

5) Your cell phone has a power output of about 0.5 watts. A typical cell phone tower receiver requires a signal of $0.2 \mu\text{W/m}^2$ to receive your phone.

(a) what is the maximum range of your cell phone?

$$S_{\text{avg}} = 0.2 \times 10^{-6} \text{ W/m}^2 = \frac{P_0}{4\pi R^2} = \frac{0.5 \text{ W}}{4\pi R^2} \quad R = \sqrt{\frac{0.5 \text{ W}}{(4\pi)(0.2 \times 10^{-6} \text{ W/m}^2)}} = 446 \text{ m}$$

(b) at the maximum range, what is the magnitude of the electric field generated by your phone?

$$S_{\text{avg}} = I = \frac{1}{2c\mu_0} E_{\text{max}}^2 \Rightarrow E_{\text{max}} = \sqrt{2Ic\mu_0} = \left[2(0.2 \times 10^{-6} \text{ W/m}^2)(3 \times 10^8 \text{ m/s})(4\pi \times 10^{-7} \text{ H/m}) \right]^{1/2}$$

$$E_{\text{max}} = 1.2 \times 10^{-2} \text{ V/m} = 12 \text{ mV/m}$$

(c) at the maximum range, what is the magnitude of the magnetic field generated by your phone?

$$B_{\text{max}} = \frac{E_{\text{max}}}{c} = 4.1 \times 10^{-11} \text{ T}$$

6) A long, conducting hollow cylinder has an inner radius of 1.0 cm and an outer radius of 5.0 cm. A current of 100 amperes, uniformly distributed across the cross section of the cylinder, flows along its length in a direction out of the page as shown here. Let r be the distance measured from the center of the cylinder. Answer the following questions.

(a) what is the magnitude of the magnetic field at $r=1.0$ cm

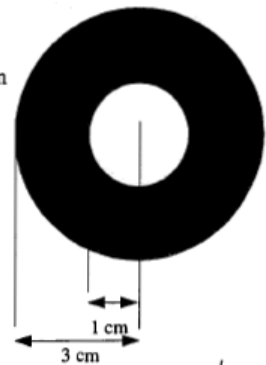
Ampere's law $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}} \quad B(2\pi R) = \mu_0 I_{\text{enc}}$
 at $r=1\text{cm} \quad I_{\text{enc}} = 0$, so $B=0$

(b) what is the magnitude of the magnetic field at $r=3.0$ cm

at $r=3\text{cm} \quad I_{\text{enc}} = 100\text{A}$, $B = \frac{\mu_0(100\text{A})}{2\pi(0.03\text{m})} \rightarrow 6.7 \times 10^{-4} \text{ T}$

(c) what is the magnitude of the magnetic field at $r=6.0$ cm

at $r=6\text{cm} \quad I_{\text{enc}} = 100\text{A}$, $B = \frac{\mu_0(100\text{A})}{2\pi(0.06\text{m})} \rightarrow 3.3 \times 10^{-4} \text{ T}$



7) A solenoid 1.35 m long and 2.90 cm in diameter carries a current of 17.0 A. The magnetic field inside the solenoid is 13.0 mT.

(i) Calculate the number of turns per unit length for this solenoid.

$$B = \mu_0 n I \quad \Rightarrow n = \frac{B}{\mu_0 I} = \frac{13.0 \times 10^{-3} \text{ T}}{17.0 \text{ A} \cdot 4\pi \times 10^{-7} \text{ H/m}} = 608.5 \text{ m}^{-1}$$

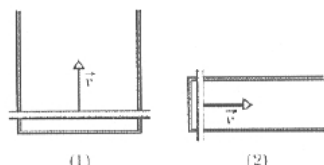
(ii) Calculate the length of the wire forming the solenoid.

Length of one winding $L_{\text{loop}} = 2\pi r = \pi d = \pi \cdot 2.9 \times 10^{-2} \text{ m}$

$$N = nL = 608.5 \text{ m}^{-1} \cdot 1.35 \text{ m} = 821.5 \text{ turns}$$

$$L_{\text{wire}} = N \cdot L_{\text{loop}} = 821.5 \cdot \pi \cdot 2.9 \times 10^{-2} \text{ m} = 74.8 \text{ m}$$

- 8) The figure shows two circuits in which a conducting bar is slid at the same speed v through the same uniform external magnetic field and along a U-shaped wire. The parallel lengths of the wire are separated by $2L$ in circuit 1 and by L in circuit 2. The current induced in circuit 1 is clockwise.



