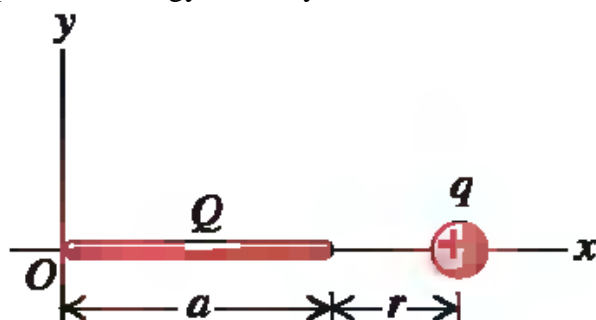


PHY102E MIDTERM-I EXAM

PART I: ANALYTIC PROBLEMS

- 1) Positive charge Q is distributed uniformly along the x -axis from $x = 0$ to $x = a$. A positive point charge q is located on the positive x -axis at $x = a + r$, a distance r to the right of the end of Q .
- (a) Calculate the electric field produced by the charge distribution Q at points on the positive x -axis where $x > a$.
- (b) Calculate the force (magnitude and direction) that the charge distribution Q exerts on q .
- (c) Calculate the potential energy of the system.



21.89. IDENTIFY: Divide the charge distribution into infinitesimal segments of length dx . Calculate E_x and E_y due to a segment and integrate to find the total field.

SET UP: The charge dQ of a segment of length dx is $dQ = (Q/a)dx$. The distance between a segment at x and the charge q is $a + r - x$. $(1 - y)^{-1} \approx 1 + y$ when $|y| \ll 1$.

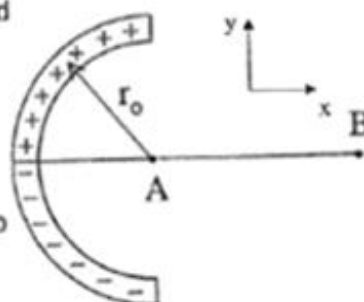
EXECUTE: (a) $dE_x = \frac{1}{4\pi\epsilon_0} \frac{dQ}{(a+r-x)^2}$ so $E_x = \frac{1}{4\pi\epsilon_0} \int_0^a \frac{Qdx}{a(a+r-x)^2} = \frac{1}{4\pi\epsilon_0} \frac{Q}{a} \left(\frac{1}{r} - \frac{1}{a+r} \right)$.

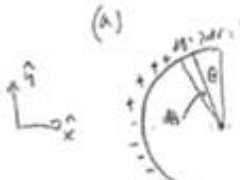
$a + r = x$, so $E_x = \frac{1}{4\pi\epsilon_0} \frac{Q}{a} \left(\frac{1}{x-a} - \frac{1}{x} \right)$. $E_y = 0$.

(b) $\vec{F} = q\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{qQ}{a} \left(\frac{1}{x-a} - \frac{1}{x} \right) \hat{i}$.

- 2) An insulating wire of negligible width is bent into a semicircle of radius r_0 . A charge of $+q$ is uniformly distributed along the upper half and a charge $-q$ is uniformly distributed along the lower half, as shown in the figure below.

- (a) What is the electric field at point A, the center of the semicircle?
- (b) What is the **direction** of the electric field at point B, a distance x_0 from A along the direction shown in the figure?
- (c) If a charge of size $+3q$ is placed at A, how much work is required to move the charge to point B?



(a)  $\lambda = \frac{Q}{\pi r_0}$ by symmetry, field component along \hat{x} cancels. $dE_y = \frac{\lambda r_0 d\theta}{r_0^2}$

$$E_y = \int_0^{\pi/2} \frac{\lambda r_0 d\theta}{r_0^2} \cos\theta = \frac{\lambda}{r_0} = \frac{2q}{\pi r_0^2} \rightarrow \text{total field from } +x \text{ - section equals } 2E_y \Rightarrow \boxed{\vec{E} = \frac{-4q}{\pi r_0^2} \hat{y}}$$

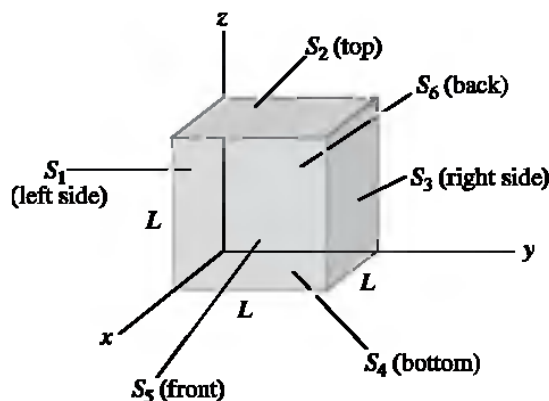
(b) By symmetry, $\boxed{\vec{E}_0 \propto -\hat{y}}$

