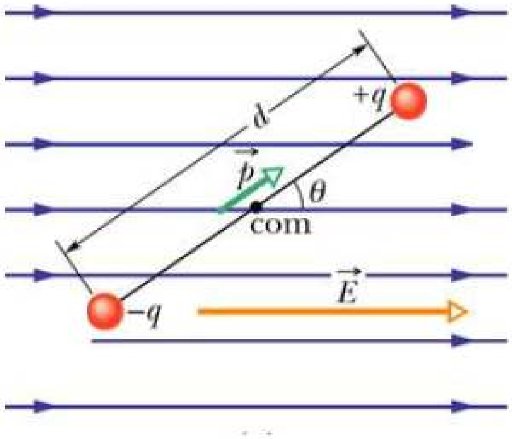
2- The figure shows an electric dipole in a electric field. Use $q = 6$ nC, $d = 4$ nm, and $\theta = 40^{\circ}$.
The magnitude of the electric field is 290 kV/m.



(a) Draw the dipole moment on the figure.

$|\mathbf{p}| = qd$, from the negative charge to the positive charge

4- The rectangle below has side lengths of 5.0 cm and 15 cm. The charges are $q_1 = -5.0$ μC and $q_2 = +2.0$ μC. Let $V = 0$ at infinity.

(a) Calculate the electric potential at point A.

$V_A = V_1 + V_2 = k\, q_1/d_1 + k\, q_2/d_2 = k(q_1/d_1 + q_2/d_2) = 8.99\ 10^9\ \mathrm{Nm^2/C^2}\ (-5\ 10^{-6}\ \mathrm{C}/0.15\ \mathrm{m} + 2\ 10^{-6}\ \mathrm{C}/\ 0.05\mathrm{m}) = 60\ \mathrm{kV}$

(b) Calculate the electric potential at point B.

$V_B = V_1 + V_2 = k\, q_1/d_2 + k\, q_2/d_1 = k(q_1/d_2 + q_2/d_1) = 8.99\ 10^9\ \mathrm{Nm^2/C^2}\ (-5\ 10^{-6}\ \mathrm{C}/0.05\ \mathrm{m} + 2\ 10^{-6}\ \mathrm{C}/\ 0.15\mathrm{m}) = -779\ \mathrm{kV}$

(c) How much work is required to move a third charge $q_3 = +3.0$ μC from B to A along a diagonal of the rectangle?

$W_{ext} = \Delta V\, q_3 = (V_A - V_B)\, q_3 = (60\ \mathrm{kV} - (-779\ \mathrm{kV}))\ 3\ 10^{-6}\ \mathrm{C} = 2.5\ \mathrm{J}$

(b) Calculate the electric potential energy of the electric dipole in the configuration shown in the figure.

$U = -\mathbf{p} \cdot \mathbf{E} = -|\mathrm{p}|\,|\mathrm{E}| \cos\theta = -q\, d\, E \cos\theta = -6\ 10^{-9}\ \mathrm{C} \times 4\ 10^{-9}\ \mathrm{m} \times 290\ 10^3\ \mathrm{V/m} \times \cos 40^{\circ} = -5.33\ 10^{-12}\ \mathrm{J}$

(b) What would be the electric potential energy of the electric dipole if it were turned 180°?

New angle between dipole and electric field: $\theta = 40^{\circ} + 180^{\circ} = 220^{\circ}$

New potential energy:
$U = -\mathbf{p} \cdot \mathbf{E} = -|\mathrm{p}|\,|\mathrm{E}| \cos\theta = -q\, d\, E \cos\theta = -6\ 10^{-9}\ \mathrm{C} \times 4\ 10^{-9}\ \mathrm{m} \times 290\ 10^3\ \mathrm{V/m} \times \cos 220^{\circ} = +5.33\ 10^{-12}\ \mathrm{J}$

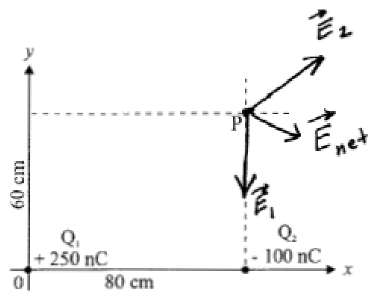
(c) Calculate the work required to turn the electric dipole 180°.

$W_{ext} = \Delta U = U_f - U_0 = +5.33\ 10^{-12}\ \mathrm{J} - (-5.33\ 10^{-12}\ \mathrm{J}) = 1.07\ ^{-11}\ \mathrm{J}$

8- Two point charges are fixed in place in an *xy* coordinate system as shown in the figure. There is no charge located at the point P.

a) Sketch, at point P on the figure, three vectors, representing the electric field due to Q_1 alone, due to Q_2 alone, and the net field due to both.

b) Calculate the net electric field vector (expressed in unit vector form) at P due to the two charges.



$$E_1 = k\frac{Q_1}{R_1^2} = \left(9\times10^9\,\frac{N\cdot m^2}{C^2}\right)\frac{100\times10^{-9}C}{(.6m)^2}$$

