell x) + \ell x\frac{dB}{dt} \qquad \frac{dB}{dt} = c \quad c > 0$$

$$\mathcal{E} = B\ell\frac{dx}{dt} + \ell c x = B\ell v_x + \ell x c = \ell(Bv_x + cx)$$

b) induced current clockwise.

c) it will move in the direction of F, to the left.

d) if $c < 0$, then the direction would be reversed. the current would flow counterclockwise and the motion would be to the right.

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11. The circuit in Fig. 29-13 has $\mathcal{E}_0 = 12$ V, $R = 25\ \Omega$, $L = 0.48$ H. The switch is closed at $t = 0$. Determine *(a)* the inductive time constant, *(b)* the current at $t = 25$ ms, *(c)* the current at 1.0 s. *(d)* What is the asymptotic value of the current?

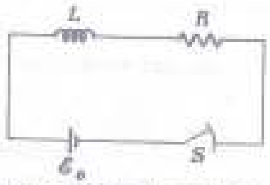

Figure 29-13. Exercise 11.

a) $$\tau_L = \frac{L}{R} = \frac{0.48\,H}{25\,\Omega} = 19.2\text{ ms}$$

b) $$i = i_0\left(1 - e^{-\frac{t}{\tau_L}}\right) = \frac{\mathcal{E}_0}{R}\left(1 - e^{-\frac{t}{\tau_L}}\right)$$

$$i = \frac{12\,V}{25\,\Omega}\left(1 - e^{-\frac{25}{19.2}}\right) = 0.35\,A$$

c) At $t = 1$ s

$$i \simeq \frac{12}{25} = 0.48\,A$$

d) $$i = \frac{\mathcal{E}_0}{R} = \frac{12\,V}{25\,\Omega} = 0.48\,A$$

6. ***An LR circuit.*** The current in the *LR* circuit of Fig. 29-22 is zero at $t = 0$ when the switch first closes at position *A*. The switch remains at position *A* for 5.0 s and then is quickly changed to position *B* for the next 5.0 s. *(a)* Determine the current in the circuit at $t = 5.0$ s just before the switch is changed to *B*. *(b)* Determine the current at $t = 10$ s. *(c)* If the switch is changed back to *A* at this instant, determine the current at $t = 15$ s and explain why this answer is different from the answer to part *(a)*.

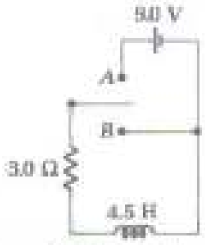

Figure 29-22. Problem 6.

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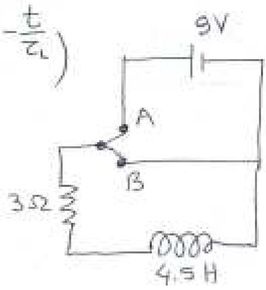
a) $i(t) = \frac{\mathcal{E}_0}{R}\left(1 - e^{-\frac{t}{\tau_L}}\right)$



$\tau_L = \frac{L}{R} = \frac{4{,}5H}{3\Omega} = 1{,}5s$

$i(5s) = \frac{\mathcal{E}_0}{R}\left(1 - {}^{-\frac{5}{1{,}5}}\right)$

$= \frac{9}{3}\left(1 - e^{-\frac{5}{1{,}5}}\right) = 2.893A$ ③

b) The switch is changed to B ③

$i = i_0 e^{-\frac{t}{\tau_L}} = i(5s)\, e^{-\frac{5}{1{,}5}} = 0{,}1032A$

c) back to A

$\mathcal{E}_0 - iR - L\frac{di}{dt} = 0$

$\mathcal{E}_0 - iR = L\frac{di}{dt}$

$\int \frac{dt}{L} = \int \frac{di}{\mathcal{E}_0 - iR}$ $\qquad \mathcal{E}_0 - iR = u$

$\qquad -R\,di = du$

$\frac{t}{L} = -\frac{1}{R}\int \frac{du}{u} \Rightarrow -\frac{Rt}{L} = \ln u - \ln K$ $\qquad$ K is a constant