(c) $W = 3q \Delta\phi = -3q \int \vec{E} \cdot d\vec{s}$ but, $\vec{E} \parallel \hat{y}$, $d\vec{s} \parallel \hat{x} \Rightarrow \vec{E} \cdot d\vec{s} = 0 \Rightarrow \boxed{W=0}$

3) A cube has sides of length $L = 0.300$ m. It is placed with one corner at the origin as shown in Figure. The electric field is not uniform but is given by $\vec{E} = (-5.00 \text{ N/C} \cdot \text{m})x\hat{i} + (3.00 \text{ N/C} \cdot \text{m})z\hat{k}$.

(a) Find the electric flux through each of the six cube faces S_1 , S_2 , S_3 , S_4 , S_5 , and S_6 .

(b) Find the total electric charge inside the cube.



22.4. IDENTIFY: Use Eq.(22.3) to calculate the flux for each surface. Use Eq.(22.8) to calculate the total enclosed charge.

SET UP: $\vec{E} = (-5.00 \text{ N/C} \cdot \text{m})x\hat{i} + (3.00 \text{ N/C} \cdot \text{m})z\hat{k}$. The area of each face is L^2 , where $L = 0.300$ m.

EXECUTE: $\hat{n}_{S_1} = -\hat{j} \Rightarrow \Phi_1 = \vec{E} \cdot \hat{n}_{S_1} A = 0$.

$\hat{n}_{S_2} = +\hat{k} \Rightarrow \Phi_2 = \vec{E} \cdot \hat{n}_{S_2} A = (3.00 \text{ N/C} \cdot \text{m})(0.300 \text{ m})^2 z = (0.27 \text{ (N/C)} \cdot \text{m})z$.

$\Phi_2 = (0.27 \text{ (N/C)} \cdot \text{m})(0.300 \text{ m}) = 0.081 \text{ (N/C)} \cdot \text{m}^2$.

$\hat{n}_{S_3} = +\hat{j} \Rightarrow \Phi_3 = \vec{E} \cdot \hat{n}_{S_3} A = 0$.

$\hat{n}_{S_4} = -\hat{k} \Rightarrow \Phi_4 = \vec{E} \cdot \hat{n}_{S_4} A = -(0.27 \text{ (N/C)} \cdot \text{m})z = 0$ (since $z = 0$).

$\hat{n}_{S_5} = +\hat{i} \Rightarrow \Phi_5 = \vec{E} \cdot \hat{n}_{S_5} A = (-5.00 \text{ N/C} \cdot \text{m})(0.300 \text{ m})^2 x = -(0.45 \text{ (N/C)} \cdot \text{m})x$.

$\Phi_5 = -(0.45 \text{ (N/C)} \cdot \text{m})(0.300 \text{ m}) = -(0.135 \text{ (N/C)} \cdot \text{m}^2)$.

$\hat{n}_{S_6} = -\hat{i} \Rightarrow \Phi_6 = \vec{E} \cdot \hat{n}_{S_6} A = +(0.45 \text{ (N/C)} \cdot \text{m})x = 0$ (since $x = 0$).

(b) Total flux: $\Phi = \Phi_2 + \Phi_5 = (0.081 - 0.135) \text{ (N/C)} \cdot \text{m}^2 = -0.054 \text{ N} \cdot \text{m}^2/\text{C}$. Therefore,

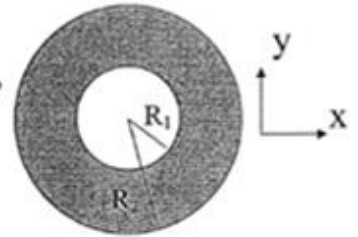
$q = \epsilon_0 \Phi = -4.78 \times 10^{-13} \text{ C}$.

EVALUATE: Flux is positive when \vec{E} is directed out of the volume and negative when it is directed into the volume.

- 4) A hollow cylinder with its axis along the z-axis has inner radius R_1 and outer radius R_2 . Charge is distributed nonuniformly within the cylinder such that $\rho = CR_1/r$.

