FIZ102E PHYSICS-II

PART I. Analytic Problems

1) A long coaxial cable in Figure 3 consists of a long cylindrical inner conductor with radius a and an outer hollow cylindrical conductor with inner and outer radii b, c respectively. The wire carries opposite currents I through its inner and outer conductors with densities $J_{in}=\alpha r$ and $J_{out}=\beta r$ respectively, where r is the radial distance from the central axis and α, β are some proportionality constants.

16-What is the value of the constant β ?

a) $\frac{I}{\pi(c^3-b^3)}$ b) $\frac{I}{2\pi(c^3-b^3)}$ c) $\frac{2I}{\pi(a^3-c^3)}$ d) $\frac{I}{\pi(b^3-a^3)}$ e) $\frac{3I}{2\pi(c^3-b^3)}$

17-What is the magnetic field inside the inner conductor at a radial distance r ($0 \le r < a$)?

a) $\frac{\alpha r \mu_0 a^2}{2(a-r)}$ b) $\mu_0 \alpha r^2 - \mu_0 I$ c) $\mu_0 \alpha r^2 - \mu_0 \beta (b^2 - r^2)$ d) $\frac{1}{3}\mu_0 \alpha r^2$ e) $\frac{\alpha r \mu_0 a^2}{2\pi a}$

18-What is the magnetic field inside the outer conductor at a radial distance r ($b<r<c$)?

a) $\mu_0(\frac{1}{3}\alpha r^2 - \frac{I}{2\pi r})$ b) $\mu_0 \frac{1}{2\pi(r-b)}$ c) $\mu_0 \beta r^2 - \frac{\mu_0 I}{2\pi b}$ d) $\mu_0(\frac{I}{2\pi r} - \frac{2}{3}\alpha r^2)$ e) $\mu_0(\frac{1}{2\pi r} - \frac{1}{3r}\beta(r^3-b^3))$

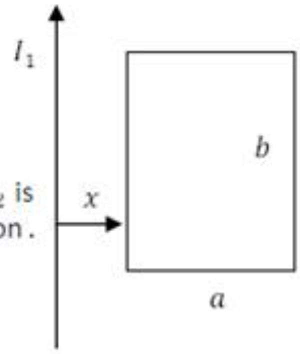
19-What is the magnetic field outside the coaxial cable at a radial distance r ($r>c$)?

a) Zero b) $\frac{\mu_0}{2\pi r}(\alpha a^2 + \beta(c^2-b^2))$ c) $\mu_0 \beta (c^2-b^2) - \frac{\mu_0 I}{2\pi a}$ d) $\mu_0 \frac{1}{\pi r}$ e) $\frac{\mu_0}{2r}(\alpha a^2 - \beta(c^2-b^2))$

20-What is the magnetic flux per unit length through the space between the conductors ($a<r<b$)?

a) zero b) $\frac{\mu_0 I}{2\pi} ln\frac{b}{a}$ c) $\mu_0 I ln\frac{b}{a}$ d) $\frac{1}{2}\mu_0 \alpha\beta(b^2-a^2) ln\frac{b}{a}$ e) $\frac{1}{2}\mu_0 \alpha (b^2-a^2) ln\frac{b}{a}$

2) A long straight wire carries a current I as shown. A square loop of dimensions $a \times b$ is positioned a distance x away as shown.



(a) What is the magnetic flux inside the square loop? Express it in terms of I_1, a, b, x, μ_0. Is it into or out of the page?

(b) If the current is $I_1(t) = I_0 t$ and the square loop has resistance R, what current I_2 is generated in the square loop as a function of time? Give the magnitude and direction.

(c) Suppose the current I_1 in the long wire is constant, but the square loop is pulled to the right at a constant speed v. What current I_2 flows in the square loop? Give the magnitude and direction. [*Hint*: $\frac{d}{dt} = v\frac{d}{dx}$. If you can't solve this part, you can still use the symbol I_2 in the following parts.]

(d) What power is needed to pull the loop at speed v when it is a distance x from the long wire? [*Note*: You can refer to previous symbolic results without rewriting them, even if you did not solve part of the problem.]

(a) $\Phi_m = \int_x^{x+a} B(r)bdr = \frac{\mu_0 b I_1}{2\pi}\int_x^{x+a}\frac{dr}{r} = \boldsymbol{\frac{\mu_0 b I}{2\pi}\ln\frac{x+a}{x}}$. The flux is **into the page**.

(b) $I_2 = \frac{1}{R}\frac{d\Phi_m}{dt} = \boldsymbol{\frac{\mu_0 b I_0}{2\pi R}\ln\frac{x+a}{x}}$.

