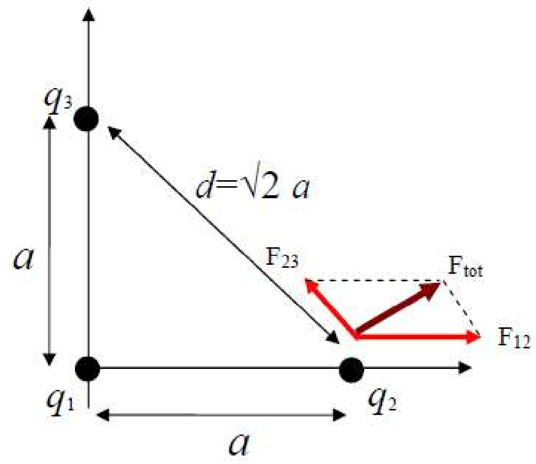
1- Three point charges, $q_1 = -1.2 \times 10^{-8}$ C, $q_2 = -2.6 \times 10^{-8}$ C and $q_3 = +3.4 \times 10^{-8}$ C, are held at the positions shown in the figure, where $a = 0.16$ m.



(a) Draw the forces acting on q_2 (including the total force, approximately)

(b) Calculate the x-component of the total force acting on q_2 due to the other two charges.

$|F_{12}| = kq_1q_2/a^2 = 8.99\ 10^9\ \mathrm{Nm^2/C^2} \times (1.2\ 10^{-8}\ \mathrm{C}) \times (2.6\ 10^{-8}\ \mathrm{C})/(0.16\mathrm{m})^2 = 1.10\ 10^{-4}\ \mathrm{N}$

F_{12} is in the x-direction, so $F_{12x} = +|F_{12}| = +1.096\ 10^{-4}$ N

$|F_{23}| = kq_2q_3/d^2 = 8.99\ 10^9\ \mathrm{Nm^2/C^2} \times (2.6\ 10^{-8}\ \mathrm{C}) \times (3.4\ 10^{-8}\ \mathrm{C})/(\sqrt{2} \times 0.16\mathrm{m})^2 = 1.55\ 10^{-4}\ \mathrm{N}$

$F_{23x} = -|F_{23}|\cos 45° = -1.098\ 10^{-4}$ N

$F_{tot\,x} = F_{12x} + F_{23x} = -1.9\ 10^{-7}\ \mathrm{N} \sim 0$

(c) Calculate the y-component of the total force acting on q_2 due to the other two charges.

$F_{12y} = 0$

$F_{23y} = +|F_{23}|\sin 45° = 1.098\ 10^{-4}$ N

$F_{tot\,y} = F_{12y} + F_{23y} = 1.098\ 10^{-4}$ N

(d) Calculate the magnitude of the total force acting on q_2 .

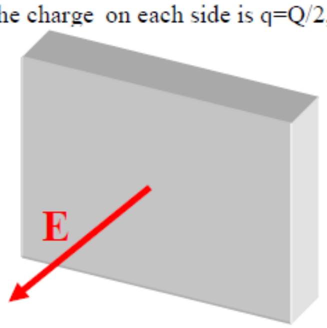
$|Ftot| = (F_{totx}{}^2 + F_{toty}{}^2)^{1/2} = 1.1\ 10^{-4}$ N

3- A square **metal** sheet with side lengths d = 20 cm is charged with a total charge $Q = +15$ mC.

(a) Calculate the surface change density σ on *each* surface of the sheet.

On a conductor, the charge will spread out on both sides. The charge on each side is q=Q/2, and the charge density on each side is

$\sigma = q/A = (Q/2)/d^2 = (15\ 10^{-3}\ C/2)/\ (0.2\ m)^2 = 0.19\ C/m^2$



(b) What is the direction and magnitude of the electric field to the left of the sheet, close to the surface of the sheet?

The electric field points away from positive charges, so it weill point away from the sheet.

