ITU Computer and Informatics Faculty
BLG 202E Numerical Methods in CE
2018 - 2019 Spring
Homework 4

Due 17.05.2019 23:00

1. Please submit your report through Ninova e-Learning System. Another way of submission will not be accepted.
2. No late submissions will be accepted.
3. In Case of Cheating and Plagiarism Strong **disciplinary action will be taken**.
4. For any questions about the Homework 4, contact Beyza EKEN (beyzaeken@itu.edu.tr).

Questions:

1. Joe had decided to buy stocks of a particularly promising Internet company. The price per share was $100, and Joe subsequently recorded the stock price at the end of each week. With the abscissae measured in days, the following data were acquired: (0, 100), (7, 98), (14, 101), (21, 50), (28, 51), (35, 50).

   In attempting to analyze what happened, it was desired to approximately evaluate the stock price a few days before the crash.

   a. Pass a linear interpolant through the points with abscissae 7 and 14. Then add to this data set the value at 0 and (separately) the value at 21 to obtain two quadratic interpolants. Evaluate all three interpolants at *x* = 12. Which one do you think is the most accurate? Explain.
   b. Plot the two quadratic interpolants above, together with the data (without a broken line passing through the data) over the interval [0, 21]. What are your observations?

2. Suppose we want to approximate the function e^x on the interval [0,1] by using polynomial interpolation with $x_0 = 0, x_1 = 1/2$ and $x_2 = 1$. Let $p_2(x)$ denote the interpolating polynomial.

   a. Find an upper bound for the error magnitude
   $$\max_{0 \le x \le 1} |e^x - p_2(x)|$$
   b. Find the interpolating polynomial using your favorite technique.
   c. Plot the function e^x and the interpolant you found, both on the same figure, using the commands `plot`.
   d. Plot the error magnitude $|e^x - p_2(x)|$ on the interval using logarithmic scale (the command `semilogy`) and verify by inspection that it is below the bound you found in part (a).

3. Derive a difference formula for the fourth derivative of f at x_0 using Taylor's expansions at $x_0 \pm h$ and $x_0 \pm 2h$. How many points will be used in total and what is the expected order of the resulting formula?

4. Let denote $x_{\pm 1} = x_0 \pm h$ and $f(x_i) = f_i$. It is known that the difference formula is

$$f_{pp_0} = (f_1 - 2f_0 + f_{-1})/\, h^2$$

provides a second order method for approximating the second derivative of f at x_0, and also that roundoff error increases like h^{-2}.

Write a MATLAB script using default floating point arithmetic to calculate and plot the actual total error for approximating $f''(1.2)$, with $f(x) = sinx$. Plot the error on a log-log scale for $h = 10^{-k}$, $k = 0{:}\,0.5{:}\,8$. Observe the roughly V shape of the plot and explain it. What is (approximately) the observed optimal h?