Since the flux is increasing into the page, the current will flow **counter-clockwise** to counteract this increase

(c) Now, the rate of change of the flux is due to the motion, and $v = \frac{dx}{dt}$, so

$$\frac{d\Phi_m}{dt} = v\frac{d\Phi_m}{dx} = \frac{\mu_0 v b I_1}{2\pi}\frac{d}{dx}\ln\frac{x+a}{x} = \frac{\mu_0 v b I_1}{2\pi}\left(\frac{1}{x+a}-\frac{1}{x}\right) = -\frac{\mu_0 v a b I_1}{2\pi x(x+a)}$$

The magnitude of the current is then $I_2 = \boldsymbol{\frac{\mu_0 v a b I_1}{2\pi R x(x+a)}}$.

Since the magnetic flux into the page is decreasing as x increases, the current is **clockwise** to counteract this.

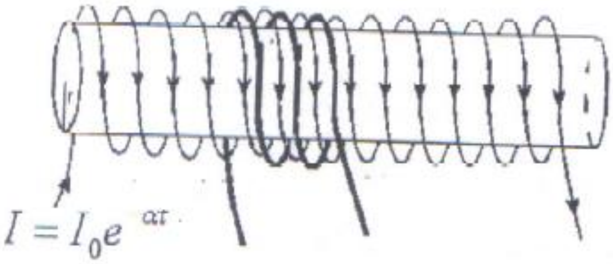
(d) The power put in by pulling the loop must equal that converted to heat in the resistance of the wire, by energy conservation. Therefore, $P = \boldsymbol{R I_2^2}$.

An equivalent solution can be obtained by calculating the forces between the wires. Parallel currents separated by a distance r attract with a force per unit length of

$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$. The net force is then attractive, with magnitude $F = \frac{\mu_0 b I_1 I_2}{2\pi}\left(\frac{1}{x}-\frac{1}{x+a}\right) = \frac{\mu_0 a b I_1 I_2}{2\pi x(x+a)}$

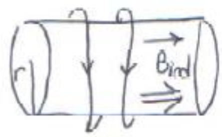
This gives $P = Fv = R I_2^2$, as above, as can be seen using the expression for I_2.

3) A very long solenoid of radius r made from n turns of wire per unit length carries a time-dependent current $I = I_0 e^{\alpha t}$ where α is a positive constant. There is another circular loop with 3 turns (shown as darker) and it is far away from the ends of the solenoid. The loop with 3 turns has resistance of R.



a) Write down the time dependent magnetic field inside the solenoid.
b) Determine the total flux on the circular loop.
c) Write Determine the emf induced on the circular loop.
d) Determine the direction of the induced current on the circular loop.
e) Find the magnitude of the induced current on the circular loop.

a) $B = \mu_0 n I = \mu_0 n I_0 . e^{-\alpha t}$



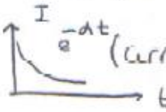
b) $\Phi = NB.A = 3\mu_0 n I_0 e^{-\alpha t} \pi r^2$

c) $\epsilon = -\frac{d\Phi}{dt} = -3\alpha\mu_0 n I_0 e^{-\alpha t} \pi r^2$

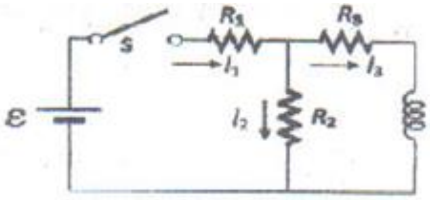
$\epsilon = -3\alpha\mu_0 n I \pi r^2$, $I = I_0 . e^{-\alpha t}$

c) $\epsilon = I.R$ $|\epsilon| = I_{ind}.R$ $3\alpha\mu_0 n I \pi r^2 = I_{ind}.R$ $\rightarrow I_{ind} = \frac{3\alpha\mu_0 n I \pi r^2}{R}$

d) $t=0 \rightarrow I = I_0$
$t=t \rightarrow I = I_0 . e^{-\alpha t}$



(current decreases.) On the same direction with the first coil



4) In the circuit shown in the figure, $R_1 = 20\ \Omega$, $R_2 = 20\ \Omega$, $R_3 = 40\ \Omega$, $L = 10\ H$ and $\varepsilon = 100V$.

I- What are the currents i_1 and i_2 immediately after the switch S is closed?

a) $i_1 = 2.5$ A, $i_2 = 2.5$ A b) $i_1 = 0$ A, $i_2 = 0$ A c) $i_1 = 3$ A, $i_2 = 2$ A d) $i_1 = 1$ A, $i_2 = 0$ A e) $i_1 = 0$ A, $i_2 = 1$ A

II- What are the currents i_1 and i_2 long time after the switch S has been closed?

a) $i_1 = 2.5$ A, $i_2 = 2.5$ A b) $i_1 = 0$ A, $i_2 = 0$ A c) $i_1 = 3$ A, $i_2 = 2$ A d) $i_1 = 1$ A, $i_2 = 0$ A e) $i_1 = 0$ A, $i_2 = 1$ A

III After the switch has been closed for a long time, it is re-opened. What are the magnitudes of the currents i_1 and i_2 just after re-opening the switch S?

