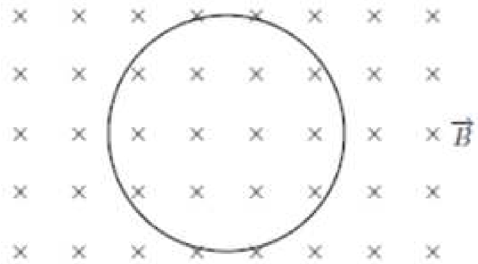
4) A circular loop of radius 2 cm is placed in a uniform magnetic field as seen in figure. The magnetic field is changed uniformly from 0.2 T to 0.8 T in a time interval of 1 s, beginning at $t = 0$.

a) Find the magnitude of magnetic field $t = 0$, $t = 0.5$ and $t = 1$ s.

b) What emf is induced in the loop at $t = 0.5$ s?

c) Find the direction of induced current in the loop. Explain clearly your answer.



a) $B = 0.2 + 0.6t$

$\Rightarrow \quad B(0) = 0.2\text{T},\ B(0.5) = 0.5\text{T},\ B(1) = 0.8\text{T}.$

b) $\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(BA) = -A\frac{dB}{dt}$

$= -\pi(2 \times 10^{-2} m)^2\, 0.6T/s = -7.54 \times 10^{-4}\text{V}$

c) The magnetic field is into the page and increasing.
By Lenz's law, induced current must decrease it.
So the direction of the induced current is counter clock-wise.

5) a) At what rate must the potential difference between the plates of a parallel-plate capacitor with a 2 μF capacitance be changed to produce a displacement current of 1.5 A?

b) Calculate the intensity of a plane traveling electromagnetic wave if B_m is 2.0×10^{-4} T.

c) 3G cellphones use electromagnetic waves of frequency 2100 MHz. What is the corresponding wavelength? Explain clearly your answer.

a) $i_d = \mathcal{E}_0 \frac{d\Phi_E}{dt} = \mathcal{E}_0 \frac{dEA}{dt} = \frac{\mathcal{E}_0 A}{d}\frac{dV}{dt} = C\frac{dV}{dt}$

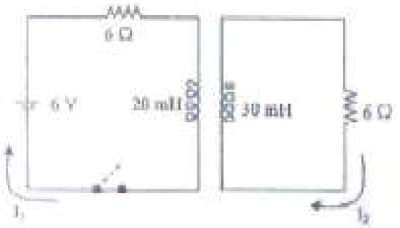
$$\frac{dV}{dt} = \frac{i_d}{C} = \frac{1.5\ A}{2 \times 10^{-6}\ F} = 7.5 \times 10^5\ V/s$$

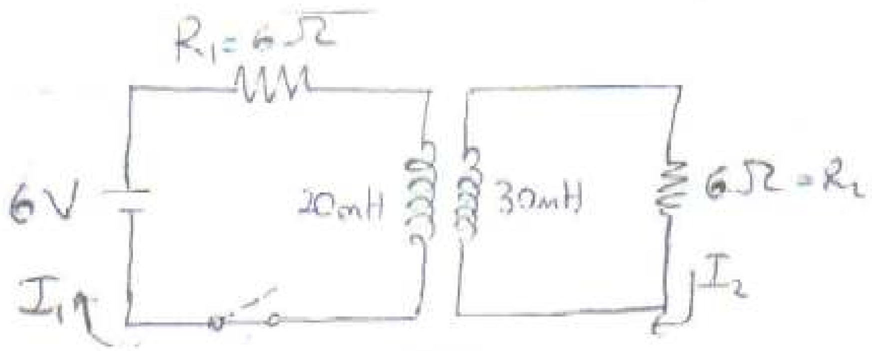
b) $I = \frac{1}{c\mu_0}E_{rms}^2 = \frac{1}{c\mu_0}\frac{E_m^2}{2} = \frac{c}{\mu_0}\frac{B_m^2}{2} = \frac{3 \times 10^8\, m/s}{4\pi \times 10^{-7}\ T.m/a}\frac{(2 \times 10^{-4}T)^2}{2}$

$= 4.777 \times 10^6\ W/m^2$

c) $\lambda = \frac{c}{f} = \frac{3 \times 10^8\ m/s}{2100 \times 10^6\ s^{-1}} = 0.14m$

8) The two identical coils in the circuit are placed close to each other and their mutual inductance is 0.7 mH. Suppose that the switch has been closed for a long time and is then opened at t=0. Calculate the current in the circuit at t = 18 ms.





Before the switch is opened $I_1 = \frac{E}{R_1} = \frac{6\,V}{6\,\Omega} = 1\,A$ $\quad I_2 = 0$

The flux through L_2 is

$$\Phi_{B21} = MI_1 = (0.7\,mH)(1\,A) = 0.7\,mWb$$

When the switch is opened the induced emf in L_2 wants to maintain this flux at $t=0$ the initial current in L_2 is

$L = \frac{N\Phi_B}{I}$ $\quad I_{20} = \frac{\Phi_{B21}}{L_2} = \frac{(0.7\,mWb)}{30\,mH} = 0{,}023\,A = 23\,mA$

This current reduces exponentially $\quad I_2 = I_{20}\, e^{-\frac{R_2 t}{L_2}} = (0.023\,A)\, e^{-\frac{(6\,\Omega)(18\,ms)}{30\,mH}}$

$= 0{,}63\,mA$

10) A red laser beam with a wavelength of 700 nm shines on a dark target which absorbs the beam's energy. The beam has a radius of 1.00 mm and power is absorbed in the target at a rate of 150 mW.

(a) What is the frequency of the laser light (in Hz = cycles per second)?

$$f = \frac{c}{\lambda} = \frac{3\times 10^{8}\text{m/s}}{700\times 10^{-9}\text{m}} = \mathbf{4.29\times 10^{14}\,Hz}.$$

(b) What is the amplitude of the electric field in the laser beam?

$$S_{\max} = \frac{E^2}{\mu_0 c} = 2S_{avg} = \frac{2P_{avg}}{A} = \frac{2(0.150\text{ W})}{\pi(0.001\text{m})^2} = \mathbf{9.55\times 10^4\,\frac{W}{m^2}}.$$

$$E = \sqrt{\mu_0 c S_{\max}} = \sqrt{(4\pi\times 10^{-7})(3\times 10^{8})(9.55\times 10^{4})}\frac{V}{m} = \mathbf{6000\,\frac{V}{m}}.$$

(c) What is the amplitude of the magnetic field in the laser beam?

$$B = \frac{E}{c} = \frac{6000}{3\times 10^{8}}\text{ T} = \mathbf{2.00\times 10^{-5}\,T}\ (= 0.20\text{ Gauss}).$$

12) An electromagnetic wave has a frequency of 100 MHz and is traveling in a vacuum. The magnetic field is given by $\vec{B}(z,t) = (10^{-8}\text{T})\cos(kz - \omega t)\hat{i}$

(a) Find the wavelength and direction of propogation of this wave.

direction of propogation is $\hat{k}$ (z direction)

$$\lambda = \frac{c}{f} = \frac{3\times 10^{8}\,m/s}{100\times 10^{6}\,Hz} = \frac{3\times 10^{8}\,m/s}{10^{8}\,Hz} \quad \boxed{\lambda = 3\,m}$$

(b) Find the direction and magnitude of the $\vec{E}$ field.