E_1 in $-\hat{j}$ direction so $\vec{E}_1 = 2500(-\hat{j})$ V/m

$$E_2 = k\frac{Q_2}{R_2^2} = \left(9\times10^9\,\frac{N\cdot m^2}{C^2}\right)\frac{(250\times10^{-9}C)}{(.6m)^2+(.8m)^2} = 2250\ V/m$$

E_1 in $.8\hat{i}+.6\hat{j}$ direction, so $\vec{E}_2 = 1800\hat{i} + 1350\hat{j}$ V/m

$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 = 1800\hat{i} - 1150\hat{j}\ V/m$$

c) Calculate the net electric potential at P due to the two charges

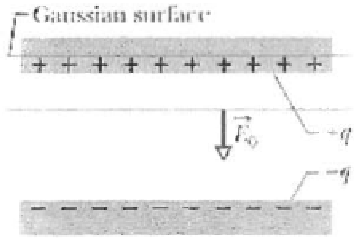
$$V_P = V_1 + V_2 = k\frac{Q_1}{R_1} + k\frac{Q_2}{R_2} \quad \text{scalar}$$

$$V_P = \left(9\times10^9\,\frac{N\cdot m^2}{C^2}\right)\left\{\left[\frac{-100\times10^{-9}C}{.6m}\right] + \left[\frac{+250\times10^{-9}C}{\sqrt{(.8m)^2+(.6m)^2}}\right]\right\} = +750\,V = (-1500V) + (+2250V)$$

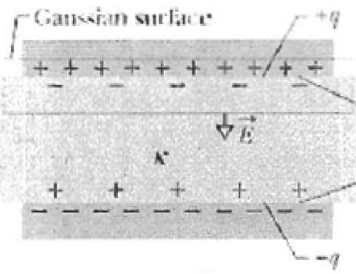
d) Calculate the amount of work that the **electric field** would do on a +5.0 nC charge brought to point P from infinitely far away.

$$W = -q\,\Delta V = -q\,V_P = -(5\times10^{-9}C)(750V - 0V) = -3.8\times10^{-6}\,J$$

11- The figure shows two views of parallel plate capacitor whose plate area $A = 110\ \text{cm}^2$ and plate separation $d = 1.35$ cm. With a battery, a potential difference of $V_0 = 60.0$ V is applied between the plates. Suppose that the battery remains connected while the dielectric slab of dielectric constant $\kappa = 2.90$, that completely fills the space between the plates, is being introduced. Calculate the following values.



(a)



(b)

(a) the capacitances both before and after the dielectric has been inserted.

for parallel plate capacitors $C = \frac{Q}{V} = \frac{\kappa \epsilon_0 A}{d}$

$$C_0 = \frac{(1)(8.854 \times 10^{-12}\ F/m)(0.011\ m^2)}{0.0135\ m} = 7.2 \times 10^{-12}\ F$$

$$C_1 = KC_0 = (2.9)(7.2 \times 10^{-12} F) = 21 \times 10^{-12}\ F$$

(b) the charges on the capacitor plates both before and after the dielectric has been inserted.

$$Q_0 = C_0 V_0 \Rightarrow Q_0 = (7.2 \times 10^{-12} F)(60V) = 4.3 \times 10^{-10}\ C$$

$$Q_1 = C_1 V_1,\ \text{but } V_1 = V_0,\ Q_1 = (21 \times 10^{-12} F)(60V) = 1.3 \times 10^{-9}\ C$$

(c) the electric field, $\boldsymbol{E}$, in the dielectric, after the dielectric is in place.

$$\int \vec{E} \cdot d\vec{l} = \Delta V,\ \vec{E} \text{ is uniform, so } \vec{E} \int d\vec{l} = \Delta V$$

$$\text{and } \vec{E} \cdot \int d\vec{l} = Ed,\ \text{so } E = \frac{\Delta V}{d}$$

$$E = \frac{60\ V}{.0135\ m} = 4400\ V/m \text{ or } N/C$$

(d) the work done (by the "inserter") to insert the dielectric into the capacitor. Be sure to clearly indicate if the work is positive or negative.

$$W = \Delta U_c = U_{c_f} - U_{c_i} = \frac{1}{2} C_f V_f^2 - \frac{1}{2} C_0 V_0^2,\ \text{but } V_f = V_0 = 60V$$

$$W = \frac{1}{2}(60V)^2 \left[21 \times 10^{-12} F - 7.2 \times 10^{-12} F\right] = 2.5 \times 10^{-8}\ J$$

21- Consider a system of two positive point charges of magnitude q on the y axis at coordinates $(0, a)$ and $(0, -a)$, as shown in Fig.3. Find the electric potential at a point $P(x, y)$ and evaluate the electric field using the knowledge of the potential field.

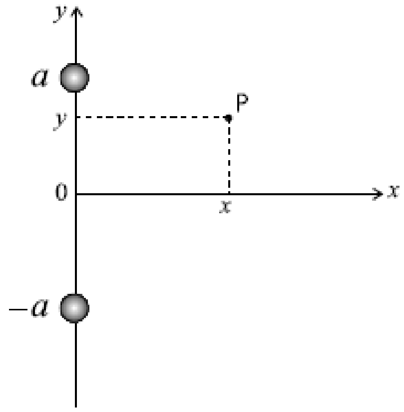


FIG. 3: A system of two point charges of magnitude q.