$-\frac{t}{\tau_L} = \ln\frac{u}{K} = \ln\frac{(\mathcal{E}_0 - iR)}{K}$

$e^{-\frac{t}{\tau_L}} = \frac{\mathcal{E}_0 - iR}{K} \Rightarrow Ke^{-\frac{t}{\tau_L}} = \mathcal{E}_0 - iR$

$i = \frac{\mathcal{E}_0}{R} - \frac{K}{R}e^{-\frac{t}{\tau_L}}$ $\qquad$ at $t = 0$

$\qquad i = i_0 = 0{,}1032$ A

$i(0) = i_0 = \frac{\mathcal{E}_0}{R} - \frac{K}{R}$

$i_0 R = \mathcal{E}_0 - K \Rightarrow K = \mathcal{E}_0 - i_0 R$

$i = \frac{\mathcal{E}_0}{R} - \left(\frac{\mathcal{E}_0 - i_0 R}{R}\right)e^{-\frac{t}{\tau_L}}$

$= \frac{\mathcal{E}_0}{R}\left(1 - e^{-\frac{t}{\tau_L}}\right) + i_0 e^{-\frac{t}{\tau_L}}$

$i = 3\left(1 - e^{-\frac{5}{1{,}5}}\right) + 0{,}103\, e^{-\frac{5}{1{,}5}}$

$i = 2{,}896$ A

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9. ***Circuits coupled with mutual inductance.*** The two circuits shown in Fig. 29-23 interact through their mutual induc-

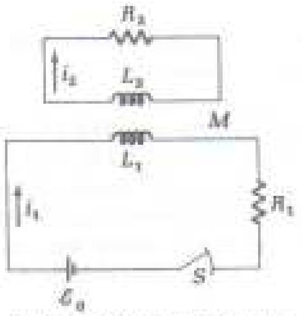


Figure 29-23. Problem 9.

tance M. *(a)* Show that the loop rule applied to each circuit gives

$$L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} + R_1 i_1 = \mathcal{E}_0$$

$$L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} + R_2 i_2 = 0$$

(b) If L_1, L_2, and M are all comparable with $R_2 \gg R_1$, then di_2/dt may be neglected in comparison with di_1/dt. Solve the equations in this approximation. Use the initial conditions $i_1(0) = i_2(0) = 0$. *(c)* Construct graphs of $i_1(t)$ and $i_2(t)$ versus t for $0 \leq t \leq 1.0$ s. Use the values $L_1 = 1.2$ H, $R_1 = 6.0\ \Omega$, $M = 0.80$ H, $L_2 = 1.4$ H, $R_2 = 600\ \Omega$, $\mathcal{E}_0 = 48$ V. *(d)* Compare the values of the maximum potential difference across R_1 and across R_2.

a) Induced emf's in both loops

$\mathcal{E}_{L_1} = L_1 \dfrac{di_1}{dt}$ self-induced emf in lower loop

$\mathcal{E}_{M2} = M \dfrac{di_1}{dt}$ mutually-induced emf in upper loop

$\mathcal{E}_{L_2} = L_2 \dfrac{di_2}{dt}$ in the upper loop

$\mathcal{E}_{M_1} = M \dfrac{di_2}{dt}$ in the lower loop

$\mathcal{E}_0 - \mathcal{E}_{L_1} - \mathcal{E}_{M_1} - i_1 R_1 = 0$ lower loop

$\mathcal{E}_0 - L_1 \dfrac{di_1}{dt} - M\dfrac{di_2}{dt} - i_1 R_1 = 0$

$L_2 \dfrac{di_2}{dt} + M \dfrac{di_1}{dt} + i_2 R_2 = 0$ upper loop

b) if $\left|\dfrac{di_2}{dt}\right| < \left|\dfrac{di_1}{dt}\right|$

then $L_1 \dfrac{di_1}{dt} + i_1 R_1 = \mathcal{E}_0$ lower loop

$M\dfrac{di_1}{dt} + i_2 R_1 = 0$ upper loop

$$i_1 \cong \frac{\mathcal{E}_0}{R_1}\left(1 - e^{-\frac{R_1 t}{L_1}}\right)$$

$$i_2 \cong -\frac{M}{R_1}\frac{di_1}{dt} = -\frac{M\mathcal{E}_0}{R_1 L_1} e^{-\frac{R_1 t}{L_1}}$$

The latter equation does not satisfy $i_2(0)=0$ so that approximation cannot be valid for short times.