$E = cB$ (for maximum value of E and B) $E = (3\times 10^{8}\,m/s)(10^{-8}\,T) = 3\,V/m$

$\vec{E} \perp \vec{B} \perp \vec{S} \Rightarrow \vec{E} = 3\,V/m \cos(kz-\omega t)\,\hat{j}$ $\quad k = \frac{2\pi}{\lambda} = \frac{2\pi}{3}\,m^{-1}$, $\omega = 2\pi\times 100\,MHz$

(c) Find the intensity of the wave.

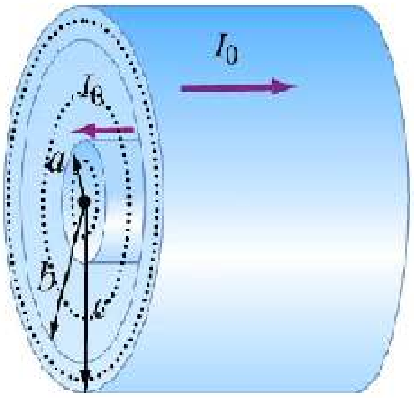
$$I = \langle S_{av}\rangle = \frac{\langle \vec{E}\times\vec{B}\rangle}{\mu_0} = \frac{1}{2}\frac{EB}{\mu_0} = \frac{1}{2}\frac{E^2}{\mu_0 c} = \frac{1}{2}\frac{E^2}{(377\Omega)}$$

$$I = \frac{1}{2}\frac{3^2}{(377)} = 1.2\times 10^{-2}\,\frac{W}{m^2}$$

(d) Find the associated radiation pressure.

$P_r = \frac{I}{c} = 3.7\times 10^{-11}$ Pa $\Leftarrow$ Pascals (N/m^2)

Problem 2 A coaxial cable consists of a solid inner conductor of radius a, surrounded by a concentric cylindrical tube of inner radius b and outer radius c. The conductors carry equal and opposite currents I_0 *distributed uniformly across their cross-sections*. Determine the magnetic field at a distance r from the axis for the following ranges of radii. On the figure below, draw the Amperian loop you use in each case.



We will walk counterclockwise around each of the 3 above loops.

(a) $r < a$

$$\oint \vec{\mathbf{B}} \cdot d\vec{s} = B2\pi r = \mu_0 I_0 \frac{r^2}{a^2}$$

$$\Rightarrow \quad \vec{\mathbf{B}} = \frac{\mu_0 I_0 r}{2\pi a^2} \text{ counterclockwise}$$

(b) $a < r < b$

$$\oint \vec{\mathbf{B}} \cdot d\vec{s} = B2\pi r = \mu_0 I_0$$

$$\Rightarrow \quad \vec{\mathbf{B}} = \frac{\mu_0 I_0}{2\pi r} \text{ counterclockwise}$$

(c) $b < r < c$

$$\oint \vec{\mathbf{B}} \cdot d\vec{s} = B2\pi r = \mu_0 I_0 \left(1 - \frac{r^2 - b^2}{c^2 - b^2}\right)$$

$$\Rightarrow \quad \vec{\mathbf{B}} = \frac{\mu_0 I_0}{2\pi r}\left(1 - \frac{r^2 - b^2}{c^2 - b^2}\right) \text{ counterclockwise}$$

a) $|d\vec{B}| = \frac{\mu_0 I}{4\pi} \frac{|d\vec{s} \times \hat{r}|}{r^2} = \frac{\mu_0 I}{4\pi} \frac{dl}{R^2}$ $\left(|d\vec{s} \times \hat{r}| = dl \quad r = R^2 \right)$

$d\vec{B}$ points out of the page at point P

b) $|\vec{B}|_{\text{right semi-circle}} = \frac{\mu_0 I}{4\pi R^2} \int_0^{\pi R} dl = \frac{\mu_0 I}{4\pi R^2} (\pi R) = \frac{\mu_0 I}{4R}$ out of page

c) Total $= \vec{B}_{\text{right semi-circle}} + \vec{B}_{\text{left semi-circle}}$ $\quad |\vec{B}| = \frac{\mu_0 I}{4R} + \frac{\mu_0 I}{4(R/2)} = \frac{3\mu_0 I}{4R}$

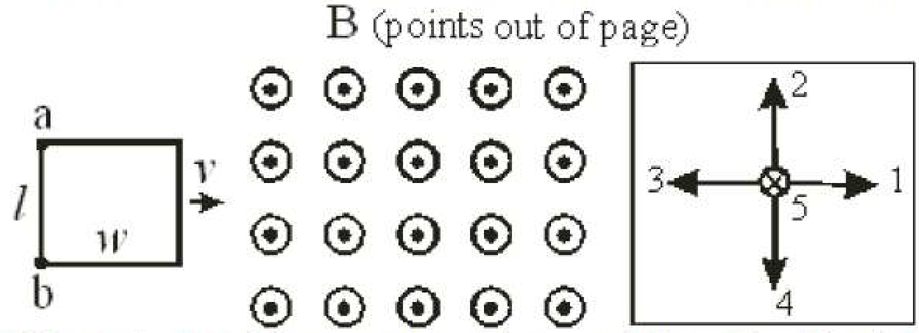
$\vec{B}$ out of page

d) $|\vec{\mu}| = I(\text{area}) = I\left(\frac{\pi R^2}{2} + \frac{\pi (R/2)^2}{2}\right) = \frac{5\pi R^2}{8} I$

current is counterclockwise, so

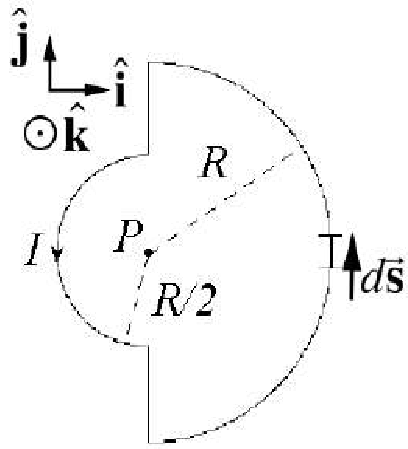
$\vec{\mu}$ direction is out of page

19. A rectangular coil moving at a constant speed v enters a region of uniform magnetic field from the left. While the coil is entering the field, the direction of the magnetic force is