- What are the magnitude $|\vec{E}|$ and direction of the electric field as a function of r for $r < R_1$?
- What are the magnitude and direction of \vec{E} for $R_2 > r > R_1$?
- What are the magnitude and direction of \vec{E} for $r > R_2$?
- What is the potential difference between the outer and inner surface of the cylinder

$$V = \phi(R_2) - \phi(R_1)?$$



4. a) Use fact that \vec{E} must be a vector pointing radially away from z-axis. Draw Gaussian surface of radius r and length l along z-axis. Only contribution to flux comes from curved surface because \vec{E} is \parallel to end caps. $\int \vec{E} \cdot d\vec{A} = |\vec{E}(r)| \cdot \underbrace{2\pi r l}_{\text{area of curved surf}}$

$$a) \int \vec{E} \cdot d\vec{A} = |\vec{E}(r)| 2\pi r l = 4\pi Q_{\text{enclosed}} = 0 \text{ for } r < R_1$$

$$\vec{E} = 0 \text{ for } r < R_1$$

$$b) |\vec{E}(r)| 2\pi r l = 4\pi \int_{R_1}^r dr' \cdot \underbrace{2\pi r' l \cdot C \cdot \frac{R_1}{r'}}_{\substack{\text{volume of shell of radius } r' \text{ and length } l}} \cdot \underbrace{\rho(r')}_{\rho(r')} \quad |\vec{E}(r)| \cdot 2\pi r l = 4\pi \cdot 2\pi l \cdot C R_1 \int_{R_1}^r dr'$$

$$|\vec{E}(r)| = 4\pi C R_1 \cdot \frac{r - R_1}{r} \text{ directed out radially assuming } C > 0$$

$$c) |\vec{E}(r)| \cdot 2\pi r l = 4\pi \cdot 2\pi l C R_1 (R_2 - R_1) \quad |\vec{E}(r)| = \frac{4\pi}{r} \cdot C R_1 (R_2 - R_1)$$

also out radially note $\lambda = 2\pi C R_1 (R_2 - R_1)$ is effective charge per length

$$d) V = \phi(R_2) - \phi(R_1) = - \int_{R_1}^{R_2} \vec{E} \cdot d\vec{s} \quad \text{parallel so dot product is product of magnitudes}$$

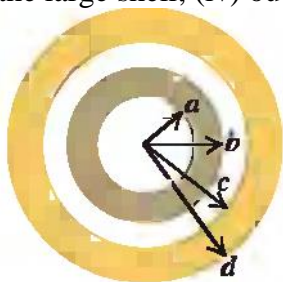
$$V = - \int_{R_1}^{R_2} dr \cdot 4\pi C R_1 \left(\frac{r - R_1}{r} \right) = - 4\pi C R_1 \int_{R_1}^{R_2} dr \left(1 - \frac{R_1}{r} \right)$$

$$= - 4\pi C R_1 \left[(R_2 - R_1) + R_1 \ln \frac{R_2}{R_1} \right]$$

- 5) A small conducting spherical shell with inner radius a and outer radius b is concentric with a larger conducting spherical shell with inner radius c and outer radius d . The inner shell has total charge $+2q$, and the outer shell has charge $+4q$.

(a) Calculate the electric field (magnitude and direction) in terms of q and the distance r from the common center of the two shells for (i) $r < a$; (ii) $a < r < b$; (iii) $b < r < c$; (iv) $c < r < d$; (v) $r > d$.

(b) What is the total charge on the (i) inner surface of the small shell; (ii) outer surface of the small shell; (iii) inner surface of the large shell; (iv) outer surface of the large shell?



22.45. IDENTIFY: Apply Gauss's law to a spherical Gaussian surface with radius r . Calculate the electric field at the surface of the Gaussian sphere.

(a) SET UP: (i) $r < a$: The Gaussian surface is sketched in Figure 22.45a.

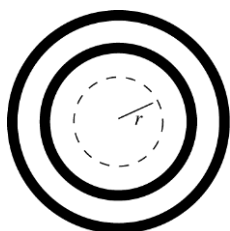


Figure 22.45a

EXECUTE: $\Phi_E = EA = E(4\pi r^2)$

$Q_{\text{encl}} = 0$; no charge is enclosed

$$\Phi_E = \frac{Q_{\text{encl}}}{\epsilon_0} \quad \text{says}$$

$$E(4\pi r^2) = 0 \text{ and } E = 0.$$

(ii) $a < r < b$: Points in this region are in the conductor of the small shell, so $E = 0$.

(iii) **SET UP:** $b < r < c$: The Gaussian surface is sketched in Figure 22.45b.

Apply Gauss's law to a spherical Gaussian surface with radius $b < r < c$.