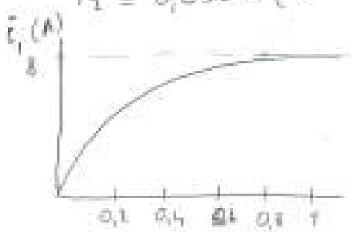
$$L_2 \frac{di_2}{dt} + i_2 R_2 \cong -M\frac{di_1}{dt} \cong -\frac{M\mathcal{E}_0}{L_1} e^{-\frac{R_1 t}{L_1}}$$

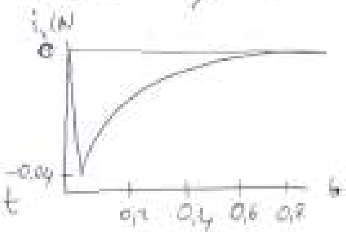
where the right-hand side can be regarded as a constant

$$i_2 \cong \frac{M\mathcal{E}_0}{R_2 L_1}\left(e^{-\frac{R_2 t}{L_2}} - e^{-\frac{R_1 t}{L_1}}\right)$$

c) $i_1 = 8\,\text{A}\,(1 - e^{-5t})$

$i_2 = 0{,}053\,\text{A}\,(e^{-429t} - e^{-5t})$





d) $V_1 = i_1 R_1$ $V_{1max} = i_{1max} R_1$

$V_{max} = 8\,\text{A} \cdot 6\,\Omega = 48$ V

$i_2 = 0{,}053\,(e^{-430t} - e^{-5t})$

$\dfrac{di_2}{dt} = 0 \Rightarrow -430\,e^{-430t} + 5e^{-5t} = 0$

$\dfrac{430}{5} = \dfrac{e^{-5t}}{e^{-430t}} = e^{425t}$

$\ln\left(\dfrac{430}{5}\right) = 425t \Rightarrow t = 0{,}01$ s

$i_2 = 0{,}053\,(0{,}011 - 0{,}949)$

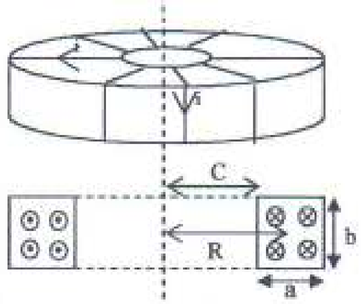
$= -0{,}05$ A

$|V_2|_{max} = |i_{2max}| \cdot R_2 = 29{,}8$ V

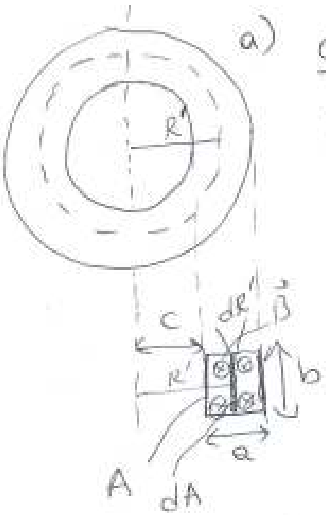
Question 1 : For a N turn toroid shown in the figure

a) evaluate the magnetic flux for the rectangular cross section of area ab

b) determine the self-inductance of the toroid



1)



a) $\oint \vec{B}\, d\vec{\ell} = \mu_0 N i$

$B 2\pi R' = \mu_0 N i$

$B = \dfrac{\mu_0 N i}{2\pi R'}$

$\int \vec{B} d\vec{A} = \Phi_B$

$\Phi_B = \int \dfrac{\mu_0 N i}{2\pi R'} dA$

$dA = b\, dR'$

$\Phi_B = \int_c^{c+a} \dfrac{\mu_0 N i\, b\, dR'}{2\pi R'}$

$\Phi_B = \dfrac{\mu_0 N i b}{2\pi} \ln \dfrac{(c+a)}{c}$

b) $L i = N \Phi_B$

$L i = \dfrac{N^2 \mu_0 i b \ln\left(\frac{c+a}{c}\right)}{2\pi}$

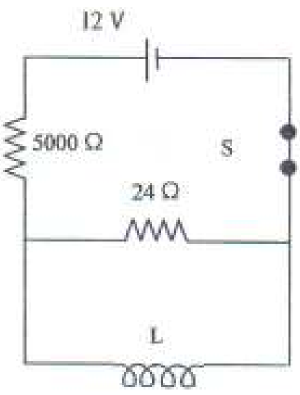
$L = \dfrac{\mu_0 N^2 b \ln\left(\frac{c+a}{c}\right)}{2\pi}$

Question 3 : The switch in the circuit has been closed for a long time.