Solution The electric potential at (x, y) is given by the superposition

$$V(x,y) = kq\left(\frac{1}{\sqrt{x^2+(y-a)^2}} + \frac{1}{\sqrt{x^2+(y+a)^2}}\right)$$

from which we can evaluate $E_x = -dV/dx$ to be

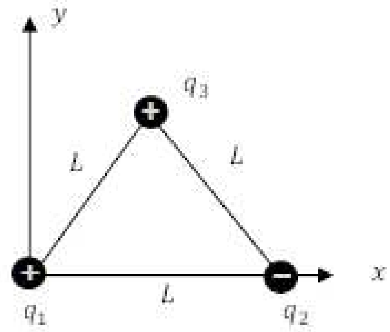
$$E_x(x,y) = kqx\left(\frac{1}{(x^2+(y-a)^2)^{3/2}} + \frac{1}{(x^2+(y+a)^2)^{3/2}}\right).$$

Similarly $E_y = -dV/dy$ becomes,

$$E_y(x,y) = kq\left(\frac{y-a}{(x^2+(y-a)^2)^{3/2}} + \frac{y+a}{(x^2+(y+a)^2)^{3/2}}\right).$$

4. Three charges $q_1 = +4.00\ \mu C, q_2 = -6.00\ \mu C,\ q_3 = +8.00\mu C$

are placed at the corners of an equilateral triangle as shown, with edge $L = 0.650$ m.



(a) Calculate the electric field at the position of charge q_1 due to q_2 and q_3. Give its magnitude and direction relative to the $+x$ axis.

The electric field due to the charge q_3 is $\vec{E}_1 = \frac{k_e q_3}{L^2}(-\hat{\imath}\cos 60^o - \hat{\jmath}\sin 60^o)$

and the electric field due to the charge q_2 is $\vec{E}_2 = -\frac{k q_2}{L^2}\hat{\imath}$.

Combining them gives a total electric field

$$\vec{E} = -\frac{9\times 10^9}{(0.650)^2}\left\{\hat{\imath}\left(-6.0+\frac{8.0}{2}\right)+\hat{\jmath}\ \frac{8.0\sqrt{3}}{2}\right\}\frac{\mu N}{C} = (42.6\ \hat{\imath} - 147.6\ \hat{\jmath})\frac{kN}{C}.$$

$$E = \sqrt{E_x^2+E_y^2} = \mathbf{153.6\frac{kN}{C}},\qquad \theta = \text{atan}\frac{E_y}{E_x} = \mathbf{-73.9^o}.$$

(b) What is the potential energy of the set of three charges (relative to a situation where they are widely separated)?

$$U = k_e\left(\frac{q_1q_2}{r}+\frac{q_1q_3}{r}+\frac{q_2q_3}{r}\right) = \frac{9\times 10^9}{0.650}(-24+32-48)\times 10^{-12}\text{J} = \mathbf{-0.554\ J}.$$

(c) Suppose charge q_1 is free to move but the other charges are held fixed, and has a mass

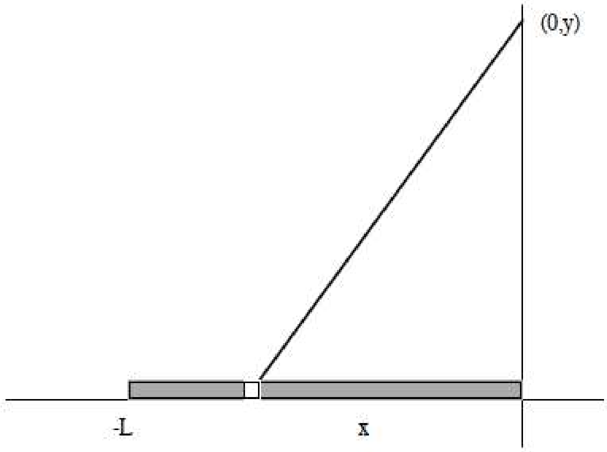
$m_1 = 1.20$ g. Find its speed when far away from the other charges, assuming it is released at rest from the origin.

The electric potential at the location of q_1 is $U_1 = k_e\left(\frac{q_1q_2}{L}+\frac{q_1q_3}{L}\right) = \frac{9\times 10^9}{0.650}(8\times 10^{-12})\text{ J} = 0.111\text{ J}.$

Energy conservation implies that

$$\frac{1}{2}mv_f^2 = U_1,\qquad v_f = \sqrt{\frac{2(0.111\text{ J})}{1.2\times 10^{-3}\text{kg}}} = \mathbf{13.6\frac{m}{s}}.$$

2. Consider the charged rod below. Assume that it is uniformly charged with charge per unit length λ.



a) Write an expression for the electric potential due to a small charge dq at the point p?

$$dV = \frac{dq}{4\pi\varepsilon_0 r} = \frac{dq}{4\pi\varepsilon_0\sqrt{x^2+y^2}}$$

b) Write an expression for the dq and the r in terms of x and the distance y.