a) $i_1 = 0$ A, $i_2 = 0$ A b) $i_1 = 0$ A, $i_2 = 1$ A c) $i_1 = 1$ A, $i_2 = 0$ A d) $i_1 = 2.5$ A, $i_2 = 2.5$ A e) $i_1 = 3$ A, $i_2 = 2$ A

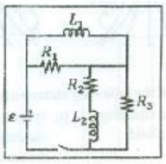
IV- After the re-opening, how much time in seconds does it take for the current on the inductor to decrease to half of its initial value?

a) $t = \frac{\ln 2}{e}$ b) $t = \frac{\ln 2}{3}$ c) $t = \frac{\ln 2}{2}$ d) $t = \frac{\ln 2}{6}$ e) $t = e \ln 2$

V- What are the currents i_2 and i_3 long time after the switch S has been re-opened?

a) $i_2 = 1$ A, $i_3 = 1$ A b) $i_2 = 2$ A, $i_3 = 1$ A c) $i_2 = 0$ A, $i_3 = 1$ A d) $i_2 = 1.5$ A, $i_3 = 1.5$A e) $i_2 = 0$ A, $i_3 = 0$ A

5) In the circuit in Figure 4 the switch is closed at t=0. Electric currents passing through the resistors R_1, R_2 and R_3 are i_1, i_2 and i_3 respectively.



21- Determine i_1 immediately after the switch is closed.

a) $\frac{\varepsilon}{R_1 + R_3}$ b) $\frac{\varepsilon}{R_3}$ c) zero d) $\frac{\varepsilon}{R_1 + \frac{R_2 R_3}{R_2 + R_3}}$ e) $\frac{R_2 + R_3}{R_2 R_3}\varepsilon$

22- Determine i_1 long time after the switch is closed.

a) $\frac{\varepsilon}{R_2}$ b) $\frac{\varepsilon}{R_1 + R_3}$ c) $\frac{\varepsilon}{R_1 + \frac{R_2 R_3}{R_2 + R_3}}$ d) Zero e) $\frac{\varepsilon}{R_2 + R_3}$

23- Determine i_2 immediately after the switch is closed.

a) zero b) $\frac{\varepsilon}{R_1 + \frac{R_2 R_3}{R_2 + R_3}}$ c) $\frac{R_3\varepsilon}{R_1 R_2 + R_1 R_3 + R_2 R_3}$ d) $\frac{\varepsilon}{R_2}$ e) $\frac{R_2 + R_3}{R_2 R_3}\varepsilon$

24- What is the voltage drop across the inductor L_2 after long time the switch has been closed ?

a) $\frac{\varepsilon}{R_2}$ b) zero c) $\frac{\varepsilon}{R_1}$ d) $\frac{\varepsilon}{R_1 + R_2}$ e) $\frac{\varepsilon}{R_3}$

25- The switch is reopened long time after the switch has been closed. Determine the current through the inductor L_1 as a function of time after the switch is reopened at t=0.

a) $\frac{R_2 + R_3}{R_2 R_3}\varepsilon(1 - e^{-\frac{tR_1}{L_1}})$ b) $\frac{\varepsilon}{R_1}e^{-\frac{tR_1}{L_2}}$ c) $\frac{R_2 + R_3}{R_2 R_3}\varepsilon e^{-\frac{tR_1}{L_1}}$ d) $\frac{\varepsilon}{R_1}e^{-\frac{tR_1}{L_1}}$ e) $\frac{\varepsilon}{R_1}(1 - e^{-\frac{tR_1}{L_1}})$

6) Assume that the intensity of solar radiation incident on the cloudtops of the Earth is 1 340 W/m^2.

(a) Calculate the total power radiated by the Sun, taking the average Earth–Sun separation to be 1.496 x 10^{11} m.

(b) Determine the maximum values of the electric and magnetic fields in the sunlight at the Earth's location.

c)The intensity of solar radiation at the top of the Earth's atmosphere is 1 340 W/m^2. Assuming that 60% of the incoming solar energy reaches the Earth's surface and assuming that you absorb 50% of the incident energy, make an order-of-magnitude estimate of the amount of solar energy you absorb in a 60-min sunbath.

Additional Problems

P34.47 (a) $\mathscr{P} = SA$: $\mathscr{P} = \left(1\,340\ \text{W/m}^2\right)\left[4\pi\left(1.496\times10^{11}\ \text{m}\right)^2\right] = \boxed{3.77\times10^{26}\ \text{W}}$

(b) $S = \dfrac{cB_{\text{max}}^2}{2\mu_0}$ so $B_{\text{max}} = \sqrt{\dfrac{2\mu_0 S}{c}} = \sqrt{\dfrac{2\left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(1\,340\ \text{W/m}^2\right)}{3.00\times10^8\ \text{m/s}}} = \boxed{3.35\ \mu\text{T}}$

$S = \dfrac{E_{\text{max}}^2}{2\mu_0 c}$ so $E_{\text{max}} = \sqrt{2\mu_0 cS} = \sqrt{2\left(4\pi\times10^{-7}\right)\left(3.00\times10^8\right)\left(1\,340\right)} = \boxed{1.01\ \text{kV/m}}$

