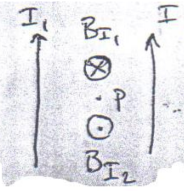
SOURCE OF MAGNETIC FIELD & BIOT SAVART-AMPERE'S LAWS

1- a)

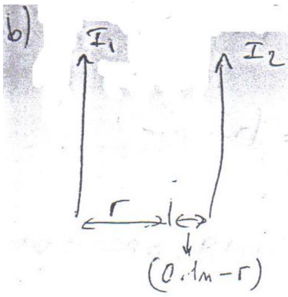


$$B_P = B_{I_1} - B_{I_2} \qquad B = 2\times10^{-7}\frac{N}{A^2}\frac{I}{r}$$

$$= 2\times10^{-7}\frac{N}{A^2}\left(\frac{12A}{0.05m} - \frac{7A}{0.05m}\right) = 2\times10^{-5}\frac{N}{Am}$$

$B_p = 2\times10^{-5}$ T directed into the page

b)



Measure distances from I_1.

$$B_{I_1} = B_{I_2}$$

$$2\times10^{-7}\frac{N}{A^2}\frac{12A}{r} = 2\times10^{-7}\frac{N}{A^2}\frac{7A}{(0.1m-r)}$$

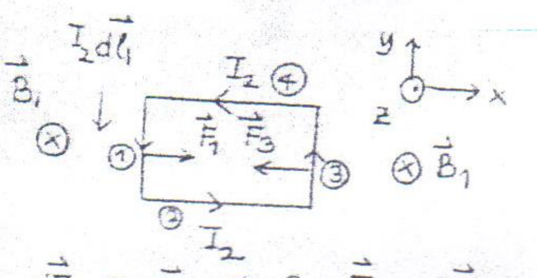
$$1.2m - 12r = 7r$$

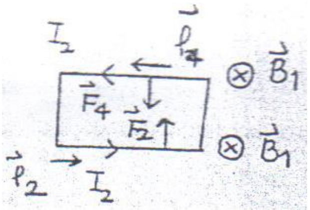
$$r = \frac{1.2m}{19} = 0.063\,m$$ to right of the 12 A current.

2- Magnetic field produced by the wire carrying the current I_1 : $B_1 = \frac{\mu_0 I_1}{2\pi r}$;

$$B_1(y_1) = \frac{\mu_0 I_1}{2\pi y_1}$$

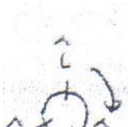
$$B_1(y_1+y_2) = \frac{\mu_0 I_1}{2\pi(y_1+y_2)}$$





$$d\vec{F}_1 = I_2\, d\vec{l}_1 \times \vec{B}_1$$
$$d\vec{F}_3 = I_2\, d\vec{l}_3 \times \vec{B}_1$$

$\vec{F}_1$ and $\vec{F}_3$ cancel out eachother!



$$\vec{F}_2 = I_2 \vec{l}_2 \times \vec{B}_1(y_1+y_2) = I_2(a\hat{i})\times(-B_1(y_1+y_2)\hat{k})$$
$$= \underline{a I_2 B_1(y_1+y_2)\hat{j}}$$

$$\vec{F}_4 = I_2 \vec{l}_4 \times \vec{B}_1(y_1) = I_2(-a\hat{i})\times(-B_1(y_1)\hat{k}) = \underline{-a I_2 B_1(y_1)\hat{j}}$$

net force: $$\Sigma\vec{F} = \vec{F}_2 + \vec{F}_4 = a I_2 \frac{\mu_0 I_1}{2\pi(y_1+y_2)}\hat{j} - a I_2 \frac{\mu_0 I_1}{2\pi y_1}\hat{j}$$

$$\boxed{\Sigma\vec{F} = \frac{\mu_0 a I_1 I_2}{2\pi}\left(\frac{1}{y_1+y_2} - \frac{1}{y_1}\right)\hat{j}}$$

3-

a) $r<a$ $\quad \oint \vec{B}\cdot d\vec{l} = \mu_0 I(r) = 0 \Rightarrow B(r) = 0$

b) $a<r<b$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I(r) = \mu_0 I\left(\frac{r^2-a^2}{b^2-a^2}\right)$$

$$B = \frac{\mu_0 I}{2\pi r}\left(\frac{r^2-a^2}{b^2-a^2}\right)$$

$$J = \frac{I}{\pi(b^2-a^2)}$$

$$I(r) \equiv J\cdot A(r) = \pi(r^2-a^2)\cdot\frac{I}{\pi(b^2-a^2)} = \frac{I(r^2-a^2)}{(b^2-a^2)}$$

c) $r>b$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I(r) = \mu_0 I$$

$$B\,2\pi r = \mu_0 I \Rightarrow \quad B = \frac{\mu_0 I}{2\pi r}$$

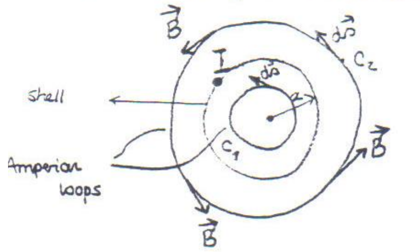
4-(a) $|\vec{B}| = \dfrac{\mu_0 I}{2\pi a}$

$$\vec{B}_{Left} = \frac{\mu_0 I}{2\pi a}(\hat{j}) \qquad \vec{B}_{Right} = \frac{\mu_0 I}{2\pi a}(-\hat{j})$$

$$\vec{B}_{lower} = \frac{\mu_0 I}{2\pi a}(\hat{i}) \qquad \vec{B}_{Total} = \vec{B}_{Left}+\vec{B}_{Right}+\vec{B}_{lower} = \frac{\mu_0 I}{2\pi a}\hat{i}$$