$$dq = \lambda dx \qquad r = \sqrt{x^2+y^2}$$

c) What is the potential at the point indicated? You may need the integral

$$\int_{-L}^{0}\frac{dx}{\sqrt{x^2+y^2}} = \ln[y] - \ln[-L+\sqrt{L^2+y^2}]$$

$$V = \int_{-L}^{0}\frac{\lambda\,dx}{4\pi\varepsilon_0\sqrt{x^2+y^2}} = \frac{\lambda}{4\pi\varepsilon_0}\int_{-L}^{0}\frac{dx}{\sqrt{x^2+y^2}} = \frac{\lambda}{4\pi\varepsilon_0}\cdot(\ln[y]-\ln[-L+\sqrt{L^2+y^2}])$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\ln\frac{y}{-L+\sqrt{L^2+y^2}}$$

P4. The electric potential at points in a space is given by function $V=2x^2-3y^2+5z^3$. What is the magnitude and direction of the electric field at the point (3, 2, -1)?

1). $V(x,y,z)$ is given

2). $\boxed{\vec{E} = -\vec{\nabla} V}$

3). $\frac{\partial V}{\partial x} = 4 \cdot x$

$\frac{\partial V}{\partial y} = -6 \cdot y$

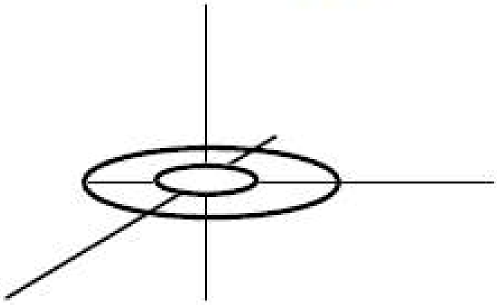
$\frac{\partial V}{\partial z} = 15 \cdot z^2$

$\vec{E}(3,2,-1) = -4 \cdot (3) \cdot \hat{x} + 6 \cdot (2) \cdot \hat{y} - 15 \cdot (-1)^2 \cdot \hat{z}$

$\vec{E}(3,2,-1) = -12 \cdot \hat{x} + 12 \cdot \hat{y} - 15 \cdot \hat{z}$

$\|\vec{E}\| = \sqrt{12^2 + 12^2 + 15^2} = 22.6 \ \frac{N}{C}$

1. Consider a ring of charge that has an inner radius a and an outer radius b. It has a charge density σ



a) Derive an expression for the potential along its axis.

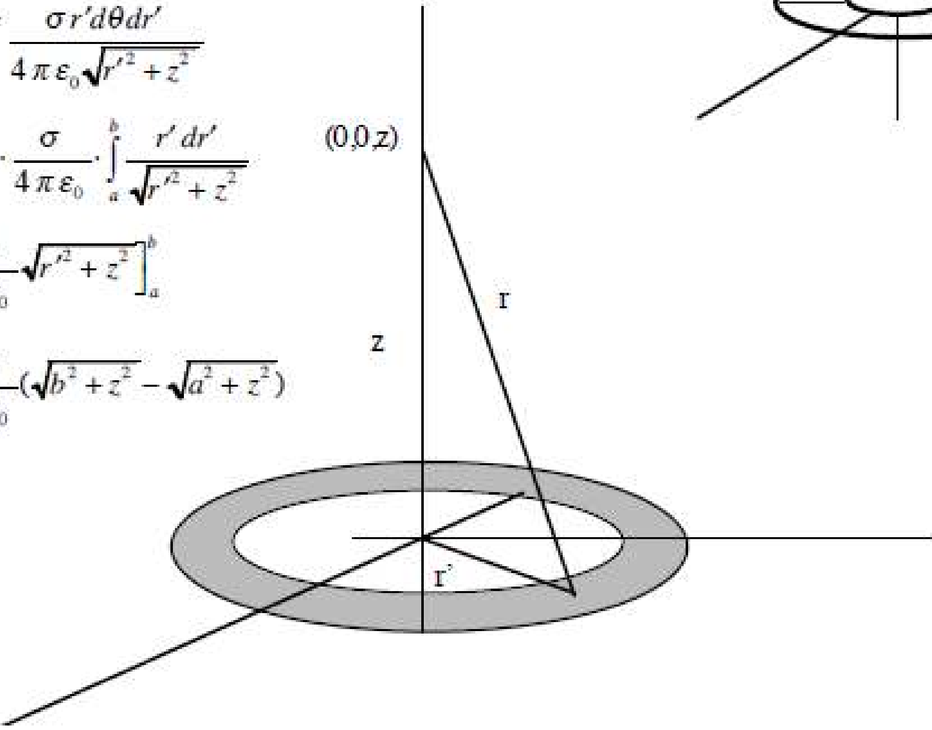
$$V = \int \frac{dq}{4\pi\varepsilon_0 r} \qquad dq = \sigma r' d\theta dr' \qquad r = \sqrt{r'^2 + z^2}$$

$$V = \int_{a,0}^{b,2\pi} \frac{\sigma r' d\theta dr'}{4\pi\varepsilon_0 \sqrt{r'^2 + z^2}}$$