- (a) Is the direction of the external magnetic field into or out of the page?

i_1 clockwise means \vec{B}_{ind} is into paper. Φ_B is increasing because area is increasing. Thus \vec{B}_{ind} is opposite to \vec{B}_{ext} .
so \vec{B}_{ext} out of paper

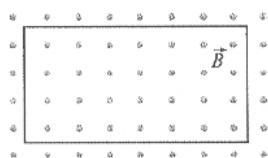
- (b) Is the direction of the current induced in circuit 2 clockwise or counterclockwise?

Φ_B is also increasing, through circuit 2 $\Rightarrow i_{ind}$ clockwise

- (c) Is the emf induced in circuit 1 larger than, the same as, or smaller than that in circuit 2?

$$\mathcal{E} = BLv \quad \mathcal{E}_1 = BL_1v = B(2L)v \quad \mathcal{E}_2 = BL_2v = B(L)v \quad \rightarrow \text{so } \mathcal{E}_1 > \mathcal{E}_2$$

- 9) The figure shows a rectangular loop of height $h = 20$ cm and width $w = 40$ cm, which is perpendicular to a uniform magnetic field \vec{B} directed out of the page. The resistance of the loop is $R = 2.0 \, \Omega$. At time $t = 0$, the magnitude of the magnetic field starts to change according to $B(t) = 0.1t^2 + 0.5$, where B is measured in tesla, and t is measured in seconds.



\uparrow has units of $\frac{T}{s^2}$

- (a) Calculate the magnitude of the emf induced in the circuit at time $t = 1.0$ s.

$$\Phi_B(t) = \int \vec{B} \cdot d\vec{A} = B(t)A \quad \text{since } B \text{ uniform. } \mathcal{E} = -\frac{d\Phi_B}{dt} = -(2 \cdot 0.1t) \cdot A \quad |\mathcal{E}| = 0.2|t| \cdot hw$$

$$|\mathcal{E}|_{t=1.0} = 0.2 \frac{T}{s^2} \cdot (2.0s) \cdot 0.2m \cdot 0.4m = \underline{\underline{0.016V}}$$

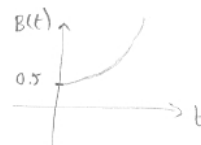
- (b) Find the magnitude and the direction of the current i induced in the loop at time $t = 1.0$ s.

Indicate the direction on the figure.

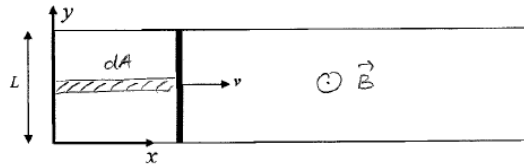
$$i = \frac{\mathcal{E}}{R} = \frac{0.016V}{2.0\Omega} = \underline{\underline{0.008A}} \quad \text{Direction } \Phi_B \text{ increases from } \vec{B} \odot \rightarrow \vec{B}_{ind} \otimes \text{ into paper } i_{ind} \text{ clockwise}$$

- (c) Will the direction of the induced current stay the same for all times after $t = 0$? (Answer yes or no, and explain).

$B(t)$ increases monotonically after $t = 0$ so Φ_B always increases after $t = 0$.
 \vec{B}_{ind} opposes change in Φ_B . This means direction of induced current stays the same



13) A rod of length $L = 10 \text{ cm}$ is forced to move at a constant speed $|v|$ to the right of 4 m s^{-1} along a pair of conducting rails. The rod has a resistance of 0.3Ω while the rails and connecting strip can be assumed to have no resistance. The apparatus, which forms a loop, lies in a non-uniform magnetic field \vec{B} that is constant in time and points out of the page everywhere. The magnitude of the magnetic field is given by $|\vec{B}| = 5.0 \times 10^{-3} y^2 \text{ T}$ when y is measured in meters



(a) What is the magnitude of the induced EMF in the loop?

$$|\mathcal{E}| = \left| -N \frac{d\Phi_B}{dt} \right| = \left| \frac{d\Phi_B}{dt} \right| \quad \Phi_B(t) = \int \vec{B} \cdot d\vec{A} = \int_0^L B(y) x(t) dy$$