Close to the surface of the conductor,

$E = \sigma/\varepsilon_0 = 0.19\ C/m^2\ /\ 8.85\ 10^{-12}\ C^2/Nm^2 = 2.1\ 10^{10}\ N/C$

(c) Calculate the electrical force on a point charge $q = +2\ \mu C$, located 0.2 mm to the left of the sheet (i.e. close to the surface).

$|\mathbf{F}| = q\,|\mathbf{E}| = 2\ 10^{-6}\ C \times 2.1\ 10^{10}\ N/C = 4.3\ 10^4\ N$

(d) Would the answer to the question in (c) be the same, if the point charge was located 20 m to the left of the sheet (i.e. far away from the sheet)? (Answer yes or no, and why).

No: since 20m is much larger than the sheet side (20cm), the sheet would look more like a point charge, and $|\mathbf{F}| \sim kqQ/r^2$ (smaller).

5- A thin cylindrical conducting tube of radius R=5.0 cm carries a surface charge density of 9.5 $\mu C/m^2$, as shown above.

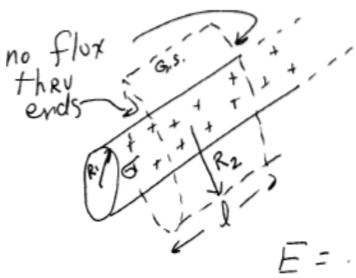
(a) use Gauss' Law to show that the electric field inside the tube is zero.

$\oint \vec{E}\cdot d\vec{A} = q_{encl}/\epsilon_0$ by $q_{encl} = 0$

so $\oint \vec{E}\cdot d\vec{A} = 0$, and, by symmetry, any $\vec{E}$ would be directed radially so for $\oint \vec{E}\cdot d\vec{A}$ to be 0

E must be 0

(b) Use Gauss' Law to determine the magnitude of the electric field at a distance from the axis r=7.0 cm (outside the tube).

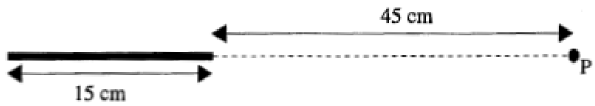


$\oint \vec{E}\cdot d\vec{A} = \frac{q_{encl}}{\epsilon_0}$ $\quad q_{encl} = \sigma(2\pi R_1)\ell$

$\oint_{GS} \vec{E}\cdot d\vec{A} = E A_{G.S.} = E(2\pi R_2 \ell)$

$$E = \frac{\cancel{2\pi} R_1 \sigma \cancel{\ell}}{\cancel{2\pi} R_2 \cancel{\ell} \epsilon_0} = \frac{\sigma}{\epsilon_0}\left(\frac{R_1}{R_2}\right) = \left(\frac{9.5\times10^{-6}\ C/m^2}{8.854\times10^{-12}\ F/m}\right)\left(\frac{5\ cm}{7\ cm}\right) = 7.7\times10^{5}\ N/C$$

6- A thin, non-conducting rod carries a net charge of 28 nC distibuted uniformly along its length. Calculate the magnitude if the electric field at the point P, which is located 45 cm from the end of the rod (as shown)



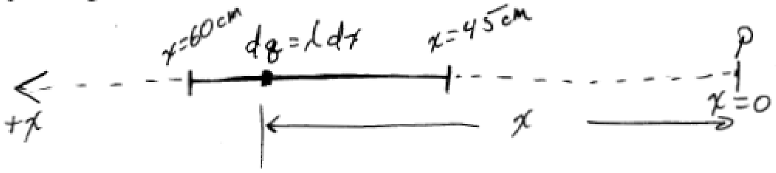
(a) calculate the linear charge density (λ) on the rod

$$\lambda = \frac{Q}{L} = \frac{28 \times 10^{-9}\,C}{.15\,m} = 1.87 \times 10^{-7}\ C/m$$

(b) assuming the rod lies on the x-axis, what is the form of the incremental charge (dq) for the integration?

$$dq = \left(\lambda \tfrac{C}{m}\right)(dx\ m) = \lambda\, dx\ C$$

(c) set up the integral to determine the electric field and solve.