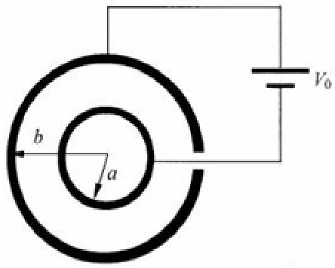
$$= 2\pi \cdot \frac{\sigma}{4\pi\varepsilon_0} \cdot \int_a^b \frac{r' dr'}{\sqrt{r'^2 + z^2}}$$

$$= \frac{\sigma}{2\varepsilon_0} \sqrt{r'^2 + z^2}\Big]_a^b$$

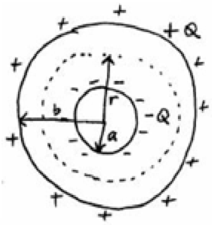
$$= \frac{\sigma}{2\varepsilon_0} (\sqrt{b^2 + z^2} - \sqrt{a^2 + z^2})$$



QUESTION 6 A spherical capacitor is formed from two concentric spherical conducting shells separated by air as shown in the figure. Inner sphere has radius $a = 5$ cm and outer has radius $b = 10$ cm. The capacitor is charged to a potential difference $V_0 = 90$ V.



(a) What is the capacitance of the capacitor?



Gauss law: $\varepsilon_0 \oint \vec{E}\cdot d\vec{s} = q_{enc}$

$\varepsilon_0 E(4\pi r^2) = Q \rightarrow E = \frac{kQ}{r^2}$

Potential diff. between conductors: $V_{ab} = -\int_a^b \vec{E}\cdot d\vec{l} = -\int_a^b E\,dr = -\int_a^b \frac{kQ}{r^2}dr$

$= kQ\left(\frac{b-a}{ab}\right)$

Capacitance:

$$C = \frac{Q}{V_{ab}} = \frac{Q}{kQ\left(\frac{b-a}{ab}\right)} = \frac{ab}{k(b-a)} = \frac{(0.05)(0.1)}{9\times10^9(0.1-0.05)} = 1.1\times10^{-11}\,F$$

$\boxed{C = 1.1\times10^{-11}F}$

(b) What charge is collected in outer surface?

$$Q = CV_0 = (1.1\times10^{-11})(90) = 1\times10^{-9}C = 1nC$$

$\boxed{Q = 1\times10^{-9}\,C}$

(c) What is the stored energy in the capacitor?

$$U = \frac{1}{2}CV_0^2 = \frac{1}{2}(1.1\times10^{-11})(90)^2 = 4.5\times10^{-8}J$$

$\boxed{U = 4.5\times10^{-8}J}$

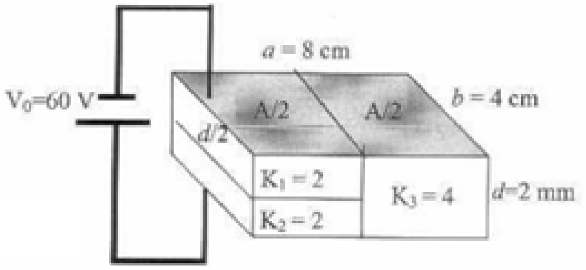
(d) What is the average energy density (stored energy per unit volume) in the capacitor?

$$u = \frac{Energy}{Volume} = \frac{U}{\frac{4}{3}\pi(b^3-a^3)} = \frac{4.5\times10^{-8}}{\frac{4}{3}\pi([0.1]^3-[0.05]^3)} = 1.2\times10^{-7}\frac{J}{m^3}$$

$\boxed{u = 1.2\times10^{-7}\,J/m^3}$

QUESTION 17

A rectangular parallel plate capacitor having sides a and b and separation d is filled with three dielectric materials as seen in the Figure.



(a) Find the equivalent capacitance value of the system.

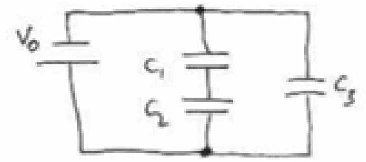


Plate area : $A = a\,b = (0.08)(0.04) = 3.2\times10^{-3}\ m^2$

$$C_1 = \frac{\varepsilon_0 K_1 (A/2)}{d/2} = \frac{\varepsilon_0 K_1 A}{d} = \frac{(8.85\times10^{-12})(2)(3.2\times10^{-3})}{2\times10^{-3}} = 2.8\times10^{-11}\,F$$

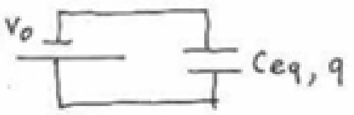
$$C_2 = \frac{\varepsilon_0 K_2 (A/2)}{d/2} = \frac{\varepsilon_0 K_2 A}{d} = \frac{(8.85\times10^{-12})(2)(3.2\times10^{-3})}{2\times10^{-3}} = 2.8\times10^{-11}\,F$$

$$C_3 = \frac{\varepsilon_0 K_3 (A/2)}{d} = \frac{\varepsilon_0 K_3 A}{2d} = \frac{(8.85\times10^{-12})(4)(3.2\times10^{-3})}{(2)(2\times10^{-3})} = 2.8\times10^{-11}\,F$$

$$C_{eq} = \frac{C_1 C_2}{C_1+C_2} + C_3 = \frac{C_1}{2} + C_3 = \frac{3C_1}{2} = \frac{3}{2}(2.8\times10^{-11}F) = 4.2\times10^{-11}F$$

$\boxed{C_{eq} = 4.2\times10^{-11}\,F}$

(b) Find the charge on each capacitor (dielectric).