A) 1
B) 2
C) 3
D) 4
E) 5

Problem 3; A current I flows around a continuous path that consists of portions of two concentric circles of radii R and $R/2$, respectively, and two straight radial segments. The point P is at the common center of the two circle segments.



a) Use the Biot-Savart Law to calculate $d\vec{\mathbf{B}}$ at P due only to that segment of the path $d\vec{s}$ shown in the sketch. Indicate on the sketch the unit vector $\hat{\mathbf{r}}$ you use and the direction of $d\vec{\mathbf{B}}$. Give the magnitude of $d\vec{\mathbf{B}}$ in terms of I, R, $|d\vec{s}| = dl$, and μ_0.

b) Using your expression in (a), find the magnetic field at P due to the right circle segment only. Give its magnitude and direction.

c) What is the total field $\vec{\mathbf{B}}$ at P? Give its magnitude and direction.

d) What is the magnetic dipole moment $\vec{\mu}$ of this current loop? Give its magnitude and direction.

a) $|d\vec{B}| = \frac{\mu_0 I}{4\pi} \frac{|d\vec{s} \times \hat{r}|}{r^2} = \frac{\mu_0 I}{4\pi} \frac{dl}{R^2}$ $\left(\begin{matrix} |d\vec{s} \times \hat{r}| = dl \\ r = R^2 \end{matrix} \right)$

$d\vec{B}$ points out of the page at point P

b) $|\vec{B}|_{\text{right semi-circle}} = \frac{\mu_0 I}{4\pi R^2} \int_0^{\pi R} dl = \frac{\mu_0 I}{4\pi R^2} (\pi R) = \frac{\mu_0 I}{4R}$ out of page

c) Total $= \vec{B}_{\text{right semi-circle}} + \vec{B}_{\text{left semi-circle}}$ $\quad |\vec{B}| = \frac{\mu_0 I}{4R} + \frac{\mu_0 I}{4(R/2)} = \frac{3\mu_0 I}{4R}$

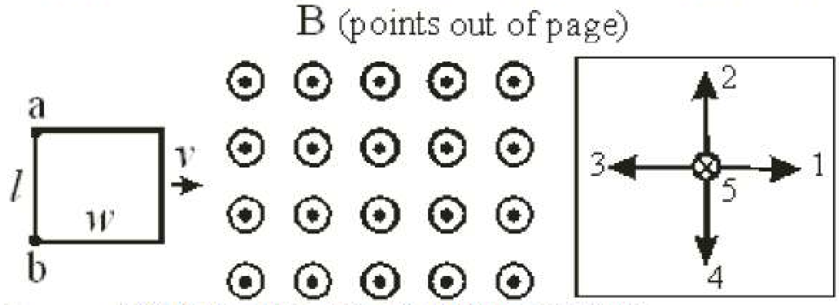
$\vec{B}$ out of page

d) $|\vec{\mu}| = I\,(\text{area}) = I\left(\frac{\pi R^2}{2} + \frac{\pi (R/2)^2}{2}\right) = \frac{5\pi R^2}{8} I$

current is counterclockwise, so

$\vec{\mu}$ direction is out of page

19. A rectangular coil moving at a constant speed v enters a region of uniform magnetic field from the left. While the coil is entering the field, the direction of the magnetic force is



A) 1
B) 2
C) 3
D) 4
E) 5

1. What do magnetic field lines depict?

a. The direction of the force exerted on a magnetic north pole.

b. The direction of the force exerted on a positive charge.

c. The direction of the force exerted on a magnetic south pole.

d. The direction of the force exerted on a negative charge.

7. A *"step-up" transformer* receives alternating current in its primary circuit and creates alternating current at a *higher voltage in its secondary* circuit. What other comparisons can you make between the step-up transformer's primary and secondary circuits?

a. The *secondary* circuit has *fewer windings* and *lower current* than the primary.

b. The *secondary* circuit has *fewer windings* and *higher current* than the primary.

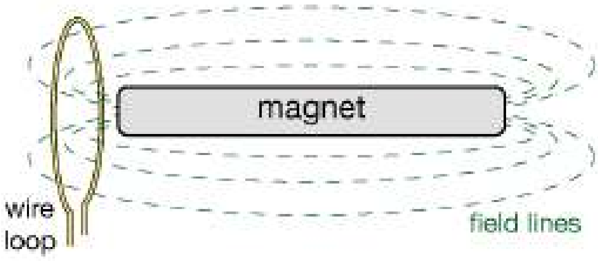
c. The *secondary* circuit has *more windings* and *lower current* than the primary.

d. The *secondary* circuit has *more windings* and *higher current* than the primary.

in a transformer, potential is proportional to the number of windings, and power is the same on both sides. In this case, the secondary is the high-voltage side, so it must have more windings than the primary. Since the power is the same on both sides and power is voltage times current, the side with the higher voltage (the secondary) has lower current.

Scenario 4. *Seismometer*

A sensor in a seismometer (a device that measures earthquake vibrations) consists of a conducting wire loop facing one pole of a permanent magnet. The wire loop is firmly attached to the ground, and the magnet is loosely suspended. When the ground shakes in an earthquake, it moves the loop toward or away from the pole of the magnet. The electric potential (voltage) around the wire loop is detected and analyzed to understand the earth's movement.



side view

end view

10. When is the *magnetic flux* through the wire loop the *greatest*?

a. The magnetic flux is greatest when the loop is *closest* to the magnet.

b. The magnetic flux is greatest when the loop is *moving*.

c. The magnetic flux is greatest when the loop is *farthest* from the magnet.

d. The magnetic flux through the loop is the *same* at all times.

Flux conceptually is the number of magnetic field lines passing through the loop. This will be greatest when the strongest field (near the pole) is at the loop.

11. When does the magnetic flux through the loop *change* the fastest?

a. The magnetic flux changes fastest when the loop is *closest* to the magnet.

b. The magnetic flux changes fastest when the loop is *moving*.

c. The magnetic flux changes fastest when the loop is *farthest* from the magnet.

d. The magnetic flux through the loop *does not change*.