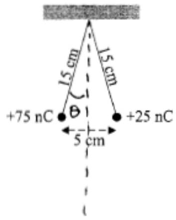
$$dE = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{dq}{r^2}\right) \quad \text{where } r = x$$

$$E = \int_{x=.45m}^{x=.60m} \left(\frac{1}{4\pi\epsilon_0}\right)\frac{\lambda\, dx}{x^2} = \left(8.99\times10^{9}\,\frac{N\cdot m^2}{C^2}\right)\left(1.87\times10^{-7}\,\frac{C}{m}\right)\int_{.45m}^{.60m}\frac{dx}{x^2}$$

$$E = 1.68 \times 10^{3}\ \frac{N\cdot m}{C}\left[-\frac{1}{x}\right]_{.45m}^{.6m} = 1.68\times10^{3}\ \frac{N\cdot m}{C}\left[\frac{1}{.45m} - \frac{1}{.6m}\right]$$

$$E = 9.3 \times 10^{2}\ N/C$$

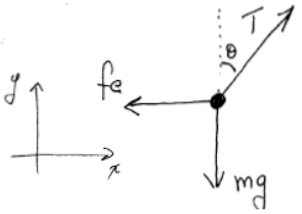
7- Two identical small spheres, having the same mass, and negligibly small radius, carry charges of 75 nC and 25 nC as shown in the figure. The spheres are attached to non-conducting, 15 cm long threads, of negligible mass. The far ends of the threads are secured to a pin in the ceiling and the spheres are allowed to hang freely as shown, coming to an equilibrium condition when they are 5.0 cm apart.



a) Calculate the magnitude of the force exerted by one sphere on the other when they are in equilibrium.

$$f_e = \frac{1}{4\pi\epsilon_0}\left(\frac{|Q_1\, Q_2|}{R^2}\right) = 9\times10^9\ \frac{N\cdot m^2}{C^2}\left[\frac{(75\times10^{-9}c)(25\times10^{-9}c)}{(.05m)^2}\right] = \underline{6.75\times10^{-3}\ \text{newton}}$$

b) In the space below, draw a free body diagram for either one of the spheres. Include all relevant forces.



$$\sin\theta = \frac{2.5cm}{15cm} \Rightarrow \theta = 9.6^\circ$$

c) Calculate the mass of either one of the spheres.

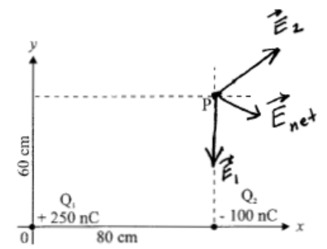
$$\left.\begin{aligned} \Sigma f_x &= -f_e + T\sin\theta = m a_x^{\,0} \\ \Sigma f_y &= -mg + T\cos\theta = m a_y^{\,0} \end{aligned}\right\} \ a_x = a_y = 0 \text{ since sphere is in equilibrium}$$

$$T = \frac{f_e}{\sin\theta} \quad \text{then} \quad -mg + \left(\frac{f_e}{\sin\theta}\right)\cos\theta = 0 \quad g = \frac{f_e}{g\tan\theta} = \frac{6.75\times10^{-3}\,N}{(9.8\times10^{0}\,m/s^2)(\tan 9.6^\circ)} = 4.1\times10^{-3}\ kg = \underline{4.1\ \text{gram}}$$

8- Two point charges are fixed in place in an xy coordinate system as shown in the figure. There is no charge located at the point P.

a) Sketch, at point P on the figure, three vectors, representing the electric field due to Q_1 alone, due to Q_2 alone, and the net field due to both.

b) Calculate the net electric field vector (expressed in unit vector form) at P due to the two charges.



$$E_1 = k\frac{Q_1}{R_1^2} = \left(9\times10^9\,\frac{N\cdot m^2}{C^2}\right)\frac{100\times10^{-9}C}{(.6m)^2}$$