P34.48 Suppose you cover a 1.7 m-by-0.3 m section of beach blanket. Suppose the elevation angle of the Sun is 60°. Then the target area you fill in the Sun's field of view is

$$(1.7\ \text{m})(0.3\ \text{m})\cos 30° = 0.4\ \text{m}^2.$$

Now $I = \dfrac{\mathscr{P}}{A} = \dfrac{U}{At}$ $U = IAt = \left(1\,340\ \text{W/m}^2\right)\left[(0.6)(0.5)\left(0.4\ \text{m}^2\right)\right]\left(3\,600\ \text{s}\right)\boxed{\sim10^6\ \text{J}}.$

7) A plane electromagnetic wave, with wavelenght $3.0\ m$, travels in vacuum in the positive y direction. The electric field component of the wave with amplitude $300\ V/m$ oscillates parallel to the z axis. The wave uniformly illuminates *a totally absorbing* surface of area $6.0\ m^2$.
(Speed of light in vacuum is $c = 3.0 \times 10^8\ m/s$, $\pi = 3$, $\mu_0 = 12 \times 10^{-7}\ T{\cdot}m/A$)

I- What is the angular frequency of the wave?

a) 1.0×10^8 rad/s b) 6.0×10^8 rad/s c) 2.0×10^8 rad/s d) 5.0×10^9 rad/s e) 12.0×10^8 rad/s

II- What is the amplitude of the magnetic field component of the wave, B_0?

a) 2.0×10^8 T b) 9×10^{10} T c) 1.0×10^8 T d) 2.0×10^{10} T e) 1.0×10^{-6} T

III- Which of the following represents the magnetic field component of the wave?

a) $B_0 \sin(2.0y - 6.0\times10^8\ t)\ \hat{j}$ b) $B_0 \sin(18.0y - 1.0\times10^8\ t)\ \hat{i}$ c) $B_0 \sin(2.0y - 1.0\times10^8\ t)\ \hat{i}$

d) $B_0 \sin(2.0y - 6.0\times10^8\ t)\ \hat{i}$ e) $B_0 \sin(18.0y - 1.0\times10^8\ t)\ \hat{j}$

IV- What is the intensity of this wave?

a) 125 W/m² b) 250 W/m² c) 500 W/m² d) 1000 W/m² e) 1500 W/m²

V- What is the average force applied by the wave to the surface?

a) 2.0×10^{-5} N b) 5.0×10^{-6} N c) 22.5×10^{-6} N d) 1.0×10^{-5} N e) 2.5×10^{-6} N

8) The electric field of a plane radio wave has the functional form $\vec{E} = (0.12V/m)\sin(kx + wt)\hat{j}$. The frequency of the transmission is 900 MHz. ($\mu_0 = 4\pi 10^{-7}$ T.m.A^{-1}, $\pi=3$)

11- In which direction is the wave propagating?

a) -y b) +y c) +z d) +x e) -x

12- What is the wavelength of the wave ?

a) 1.33 nm b) 2.38 μm c) 0.33 m d) 0.55 m e) 0.75 m

13- What is the the magnetic field vector?

a) $\vec{B} = (4\times10^{-10}T)sin(kx+\omega t)\hat{j}$ b) $\vec{B} = (10^{-10}T)cos(kx+\omega t)\hat{j}$ c) $\vec{B} = (-10^{-10}T)sin(kx+\omega t)\hat{k}$

d) $\vec{B} = (-4\times10^{-10}T)sin(kx+\omega t)\hat{k}$ e) $\vec{B} = (-10^{-10}T)sin(kx-\omega t)\hat{k}$

14- What is the average intensity (power per unit area) transported by the wave?

a) 20 μW/m² b) 36 μW/m² c) 6 μW/m² d) 28 μW/m² e) 12 μW/m²

15- What is the average force exerted to a totally absorbing surface of area 3 m² perpendicular to the x-axis?

a) 2×10^{-13} N b) 4×10^{-13} N c) 32×10^{-14} N d) 12×10^{-13} N e) 16×10^{-13} N

9) Questions 1-3 relate to the circuit shown in which the switch S had been open for a long time.