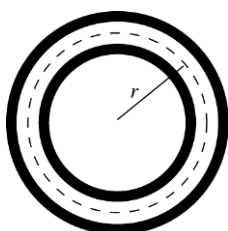


Figure 22.45b

EXECUTE: $\Phi_E = EA = E(4\pi r^2)$

The Gaussian surface encloses all of the small shell and none of the large shell, so $Q_{\text{encl}} = +2q$.

$\Phi_E = \frac{Q_{\text{encl}}}{\epsilon_0}$ gives $E(4\pi r^2) = \frac{2q}{\epsilon_0}$ so $E = \frac{2q}{4\pi\epsilon_0 r^2}$. Since the enclosed charge is positive the electric field is radially outward.

(iv) $c < r < d$: Points in this region are in the conductor of the large shell, so $E = 0$.

(v) **SET UP:** $r > d$: Apply Gauss's law to a spherical Gaussian surface with radius $r > d$, as shown in Figure 22.45c.

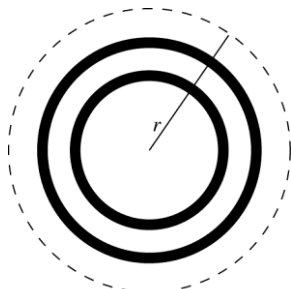


Figure 22.45c

EXECUTE: $\Phi_E = EA = E(4\pi r^2)$

The Gaussian surface encloses all of the small shell and all of the large shell, so $Q_{\text{encl}} = +2q + 4q = 6q$.

$$\Phi_E = \frac{Q_{\text{encl}}}{\epsilon_0} \text{ gives } E(4\pi r^2) = \frac{6q}{\epsilon_0}$$

$E = \frac{6q}{4\pi\epsilon_0 r^2}$. Since the enclosed charge is positive the electric field is radially outward.

The graph of E versus r is sketched in Figure 22.45d.

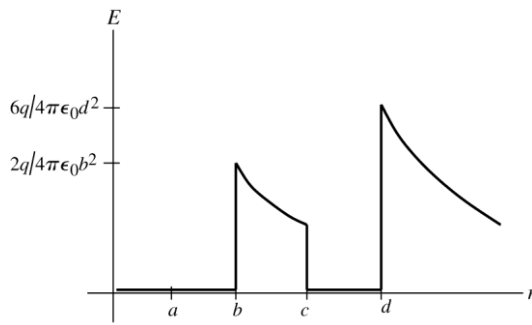


Figure 22.45d

(b) IDENTIFY and SET UP: Apply Gauss's law to a sphere that lies outside the surface of the shell for which we want to find the surface charge.

EXECUTE: (i) charge on inner surface of the small shell: Apply Gauss's law to a spherical Gaussian surface with radius $a < r < b$. This surface lies within the conductor of the small shell, where $E = 0$, so $\Phi_E = 0$. Thus by Gauss's law $Q_{\text{encl}} = 0$, so there is zero charge on the inner surface of the small shell.

(ii) charge on outer surface of the small shell: The total charge on the small shell is $+2q$. We found in part (i) that there is zero charge on the inner surface of the shell, so all $+2q$ must reside on the outer surface.

(iii) charge on inner surface of large shell: Apply Gauss's law to a spherical Gaussian surface with radius $c < r < d$. The surface lies within the conductor of the large shell, where $E = 0$, so $\Phi_E = 0$. Thus by Gauss's law $Q_{\text{encl}} = 0$. The surface encloses the $+2q$ on the small shell so there must be charge $-2q$ on the inner surface of the large shell to make the total enclosed charge zero.

(iv) charge on outer surface of large shell: The total charge on the large shell is $+4q$. We showed in part (iii) that the charge on the inner surface is $-2q$, so there must be $+6q$ on the outer surface.

EVALUATE: The electric field lines for $b < r < c$ originate from the surface charge on the outer surface of the inner shell and all terminate on the surface charge on the inner surface of the outer shell. These surface charges have equal magnitude and opposite sign. The electric field lines for $r > d$ originate from the surface charge on the outer surface of the outer sphere.

6) An insulating solid sphere of radius R has a uniform positive charge Q as shown in the Figure below.