E_1 in $-y$ direction so $\vec{E}_1 = 2500(-\hat{j})\ V/m$

$$E_2 = k\frac{Q_2}{R_2^2} = \left(9\times10^9\,\frac{N\cdot m^2}{C^2}\right)\frac{(250\times10^{-9}c)}{(.6m)^2+(.8m)^2} = 2250\ V/m$$

E_1 in $.8\hat{i} + .6\hat{j}$ direction, so $\vec{E}_2 = 1800\hat{i} + 1350\hat{j}\ V/m$

$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 = 1800\hat{i} - 1150\hat{j}\ V/m$$

c) Calculate the net electric potential at P due to the two charges

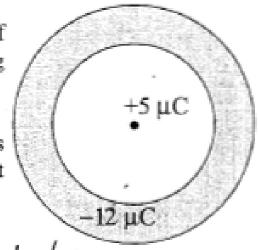
$$V_P = V_1 + V_2 = k\frac{Q_1}{R_1} + k\frac{Q_2}{R_2} \quad \text{scalar}$$

$$V_P = \left(9\times10^9\,\frac{N\cdot m^2}{C^2}\right)\left\{\left[\frac{-100\times10^{-9}C}{.6m}\right] + \left[\frac{+250\times10^{-9}C}{\sqrt{(.8m)^2+(.6m)^2}}\right]\right\} = +750\,V = (-1500V) + (+2250V)$$

d) Calculate the amount of work that the **electric field** would do on a +5.0 nC charge brought to point P from infinitely far away.

$$W = -q\,\Delta V = -q\,V_P = -(5\times10^{-9}c)(750V - 0V) = -3.8\times10^{-6}\,J$$

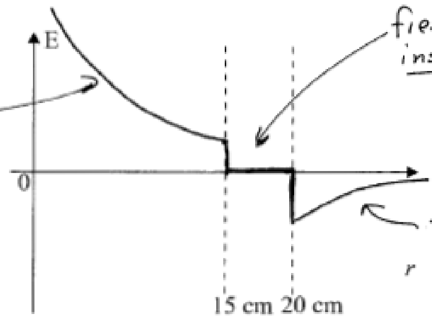
9- A hollow, **conducting**, spherical shell has an inner radius of 0.15 m and an outer radius of 0.20 m. A point charge of +5.0 μC is fixed in place at the center of the hollow cavity, and an additional charge of −12 μC is placed on the conducting shell itself.



a) Sketch, on the axes provided, an approximate graph of electric field strength versus radius. Treat outward directed field as positive, and inward as negative. You do not have to compute numerical values.

field of a +5 μC point charge

field zero inside conductor



field of −7 μC (net) point charge located at r = 0

b) Determine the amounts of charge on the inner and outer surfaces of the conducting shell.

charge on inner surface must "cancel" point charge at center, so $Q_{inner} = -5\,\mu C$

now, $-12\,\mu C = Q_{inner} + Q_{outer} \Rightarrow Q_{OUTER} = -7\,\mu C$

c) Calculate the electric potential at a point 1.0 meter from the center of the shell. What assumption did you make to perform this calculation?

outside the shell, treat it like a point charge of a net −7 μC $[+5\,\mu C + (-12\,\mu C)]$ at the center

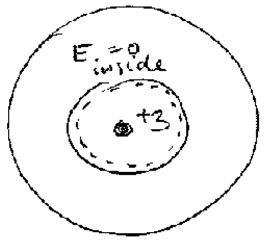
so $V = K\dfrac{Q}{r} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{r} = \left(9\times10^{9}\,\frac{N\cdot m^2}{C^2}\right)\left(\dfrac{-7\times10^{-6}\,C}{1.0\,m}\right)$

-6.3×10^{4} volt

10- An isolated conductor of arbitrary shape has a net charge of +10 μC. Inside the conductor is a cavity within which is a point charge q = +3.0 μC.

