QUESTION – 1

$$f_1(7) = c_0 + c_1 7 = 98$$

$$f_1(14) = c_0 + c_1 14 = 101$$

$$\begin{bmatrix} 1 & 7 \\ 1 & 14 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} 98 \\ 101 \end{bmatrix} \rightarrow \begin{bmatrix} c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} 1 & 7 \\ 1 & 14 \end{bmatrix}^{-1} \begin{bmatrix} 98 \\ 101 \end{bmatrix} = \begin{bmatrix} 95 \\ 0.4286 \end{bmatrix}$$

$$f_1(x) = 95 + 0.4286x$$

$$f_1(12) = 95 + 0.4286 * 12 = 100.1432$$

$$f_2(0) = c_0 = 100$$

$$f_2(7) = c_0 + c_1 7 + c_2 7^2 = 98$$

$$f_2(14) = c_0 + c_1 14 + c_2 14^2 = 101$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 7 & 7^2 \\ 1 & 14 & 14^2 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 100 \\ 98 \\ 101 \end{bmatrix} \rightarrow \begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 7 & 7^2 \\ 1 & 14 & 14^2 \end{bmatrix}^{-1} \begin{bmatrix} 100 \\ 98 \\ 101 \end{bmatrix} = \begin{bmatrix} 100 \\ -0.6429 \\ 0.0510 \end{bmatrix}$$

$$f_2(x) = 100 - 0.6429x + 0.0510x^2$$

$$f_2(12) = 100 - 0.6429 \times 12 + 0.0510 \times 12^2 \ = \ 99.6292$$

$$f_3(21) = c_0 \ + \ \text{c\_1 21} \ + \ \text{c\_2 \^{}2} = 50$$

$$f_3(7) = c_0 + c_1 7 + c_2 7^2 = 98$$

$$f_{3(14)} = c_0 + c_1 14 + c_2 14^2 = 101$$

$$\begin{bmatrix} 1 & 21 & 21^2 \\ 1 & 7 & 7^2 \\ 1 & 14 & 14^2 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 50 \\ 98 \\ 101 \end{bmatrix} \rightarrow \begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 1 & 21 & 21^2 \\ 1 & 7 & 7^2 \\ 1 & 14 & 14^2 \end{bmatrix}^{-1} \begin{bmatrix} 50 \\ 98 \\ 101 \end{bmatrix} = \begin{bmatrix} 41 \\ 12 \\ -0.5510 \end{bmatrix}$$

$$f_3(x) = 41 + 12x - 0.5510x^2$$

$$f_3(12) = 41 + 12 \times 12 - 0.5510 \times 12^2 \ = \ 105.656$$

I think the f_3 is the best choice, since it also results better for other data points which means it is much closer to giving the real at point x =12 than the others.

QUESTION – 3

$$f(x_0 \pm h) = f(x_0) \pm hf'(x_0) + \frac{h^2 f''(x_0)}{2} \pm \frac{h^3 f'''(x_0)}{6} + \frac{h^4 f^{(4)}(x_0)}{24} \pm \frac{h^5 f^{(5)}(x_0)}{120} + \frac{h^6 f^{(6)}(\xi)}{720}$$

$$\frac{h^4 f^{(4)}(x_0)}{24} = f(x_0 \pm h) - f(x_0) \mp hf'(x_0) - \frac{h^2 f''(x_0)}{2} \mp \frac{h^3 f'''(x_0)}{6} \mp \frac{h^5 f^{(5)}(x_0)}{120} - \frac{h^6 f^{(6)}(\xi)}{720}$$

① forward substition

$$f^{(4)}(x_0) = \frac{24 f(x_0+h)}{h^4} - \frac{24 f(x_0)}{h^4} - \cancel{\frac{24 f'(x_0)}{h^3}} - \frac{12 f''(x_0)}{h^2} - \cancel{\frac{4 f'''(x_0)}{h}} - \cancel{\frac{h f^{(5)}(x_0)}{5}} - \frac{h^2 f^{(6)}(\xi)}{30}$$

② Backward subsition

$$f^{(4)}(x_0) = \frac{24 f(x_0-h)}{h^4} - \frac{24 f(x_0)}{h^4} + \cancel{\frac{24 f'(x_0)}{h^3}} - \frac{12 f''(x_0)}{h^2} + \cancel{\frac{4 f'''(x_0)}{h}} + \cancel{\frac{h f^{(5)}(x_0)}{5}} - \frac{h^2 f^{(6)}(\xi)}{30}$$

$$f(x_0 \pm 2h) = f(x_0) \pm 2hf'(x_0) + \frac{4h^2 f''(x_0)}{2} \pm \frac{8h^3 f'''(x_0)}{6} + \frac{16h^4 f^{(4)}(x_0)}{24} \pm \frac{32h^5 f^{(5)}(x_0)}{120} + \frac{64h^6 f^{(6)}(\xi)}{720}$$

$$\frac{\overset{2}{\cancel{16}} h^4 f^{(4)}(x_0)}{\cancel{24}_3} = f(x_0 \pm 2h) - f(x_0) \mp 2hf'(x_0) - \frac{4h^2 f''(x_0)}{2} \mp \frac{8h^3 f'''(x_0)}{6} \mp \frac{32h^5 f^{(5)}(x_0)}{120} - \frac{64h^6 f^{(6)}(\xi)}{720}$$