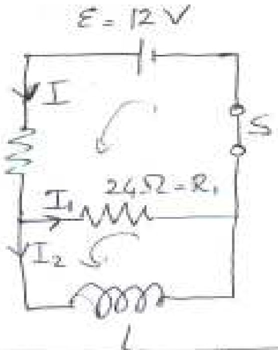
a) What is the current in each leg of the circuit

b) When the switch is opened, the current in the inductor drops by a factor of 2 in 8 μs. What is the value of the inductance?

c) What is the current passing in each leg at 12 μs.



3) a) After a long time the currents will be constant.



$I_2 = \frac{12V}{5000\Omega} = 2.4mA \quad t \to \infty$

$I_1 = 0$

---

one can get these results by applying Kirchhoff's loop rule:

$$\boxed{\begin{aligned} &12V - IR - I_1R_1 = 0 \\ &I_1R_1 - L\frac{dI_2}{dt} = 0 \\ &I = I_1 + I_2 \end{aligned}}$$

$I_1 = \frac{L}{R_1}\frac{dI_2}{dt} \Longleftarrow$

$I_2 = \frac{\varepsilon}{R}\left(1 - e^{-\frac{RR_1t}{(R+R_1)L}}\right)$

$t \to \infty \quad I_2 = \frac{\varepsilon}{R} \quad I_1 = 0$

---

b) When the switch is opened there is no current through the battery

$I_1 = I_2 = I_L \qquad I_L = I_0 e^{-\frac{R_1}{L}t}$

$I_0 = 2.4\,mA \qquad \frac{\cancel{I_0}}{2} = \cancel{I_0} e^{-\frac{R_1}{L}t_1}$

$t_1 = 8\,\mu s$

$\frac{1}{2} = e^{-\frac{R_1}{L}t_1}$

$\ln\left(\frac{1}{2}\right) = -\frac{R_1}{L}t_1$

$L = \frac{-R_1t_1}{\ln 0,5} = \frac{-(24\Omega)(8\cdot 10^{-6}s)}{\ln(0.5)}$

$L = 277\,\mu H$

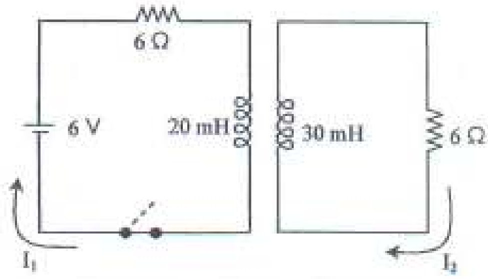
c) $I = 0 \quad I_L = I_0 e^{-\frac{R_1}{L}t} = I_1$

$I_L = I_0 e^{-\frac{R_1}{L}t_2} \qquad t_2 = 12\,\mu s$

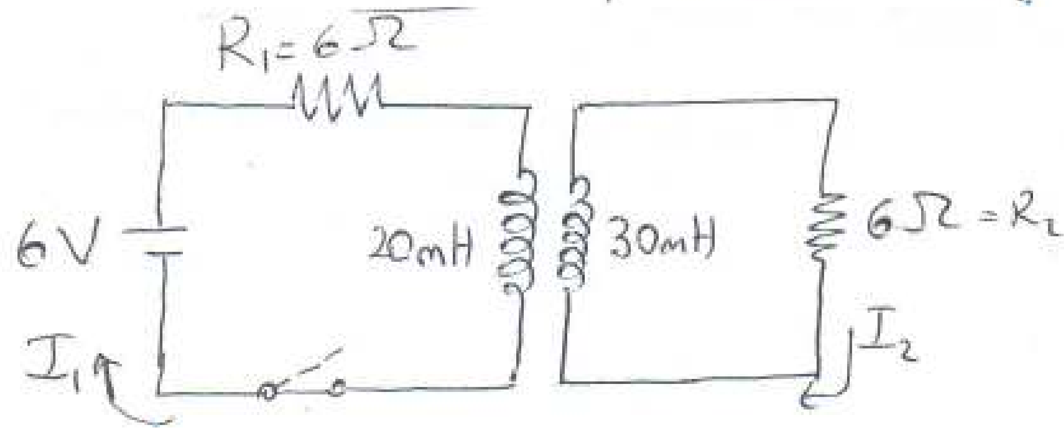
$I_L = (2.4mA)\, e^{-\frac{(24\Omega)\cdot 12\cdot 10^{-6}s}{277\cdot 10^{-6}H}}$