(i) What is the charge on the wall of the cavity? Circle the right answer.

+3.0 μC −3.0 μC +10 μC −10 μC +13 μC



(i) Use Gauss' law, with surface just inside conducting material

$q_{encl} = \epsilon_0 \oint \vec{E}\cdot d\vec{A} = 0$

since E = 0 inside metal.

So $q_{encl} = q_{point} + q_{inner} = 0$

$\Rightarrow q_{inner} = -q_{point} = -3\,\mu C$

(ii) What is the charge on the outer surface of the conductor? Circle the right answer:

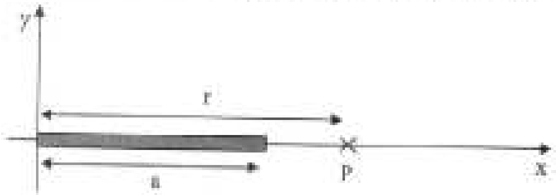
+3.0 μC –3.0 μC +10 μC +7 μC +13 μC

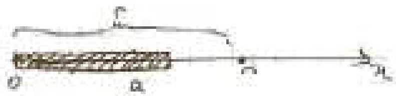
By charge conservation

$q_{TOT,\,SHELL} = 10\,\mu C = q_{inner} + q_{outer}$

$q_{outer} = 10\,\mu C - q_{inner} = 10\,\mu C - (-3\,\mu C) = \underline{+13\,\mu C}$

12- Positive charge Q is distrubuted uniformly along the x-axis from $x=0$ to $x=a$. Calculate the x and y components of the electric field produced by the charge distribution Q at the point P located on the positive x-axis at $x=r$, where $r>a$.

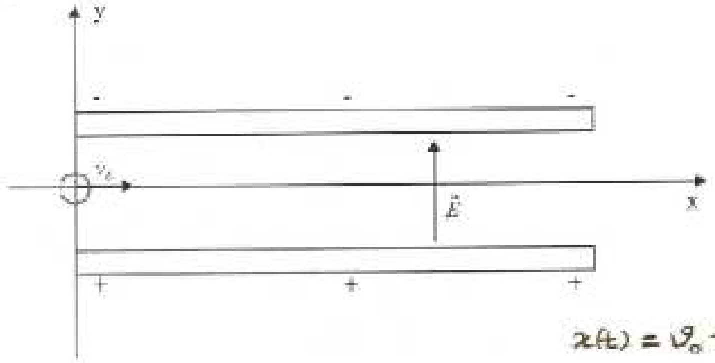




$E_x = \int_0^a \frac{1}{4\pi\epsilon_0}\frac{\lambda\,dx}{(r-x)^2}$, $\lambda = \frac{Q}{a}$

$E_x = \frac{\lambda}{4\pi\epsilon_0}(r-x)^{-1}\Big|_0^a = \frac{\lambda}{4\pi\epsilon_0}\left[\frac{1}{r-a} - \frac{1}{r}\right]$ $E_x = \frac{\lambda}{4\pi\epsilon_0}\frac{a}{(r-a)(r)}$

13- An electron is launched into a uniform electric field (magnitude E, direction shown in the figure below) with an initial horizontal velocity v_0. Initial coordinates of the electron are $x_0=0$ and $y_0=0$. Find out the equation describing the trajectory of the electron ($y=f(x)$, where f is a function of e, E, m, and v_0. The charge of an electron is $-e$.).

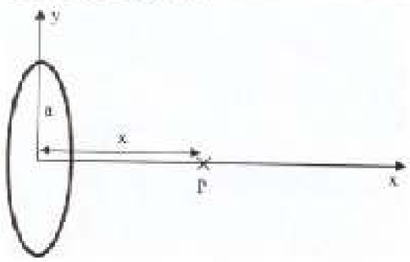


$$x(t) = v_0 t$$

$$y(t) = -\frac{1}{2}\frac{eE}{m}t^2 = -\frac{1}{2}\frac{eE}{m}\left(\frac{x}{v_0}\right)^2 \rightarrow$$

$$\boxed{y = -\frac{1}{2}\frac{eE}{m v_0^2}x^2}$$

14- A ring shaped conductor with radius a carries a total charge Q uniformly distributed around it. Find the electric field at a point P that lies on the axis of the ring at a distance x from its center.