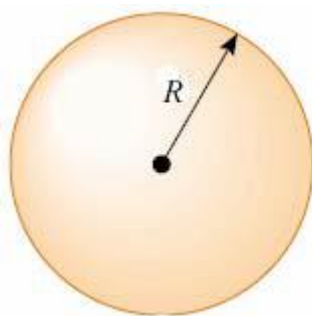
a) Find charge density ρ .

b) Find the electric potential at a point r outside the sphere ($r > R$). Take the potential to be zero at $r = \infty$.

c) Find the potential at a point r inside the sphere ($r < R$).

d) Plot potential $V(r)$ as a function of the distance from the center.

e) What can you say about the results (a), (b), and (c) if we have conducting sphere having the same amount of charge on it?



Q2 (35 points) Potential

Solution:

a) charge density $\rho = \frac{Q}{\frac{4}{3}\pi R^3}$

b) $V(r > R) = ?$



Electric field at point r from Gauss law

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0} \Rightarrow E(r) = \frac{Q}{4\pi\epsilon_0} \frac{1}{r^2}$$

Potential at point r :

$$V_r - V_{\infty} = - \int_{\infty}^r \vec{E}(r) \cdot d\vec{r} = - \int_{\infty}^r \frac{Q}{4\pi\epsilon_0} \frac{1}{r^2} dr$$

$$V(r > R) = \frac{Q}{4\pi\epsilon_0} \frac{1}{r}, \quad V(r=R) = \frac{Q}{4\pi\epsilon_0} \frac{1}{R}$$

c) $V(r < R) = ?$ Electric field inside the sphere, from Gauss law



$$\oint \vec{E} \cdot d\vec{A} = \rho \cdot \frac{4}{3}\pi r^3 / \epsilon_0 \Rightarrow E(r) = \frac{Q}{4\pi\epsilon_0 R^3} r$$

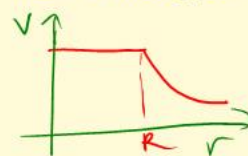
$$V_r - V_R = - \int_R^r E(r) dr \Rightarrow V(r < R) = \frac{Q}{8\pi\epsilon_0 R} \left(3 + \frac{r^2}{R^2} \right)$$



e) a) charge density $\rho = 0$ inside conductor

b) $V(r > R) = \frac{Q}{4\pi\epsilon_0} \frac{1}{r}$

c) $V(r < R) = \frac{Q}{4\pi\epsilon_0} \frac{1}{R}$



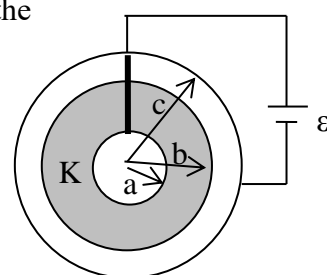
7) A spherical capacitor consists of a conducting sphere of radius "a" and a concentric conducting shell of inner radius "b=2a" and outer radius "c=3a". The space between the capacitors is filled with a dielectric medium of constant "K". The inner sphere carries +Q and the outer shell -Q.

a) Find the magnitude and direction of the electric field \vec{E} in the space between the plates.

b) Find the potential difference between the plates.

c) Find the capacitance of the device.

d) What is the energy density as a function of the radius "r" in the space between the inner sphere and the shell?



$$a) 4\pi r^2 E = \frac{Q}{\epsilon_0} \Rightarrow \vec{E} = \frac{Q}{4\pi \epsilon_0 K r^2} \hat{r}$$

$$b) \Delta V_{ba} = - \int_a^b \frac{Q}{4\pi \epsilon_0 K r^2} dr = \frac{Q}{4\pi \epsilon_0 K} \left(\frac{1}{b} - \frac{1}{a} \right) = \frac{Q}{4\pi \epsilon_0 K} \left(\frac{1}{2a} - \frac{1}{a} \right)$$

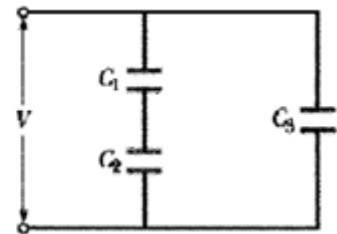
$$\Delta V_{ab} = \frac{Q}{4\pi \epsilon_0 K} \left(\frac{1}{a} - \frac{1}{b} \right) = \frac{Q}{4\pi \epsilon_0 K} \frac{1}{2a} = \frac{Q}{8\pi \epsilon_0 K a}$$

$$c) C = \frac{Q}{V_{ab}} = 4\pi \epsilon_0 K \left(\frac{ab}{b-a} \right) = 8\pi \epsilon_0 K a$$

$$d) U = \frac{K \epsilon_0}{2} E^2 = \frac{K \epsilon_0}{2} \frac{Q^2}{16\pi^2 \epsilon_0^2 K^2 r^4} = \frac{Q^2}{32\pi^2 \epsilon_0 K r^4}$$

- 8) In the figure, a potential difference $V = 5.0 \text{ V}$ is applied across a capacitor arrangement with capacitances $C_1 = 3.0 \mu\text{F}$, $C_2 = 2.0 \mu\text{F}$, and $C_3 = 1.0 \mu\text{F}$.