1. What is the instantaneous current at point X immediately after the switch is closed?

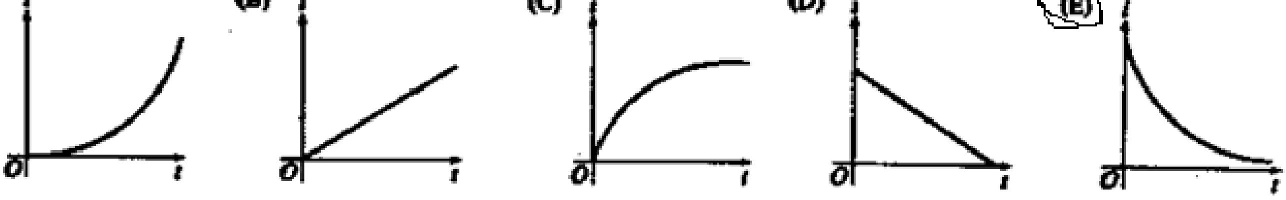
(a) 0 (b) $\frac{\varepsilon}{R}$ (c) $\frac{\varepsilon}{2R}$ (d) $\frac{\varepsilon}{LR}$ (e) $\frac{\varepsilon R}{2R}$

2. When the switch has been closed for a long time, what is the energy stored in the inductor?

(a) $\frac{L\varepsilon}{2R}$ (b) $\frac{L\varepsilon^2}{2R^2}$ (c) $\frac{L\varepsilon^2}{4R^2}$ (d) $\frac{LR^2}{2\varepsilon^2}$ (e) $\frac{\varepsilon^2 R^2}{4L}$

3. After the switch has been closed for a long time, it is opened at time $t = 0$. which of the following graphs best represents the subsequent current i at point X as a function of time t?

(A) (B) (C) (D) (E)



10) A red laser beam with a wavelength of 700 nm shines on a dark target which absorbs the beam's energy. The beam has a radius of 1.00 mm and power is absorbed in the target at a rate of 150 mW.

(a) What is the frequency of the laser light (in Hz = cycles per second)?

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8 \mathrm{m/s}}{700 \times 10^{-9}\mathrm{m}} = \mathbf{4.29 \times 10^{14} Hz.}$$

(b) What is the amplitude of the electric field in the laser beam?

$$S_{\max} = \frac{E^2}{\mu_0 c} = 2S_{avg} = \frac{2P_{avg}}{A} = \frac{2(0.150\ \mathrm{W})}{\pi(0.001\mathrm{m})^2} = \mathbf{9.55 \times 10^4 \frac{W}{m^2}}.$$

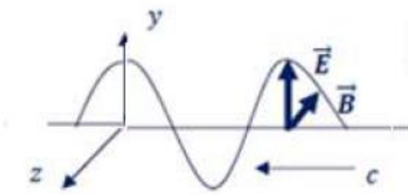
$$E = \sqrt{\mu_0 c S_{\max}} = \sqrt{(4\pi \times 10^{-7})(3 \times 10^8)(9.55 \times 10^4)}\frac{V}{m} = \mathbf{6000 \frac{V}{m}}.$$

(c) What is the amplitude of the magnetic field in the laser beam?

$$B = \frac{E}{c} = \frac{6000}{3 \times 10^8}\ \mathrm{T} = \mathbf{2.00 \times 10^{-5} T}\ (= 0.20\ \mathrm{Gauss}).$$

11) The electric field of a plane radio wave propagating in the x-direction points along the y-axis and has the functional form **Ey=(0.24V/m) cos(kx+wt).** The frequency of the transmission is 900 MHz (Hz = cycles per second).

(a) Which direction (left or right) is the wave moving along the x-axis?



The + sign in the argument means it is travelling **left.**

(b) What is the wavelength of the wave, which is typical of a cellular telephone transmission?

$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\,\text{m/s}}{9.00\times10^{8}\,\text{s}^{-1}} = \mathbf{0.333\ m}$$

(c) What is the amplitude of the magnetic field vector?

$$B_{\max} = \frac{E_{\max}}{c} = \frac{(0.24\,\text{V/m})}{3.00\times10^{8}\,\text{m/s}} = \mathbf{8.00\times10^{-10}\ T}\ (=8.00\,\mu\text{Gauss})$$

(d) When the electric field has its maximum upward value, which way does the magnetic field point?

The Poynting Vector $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$ points in the direction of the wave motion. The right-hand rule imples that when $\vec{E}$ is pointing up, $\vec{B}$ must point into the page in the figure above, in the $-z$ **direction.** Then $\vec{S}$ will point to the left.

(e) What is the average intensity (power per unit area) of the wave?

$$\frac{P_{avg}}{A} = S_{avg} = \frac{1}{2}S_{\max} = \frac{1}{2\mu_0}E_{\max}B_{\max} = \frac{(0.24)(8.00\times10^{-10})}{8\pi\times10^{-7}}\frac{\text{W}}{\text{m}^2} = 76.4\,\frac{\mu\text{W}}{\text{m}^2}\ \left(=0.764\frac{\text{W}}{\text{cm}^2}\right).$$

12) An electromagnetic wave has a frequency of 100 MHz and is traveling in a vacuum. The magnetic field is given by $\vec{B}(z,t) = (10^{-8}\text{T})\cos(kz-\omega t)\hat{i}$

(a) Find the wavelength and direction of propogation of this wave.

direction of propagation is $\hat{k}$ (z direction)

$$\lambda = \frac{c}{f} = \frac{3\times10^{8}\,m/s}{100\times10^{6}\,Hz} = \frac{3\times10^{8}\,m/s}{10^{8}\,Hz} \quad \boxed{\lambda = 3\,m}$$

(b) Find the direction and magnitude of the $\vec{E}$ field.