$I_L = 0.85\,mA$

a) **Question 4** : The two identical coils in the circuit are placed close to each other and their mutual inductance is 0.7 mH. Suppose that the switch has been closed for a long time and is then opened at t=0. Calculate the current in the circuit at t = 18 ms.



4)



Before the switch is opened

$I_1 = \dfrac{\mathcal{E}}{R_1} = \dfrac{6\,V}{6\,\Omega} = 1\,A \qquad I_2 = 0$

the flux through L_2 is

$\Phi_{B21} = M I_1 = (0.7\,mH)\cdot(1\,A) = 0.7\,mWb$

When the switch is opened the induced emf in L_2 wants to maintain this flux at $t=0$ the initial current in L_2 is

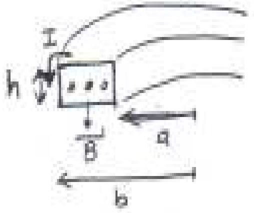
$I_{20} = \dfrac{\Phi_{B21}}{L_2} = \dfrac{(0.7\,mWb)}{30\,mH} = 0{,}023\,A = 23\,mA$

This current reduces exponentially

$I_2 = I_{20}\, e^{-\frac{R_2 t}{L_2}} = (0{,}023\,A)\, e^{-\frac{(6\Omega)(18ms)}{30\,mH}}$

$= 0{,}63\,mA$

problem) Consider the toroidal solenoid shown in the figure



Find the total energy stored in the toroidal solenoid.

Solution: First method:

From Ampere's Law within the toroid

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} \Rightarrow B(2\pi r) = \mu_0 (IN)$$

where N is total number of turns.

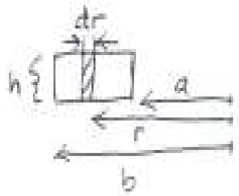
$$\Rightarrow \boxed{B = \frac{\mu_0 I N}{2\pi r}}$$

Energy density u_B is given by

$$u_B = \frac{B^2}{2\mu_0} = \frac{\mu_0 I^2 N^2}{8\pi^2 r^2}$$

Total energy in the toroid is

$U_B = \int u_B \, dV$ where dV is the volume element in the toroid. Taking



we have $dV = (2\pi r\, dr)h$

Hence

$$U_B = \int u_B \, 2\pi r\, dr\, h = \frac{\mu_0 I^2 N^2 h\, 2\pi}{8\pi^2}\int_a^b \frac{dr}{r}$$

$$\boxed{U_B = \frac{\mu_0 I^2 N^2 h}{4\pi}\ln\frac{b}{a}}$$

second method:

from Ampere's law: $B = \frac{\mu_0 I N}{2\pi r}$

Flux in the toroid is

$$\Phi_B = \int B\, ds = \int B(h\, dr) = \frac{\mu_0 I N h}{2\pi}\int_a^b \frac{dr}{r}$$

$$\Phi_B = \frac{\mu_0 I N h}{2\pi}\ln\frac{b}{a}$$

Self inductance L is given by

$$L = \frac{N\Phi_B}{I} = \frac{\mu_0 N^2 h}{2\pi}\ln\frac{b}{a}$$

Total energy in the toroid is

$$U_B = \frac{1}{2}LI^2 = \frac{\mu_0 I^2 N^2 h}{4\pi}\ln\frac{b}{a}$$

40. (II) (a) What is the magnetic field energy density inside a straight wire of radius a that carries current I uniformly over its area? (b) What is the total magnetic field energy per unit length inside the wire?