- Find the equivalent capacitance of the combination
- Find the charge on C_1 .
- Find the stored energy U_3 in capacitor C_3 .



- 8) In the figure, a potential difference $V = 5.0 \text{ V}$ is applied across a capacitor arrangement with capacitances $C_1 = 3.0 \mu\text{F}$, $C_2 = 2.0 \mu\text{F}$, and $C_3 = 1.0 \mu\text{F}$.

- Find the equivalent capacitance of the combination

$$C_{12} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{1}{\frac{1}{3} + \frac{1}{2}} = \frac{6}{5} \mu\text{F} = 1.2 \mu\text{F}$$

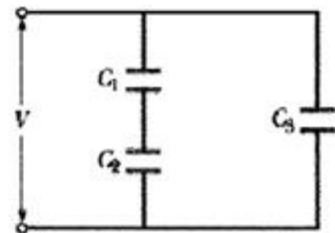
$$C_{123} = 2.2 \mu\text{F}$$

- Find the charge on C_1 .

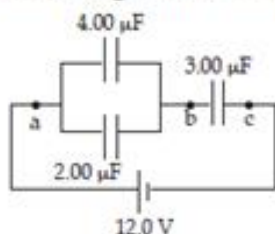
$$q = C_{12} V = 1.2 \mu\text{F} \times 5.0 \text{ V} = 6 \mu\text{C}$$

- Find the stored energy U_3 in capacitor C_3 .

$$U_3 = \frac{1}{2} C_3 V^2 = \frac{1}{2} \times 1.0 \mu\text{F} \times 25 \text{ V}^2 = 12.5 \mu\text{J}$$



- 9) (a) Find the equivalent capacitance of the group of capacitors in Figure
 (b) Find the charge on and the potential difference across each.



Solution:

Using the rules for combining capacitors in series and in parallel, the circuit is reduced in steps as shown below. The equivalent capacitor is shown to be a $2.00 \mu\text{F}$ capacitor.

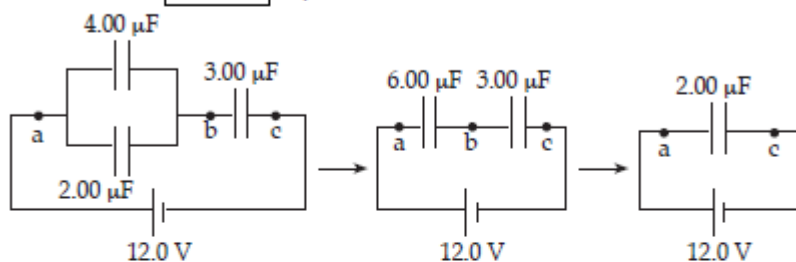


Figure 1

Figure 2

Figure 3

(b) From Figure 3: $Q_{ac} = C_{ac}(\Delta V)_{ac} = (2.00 \mu\text{F})(12.0 \text{ V}) = 24.0 \mu\text{C}$

From Figure 2: $Q_{ab} = Q_{bc} = Q_{ac} = 24.0 \mu\text{C}$

Thus, the charge on the $3.00 \mu\text{F}$ capacitor is $Q_3 = 24.0 \mu\text{C}$

Continuing to use Figure 2, $(\Delta V)_{ab} = \frac{Q_{ab}}{C_{ab}} = \frac{24.0 \mu\text{C}}{6.00 \mu\text{F}} = 4.00 \text{ V}$,

and $(\Delta V)_3 = (\Delta V)_{bc} = \frac{Q_{bc}}{C_{bc}} = \frac{24.0 \mu\text{C}}{3.00 \mu\text{F}} = 8.00 \text{ V}$