$E = cB$ (for maximum value of E and B) $E = (3\times10^{8}\,m/s)(10^{-8}\,T) = 3\,V/m$

$\vec{E}\perp\vec{B}\perp\vec{S} \Rightarrow \vec{E} = 3\,V/m\,\cos(kz-\omega t)\,(-\hat{j})$ $\quad k = \frac{2\pi}{\lambda} = \frac{2\pi}{3}\,m^{-1}$, $\omega = 2\pi\times100\,MHz$

(c) Find the intensity of the wave.

$$I = \langle S_{av}\rangle = \frac{\langle\vec{E}\times\vec{B}\rangle}{\mu_0} = \frac{1}{2}\frac{EB}{\mu_0} = \frac{1}{2}\frac{E^2}{\mu_0 c} = \frac{1}{2}\frac{E^2}{(377\Omega)}$$

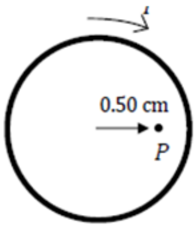
$$I = \frac{1}{2}\frac{3^2}{(377)} = 1.2\times10^{-2}\,\frac{W}{m^2}$$

(d) Find the associated radiation pressure.

$P_r = \frac{I}{c} = 3.7\times10^{-11}$ Pa $\Leftrightarrow$ Pascals $\left(\frac{N}{m^2}\right)$

13) The figure shows the cross-section of a 30 cm long solenoid with a cross-sectional area of 2.4 cm². The solenoid has 1600 turns of wire. If the time t is given in Amperes, the current in the wire is

$I = (40.0\,t^2 + 1.80)$ Amperes.



(a) What is the magnetic flux through the cross section shown at $t = 0$? Is it into or out of the page, if the current circles clockwise about the solenoid

$$\Phi_m = AB = A\mu_0\frac{NI}{l} = (2.4\times10^{-4}\ \mathrm{m^2})\left(4\pi\times10^{-7}\frac{\mathrm{Tm}}{\mathrm{A}}\right)\frac{1600\ (1.80\ \mathrm{A})}{0.30\ \mathrm{m}} = 2.90\ \mu\mathrm{Wb}.$$

(b) What is the inductance of the solenoid?

$$L = \frac{N\Phi_m}{I} = \frac{1600\ (2.90\ \mu\mathrm{Wb})}{1.80\ \mathrm{A}} = 2.58\ \mathrm{mH}.$$

(c) What emf is generated in the solenoid at time t = 0?

$$\mathcal{E} = L\frac{dI}{dt} = 0$$

(d) What is the electric field at point P, a distance of 0.50 cm from the center? Give the magnitude and direction (up, down, left, right, into or out of the page).

The emf about a circle of radius $r = 0.50$ cm inside the solenoid is (with t in seconds)

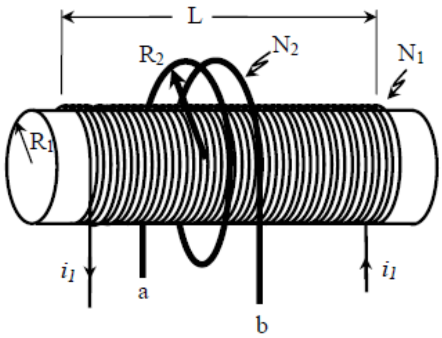
$$\oint \vec{E}\cdot d\vec{s} = 2\pi r\,E = \frac{d\Phi_m}{dt} = \frac{L\,dI}{N\,dt} = \frac{(2.58\ \mathrm{mH})(80\,t\ \mathrm{A/s})}{1600} = (129\,t)\ \mu\mathrm{V}.$$

$$E = \frac{(129\,t)\ \mu\mathrm{V}}{2\pi(0.0050\ \mathrm{m})} = (\mathbf{4.11}\,\boldsymbol{t})\frac{\mathbf{mV}}{\mathbf{m}}.$$

The direction is such that a current following the emf would oppose the increase in flux into the page. Thus, the emf is counter-clockwise, to generate flux out of the page. At point P, the electric field is directed **upward** in the figure. At $t = 0$, E vanishes. That answer would also be accepted.

14) <u>Questions 19-20</u>

A solenoid with length L contains N_1 turns of wire and has a radius of R_1. It initially carries a current i_1=1.5 A. Then, a short coil of radius R_2 and N_2 turns (heavy lines) is wrapped about center of the solenoid as shown.