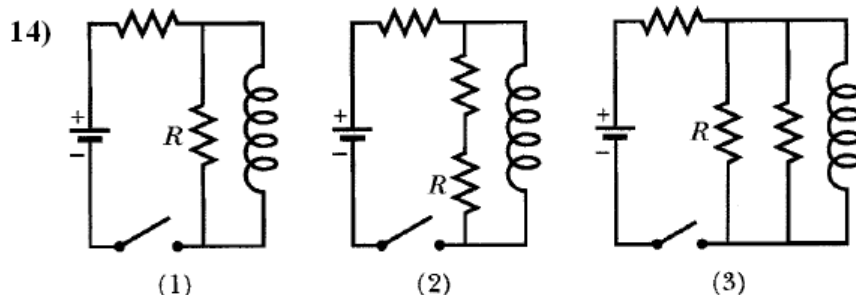
$$\Phi_B(t) = \int_0^L 5 \cdot 10^{-3} \frac{\text{T}}{\text{m}^2} y^2 x(t) dy = 5 \cdot 10^{-3} \frac{\text{T}}{\text{m}^2} \cdot x(t) \left[\frac{y^3}{3} \right]_0^L = \frac{5}{3} \cdot 10^{-3} \frac{\text{T}}{\text{m}^2} L^3 x(t)$$

$$|\mathcal{E}| = \frac{d\Phi_B}{dt} = \frac{5 \cdot 10^{-3} \text{T}}{3} L^3 \frac{dx}{dt} = \frac{5}{3} \cdot 10^{-3} \frac{\text{T}}{\text{m}^2} \cdot L^3 \cdot v = \frac{5}{3} \cdot 10^{-3} \frac{\text{T}}{\text{m}^2} \cdot (0.1 \text{ m})^3 \cdot 4 \frac{\text{m}}{\text{s}} = \underline{\underline{6.67 \cdot 10^{-6} \text{ V}}}$$

(b) What are the magnitude and direction (clockwise, counter clockwise or no current if the magnitude is zero) of the current induced in the loop?

$$i = \frac{\mathcal{E}}{R} = \frac{6.67 \cdot 10^{-6} \text{ V}}{0.3 \Omega} = \underline{\underline{2.22 \cdot 10^{-5} \text{ A}}} \quad \Phi \text{ increasing so } \vec{B}_{\text{ind}} \text{ opposite } \vec{B} \cdot \vec{B}_{\text{ind}} \otimes$$

\Rightarrow ind clockwise



Examine the three circuits above. The resistors, labeled or not, all have the same resistance. The inductors in all three circuits are the same, as are all three batteries. At the instant the switch is closed;

(a) Rank the circuits in order of the amount of **current** flowing through the **battery**, using the language $1 < 2 = 3$, $2 > 3 > 1$.

$$i_L = 0 \quad R_1 = 2R, \quad R_2 = 3R, \quad R_3 = R + \frac{1}{\frac{1}{R} + \frac{1}{R}} = R + \frac{R}{2} = 1.5R$$

$$i_3 > i_1 > i_2$$

After a time very long compared to the inductive time constant;

(b) Rank the circuits in order of the **current** flowing in the **resistor** marked R.

All current flows in inductor where $R = 0$ $i_R = 0$ for all three, all tie

(c) Rank the circuits in terms of the **magnetic energy** now stored in the **inductor**.

$$U_B = \frac{1}{2} L i_L^2 \quad \text{All current in } L \quad \text{so} \quad \underline{\underline{\text{all tie}}}$$

15) In an oscillating LC circuit in which $C = 4.00 \mu\text{F}$, the maximum potential difference across the capacitor during the oscillations is 1.50 V , and the maximum current through the inductor is 50.0 mA .

(a) Calculate the inductance L .

$$V_C(t) = V_C \cos(\omega_c t), \quad i(t) = I \sin(\omega_c t)$$

$$q(t) = C V_C(t) = C V_C \cos(\omega_c t) = Q \cos(\omega_c t)$$

Conservation of energy:

$$U_{E, \max} = U_{B, \max} \Rightarrow \frac{1}{2C} Q^2 = \frac{1}{2} L I^2$$

$$\Rightarrow L = \frac{1}{C} \frac{Q^2}{I^2} = \frac{1}{C} \frac{(C V_C)^2}{I^2} = \frac{C V_C^2}{I^2} = \frac{4 \cdot 10^{-6} \text{ F} \cdot (1.5 \text{ V})^2}{(50 \cdot 10^{-3} \text{ A})^2} = \underline{\underline{0.0036 \text{ H}}}$$

(b) Calculate the frequency f of the oscillations.

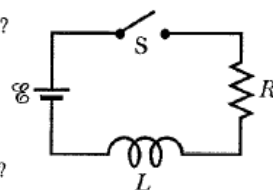
$$f = \frac{\omega}{2\pi} = \frac{1}{\sqrt{LC}} \cdot \frac{1}{2\pi} = \frac{1}{2\pi \sqrt{0.0036 \text{ H} \cdot 4 \cdot 10^{-6} \text{ F}}} = \underline{\underline{1.33 \text{ kHz}}}$$

18) The figure shows an RL circuit, where $R = 22 \Omega$, $L = 1 \text{ mH}$, and the battery voltage $\mathcal{E} = 12.0 \text{ V}$.