If the field stays constant with time, the only way to change the flux through the loop is to move the loop.

12. When is the *induced voltage* around the conducting loop the greatest?

a. The voltage around the loop is greatest when the loop is *closest* to the magnet.

b. The voltage around the loop is greatest when the loop is *moving*.

c. The voltage around the loop is greatest when the loop is *farthest* from the magnet.

d. The voltage around the loop is the *same* at all times.

Faraday's law: Induced voltage around the loop equals rate of change of flux inside the loop; $V = \Delta\Phi/\Delta t$. (That's even in the song!)

13. When an earthquake vibration pushes the wire loop toward the pole of the magnet, a current is induced in the loop. This current, in turn, creates a magnetic field. How does this magnetic field act on the magnet?

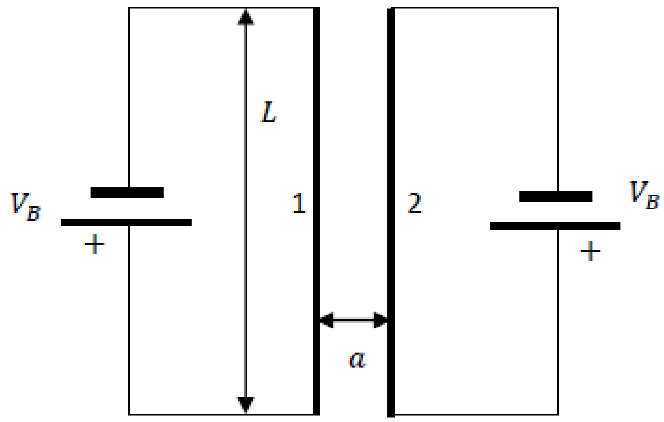
a. When the loop is pushed toward the magnet, the loop's induced magnetic field *pulls* the magnet *toward* the loop (the loop attracts the magnet).

b. When the loop is pushed toward the magnet, the loop's induced magnetic field *pushes* the magnet *away from* the loop (the loop repels the magnet).

c. When the loop is pushed toward the magnet, the loop's induced magnetic field pushes the magnet *sideways*.

Lenz's law: The induced field acts to oppose the magnetic flux change that created the field. (Ungrateful, but required by conservation of energy.)

Part 1. Two wires of length $L = 3.20$ m, labeled 1 and 2 in the figure, are connected to batteries as shown, each with emf $V_B = 9.00$ V. The resistance of the wire on the left is $R_1 = 3.05\Omega$ and the resistance R_2 of the wire on the right is unknown. The separation of the wires is $a = 5.25$ mm. And the force between the wires is $F = 4.68 \times 10^{-4}$N. Assume all wires except the two parallel ones are far apart, and all other resistance in the circuit is negligible.



(a) What is the direction of the magnetic field due to the current in the long wire on the left at the location of the long wire on the right?

At the location of wire 2, the magnetic field due to wire 1 is **into the page**.

The current flows upward through wire 1, so the field lines make circles around it that come out of the page on the left, and into the page on the right of wire 1.

(b) Is the force between the wires attractive or repulsive?

The force on wire 2 due to the magnetic field from wire 1 is

$$F_2 = i_2\vec{L} \times \vec{B}_1,$$

with $\vec{L}$ directed upward along the wire and $\vec{B}_1$ directed into the page, so that the force on wire 2 is to the left. This is an **attractive** force.

(c) If the current flowing in the circuits on the left and right are i_1 and i_2, respectively, what is the magnitude of the magnetic field due to wire 1 at the location of wire 2? Express this in terms of the appropriate symbols, and justify the result using Ampere's Law.

By Ampere's Law, applied to a circle of radius a about wire 1,

$$\oint \vec{B} \cdot d\vec{s} = 2\pi a\, B = \mu_0 i_{\text{enc}} = \mu_0 i_1.$$

Therefore, $B = \dfrac{\mu_0 i_1}{2\pi a}$.

(d) Find the unknown resistance R_2 numerically. Show your work and all equations used, and include correct units.

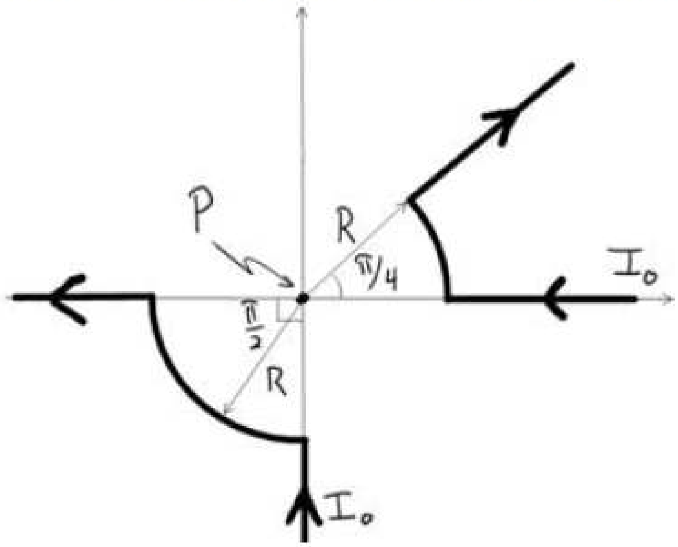
The force on wire 2 due to the magnetic field of wire 1 is $F_2 = i_2\vec{L} \times \vec{B}_1$, and $\vec{L}$ is perpendicular to $\vec{B}_1$, so

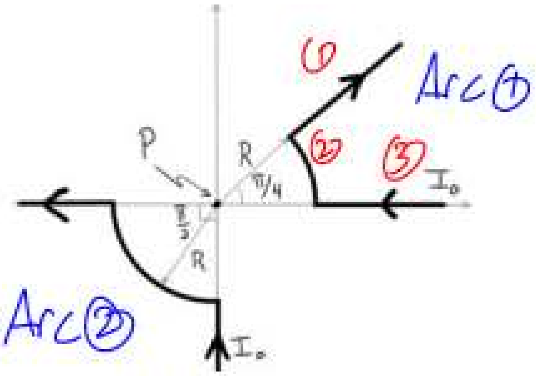
$$F = i_2 L B_1 = \frac{\mu_0 i_1 i_2 L}{2\pi a} = \frac{\mu_0 V^2 L}{2\pi a R_1 R_2},$$

Therefore,

$$R_2 = \frac{\mu_0 V^2 L}{2\pi a R_1 F} = \frac{(4\pi \times 10^{-7}\,\mathrm{N/A^2})(9.00\ \mathrm{V})^2(3.20\ \mathrm{m})}{2\pi(5.25 \times 10^{-3}\mathrm{m})(3.05\Omega)(4.68 \times 10^{-4}\mathrm{N})} = 6.92\Omega.$$

2. A current I0 is present in the two circularly arced wire segments of radius R depicted below as thick black lines. One arc subtends an angle of π/2, and the other subtends and angle of π/4. The lead wires point radially outward from point P, the location of the center of the two arced wire segments. Pay special attention to the direction of the current in the two arcs. Derive from the Biot-Savart law the magnitude and direction of the magnetic field at the point P.