19) If the current is reduced from 1.50 A to zero in 0.125 s (at a constant rate), what voltage is measured between the ends a and b of the short coil (V_a-V_b) ?

a) $12\pi\mu_0 R_1^2 N_1 N_2/L$ b) $\pi^2\mu_0 R_2^2 N_2/L$ c) $\pi\mu_0 R_2^2 N_1 N_2/L$ d) $12\pi\mu_0 R_2^2 N_2/L$

e) $\mu_0 R_2^2 N_1 N_2/L$

20) What is the mutual inductance, M of this combination?

a) $\pi\mu_0 N_1 N_2 R_1^2/L$ b) $\mu_0 N_1 N_2 R_2^2/L$ c) $\pi\mu_0 N_2 R_2^2/L$ d) $\pi\mu_0 N_1 R_1^2/L$

e) $\mu_0 N_1 N_2 R_2^2/L$

R ... outward

19) By Faraday's Law, the emf generated in the short coil is $\mathcal{E} = \Delta V = V_a - V_b = -N_2 \frac{d\Phi_B}{dt} = -N_2 \pi R_1^2 \frac{dB}{dt} = -N_2 \pi R_1^2 \mu_0 n_1 \frac{dI_1}{dt}$

where N_2 is the number of turns in short coil, $\frac{dI_1}{dt} = -\frac{1.5A}{0.125s}$

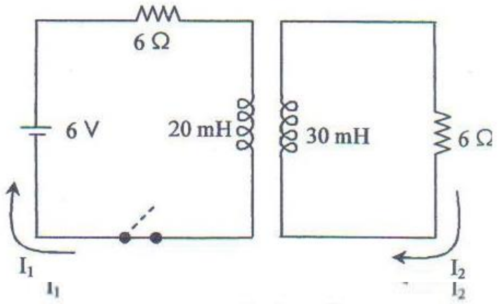
$n_1 = \frac{N_1}{L_1}$ and R_1 is the radius of the solenoid, since there is no field outside its coil. Then

$$\mathcal{E} = V_a - V_b = -N_2 \pi R_1^2 \mu_0 \frac{N_1}{L_1}(-12 A/s) = \underline{\underline{\frac{12 N_2 \pi R_1^2 \mu_0 N_1}{L_1}}}$$

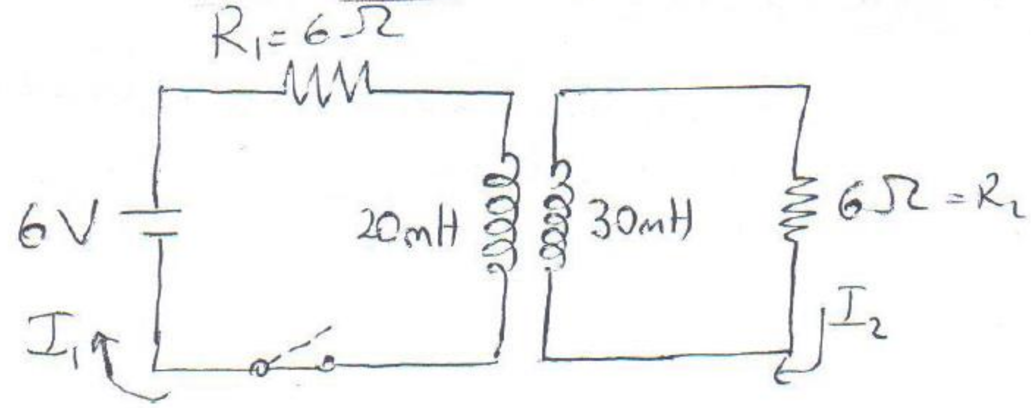
b) $$|\mathcal{E}| = \left|M \cdot \frac{dI_1}{dt}\right| = \frac{N_2 \pi R_1^2 \mu_0 N_1}{L_1} \cdot \frac{dI_1}{dt}$$

$$\Longrightarrow \boxed{M = \frac{N_2 \pi R_1^2 \mu_0 N_1}{L_1}}$$

15) The two identical coils in the circuit are placed close to each other and their mutual inductance is 0.7 mH. Suppose that the switch has been closed for a long time and is then opened at t=0. Calculate the current in the circuit at t = 18 ms.



4)



Before the switch is opened

$$I_1 = \frac{\mathcal{E}}{R_1} = \frac{6\,V}{6\Omega} = 1\,A \qquad I_2 = 0$$

The flux through L_2 is

$$\Phi_{B21} = MI_1 = (0.7\,mH)\cdot(1\,A) = 0.7\,mWb$$

When the switch is opened the induced emf in L_2 wants to maintain this flux

at $t=0$ the initial current in L_2 is

$$I_{20} = \frac{\Phi_{B21}}{L_2} = \frac{(0.7\,mWb)}{30\,mH} = 0{,}023\,A = 23\,mA$$

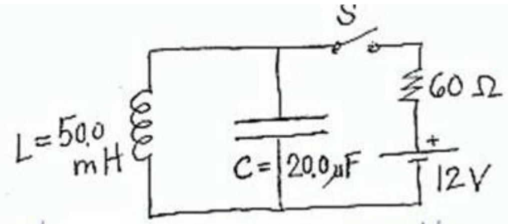
This current reduces exponentially

$$I_2 = I_{20}\, e^{-\frac{R_2 t}{L_2}} = (0{,}023\,A)\, e^{-\frac{(6\Omega)(18ms)}{30\,mH}}$$

$$= 0{,}63\,mA$$

16) The switch is closed for a long time, to energize L and C, and then re-opened.

a) At what frequency f will the circuit oscillate?