(i) What is the current in the circuit a long time after the switch S is closed?

$$i(t) = \frac{\mathcal{E}}{R} (1 - e^{-t/\tau})$$

$$i(t \rightarrow \infty) = \frac{\mathcal{E}}{R} = \frac{12.0 \text{ V}}{22 \Omega} = 5.5 \times 10^{-1} \text{ (A)}$$



(ii) How long does it take for the current to reach 1/2 of its maximum value?

$$i(t = t_{1/2}) = \frac{\mathcal{E}}{R} (1 - e^{-t_{1/2}/\tau}) = \frac{\mathcal{E}}{R} \cdot \frac{1}{2} \cdot e^{-t_{1/2}/\tau} = \frac{1}{2} \cdot e^{-t_{1/2}/\tau} = -\ln 2$$

$$\therefore t_{1/2} = \tau \ln 2 = \left(\frac{L}{R}\right) \ln 2 = \frac{1 \times 10^{-3} \text{ H}}{22 \Omega} \cdot \ln 2 = 3.15 \times 10^{-5} \text{ (sec)}.$$

(iii) At the time when current reaches 1/2 of its maximum value, what is the rate at which energy is being stored in the inductor?

$$U_B = \frac{1}{2} L i^2 \quad \frac{dU_B}{dt} = \frac{di}{dt} \cdot L \cdot i$$

$$\left. \frac{dU_B}{dt} \right|_{t=t_{1/2}} = \left. \frac{di}{dt} \right|_{t=t_{1/2}} \cdot L \cdot i(t_{1/2}) \quad ; \quad i(t_{1/2}) = \frac{1}{2} \frac{\mathcal{E}}{R}.$$

$$\left. \frac{di}{dt} \right|_{t=t_{1/2}} = \frac{\mathcal{E}}{R} \cdot \left(\frac{1}{\tau}\right) e^{-t_{1/2}/\tau} = \frac{\mathcal{E}}{R} \cdot \frac{R}{L} \cdot \frac{1}{2} = \frac{1}{2} \frac{\mathcal{E}}{L}$$

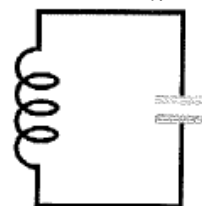
$$\left. \frac{dU_B}{dt} \right|_{t=t_{1/2}} = \frac{1}{2} \cdot \frac{\mathcal{E}}{L} \cdot L \cdot \frac{1}{2} \frac{\mathcal{E}}{R} = \frac{1}{4} \frac{\mathcal{E}^2}{R} = \frac{1}{4} \frac{(12.0 \text{ V})^2}{22 \Omega} = 1.64 \text{ (W)}$$

19) An oscillating LC circuit consisting of a $1.0 \mu\text{F}$ capacitor and a 3.0 mH coil is shown in the figure. It has a maximum voltage of 3.0 V across the capacitor.

(i) What is the maximum charge on the capacitor?

$$q_0 = CV_0 = (1.0 \times 10^{-6} \text{ F})(3.0 \text{ V}) = 3.0 \times 10^{-6} \text{ (C)}$$

(ii) If the charge on the capacitor is maximized at $t = 0$, calculate the first time the current in the circuit reaches its largest magnitude.



$$q = q_0 \cos(\omega t + \phi) \quad \text{at } t=0 \quad q=q_0 \quad \therefore \phi=0$$

$$\text{Then } i = \frac{dq}{dt} = -\omega q_0 \sin \omega t$$

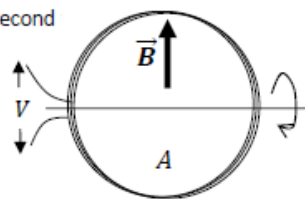
For i to be its largest magnitude for the first time, $t = \frac{\pi}{2\omega}$

$$\therefore t = \frac{\pi}{2} \cdot \sqrt{LC} = \frac{\pi}{2} \sqrt{(3.0 \times 10^{-3} \text{ H})(1.0 \times 10^{-6} \text{ F})} = 8.6 \times 10^{-5} \text{ (sec)}$$

(iii) What is the maximum energy stored in the magnetic field of the coil?

$$U_{B, \max} = U_{E, \max} = \frac{1}{2} \frac{q_0^2}{C} = \frac{1}{2} \frac{(3.0 \times 10^{-6} \text{ C})^2}{1.0 \times 10^{-6} \text{ F}} \\ = 4.5 \times 10^{-6} \text{ (J)}.$$

24) A coil of area 0.100 m^2 with 125 turns of wire is rotating at 60.0 cycles per second with the axis of rotation perpendicular to a 0.025 T magnetic field, as shown. At this instant in the picture, the top of the coil is coming toward you and the bottom is moving away from you. The two wires coming out on the left are the ends of the coil.