40. (a) From the cylindrical symmetry, we know that the magnetic field is circular. We apply Ampere's law to a circular path to find the magnetic field inside the wire:

$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 I_{\text{enclosed}}$;

$B2\pi r = \mu_0 (I/\pi a^2)\pi r^2$, which gives $B = \mu_0 I r/2\pi a^2$.

The energy density of the magnetic field is

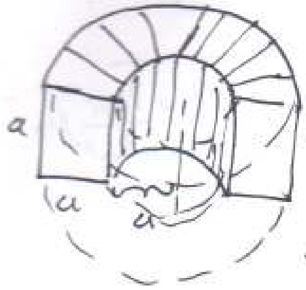
$u_B = \frac{1}{2}B^2/\mu_0 = \boxed{\mu_0 I^2 r^2/8\pi^2 a^4.}$

(b) Because the energy density is not constant, we integrate over the volume. For a differential element, we choose a cylindrical shell centered on the wire, with radius $r < a$, thickness dr, and length L. The energy per unit length is

$$\frac{U_B}{L} = \frac{1}{L}\int_0^a \frac{\mu_0 I^2 r^2}{8\pi^2 a^4} L 2\pi r\, dr = \frac{\mu_0 I^2}{4\pi a^4}\int_0^a r^3\, dr = \boxed{\frac{\mu_0 I^2}{16\pi}}$$

Question: A toroid of inner radius 'a', outer '2a height with N turns of wire carriyg is corent I.

Find the B field., induction coefficient, energy and energy density



With an Ampere loop radius $a < r < 2a$ the current encircled is NI

$$\therefore \oint \vec{B}\cdot d\vec{l} = \mu_0 NI \qquad \Big\} \boxed{B = \frac{\mu_0 NI}{2\pi r}}$$

$$= 2\pi r B$$

$$\Phi_1 = \iint \vec{B}\cdot d\vec{A} = \int_a^{2a}\int_0^a \frac{\mu_0 NI}{2\pi r}\,dz\,dr = \frac{\mu_0 NI}{2\pi} a \int_a^{2a} \frac{dr}{r}$$

$$= \frac{\mu_0 NI a}{2\pi} \ln 2$$

with N loops $\quad \Phi_n = \left(\frac{\mu_0 N^2 a \ln 2}{2\pi}\right) I$.

$$L = \Phi_n / I = \boxed{\frac{\mu_0 N^2 a \ln 2}{2\pi} = L}$$

$$U = \frac{1}{2} L I^2 = \boxed{\frac{\mu_0 N^2 a \ln 2}{4\pi} I^2 = U}$$

$$u = \frac{B^2}{2\mu_0} = \frac{\mu_0^2 N^2 I^2}{2\cdot\cancel{\mu_0}\, 4\pi^2 r^2} = \boxed{\frac{\mu_0 N^2 I^2}{8\pi^2 r^2} = u}$$

Question:

4) A solenoid of area A_{in} length l_{in} with N_{in} turns is inside a larger solenoid (A_{out}, l_{out}, N_{out}). They are coaxial. Same current I is going through both.

Find $L_{equ.}$

Forgetting inner solenoid: $\quad l_{out} B = N_{out} I \mu_0 \Rightarrow B_{out} = \frac{N_{out}}{l_{out}} \mu_0 I$

$$\Phi = \mu_0 \frac{N_{out}}{l_{out}} I A_{out}$$

$$\Phi_N = \mu_0 \frac{N_{out}^2}{l_{out}} A_{out} I \Rightarrow L_{out} = \mu_0 \frac{N_{out}^2 A_{out}}{l_{out}}$$

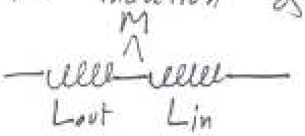
Similarly inner solenoid has $\quad L_{in} = \mu_0 \frac{N_{in}^2 A_{in}}{l_{in}}$