$q = C_{eq} V_0 = (4.2\times10^{-11})(60) = 2.5\times10^{-9}\ C$

$q_3 = C_3 V_0 = (2.8\times10^{-11})(60) = 1.7\times10^{-9}\ C$

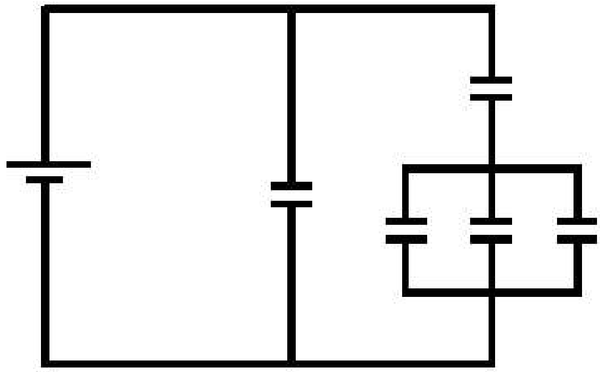
$q_1 = q_2 = q_{12}$

$q = q_{12} + q_3$

$q_{12} = q - q_3 = (2.5 - 1.7)\times10^{-9} = 0.8\times10^{-9}\,C$

$q_1 = 0.8\times10^{-9}\,C$
$q_2 = 0.8\times10^{-9}\,C$
$q_3 = 1.7\times10^{-9}\,C$

3. Consider the circuit shown below. Assume all capacitors are 1 mF and the battery is 10 V



a) What is the equivalent capacitance.

$$C_{3p} = C + C + C = 3mF \qquad \frac{1}{C_s} = \frac{1}{C_{3p}} + \frac{1}{C} \Rightarrow C_s = \frac{3}{4}mF$$

$$C_{eq} = C_s + C = \frac{7}{4}mF$$

b) What is the charge on each capacitor?

The charge on the left capacitor is the easiest.

$$q_L = CV = 1mF \cdot 10V = 10mC$$

Now compute the charge on the equivalent capacitance for the array of charges on the right.

$$q = C_{series}V = \frac{3}{4}mF \cdot 10V = \frac{30}{4}mC$$

The charge on this equivalent capacitor is the same as the charge on the series capacitors that were used to create it. So the charge on the upper right capacitor has this value We are left with the three capacitors in parallel. The voltage across the three is the same as the voltage across the equivalent capacitor. We can calculate this voltage, since we know the charge on the equivalent capacitor--it is the charge we just computed.

$$V = \frac{q}{C_{3p}} = \frac{30/4\ mC}{3mF} = 2.5V$$

Now that we know the voltage across each of these capacitors, we can find the charge on each.

$$q = CV = 1mF \cdot 2.5V = 2.5mC$$

We repeat this for each capacitor in parallel, but in this case, they are all the same...

c) How much energy is stored in the capacitors?

The easiest way to do this is to consider the equivalent capacitance

$$U = \frac{1}{2}CV^2 = \frac{1}{2} \cdot \frac{7}{4}mF \cdot (10V)^2 = 87.5J$$

5. A coaxial cable of length l has an inner conductor of radius of a which carries a charge Q. The surrounding conductor has an inner radius b and a charge of $-Q$. Assume the electrical properties of the material between the conductors are the same as empty space.

(a) Find the electric field as a function of r between the conductors, for $a < r < b$.

This is one of the geometries where the electric field can be found using Gauss's Law. Construct a Gaussian cylinder of radius r and length L between the two conductors of the cable. The electric flux through this cylinder is $\Phi = EA = 2\pi r l E$. Gausses law relates this to the charge inside:

$2\pi r l E = \Phi = \frac{Q}{\epsilon_0}$ for $\lambda = Q/l$. Therefore, $E(r) = \frac{Q}{2\pi\epsilon_0 r l}$

(b) Find the potential difference $V_a - V_b$ between the conductors if $Q = 25\,\text{nC}$, $a = 1.2\,\text{mm}$ $b = 2.4\,\text{mm}$, and $L = 30.0\,\text{m}$.

$$V_a - V_b = \int_a^b \frac{Q\,dr}{2\pi\epsilon_0 r l} = \frac{Q}{2\pi\epsilon_0 l}\ln\left(\frac{b}{a}\right) = \frac{25\times10^{-9}\ \ln 2}{2\pi(8.85\times10^{-12})(30)}\text{V} = \mathbf{10.4\ V}.$$

(c) Find the capacitance of this length of cable. [If you don't have a result for the previous question, assume $\Delta V = 12.0$ V for (c) and (d).]

$$C = \frac{Q}{\Delta V} = \mathbf{2.40\ nF}.$$

(d What electrical energy is stored in the cable?

$$U_e = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(3.38\times10^{-9})(10.4)^2| = \mathbf{366\ nJ}.$$

Problem 1: In lecture, you saw a demo with a parallel plate capacitor with capacitance C_0. The capacitor was connected to an ideal power supply with an output voltage ΔV and charged to a charge Q.The capacitor was disconnected from the power supply, still carrying the same charge Q. Then the distance between the plates was increased by a factor of 2.