(a) What is the maximum voltage measured between the two lead wires as the coil rotates?

The emf generated in the loop is

$$V(t) = \left| \frac{Nd\Phi}{dt} \right| = N \frac{d}{dt} (AB \sin \omega t) = NAB\omega \cos \omega t,$$

so the maximum emf is

$$V_{\max} = NAB(2\pi f) = (125)(0.100 \text{ m}^2)(0.025 \text{ T})(2\pi \times 60 \text{ s}^{-1}) = \mathbf{118 \text{ V}}.$$

(b) What is the orientation of the coil with respect to the magnetic field when the maximum induced voltage occurs?

The maximum emf occurs when the loop coil is **parallel** to the magnetic field.

(c) What is the magnitude and direction (clockwise, counter-clockwise, or zero) of the current flow if the coil is attached to a 100Ω resistance, at the instant shown in the figure? Neglect the coil's resistance and inductance.

$$I = \frac{V}{R} = \mathbf{1.18 \text{ A}}.$$

At the moment shown, the flux is zero but increasing away from you as the loop rotates toward you. According to Lenz's Law, a **counterclockwise** current will be generated to counteract this, by generating flux toward you.

(d) What is the average power generated in the coil over a complete revolution?

The instantaneous power is $P = VI = V_{\max} I_{\max} \cos^2 \omega t$, and the square of the cosine oscillates symmetrically about a value of $\frac{1}{2}$, so $P_{\text{avg}} = \frac{1}{2} V_{\max} I_{\max} = \mathbf{69.6 \text{ W}}.$

27) A radio station on the surface of the earth (EARTH-ROCK-FM) broadcasts with an average power of 50 kW. Assume that the transmitter radiates in all directions above the ground.

$$P = 50 \text{ kW}$$

$$I = \frac{50 \times 10^3 \text{ W}}{2\pi (100 \times 10^3 \text{ m})^2} = 6.28 \times 10^{-10} \text{ W/m}^2$$

$$I = 7.96 \times 10^{-7} \text{ W/m}^2$$

(a) Find the amplitude of the electric field detected by a satellite at a distance of 100 km from the antenna.

$$I = \frac{1}{2} \frac{E^2}{377 \Omega} \Rightarrow E = \sqrt{I(2)(377 \Omega)} = 2.45 \times 10^{-2} \text{ V/m}$$

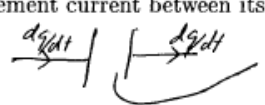
(b) Find the amplitude of the magnetic field detected by a satellite at a distance of 100 km from the antenna.

$$B = \frac{E}{c} = \frac{2.45 \times 10^{-2} \text{ V/m}}{3 \times 10^8 \text{ m/s}} = 0.82 \times 10^{-10} \text{ T}$$

(c) Is the magnitude of this magnetic field *much smaller*, *much larger*, or *about the same size* as the earth's magnetic field?

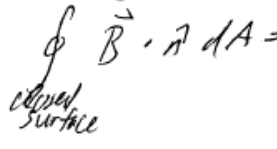
$B_{\text{earth}} \sim 10^{-4} \text{ T}$ so this field is much smaller
However note that the Earth's B field is constant

- 28) (a) Consider a parallel plate capacitor charging up at a rate $dQ/dt = 2.0 \text{ A}$. What is the conduction current and displacement current between its plates?



$$\left. \begin{array}{l} I_{\text{conduction}} = 0 \\ I_{\text{displacement}} = 2.0 \text{ A} \end{array} \right\} \begin{array}{l} I_{\text{total}} \\ = I_{\text{displacement}} \\ = \text{constant} \end{array}$$

- (b) What is Gauss' Law for magnetism? What are the consequences for magnetic monopoles?



$$\oint \vec{B} \cdot \vec{n} dA = 0$$
 closed surface

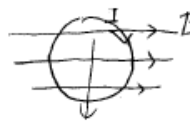
 no magnetic monopoles
 (consistent with experiment so far)

- (c) An electromagnetic radiation source radiates uniformly in all directions.

How does the magnitude of the E field vary with distance: (i) E is constant, (ii) $E \propto 1/r^2$,
 (iii) $E \propto 1/r$, $E \propto 1/r^3$.

$$I \propto |E|^2 \Rightarrow I \propto \frac{1}{r^2} \Rightarrow |E| \propto \frac{1}{r}$$

- (d) A current loop is in the plane of the paper. The B field is directed to the right. What is the direction of the torque on the current loop?