$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{s} \times \hat{r}}{r^2}$$

Arc ①

Leg ① + ③ : $d\vec{s} \parallel \hat{r} \Rightarrow d\vec{s} \times \hat{r} = 0$

$$\therefore \int d\vec{B} = \int_{②} \frac{\mu_0}{4\pi} I \frac{d\vec{s} \times \hat{r}}{r^2}, \quad d\vec{s} \perp \hat{r},\ ds = R\,d\theta,\ r = R$$

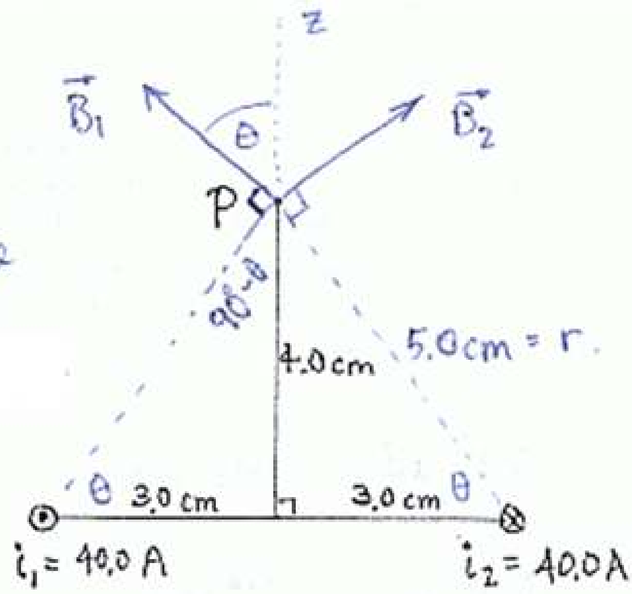
$$\Rightarrow \vec{B} = \frac{\mu_0}{4\pi} I \frac{1}{R} \int d\theta = \frac{\mu_0}{4\pi} \frac{I}{R} \frac{\pi}{4} \text{ out of page}$$

Arc 2: $\vec{B} = \frac{\mu_0}{4\pi} I \frac{1}{R}\left(\frac{\pi}{2}\right)$ into page

$$\vec{B}_{total} = \vec{B}_{①} + \vec{B}_{②} = \frac{\mu_0}{4\pi} \frac{I}{R}\left(\frac{\pi}{2} - \frac{\pi}{4}\right) = \frac{\mu_0}{4\pi} \frac{I}{R} \frac{\pi}{4} \text{ into page}$$

14. Two very long straight wires carry currents $i_1 = 40.0$ A and $i_2 = 40.0$ A in opposite directions perpendicular to the page as shown.

(a) (12 pts) Determine the magnitude and direction of the magnetic field produced at point P.



By symmetry, Net $\vec{B}$ will be up the page

$$B_1 = \frac{\mu_0 i_1}{2\pi r_1} = \frac{\left(4\pi \times 10^{-7} \frac{Tm}{A}\right)(40.0\,A)}{2\pi\,(0.05\,m)}$$

$$B_1 = B_2 = 0.00016\,T = 0.16\,mT$$

At P. $B_P = B_z = 2 B_1 \cos\theta$

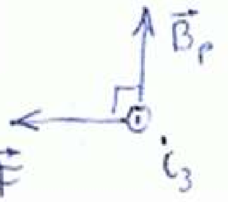
where $\cos\theta = \frac{adj}{hyp} = \frac{3}{5} = 0.6$

$$B_P = B_z = 2\,(0.16\,mT)(0.6) = \underline{0.192\,mT}$$

(b) If a wire carrying a current $i_3 = 20.0A$ out of the page is placed at P, what are the magnitude and direction of the magnetic force per unit length acting on it?

$\vec{F} = i_3 \vec{L}_3 \times \vec{B}_P \quad F = i_3 L_3 B_P \sin\phi$ where $\phi = 90°$ between $\vec{L}_3$ and $\vec{B}_1$

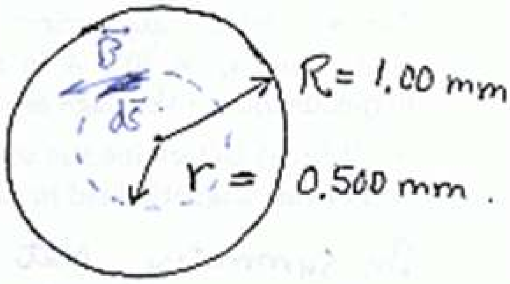
$$F/L_3 = i_3 B_P = (20.0\ \text{A})(0.192\ \text{mT}) = \underline{3.84\ \text{mN/m}}.$$



Force is <u>to the left.</u>

15. A long straight wire with a circular cross section of radius $R = 1.00$ mm has a <u>uniform current density</u>, and carries a current of 25.0 A.

Apply Ampere's Law and find the magnetic field magnitude within the wire at radius $r = 0.500$ mm from the axis of the wire.



$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc} \quad \vec{B} \parallel d\vec{s}$ around path.

$\oint B\, ds = \mu_0 i_{enc} \quad \vec{B}$ has constant magnitude around path

$B \oint ds = \mu_0 i_{enc}$

$$B \cdot 2\pi r = \mu_0 i_{enc} \Rightarrow \boxed{B = \frac{\mu_0 i_{enc}}{2\pi r}.}$$

$$i_{enc} = J \cdot \text{area} = \frac{i}{\pi R^2} \cdot \pi r^2 = i \frac{r^2}{R^2}$$