The flux of outer through inner solenoid is $\Phi = B_{out} A_{in}$

$$= \mu_0 \frac{N_{out}}{l_{out}} A_i \ -$$

$$M = \left[\frac{\mu_0 N_{out} N_{in} A_{in}}{l_{out}} \right]$$

The induction of the whole setup is



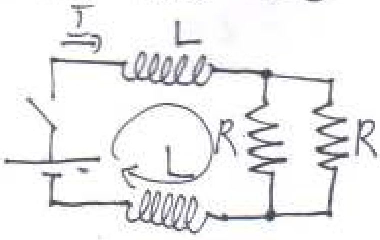
$$\mathcal{E} = -L_{out} \frac{dI}{dt} \pm M \frac{dI}{dt} \pm M \frac{dI}{dt} - L_{in} \frac{dI}{dt}$$

$$= -\underbrace{\left(L_{out} + L_{in} \mp 2M\right)}_{L_{eqv.}} \frac{dI}{dt}$$

$\pm$ Because fields can be $\uparrow\uparrow$ or $\uparrow\downarrow$.

Question:

At time $t=0$ switch is closed. Find $I(t)$



The resistances ∥ $R_{eq} = R/2$
The ind series $L_{eq} = 2L$

$$\mathcal{E} - L\frac{dI}{dt} - \frac{R}{2}I - L\frac{dI}{dt} = 0$$
$$\mathcal{E} = 2L\frac{dI}{dt} + \frac{R}{2}I$$

I must have 2 terms. 1) Proportional to its derivative (ie $e^{\alpha t}$)
2) Constant

Try $I = A + Be^{\alpha t}$, $\frac{dI}{dt} = B\alpha e^{\alpha t}$

$$\mathcal{E} = 2LB\alpha e^{\alpha t} + \frac{R}{2}A + \frac{RB}{2}e^{\alpha t}$$
$$\mathcal{E} - \frac{RA}{2} = Be^{\alpha t}\left(2L\alpha + \frac{R}{2}\right)$$

since one side is time independent & the other time dependent
oth must be zero

$\Rightarrow A = 2\mathcal{E}/R$

$e^{\alpha t}$ can't be 0, $B=0$ is trivial $\Rightarrow 2L\alpha + \frac{R}{2} = 0 \Rightarrow \alpha = -\frac{R}{4L}$

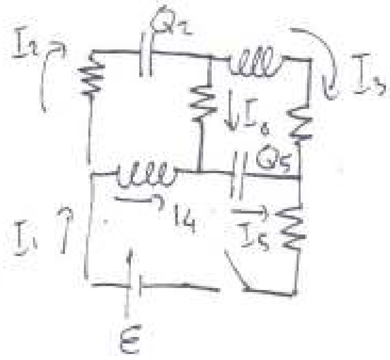
itial condition $I(0)=0$ determines B. $A+B=0 \Rightarrow B = -\frac{2\mathcal{E}}{R}$

$$\therefore \boxed{I = \frac{2\mathcal{E}}{R}\left(1 - e^{-Rt/4L}\right)}$$

Question:

a) Write circuit equations,

6) b) Find the currents at time $t=0$, & $t \to \infty$



All resistances R
All inductors L
All capacitors C
Battery $\mathcal{E}$

a) $\mathcal{E} - L\frac{dI_4}{dt} - \frac{1}{C}Q_5 - I_1 R = 0$

$-RI_2 - \frac{1}{C}Q_2 - RI_6 + L\frac{dI_4}{dt} = 0$

$RI_6 - L\frac{dI_3}{dt} - RI_3 + \frac{1}{C}Q_5 = 0$

$I_1 = I_2 + I_4$

$I_2 = I_3 + I_6$

$I_4 + I_6 = I_5$

$I_2 = dQ_2/dt$

$I_5 = dQ_5/dt$

b) at $t=0$ capacitors are empty behave like short circuits.
inductors " " open circuits.

$I_4 = I_3 = 0$, $I_1 = I_2 = I_6 = \mathcal{E}/3R$

c) as $t \to \infty$ capacitors are full behave like open circuits
inductors " " short " .

$I_2 = I_5 = 0$, $I_1 = I_4 = -I_6 = I_3 = \mathcal{E}/3R$

$V_2 = \mathcal{E}/3 \Rightarrow Q_2 = \mathcal{E}C/3$

$V_5 = 2\mathcal{E}/3 \Rightarrow Q_5 = 2\mathcal{E}C/3$