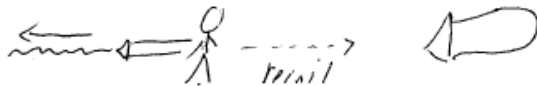


$$\vec{m} = IA \hat{n} \rightarrow \text{points into the paper}$$

$$\vec{\tau} = \vec{m} \times \vec{B} \rightarrow \text{points downward}$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$
 loop will rotate so that \vec{m} is aligned with \vec{B} field.

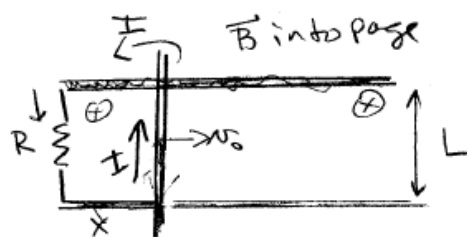
- (e) You are stranded in outer space outside your spaceship. Fortunately, you have a 100 MW laser attached to your spacesuit. How can you get back to your spaceship?



Fire in the direction opposite to the spaceship. The recoil momentum will send you back to the spaceship.

- 46) The figure below shows a pair of parallel conducting rails of negligible resistance, a distance L apart. A uniform magnetic field \vec{B} is directed into the page. A resistance R is connected across the rails, and a conducting bar of negligible resistance is being pulled across the rails with constant velocity v_0 to the right. Friction between the moving bar and rails is negligible.

(a) Determine the direction and magnitude of the current in the resistor.



The changing flux in the loop will induce an emf which will drive a current.

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \quad \Phi_B = L \times B \quad \frac{d\Phi_B}{dt} = LBv_0$$

The direction of the induced current will be such as to oppose the change of flux. The flux is increasing downward, so I will be in the direction that would produce an upward flux, I is in the counterclockwise direction. The current through R is down.

$$\mathcal{E} = IR = LBv_0 \quad I = \frac{LBv_0}{R} \text{ ccw}$$

(b) Determine the constant external force which must be applied to the bar to keep it moving with constant velocity v_0 to the right.

Once the current is established, \vec{B} exerts a force on the rod.

$$\vec{F} = I \vec{L} \times \vec{B} \quad \begin{array}{c} \uparrow I \\ \otimes \vec{B} \end{array} \quad \vec{F} \text{ is to the left, opposing the motion}$$

$$\vec{F}_B = \frac{L^2 B^2 v_0}{R} (-\hat{i}) \quad \text{force on rod by } \vec{B}$$

To keep the bar moving at constant velocity, an external force must be applied to cancel \vec{F}_B

$$\vec{F}_{\text{ext}} = \frac{B^2 L^2 v_0}{R} \hat{i}$$

(c) Determine the rate at which work must be done by the external force to maintain a constant velocity v_0 .

$$\text{Work} = \int \vec{F} \cdot d\vec{\ell} = F_{\text{ext}} \cdot \Delta x$$

$$\frac{dw}{dt} = F_{\text{ext}} \frac{dx}{dt} = F_{\text{ext}} v_0 \quad \frac{dw}{dt} = \frac{B^2 L^2 v_0^2}{R}$$

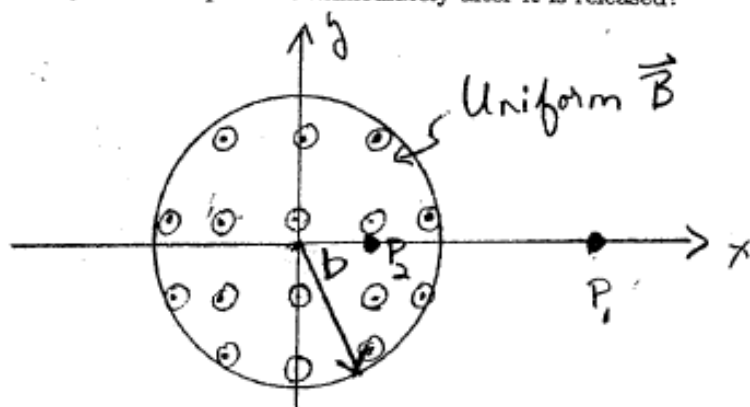
(d) Compare the rate at which the external agent does work to the power dissipated in the resistor.

$$P = I^2 R = \frac{L^2 B^2 v_0^2}{R^2} \cdot R = \frac{L^2 B^2 v_0^2}{R}$$

$$\text{Power (in } R) = \frac{L^2 B^2 v_0^2}{R} \Rightarrow \text{same as } \frac{dw}{dt} \text{ done by external force!}$$