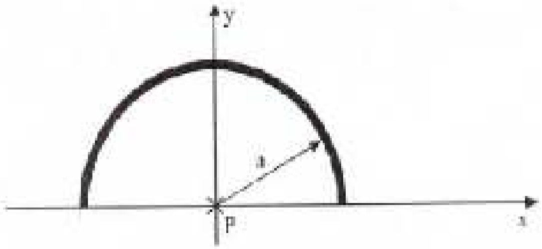


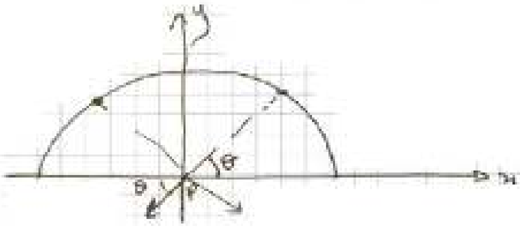
$$E_x = \frac{1}{4\pi\epsilon_0}\frac{Q}{(x^2+a^2)}\cdot\frac{x}{\sqrt{x^2+a^2}}$$

$$= \frac{Q}{4\pi\epsilon_0}\frac{x}{(x^2+a^2)^{3/2}},$$

along the $+x$ direction

15- Positive charge Q is distributed uniformly around a semicircle of radius a. Find the electric field (magnitude and direction) at the center of curvature P.



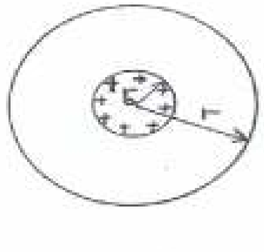


$$E_y = \int -\frac{1}{4\pi\varepsilon_0}\frac{dQ}{a^2}\sin\theta \quad , \quad dQ = r\,d\theta\,\lambda$$

$$= \int -\frac{1}{4\pi\varepsilon_0}\frac{r\lambda\,d\theta}{a^2}\sin\theta = -\frac{1}{4\pi\varepsilon_0}\frac{r\lambda}{a^2}\int_0^{\pi}\sin\theta\,d\theta = -\frac{1}{4\pi\varepsilon_0}\frac{r\lambda}{a^2}\left(-\cos\theta\right)\Big|_0^{\pi} = -\frac{1}{4\pi\varepsilon_0}\frac{r\lambda}{a^2}\,2$$

$$\boxed{E_y = -\frac{1}{2\pi\varepsilon_0}\frac{r\lambda}{a^2}}$$ along the $-y$ direction.

19- A conducting sphere of radius r_0 is located at the origin and has total charge Q. Derive the electric field $\vec{E}$ at $\vec{r} = x\,\mathrm{i} + y\,\mathrm{j} + z\,\mathrm{k}$ for $r < r_0$ and for $r > r_0$.



The charge is distributed uniformly over the surface of the conducting sphere

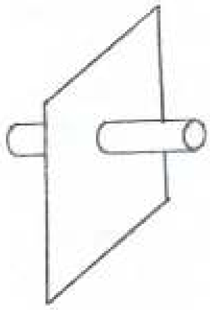
By symmetry, $\vec{E} = E(r)\,\hat{r}$.

Gauss' Law: $\displaystyle\int_{\text{sphere}} \vec{E}\cdot d\vec{s} = E\int_{\text{sphere}} ds = E\,4\pi r^2 = \frac{q_{\text{inside}}}{\varepsilon_0}$.

$r < r_0$: $\vec{E} = 0$,

$r > r_0$: $\vec{E} = \dfrac{Q}{4\pi\varepsilon_0 r^2}$.