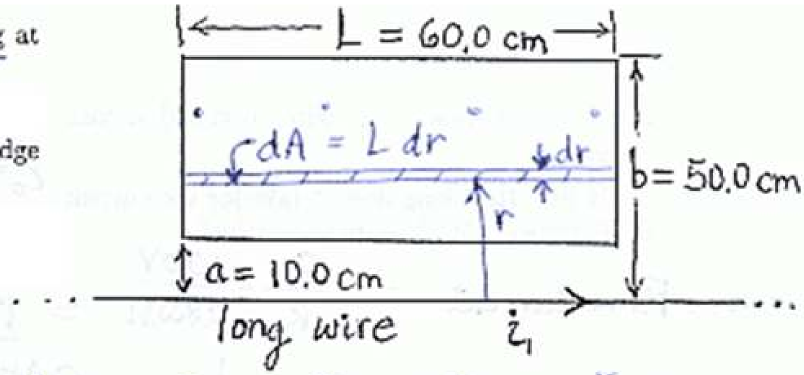
$$\boxed{B = \frac{\mu_0}{2\pi r} \cdot \frac{i r^2}{R^2} = \frac{\mu_0 i r}{2\pi R^2}.}$$

at $r = 0.500$ mm

$$B = \frac{(4\pi \times 10^{-7}\ \text{Tm/A})(25.0\text{A})(0.500 \times 10^{-3}\ \text{m})}{2\pi\ (1.00 \times 10^{-3}\ \text{m})^2} = \underline{0.0025\ \text{T}}$$

16. A long straight wire has a current increasing at the rate of 250 A/s.

(a) What emf is induced in a rectangular 40.0 cm × 60.0 cm wire loop with its near edge 10.0 cm from the wire, as shown?



It is a non-uniform B-field.

$$\Phi = \int \vec{B}\cdot d\vec{A} = \int B\, dA = \int_a^b \frac{\mu_0 i_1}{2\pi r}\cdot L\, dr$$

$$\Phi = \frac{\mu_0 i_1 L}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 i_1 L}{2\pi}\ln\left(\frac{b}{a}\right)$$ only i_1 is changing

$$\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{\mu_0 L}{2\pi}\frac{di_1}{dt}\ln\left(\frac{b}{a}\right)$$

$$= -\frac{(4\pi\times10^{-7}\,\mathrm{Tm/A})(0.60\,\mathrm{m})}{2\pi}\left(250\,\frac{\mathrm{A}}{\mathrm{s}}\right)\ln\left(\frac{0.5\,\mathrm{m}}{0.1\,\mathrm{m}}\right)$$

$$= -4.83\times10^{-5}\,\mathrm{V} = -48.3\,\mu\mathrm{V}.$$

(+) $\mathcal{E}$ corresponds to ↺ CCW, so negative is CW.

(b) The emf is in the clockwise / counterclockwise sense (circle one).

Induced current must generate $\vec{B}_i$ into page within the loop.

(c) What is the mutual inductance M between these conductors?

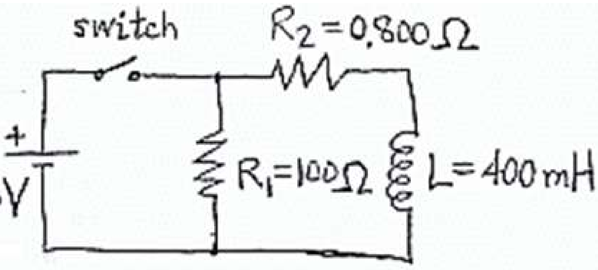
There is only 1 turn, so this is just the ratio of the flux through the loop to the current that produced it:

$$M = \frac{\Phi}{i_1} = \frac{\mu_0 L}{2\pi}\ln\left(\frac{b}{a}\right) = \left|\frac{\mathcal{E}}{di/dt}\right|$$

$$= \frac{48.3\,\mu\mathrm{V}}{250\,\mathrm{A/s}} = 1.93\times10^{-7}\,\mathrm{H} = \underline{0.193\,\mu\mathrm{H}}$$

17. In the circuit shown, the switch is closed at time $t = 0$.

(a) How long does it take for the current in R_2 to reach 50 % of its final value?



$i(\infty)$ = Final current $= \frac{\mathcal{E}_0}{R_2} = \frac{10.0\text{V}}{0.800\,\Omega}$
$= 12.5\,\text{A}$

Time constant $\tau_L = \frac{L}{R_2} = \frac{0.400\text{H}}{0.800\,\Omega} = 0.500\text{ s}$

$i = i(\infty)(1 - e^{-t/\tau_L})$ $\quad 0.5 \cdot i(\infty) = i(\infty)(1 - e^{-t/\tau_L})$ $\quad 0.5 = e^{-t/\tau_L}$

$t = -\tau_L \ln(0.5) = -(0.500\text{s}) \ln(\tfrac{1}{2}) = 0.347\text{ s}$

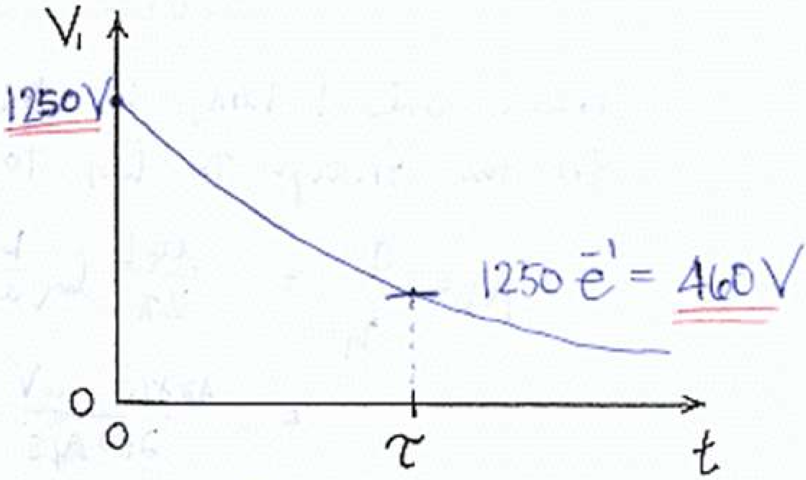
(b) After a long time (say, 1 hour), the switch is opened. Find the magnitude and direction of the current in R_1 just after the switch is opened.

L tries to maintain the current it had before S was opened,
which was $i(\infty) = 12.5\text{ A}$.
So 12.5 A will flow upward thru R_1,
instantaneously. Subsequently, it will decay away.

(c) Sketch on the provided axes the voltage across R_1 versus time t after S was opened. Be sure to include the values at the initial time and one time constant later.

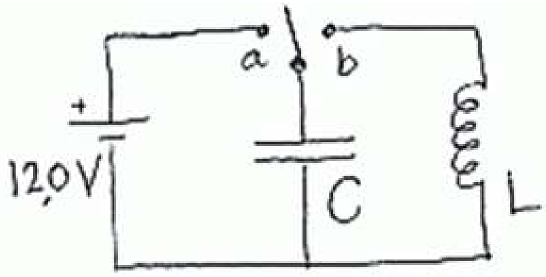
Current decays passing thru
$R = R_1 + R_2 = 100.8\,\Omega$.
$\tau = \frac{L}{R} = \frac{0.400\text{H}}{100.8\,\Omega} = 3.97\,\text{ms}$.