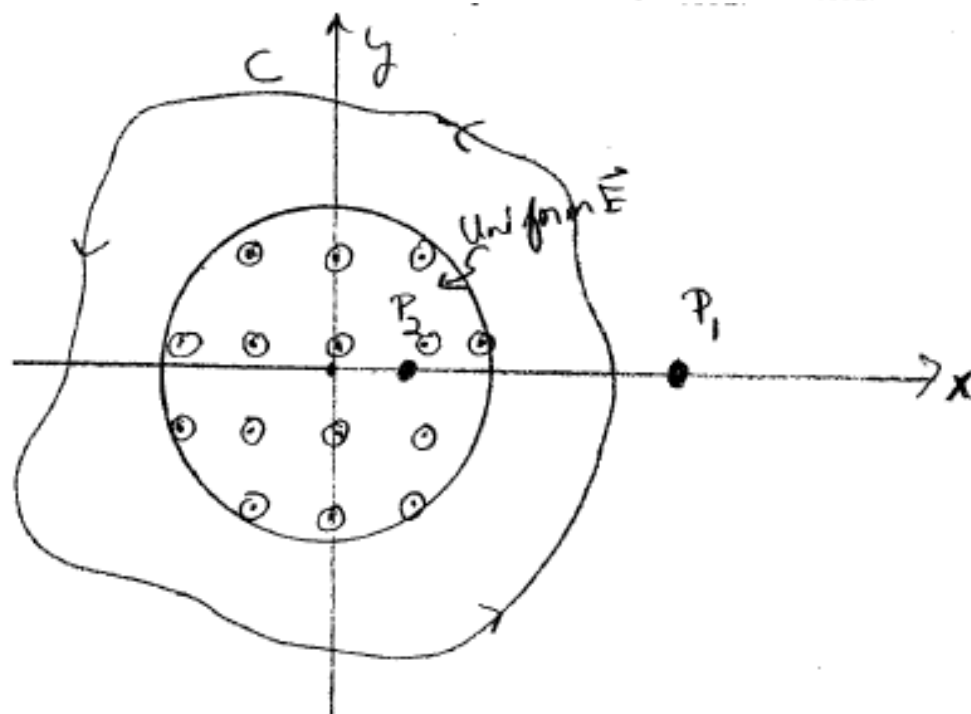
- 47) A cylindrical region of space has a uniform magnetic field \vec{B} pointing up out of the page as shown in the figure below. The coordinate r measures the distance from the axis of the cylindrical region. For purposes of this problem, we will neglect fringe field effects, so that we can assume the field drops abruptly to zero at $r = b$. Although the field is uniform in space, its magnitude is increasing at a constant rate $\frac{d|\vec{B}|}{dt} = \alpha$.

- (a) Determine the electric field \vec{E} everywhere in space due to the changing magnetic field. Be sure to indicate both the direction and magnitude of the field. Plot the magnitude of the field $|\vec{E}(r)|$ vs. r .
 (b) If an electron is released from rest at the point P_1 ($x = 2b, y = 0$) what acceleration if any does it experience immediately after it is released? If an electron is released from rest at the point P_2 ($x = \frac{b}{2}, y = 0$) what acceleration if any does it experience immediately after it is released?

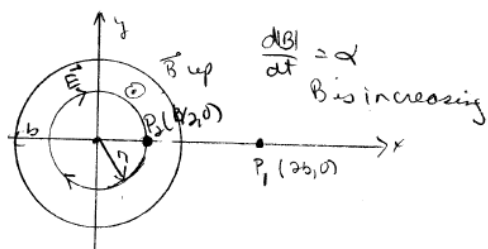


Now consider a similar situation, except that instead of a uniform magnetic field in the cylindrical region of space we now have a uniform electric field. As before, the magnitude of the electric field is increasing with time at a constant rate $\frac{d|\vec{E}|}{dt} = \alpha$.

- (c) Determine the magnetic field \vec{B} everywhere in space due to the changing electric field. Be sure to indicate both the direction and magnitude of the field. Plot the magnitude of the field $|\vec{B}(r)|$ vs. r .
 (d) Determine the displacement current I_D that passes through the region enclosed by the contour C shown in the figure below.
 (e) If an electron is released from rest at the point P_1 ($x = 2b, y = 0$), what acceleration if any does it experience immediately after it is released? If an electron is released from rest at the point P_2 ($x = \frac{b}{2}, y = 0$), what acceleration if any does it experience immediately after it is released?