(b) $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl} = \mu_0(I+I-I) = \mu_0 I$

5- (i) View from the x-axis



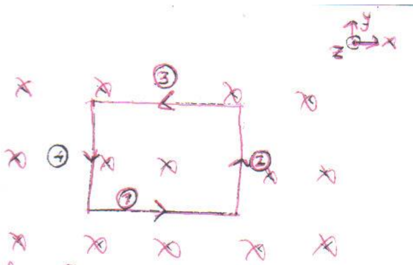
Inside: $\vec{B} = 0$ - $i_{enc}(C_1) = 0$!

Outside: $\underbrace{\oint_{C_2} \vec{B}\cdot d\vec{s}}_{B.2\pi r} = \mu_0 \underbrace{i_{enc}(C_2)}_{I}$

$$\Rightarrow \boxed{B(r>a) = \mu_0 I} .$$

.figure

(ii)



nonuniform B produced by the shell

$$\vec{F}_1 = i\vec{L}_1 \times \vec{B}_1 = (i\,b\hat{x}) \times \left(\frac{\mu_0 I}{2\pi\, 2b}\hat{z}\right)$$

$$\Rightarrow \boxed{\vec{F}_1 = \frac{\mu_0 i I}{4\pi}(-\hat{y})} .$$

$$\vec{F}_3 = i\vec{L}_3 \times \vec{B}_3 = i(-b\hat{x}) \times \left(\frac{\mu_0 I}{2\pi\, 3b}\hat{z}\right) \Rightarrow \boxed{\vec{F}_3 = \frac{\mu_0 i I}{6\pi}\hat{y}} .$$

$\boxed{\vec{F}_2 + \vec{F}_4 = 0}$ - magnitudes are the same but their directions are opposite; this is obvious from the symmetry.

$$\Rightarrow \quad \vec{F}_{net} = \vec{F}_1 + \dots + \vec{F}_4$$

$$\boxed{\vec{F}_{net} = -\left(\frac{\mu_0 i I}{12\pi}\right)\hat{y}} .$$

6-

a) $\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{in}$

$I_{in} = 3I_0 - I_0 = 2I_0$

$\oint \vec{B}\cdot d\vec{\ell} = 2\mu_0 I_0$

b) $\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{in}$

$B 2\pi r = \mu_0 I_0$

$B = \dfrac{\mu_0 I_0}{2\pi r}$

c) $J = \dfrac{I_0}{\pi a^2}$ $\quad I_{in} = J\pi r^2$ $\quad I_{in} = I_0 \left(\dfrac{r}{a}\right)^2$

$B 2\pi r = \mu_0 I_{in} = \mu_0 I_0 \left(\dfrac{r}{a}\right)^2 \Rightarrow B = \dfrac{\mu_0 I_0}{2\pi a^2} r$

d) $J = \dfrac{3I_0}{\pi(c^2-b^2)}$ $\quad I_{in} = J\pi(r^2-b^2) - I_0$

$= 3I_0 \dfrac{r^2-b^2}{c^2-b^2} - I_0$

$B 2\pi r = \mu_0 I_{in}$

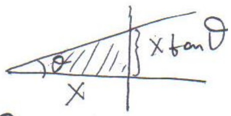
$B = \dfrac{\mu_0 I_0}{\ } \left(3 \dfrac{r^2-b^2}{c^2-b^2} - 1\right)$

ELECTROMAGNETIC INDUCTION & FARADAY'S LAW

7-

a) $|\mathcal{E}| = \left|\dfrac{d\Phi_B}{dt}\right|$ $\quad \Phi_B = \int \vec{B}\cdot d\vec{A}$ $\quad \vec{B} \parallel d\vec{A}$

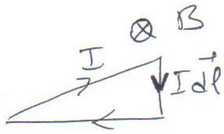
$\Phi_B = B\,A$ $\quad A = \dfrac{x^2}{2}\tan\theta$



$\mathcal{E} = \left|\dfrac{d\Phi_B}{dt}\right| = \dfrac{B}{2}\, 2x \dfrac{dx}{dt}\tan\theta = B x v \tan\theta$

b) $i = \dfrac{\mathcal{E}}{R}$ $\quad R = R_0 \dfrac{x\tan\theta}{L_0}$

$i = \dfrac{B v L_0}{R_0}$, clockwise



c) $\vec{F} = \int I d\vec{\ell} \times \vec{B}$ $\quad d\vec{\ell} \perp \vec{B}$ and $\vec{B}$ constant

$|\vec{F}| = IBL$ $\quad L = x\tan\theta$

$= \dfrac{B^2 v L_0}{R_0} x\tan\theta$, $+x$ direction