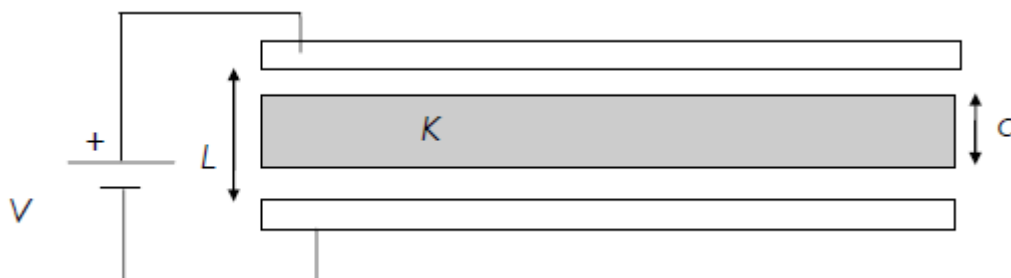
From Figure 1, $(\Delta V)_4 = (\Delta V)_2 = (\Delta V)_{ab} = 4.00 \text{ V}$

and $Q_4 = C_4(\Delta V)_4 = (4.00 \mu\text{F})(4.00 \text{ V}) = 16.0 \mu\text{C}$

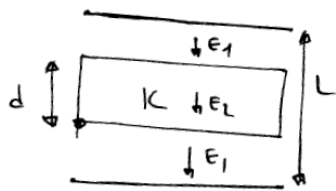
$Q_2 = C_2(\Delta V)_2 = (2.00 \mu\text{F})(4.00 \text{ V}) = 8.00 \mu\text{C}$

- 10) A dielectric slab of thickness d and dielectric constant K is inserted inside a capacitor with area A and thickness L . A voltage difference V is applied across the plates.

- a) Find the electric fields inside the air gap and inside the dielectric.
 b) Find the surface charge density on each of the plates.
 c) Use the information in (b) to obtain the capacitance.
 d) Find the capacitance of the system using capacitors connected in series, and compare your result with part (c).



Q3: Solution (A)

 E_1 : E-FIELD IN AIR E_2 : " IN DIELECTRIC

$$E_2 = \frac{E_1}{K} \quad (1)$$

$$\Delta V = - \int \vec{E} \cdot d\vec{\ell} \Rightarrow \Delta V = E_2 \cdot d + E_1(L-d) \quad (2)$$

Use (1) in (2)

$$\Delta V = V = \frac{E_1}{K} \cdot d + E_1(L-d) = E_1 \left(\frac{d}{K} + (L-d) \right)$$

$$E_1 = \frac{V}{\frac{d}{K} + (L-d)} \quad (3)$$

Use (3) in (1)

$$E_2 = \frac{V}{d + K(L-d)} \quad (4)$$

$$(B) \quad E_1 = \frac{\sigma}{\epsilon_0} \quad (5) \quad \text{use (3) in (5)}$$

$$\sigma = \epsilon_0 \cdot \frac{V}{\frac{d}{K} + (L-d)} \quad (6)$$

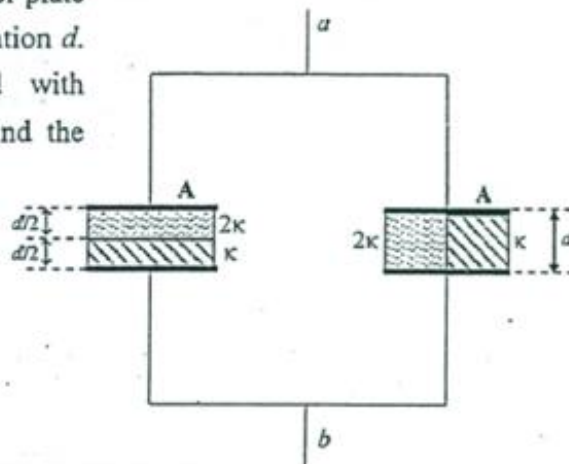
$$(C) \quad Q = \sigma \cdot A = \frac{\epsilon_0 \cdot V \cdot A}{\frac{d}{K} + (L-d)} \quad Q = CV \Rightarrow C = \frac{Q}{V} = \frac{\epsilon_0 \cdot A}{\frac{d}{K} + (L-d)} \quad (7)$$

$$(D) \quad \begin{array}{l} \text{AIR} \\ \text{DIELECT.} \end{array} \quad \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \quad \begin{array}{c} C_1 \\ C_2 \end{array} \quad \begin{array}{l} C_1 = \frac{\epsilon_0 \cdot A}{L-d} \\ C_2 = \frac{K \cdot \epsilon_0 \cdot A}{d} \end{array}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{\epsilon_0 \cdot A} \cdot \left(\frac{d}{K} + (L-d) \right)$$

$$\Rightarrow C_{eq} = \frac{\epsilon_0 \cdot A}{\frac{d}{K} + (L-d)} \quad (8)$$

- 11) The circuit shown in the figure consists of parallel-plate capacitors with plates of area A and plate separation d . The regions between the plates are filled with dielectrics of dielectric constants κ and 2κ . Find the equivalent capacitance of the system.