20- The surface of a metal is along the xz plane and has surface charge density ρ_s (unit = Coulomb/m^2). Derive the electric field $\vec{E}$ at $\vec{r} = x\,\mathrm{i} + y\,\mathrm{j} + z\,\mathrm{k}$ outside and inside the metal.



Let the metal be at $y<0$.

$\vec{E} = 0$ inside the metal ($y<0$).

By symmetry, $\vec{E} = E(y)\,\hat{j}$ outside the metal.

Gauss' Law: $\displaystyle\int_{\text{cylinder}} \vec{E}\cdot d\vec{s} = E\int_{\substack{\text{outside}\\ \text{base}}} ds = E\pi r^2 = \frac{\rho_s \pi r^2}{\varepsilon_0}$ $\quad E = \dfrac{\rho_s}{\varepsilon_0}$.

$\vec{E}\,(\text{outside}) = \dfrac{\rho_s}{\varepsilon_0}\,\hat{j}$.

21- Consider a system of two positive point charges of magnitude q on the y axis at coordinates $(0, a)$ and $(0, -a)$ shown in Fig.3. Find the electric potential at a point $P(x, y)$ and evaluate the electric field using the knowle of the potential field.

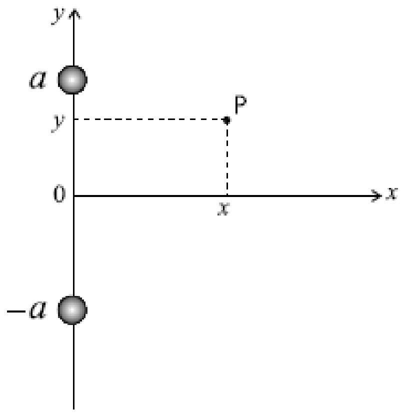


FIG. 3: A system of two point charges of magnitude q.

Solution The electric potential at (x, y) is given by the superposition

$$V(x, y) = kq\left(\frac{1}{\sqrt{x^2 + (y-a)^2}} + \frac{1}{\sqrt{x^2 + (y+a)^2}}\right)$$

from which we can evaluate $E_x = -dV/dx$ to be

$$E_x(x, y) = kqx\left(\frac{1}{(x^2 + (y-a)^2)^{3/2}} + \frac{1}{(x^2 + (y+a)^2)^{3/2}}\right).$$

Similarly $E_y = -dV/dy$ becomes,

$$E_y(x, y) = kq\left(\frac{y-a}{(x^2 + (y-a)^2)^{3/2}} + \frac{y+a}{(x^2 + (y+a)^2)^{3/2}}\right).$$

22- Consider a system of two positive point charges of magnitude q on the y axis at coordinates $(0, a)$ and $(0, -a)$ as shown in Fig.4. Calculate the electric field at a point $P(x, 0)$ and determine the electric potential from the knowledge of the electric field.

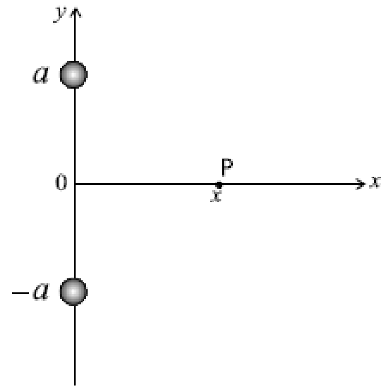


FIG. 4: A system of two point charges of magnitude q.

Solution Electric field at a point $(x, 0)$ is calculated directly as

$$\vec{E}(x) = k\frac{2qx}{(x^2 + a^2)^{3/2}}\hat{x}.$$

Using the electric field we evaluate the potential at the same point relative to a reference point at infinity through

$$\begin{aligned} V(x) &= \int_x^\infty E(x)dx = 2kq\int_x^\infty \frac{xdx}{(x^2 + a^2)^{3/2}}, \\ &= 2kq\frac{1}{\sqrt{x^2 + a^2}}. \end{aligned}$$

In evaluating the integral the change of variable $u = x^2 + a^2$ may be used.