$i_1 = i_L = i(0)\, e^{-t/\tau}$
$V_1 = i_1 R_1 = (100\,\Omega)(12.5\text{A})\, e^{-t/\tau}$
$V_1 = (1250\text{V})\, e^{-t/\tau}$



$\tau = 3.97\,\text{ms}$

9. A 2.65 Henry inductor is to be connected with a capacitance C to form a low frequency oscillator with $f = 12.0$ Hz. The circuit is energized by first charging C with a 12.0 V battery (switch at a), then connecting L and C together at time $t = 0$ (switch at b).



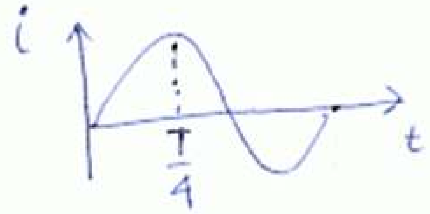
(a) What value for C will give the desired frequency?

$$2\pi f = \omega = \frac{1}{\sqrt{LC}} \qquad \omega^2 = \frac{1}{LC}$$

$$C = \frac{1}{\omega^2 L} = \frac{1}{(2\pi f)^2 L} = \frac{1}{(2\pi \times 12.0/s)^2 (2.65 H)} = 6.64 \times 10^{-5}\ F = \underline{66.4\ \mu F}$$

(b) After how much time will the inductor current first reach a maximum?

The current starts at zero, then rises to its first peak after a quarter period.



$$t = \frac{1}{4}T = \frac{1}{4}\cdot\frac{1}{f} = \frac{1}{4(12.0/s)}$$

$$t = \frac{1}{48}s = \underline{0.0208 s = 20.8\ ms}$$

(c) What will that maximum current be?

Conserve the energy — that initially stored in C eventually is converted to magnetic energy in L

tal ergy $U = \frac{1}{2}CV_0^2 = \frac{1}{2}LI^2$ (where $V_0 = 12.0\ V = \frac{q_0}{C}$)

$$U = \frac{1}{2}(66.4\ \mu F)(12 V)^2 = 4.78\ mJ$$

$$I = \sqrt{\frac{2U}{L}} = \sqrt{\frac{2(4.78\times10^{-3} J)}{2.65 H}} = 0.0601 A = 60.1\ mA$$

or simply $I = \sqrt{\frac{CV_0^2}{L}}$

13. Suppose your cell phone is isotropically emitting radio waves of frequency 880 MHz with a total power of 0.25 W.

(a) What is the wavelength of its electromagnetic radiation?

$$c = f\lambda \qquad \lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ m/s}{880\times10^{6}\ Hz} = 0.34\ m = 34\ cm$$

(b) What intensity does the radiation have at a distance of 5.00 km from the phone?

The power spreads over a sphere of radius 5.00 km.

$$I = \frac{P}{4\pi r^2} = \frac{0.25\ W}{4\pi\,(5.00\times10^{3}\ m)^2} = 7.96\times10^{-10}\ \frac{W}{m^2}$$

2 sig. figs. $\rightarrow$ $8.0\times10^{-10}\ W/m^2$

(c) How long did the signal take to travel 5.00 km?

$$x = ct \qquad t = \frac{x}{c} = \frac{5000\ m}{3.00\times10^{8}\ m/s} = 1.67\times10^{-5}\ s = 16.7\ \mu s$$

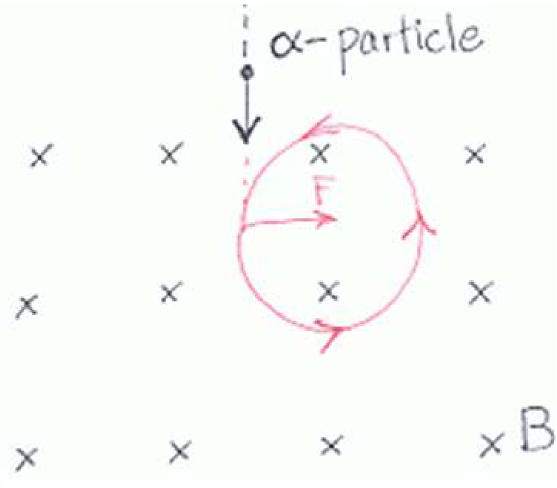
(d) What electric field amplitude do the waves have 5.00 km from the phone?

$$I = \frac{E_{rms}^2}{c\mu_0} = \frac{(E_m/\sqrt{2})^2}{c\mu_0} = \frac{E_m^2}{2c\mu_0}$$

Then $E_m = \sqrt{2c\mu_0 I} = \sqrt{2\left(3.00\times10^{8}\frac{m}{s}\right)\left(4\pi\times10^{-7}\frac{Tm}{A}\right)\left(7.96\times10^{-10}\frac{W}{m^2}\right)}$

$$= 7.75\times10^{-4}\ V/m = 0.775\ mV/m$$

P4. An α-particle ($q = +2e$, $m = 4.0026u = 6.646 \times 10^{-27}$ kg) is accelerated through a potential difference of 12,000 V, and then enters a region with a uniform magnetic field, $B = 0.50$ T, where it performs cyclotron motion.



(a) Sketch the direction of its orbital motion on the diagram.

see diagram. CCW motion.

(b) What is the frequency f of its circular motion?

$$F = qvB = m\frac{v^2}{r} \quad \text{then} \quad qB = m\frac{v}{r} = m\omega$$

$$\omega = 2\pi f = \frac{qB}{m} = \frac{2(1.6\times10^{-19}\text{C})(0.50\text{T})}{6.646\times10^{-27}\text{kg}}$$

$$= 2.4075\times10^{7} \text{ rad/s}$$

$$f = \frac{\omega}{2\pi} = \frac{2.4075\times10^{7}/\text{s}}{2\pi} = 3.83\times10^{6}\text{ Hz} = 3.83 \text{ MHz}$$

(c) Determine its velocity.

Work done by $\vec{E}$-field is converted to kinetic energy.