③ Forward Substition

$$f^{(4)}(x_0) = \frac{3 f(x_0+2h)}{2h^4} - \frac{3 f(x_0)}{2h^4} - \cancel{\frac{3 f'(x_0)}{h^3}} - \frac{3 f''(x_0)}{h^2} - \cancel{\frac{2 f'''(x_0)}{h}} - \frac{4}{5} h f^{(5)}(x_0) - \frac{2}{15} h^2 f^{(6)}(\xi)$$

④ Backward Substition

$$f^{(4)}(x_0) = \frac{3 f(x_0-2h)}{2h^4} - \frac{3 f(x_0)}{2h^4} + \cancel{\frac{3 f'(x_0)}{h^3}} - \frac{3 f''(x_0)}{h^2} + \cancel{\frac{2 f'''(x_0)}{2}} + \frac{4}{5} h f^{(5)}(x_0) - \frac{2}{15} h^2 f^{(6)}(\xi)$$

We sum ①, ②, ③ and ④.

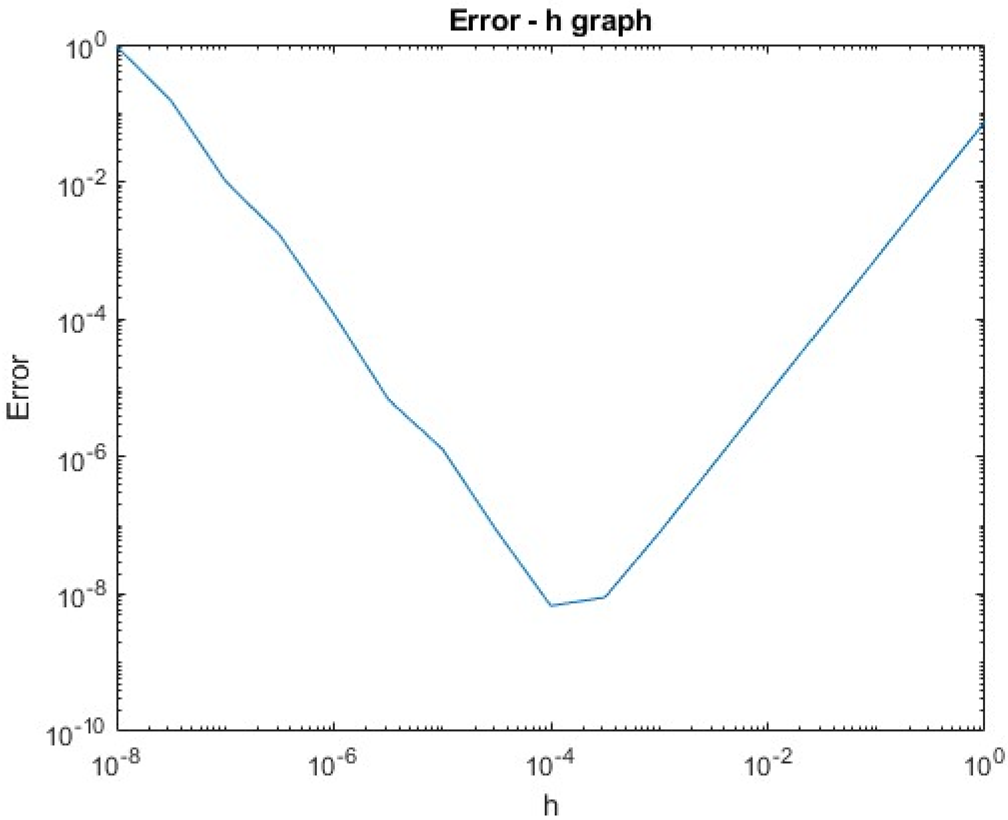
$$4 f^{(4)}(x_0) = \frac{48[f(x_0+h) - f(x_0-h)] + 3[f(x_0+2h) - f(x_0-2h)]}{2h^4} - \frac{51 f(x_0)}{h^4} - \frac{30 f''(x_0)}{h^2} - \frac{10 h^2 f^{(6)}(\xi)}{30}$$

$$f^{(4)}(x_0) = \frac{48[f(x_0+h) - f(x_0-h)] + 3[f(x_0+2h) - f(x_0-2h)]}{8h^4} - \frac{51 f(x_0)}{4h^4} - \frac{15 f''(x_0)}{2h^2} - \frac{h^2 f^{(6)}(\xi)}{12}$$

$$\underbrace{f(x_0+h) - f(x_0-h) - f(x_0+2h) - f(x_0-2h) - \overset{f''(x_0)}{f(x_0)} -}_{5 \text{ points required.}}$$

As it can be seen from the last term, order of the formula is $O(h^2)$.

QUESTION – 4



As h decreases, error decreases until the point at 10^{-4} after that error starts to increase because of the roundoff error.

Optimal h is 10^{-4} which is the lowest point on the graph which has lowest error.

```
function q4

    function x_trun = trunc(x, n)
        x_trun = x - rem(x,10^-n);
    end

    function result = fppo(x0, h)
    result = (sin(x0 + h) - 2 * sin(x0) + sin(x0 - h)) / h.^2;
    end

    function error = total_error(x0, h)
    error = abs(-sin(x0) - fppo(x0, h)) + abs(fppo(x0, h) -
trunc(fppo(x0, h), 10));
    % error = abs(fppo(x0, h) - trunc(fppo(x0, h), 8));

    end

    x0 = 1.2;
    error = [];
    for k = 0:0.5:8
        error = [error, total_error(x0, 10.^-k)];
    end
    k = 0:0.5:8;
    loglog(10.^-k, error);
end
```