1st series connection

$$\frac{1}{C_{eq1}} = \frac{1}{C_a} + \frac{1}{C_b}$$

$$C_a = 2\kappa C_0 \quad C_0 = \epsilon_0 \frac{A}{d/2}$$

$$C_b = \kappa C_0$$

$$\frac{1}{C_{eq1}} = \frac{d}{2\epsilon_0 A 2\kappa} + \frac{d}{2\epsilon_0 A \kappa}$$

$$\frac{1}{C_{eq1}} = \frac{d}{2\epsilon_0 A \kappa} \left(\frac{1}{2} + 1 \right)$$

$$C_{eq1} = \frac{4}{3} \frac{\epsilon_0 A \kappa}{d}$$

2nd parallel connection

$$C_{eq2} = C_e + C_f$$

$$C_{eq2} = C_e + C_f$$

$$C_{eq2} = 2\kappa \epsilon_0 \frac{A}{2d} + \kappa \epsilon_0 \frac{A}{2d}$$

$$C_{eq2} = \frac{3}{2} \kappa \frac{\epsilon_0 A}{d}$$

$$C_e = 2\kappa C_0 \quad C_0 = \epsilon_0 \frac{A/2}{d}$$

$$C_f = \kappa C_0$$

the equivalent capacitance of the system

$$C_{eq} = C_{eq1} + C_{eq2} = \left(\frac{4}{3} + \frac{3}{2} \right) \kappa \frac{\epsilon_0 A}{d}$$

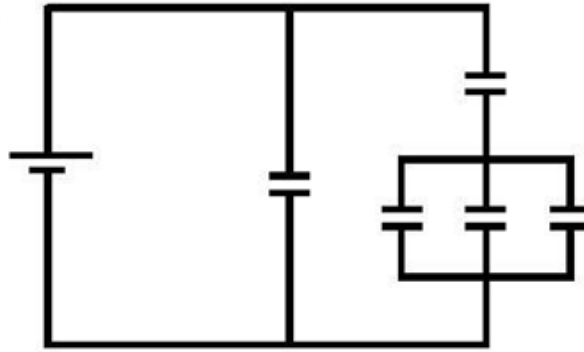
$$= \frac{17}{6} \kappa \frac{\epsilon_0 A}{d}$$

12) Consider the circuit shown below. Assume all capacitors

are 1 mF and the battery is 10 V

a) What is the equivalent capacitance.

b) What is the charge on each capacitor?



THIS IS ONE OF THE QUESTION UPLOADED TO NINOA. CHECKED THE ANSWER FROM THE NINOA !!

② A 3.55 mF capacitor C_1 is charged to a potential difference $V_0 = 6.30$ V, using a battery. The charging battery is then removed, and the capacitor is connected as in figure to an uncharged 8.95 mF capacitor C_2 . After the switch S is closed, charge flows from C_1 to C_2 until an equilibrium is established with both capacitors at the same potential difference V .

a) What is the common potential difference?

b) What is the energy stored in the electric field before and after the switch S is thrown?

Solution:

a) The original charge q_0 is shared by both capacitors, and $q_0 = q_1 + q_2$
 applying the relation $q = C \cdot V$ to both
 $C_1 V_0 = C_1 V + C_2 V$

$$\Rightarrow V = V_0 \cdot \frac{C_1}{C_1 + C_2} = \frac{(6.30 \text{ V})(3.55 \text{ mF})}{(3.55 \text{ mF} + 8.95 \text{ mF})} = 1.79 \text{ V}$$

b) The initial stored energy: $U_i = \frac{1}{2} C_1 V_0^2 = \frac{1}{2} (3.55 \times 10^{-6} \text{ F})(6.30 \text{ V})^2$
 $U_i = 70.5 \text{ mJ} = 7.05 \times 10^{-5} \text{ J}$
 The final energy: $U_f = \frac{1}{2} C_1 V^2 + \frac{1}{2} C_2 V^2$
 $\Rightarrow U_f = \frac{1}{2} (C_1 + C_2) V^2 = \frac{1}{2} (3.55 \times 10^{-6} \text{ F} + 8.95 \times 10^{-6} \text{ F})(1.79 \text{ V})^2$
 $U_f = 2 \times 10^{-5} \text{ J} = 20 \text{ mJ}$
 $U_f < U_i$ by about 72%.

\Rightarrow This is not a violation of energy conservation. The missing energy appears as thermal energy in the connecting wires.