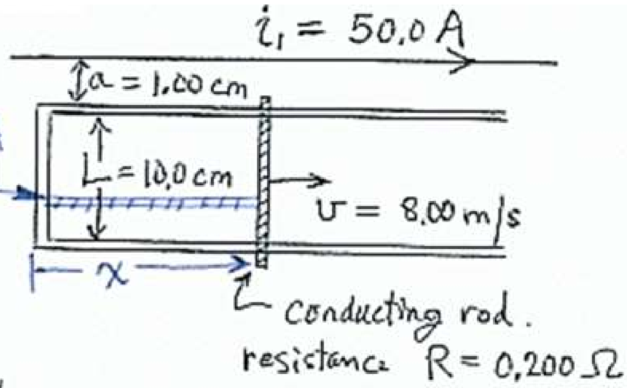
$$qV = \frac{1}{2}mv^2 = K \qquad \text{use } q = 2e.$$

$$v = \sqrt{\frac{2qV}{m}} = \sqrt{\frac{2(2)(1.6\times10^{-19}\text{C})(12000\text{V})}{6.646\times10^{-27}\text{kg}}} = 1.075\times10^{6} \text{ m/s}$$

(d) Determine the radius of its orbit.

$$\omega = \frac{v}{r} \quad \text{so} \qquad r = \frac{v}{\omega} = \frac{1.075\times10^{6}\text{ m/s}}{2.4075\times10^{7}/\text{s}} = 0.04465 \text{ m} = 4.46 \text{ cm}$$

A rod of length $L = 10.0$ cm moves at constant speed $v = 8.00$ m/s on conducting rails of negligible resistance. It moves in the nonuniform magnetic field of a nearby wire as shown.



a) Calculate the emf induced in the circuit.

$$\Phi = \int \vec{B}\cdot d\vec{A} = \int_a^{a+L} \left(\frac{\mu_0 i_1}{2\pi r}\right)\cdot(x\,dr) = \frac{\mu_0 i_1 x}{2\pi} \ln\left(\frac{a+L}{a}\right)$$

For constant speed, $x = vt$, or $\frac{dx}{dt} = v$.

$$\mathcal{E} = -\frac{d\Phi}{dt} = \frac{-\mu_0 i_1}{2\pi}\frac{dx}{dt}\ln\left(\frac{a+L}{a}\right) = \boxed{-\frac{\mu_0 i v}{2\pi}\ln\left(\frac{a+L}{a}\right)}$$

$$\mathcal{E} = -\frac{(4\pi\times10^{-7}\,\mathrm{Tm/A})(50\,\mathrm{A})(8.00\,\mathrm{m/s})}{2\pi}\ln\left(\frac{1+10}{1}\right) = 0.000192\ \mathrm{V}$$
$$= \underline{0.192\ \mathrm{mV}}$$

b) Find the current in the rod, and the force needed to move it at constant speed.

$$i = \mathcal{E}/R = 0.192\,\mathrm{mV}/0.200\,\Omega = \underline{0.959\ \mathrm{mA}}$$

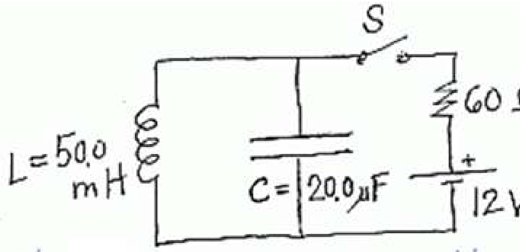
$$F = F_B = \int_a^{a+L} iB\,dr = i\int_a^{a+L}\frac{\mu_0 i_1}{2\pi r}\,dr = \frac{\mu_0 i i_1}{2\pi}\ln\left(\frac{a+L}{a}\right) = \frac{(4\pi\times10^{-7}\,\frac{\mathrm{Tm}}{\mathrm{A}})(0.959\,\mathrm{mA})(50\,\mathrm{A})}{2\pi}\ln(11)$$
$$= \underline{2.30\times10^{-8}\ \mathrm{N}}$$

c) Find the mechanical power applied to the rod.

$$P_m = F\cdot v = (2.30\times10^{-8}\,\mathrm{N})(8\,\mathrm{m/s}) = 1.84\times10^{-7}\,\mathrm{W} = 0.184\,\mu\mathrm{W}.$$

Compare $P_i = i^2 R = (0.959\,\mathrm{mA})^2(0.2\,\Omega) = \underline{0.184\,\mu\mathrm{W}}.$

1. The switch is closed for a long time, to energize L and C, and then re-opened.



a) At what frequency f will the circuit oscillate?

$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.05\,H)(20\times10^{-6}F)}} = 1000$ rad/s $\quad f = \frac{\omega}{2\pi} = \frac{1000 \text{ rad/s}}{2\pi \text{ rad}} = 159$ Hz.

b) When the switch is re-opened, the initial capacitor charge is:

Since the inductor acts like a short across C, the capacitor voltage is zero.

the initial current from L into top electrode of C must be:

The inductor acts like a short ckt. So the current is

$i_0 = \frac{12V}{60\Omega} = 0.20$ A But really

$i_0 = -0.20$ A (flowing out of top C-plate)

c) The total energy in the electrical oscillations is:

$U = \frac{1}{2}Li_0^2 + \frac{1}{2}\frac{q_0^2}{C} = \frac{1}{2}(0.050H)(0.20A)^2 = 0.0010$ J $= 1.0$ mJ.

d) Find the peak voltage that will appear across C during the oscillations.

Conserve energy. $U = \frac{1}{2}\frac{Q^2}{C}$ $\quad Q = \max. q$.

$Q = \sqrt{2CU} = \sqrt{2(20\times10^{-6}F)(0.001J)} = 0.00020$ C

Voltage is $V = Q/C = 10.0$ Volts.

10. A charged capacitor is connected across an inductor to form an LC circuit. When the charge on the capacitor is 0.25 C the current is 4 A. If the maximum current is 5 A, what is the period of LC oscillations in seconds?

In an LC circuit, the total energy must remain constant: $U = \frac{1}{2}Li_1^2 + \frac{q_1^2}{2C}$

The subscript "1" denotes the values for the current and charge at the first time. At a later time, all the energy is in the inductor (when there is maximum current), so:

$$U = \frac{1}{2}Li_{\max}^2$$

Equating these energies gives us:

$$U = \frac{1}{2}Li_1^2 + \frac{q_1^2}{2C} = \frac{1}{2}Li_{\max}^2 \Rightarrow i_{\max}^2 - i_1^2 = \frac{q_1^2}{LC} \Rightarrow LC = \frac{q_1^2}{i_{\max}^2 - i_1^2} = \frac{1}{\omega^2} = \left(\frac{T}{2\pi}\right)^2$$

$$\Rightarrow T = 2\pi\sqrt{\frac{q_1^2}{i_{\max}^2 - i_1^2}} = 0.52 \text{ s}$$