$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.05\,H)(20\times10^{-6}\,F)}} = 1000$ rad/s $\quad f = \frac{\omega}{2\pi} = \frac{1000\text{ rad/s}}{2\pi\text{ rad}} = 159$ Hz.

b) When the switch is re-opened, the initial capacitor charge is:

Since the inductor acts like a short across C, the capacitor voltage is zero.

the initial current from L into top electrode of C must be:

The inductor acts like a short ckt. So the current is

$i_0 = \frac{12V}{60\Omega} = 0.20$ A But really

$i_0 = -0.20$ A (flowing out of top C-plate)

c) The total energy in the electrical oscillations is:

$U = \frac{1}{2}Li_0^2 + \frac{1}{2}\frac{q_0^2}{C} = \frac{1}{2}(0.050H)(0.20A)^2 = 0.0010\text{ J} = 1.0\text{ mJ}$

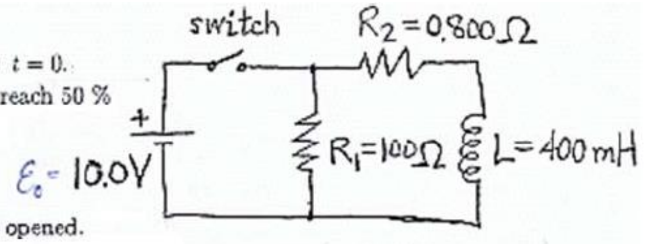
d) Find the peak voltage that will appear across C during the oscillations.

Conserve energy. $U = \frac{1}{2}\frac{Q^2}{C}$ $\quad Q = \max.\ q$.

$Q = \sqrt{2CU} = \sqrt{2(20\times10^{-6}F)(0.001J)} = 0.00020$ C

Voltage is $V = Q/C = 10.0$ Volts.

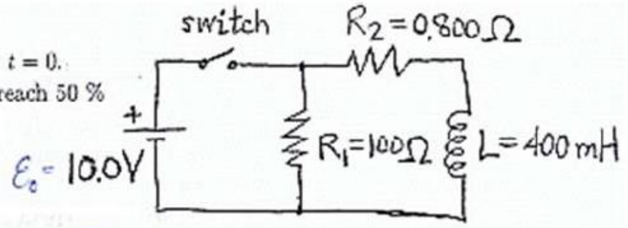
16) In the circuit shown, the switch is closed at time $t = 0$.
(a) How long does it take for the current in R_2 to reach 50 % of its final value?



(b) After a long time (say, 1 hour), the switch is opened.
Find the magnitude and direction of the current in R_1 just after the switch is opened.
(c) Sketch on the provided axes the voltage across R_1 versus time t after S was opened.
Be sure to include the values at the initial time and one time constant later.

16) In the circuit shown, the switch is closed at time $t = 0$.

(a) How long does it take for the current in R_2 to reach 50 % of its final value?



$i(\infty)$ = Final current $= \dfrac{\mathcal{E}_0}{R_2} = \dfrac{10.0\text{V}}{0.800\,\Omega}$
$= \underline{12.5\text{A}}$

Time constant $\tau_L = \dfrac{L}{R_2} = \dfrac{0.400\text{H}}{0.800\,\Omega} = \underline{0.500\text{ s}}$

$i = i(\infty)\left(1 - e^{-t/\tau_L}\right)$ $\quad 0.5\, i(\infty) = i(\infty)\left(1 - e^{-t/\tau_L}\right)$ $\quad 0.5 = e^{-t/\tau_L}$

$t = -\tau_L \ln(0.5) = -(0.500\text{s}) \ln\left(\tfrac{1}{2}\right) = \underline{0.347\text{ s}}$

(b) After a long time (say, 1 hour), the switch is opened. Find the magnitude and direction of the current in R_1 just after the switch is opened.

L tries to maintain the current it had before S was opened,
which was $i(\infty) = 12.5$ A.
So <u>12.5 A will flow upward thru R_1,</u>
instantaneously. Subsequently, it will decay away

(c) Sketch on the provided axes the voltage across R_1 versus time t after S was opened. Be sure to include the values at the initial time and one time constant later.

Current decays passing thru
$R = R_1 + R_2 = 100.8\,\Omega$.
$\tau = \dfrac{L}{R} = \dfrac{0.400\text{H}}{100.8\,\Omega} = \underline{3.97\text{ ms}}$

$i_1 = i(0)\, e^{-t/\tau}$
$V_1 = i_1 R_1 = (100\,\Omega)(12.5\text{A})\, e^{-t/\tau}$
$V_1 = \underline{(1250\text{ V})}\, e^{-t/\tau}$