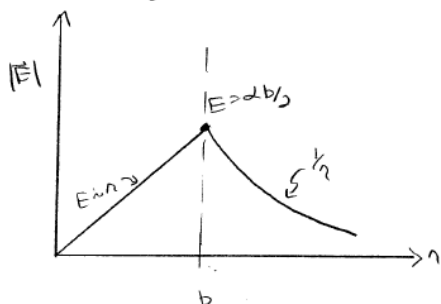
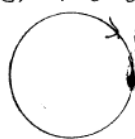
47)

(a) From Faraday's law, $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$

From cylindrical symmetry, \vec{E} is constant along a contour at constant r . The direction of \vec{E} is given by Lenz's Law, since Φ_B is increasing upwards, \vec{E} forms closed loops in the clockwise sense.

For $r < b$, $\Phi_B = \pi r^2 B$ $\frac{d\Phi_B}{dt} = \pi r^2 \frac{dB}{dt} = \pi r^2 \alpha$ $\oint \vec{E} \cdot d\vec{l} = 2\pi r |\vec{E}| = \pi r^2 \alpha$
 $|\vec{E}| = \frac{\alpha r}{2}$ - direction is cw loops, $r < b$

For $r > b$, $\Phi_B = \pi b^2 B$ $\frac{d\Phi_B}{dt} = \pi b^2 \alpha$ $2\pi r |\vec{E}| = \pi b^2 \alpha$ $|\vec{E}| = \frac{b^2 \alpha}{2r}$ - cw loops $r > b$

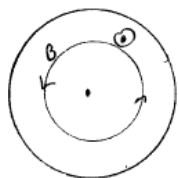
(b) An electron is released from rest at $x=2b, y=0$. $\vec{E} = -\vec{r} E$ acceleration is upward.

$\vec{a} = \frac{e b \alpha}{4 m_e} \hat{j}$ at P_1 At $P_2 (b/2, 0)$ $|\vec{E}|(b/2) = \frac{d(\Phi_B)}{2} = \frac{b^2 \alpha}{4}$

$\vec{a} = \frac{e b \alpha}{4 m_e} \hat{j}$ at P_2

$\vec{a} = \frac{e b \alpha}{4 m_e} \hat{j}$ at P_2

Note that since the electron is released from rest, it is not accelerated by the \vec{B} field.

(c) Now we have a charging electric field $\frac{dE}{dt} = \alpha$ 

from the extended form of Ampere's Law:

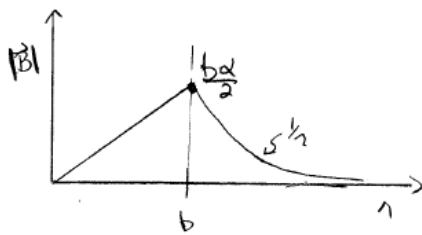
$\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$ By symmetry \vec{B} forms concentric loops. The direction is counter clockwise.

For $r < b$ $\Phi_E = \pi r^2 E$ $\frac{d\Phi_E}{dt} = \pi r^2 \frac{dE}{dt} = \pi r^2 \alpha$

$\oint \vec{B} \cdot d\vec{l} = 2\pi r B = \mu_0 \epsilon_0 \pi r^2 \alpha$ $|\vec{B}| = \frac{\mu_0 \epsilon_0 \alpha r}{2}$ direction is ccw loops for $r < b$

for $r > b$, $\Phi_E = \pi b^2 E$ $\frac{d\Phi_E}{dt} = \pi b^2 \alpha$ $\int \vec{B} \cdot d\vec{l} = 2\pi r B = \mu_0 \epsilon_0 \pi b^2 \alpha$

$|\vec{B}| = \frac{\mu_0 \epsilon_0 \alpha b^2}{2r}$ - ccw loops for $r > b$



(d) $I_D = \epsilon_0 \frac{d\Phi_E}{dt} = \epsilon_0 \pi b^2 \frac{dE}{dt}$

$I_D = \epsilon_0 \pi b^2 \alpha$

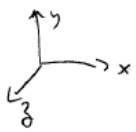
(e) If an electron is released from $x = 2b, y = 0$, it is out of the region of \vec{E} , and since it is released

from rest, it is not accelerated by \vec{B} ! $\boxed{\text{At } P_1, \vec{a} = 0}$

At $P_2 (x = b/2, y = 0)$, \vec{a} due to \vec{B} is still zero since the electron is released from rest.

But $\vec{E} \neq 0$ here & the electron will be accelerated in the direction \perp the page.

$\vec{F}_{\text{net}} = -e \vec{E} = -e E(t) \hat{k} = m_e \vec{a}$



$\vec{a} = \frac{-e E(t)}{m_e} \hat{k}$ acceleration is into the page.