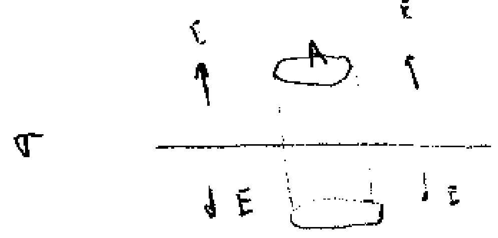
(a) How big is the potential difference between the capacitor plates after they have been moved apart?

(b) How big is the stored electrical energy in the capacitor after the plates have been moved apart?

(c) Explain how energy was conserved when moving the plates apart (1 sentence).

(d) Suppose we had not disconnected the power supply before moving the plates apart. How big would the stored energy be after the plates have been moved apart in this case?

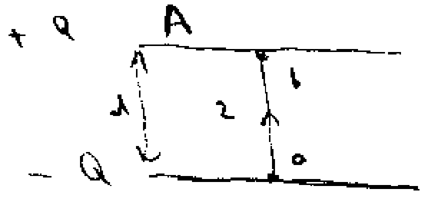
1 -



$\oint E\cdot da = Q_{in}/\epsilon_0$

$E\cdot 2A = \sigma A/\epsilon_0$

$E = \frac{\sigma}{2\epsilon_0}$



$\uparrow E_+ \ \downarrow E_- \Rightarrow \vec{E} = 0$

$\downarrow E_+ \ \downarrow E_- \Rightarrow E = \frac{\sigma}{\epsilon}$

$\downarrow E_+ \ \uparrow E_- \Rightarrow E = 0$

$$V_{ab} = -\int_a^b E\cdot ds = -\int_0^b \frac{\sigma}{\epsilon_0}(-\hat{z})\cdot d\hat{z} = \frac{\sigma}{\epsilon_0}d = \frac{Q}{A\epsilon_0}d$$

$Q = CV \Rightarrow C = \frac{A\epsilon_0}{d}$

a) $d \to 2d$ $\quad Q \to Q$

$C \to C_0/2$ $\quad V \to 2\Delta V$

b) $P = IV = \frac{dQ}{dt} V$

$$U = \int P\,dV = \int_0^Q \frac{dQ'}{dt}\frac{Q'}{C} = \frac{1}{2}\frac{Q^2}{C}$$

$Q \to Q$ $\quad C \to C/2$

$U \to 2U = \frac{Q^2}{C_0}$

c) Energy was added to the sytem by doing work against the electrical force to move the plates.

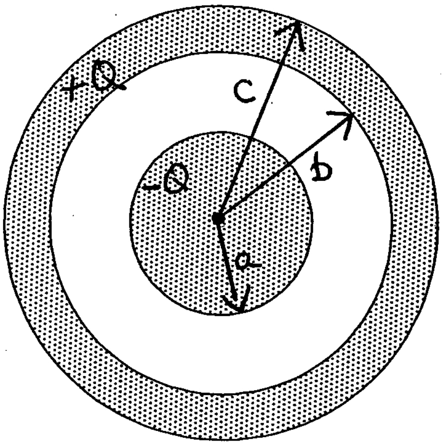
d) $U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$

$V \to V_0$ $\quad C \to C_0/2$ $\quad U = U_0/2 = \frac{Q^2}{4C_0}$

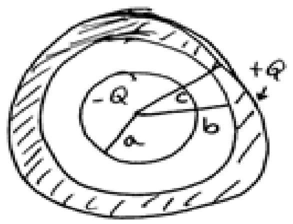
Notice: E remains the same in parts a-c but the volume it is affecting is doubled. In part d, E reduces to half its value.

I. A spherical capacitor consists of an inner solid conducting sphere of radius a surrounded by a spherical conducting shell of inner radius b and outer radius c. The capacitor is charged with $-Q$ on the inner sphere and $+Q$ on the outer spherical shell. The coordinate r measures the distance from the center of the solid sphere.

(a) Determine the electric field $\vec{E}(r)$ everywhere in space due to this charge configuration, indicating both direction and magnitude. Sketch $E(r)$ vs. r for all r.
(b) In part (c) you will calculate the potential difference ΔV between the inner and outer conductors. Before you calculate the value, indicate what the *sign* (positive or negative) of $\Delta V = V_b - V_a$ must be and give a clear explanation as to why.
(c) Determine the potential difference $\Delta V = V_b - V_a$ between the inner and outer conductors.
(d) From the result in (c), determine the capacitance of this device and the energy stored in it.
(e) Taking the electrostatic potential to be zero at infinity, determine $V(r)$ for all r and sketch $V(r)$ vs. r.
(f) Determine the energy density in the electric field in the region between a and b. Integrate the energy density over the volume between a and b and compare to your answer in (d).



