61. A parallel-plate capacitor is constructed by filling the space between two square plates with blocks of three dielectric materials, as in Figure P26.61. You may assume that $\ell \gg d$. (a) Find an expression for the capacitance of the device in terms of the plate area A and d, κ_1, κ_2, and κ_3. (b) Calculate the capacitance using the values $A = 1.00\ \text{cm}^2$, $d = 2.00$ mm, $\kappa_1 = 4.90$, $\kappa_2 = 5.60$, and $\kappa_3 = 2.10$.

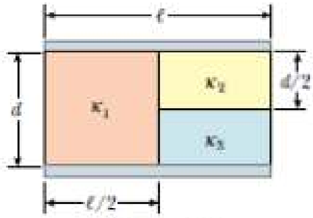

Figure P26.61

P26.61 (a)
$$C_1 = \frac{\kappa_1 \epsilon_0 A/2}{d};\ C_2 = \frac{\kappa_2 \epsilon_0 A/2}{d/2};\ C_3 = \frac{\kappa_3 \epsilon_0 A/2}{d/2}$$

$$\left(\frac{1}{C_2}+\frac{1}{C_3}\right)^{-1} = \frac{C_2 C_3}{C_2+C_3} = \frac{\epsilon_0 A}{d}\left(\frac{\kappa_2\kappa_3}{\kappa_2+\kappa_3}\right)$$

$$C = C_1 + \left(\frac{1}{C_2}+\frac{1}{C_3}\right)^{-1} = \boxed{\frac{\epsilon_0 A}{d}\left(\frac{\kappa_1}{2}+\frac{\kappa_2\kappa_3}{\kappa_2+\kappa_3}\right)}$$

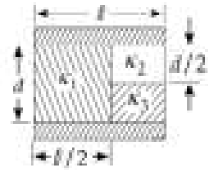

FIG. P26.61

(b) Using the given values we find: $C_{\text{total}} = 1.76 \times 10^{-12}\ \text{F} = \boxed{1.76\ \text{pF}}$.

64. A capacitor is constructed from two square plates of sides ℓ and separation d. A material of dielectric constant κ is inserted a distance x into the capacitor, as shown in Figure P26.64. Assume that d is much smaller than x. (a) Find the equivalent capacitance of the device. (b) Calculate the energy stored in the capacitor, letting ΔV represent the potential difference. (c) Find the direction and magnitude of the force exerted on the dielectric, assuming a constant potential difference ΔV. Ignore friction. (d) Obtain a numerical value for the force assuming that ℓ = 5.00 cm, ΔV = 2 000 V, d = 2.00 mm, and the dielectric is glass (κ = 4.50). (*Suggestion:* The system can be considered as two capacitors connected in parallel.)

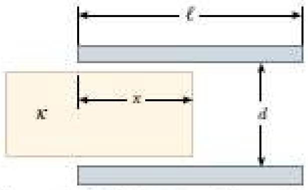

Figure P26.64 Problems 64 and 65.

P26.64 (a) $$C = \frac{\epsilon_0}{d}\left[(\ell - x)\ell + \kappa \ell x\right] = \boxed{\frac{\epsilon_0}{d}\left[\ell^2 + \ell x(\kappa - 1)\right]}$$

(b) $$U = \frac{1}{2}C(\Delta V)^2 = \boxed{\frac{1}{2}\left(\frac{\epsilon_0 (\Delta V)^2}{d}\right)\left[\ell^2 + \ell x(\kappa - 1)\right]}$$

(c) $$\mathbf{F} = -\left(\frac{dU}{dx}\right)\hat{\mathbf{i}} = \boxed{\frac{\epsilon_0 (\Delta V)^2}{2d}\ell(\kappa - 1) \text{ to the left}} \text{ (out of the capacitor)}$$

(d) $$F = \frac{(2\,000)^2\left(8.85 \times 10^{-12}\right)(0.050\,0)(4.50 - 1)}{2\left(2.00 \times 10^{-3}\right)} = \boxed{1.55 \times 10^{-3}\text{ N}}$$