I. (a)



(a) Since we have two conductors, the electric field inside the conductors is zero. All charge resides on surface of conductors.

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

$r < a$ $E = 0$ No charge enclosed

$b < r < c$ $E = 0$

$a < r < b$ $\oint \vec{E} \cdot d\vec{A} = -E \cdot 4\pi r^2 = \frac{Q_{enc}}{\varepsilon_0} = -\frac{Q}{\varepsilon_0} \Rightarrow \vec{E} = \frac{-Q}{4\pi\varepsilon_0 r^2}\hat{r}$

$r > c$ $E = 0$ since $Q_{enc} = 0$

$E = 0$ for all space except for $a < r < b$ where $\vec{E} = \frac{-Q}{4\pi\varepsilon_0 r^2}\hat{r}$

(b) $\Delta V = V_b - V_a$ Since $\vec{E}$ points from high potential to low potential $\Rightarrow V_b > V_a$ so $\boxed{\Delta V = V_b - V_a > 0 \quad \text{positive}}$

(c) $\Delta V = -\int_a^b \vec{E}\cdot d\vec{l} = -\int_a^b \left(\frac{-Q}{4\pi\varepsilon_0 r^2}\right) dr = \int_a^b \frac{Q}{4\pi\varepsilon_0 r^2}\, dr$

$$\Delta V = \frac{Q}{4\pi\varepsilon_0}\int_a^b \frac{dr}{r^2} = \frac{-Q}{4\pi\varepsilon_0}\left(\frac{1}{r}\Big|_a^b\right) = \frac{-Q}{4\pi\varepsilon_0}\left(\frac{1}{b}-\frac{1}{a}\right) = \underline{\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)}$$

(d) from (c)

$\Delta V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$ $\quad$ Note $\Delta V > 0$ $\quad C \equiv \frac{Q}{\Delta V} = 4\pi\varepsilon_0\left(\frac{1}{a}-\frac{1}{b}\right)^{-1}$

$\frac{1}{a}-\frac{1}{b} = \frac{b-a}{ab} \Rightarrow \boxed{C = \frac{4\pi\varepsilon_0\, ab}{b-a}}$ $\quad \boxed{U = \frac{1}{2}QV = \frac{1}{2}Q^2\left(\frac{b-a}{4\pi\varepsilon_0 ab}\right)}$ $\quad \boxed{U = \frac{Q^2(b-a)}{8\pi\varepsilon_0 ab}}$

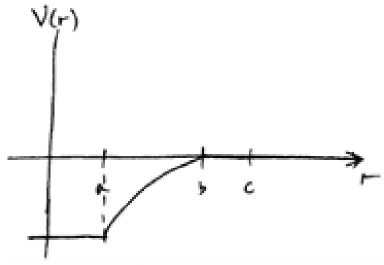
(e) $\Delta V_{\infty,r} = -\int_\infty \vec{E}\cdot d\vec{l} \Rightarrow \underline{\Delta V_{\infty,r} = V(r) - V(\infty)}$ $\quad \Delta V_{\infty,r}(r>c) = -\int_\infty^r 0\, dl = 0 \Rightarrow V(r) = 0$

$\Delta V_{\infty,r}(b<r<c) = -\int_\infty^r \vec{E}\cdot d\vec{l} = -\int_\infty^r 0\, dl = 0 \Rightarrow V(r) = 0$

$\Delta V_{\infty,r}(a<r<b) = -\int_\infty^r \vec{E}\cdot d\vec{l} = -\int_\infty^b 0\, dr + -\int_b^r \left(\frac{-Q}{4\pi\varepsilon_0 r^2}\right) dr$ $\quad \Delta V_{\infty,r}(a<r<b) = +\int_b^r \frac{Q}{4\pi\varepsilon_0 r^2}\, dr = \frac{-Q}{4\pi\varepsilon_0}\left(\frac{1}{r}-\frac{1}{b}\right)$

$\Delta V_{\infty,r}(r<a) = \frac{-Q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$

(Note: @ $r=b$ $\Delta V = 0$)



(f) $u_E = \frac{1}{2}\varepsilon_0 |E|^2 = \frac{1}{2}\varepsilon_0 \left|\frac{-Q}{4\pi\varepsilon_0 r^2}\right|^2$ $\quad a<r<b$ $\quad u_E = \frac{1}{2}\frac{\varepsilon_0 Q^2}{16\pi^2\varepsilon_0^2 r^4} = \frac{Q^2}{32\pi^2\varepsilon_0 r^4}$

$U = \int_a^b u_E\, dV$ $\quad dV \equiv$ infinitesimal volume of sphere $\quad dV = 4\pi r^2\, dr$

$U = \frac{Q^2}{32\pi^2\varepsilon_0}\cdot 4\pi \int_a^b \frac{dr}{r^2} = \frac{Q^2}{8\pi\varepsilon_0}\left(-\frac{1}{r}\Big|_a^b\right)$ $\quad \boxed{U = \frac{Q^2}{8\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right) = \frac{Q^2}{8\pi\varepsilon_0}\left(\frac{b-a}{ab}\right)}$ Same as in (d).