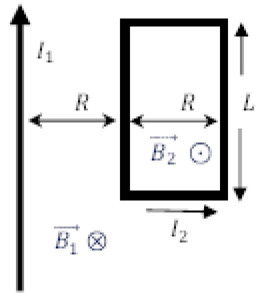
3. In the figure at right, the current in the long, straight wire is $I_1 = 20$ A and the wire lies in the plane of the rectangular loop, which carries the current $I_2 = 12$ A. The dimensions are $R = 0.10$ m and $L = 0.30$ m.



(a) What is the direction of the magnetic field due to the long straight wire at any point to its right? [In all parts of this problem, specify direction as "left", "right", "up", "down", "into the page", or "out of the page".]

The direction of the magnetic field due to I_1 can be found by pointing your right thumb in the direction of I_1 and curling your fingers around the wire. The fingers point in the direction of the magnetic field $\vec{B_1}$, which is **into the page** to the right of the wire.

(b) What is the direction of the magnetic field due to the rectangular loop at any point inside the loop?

The same right-hand rule shows that the contribution from each side of the rectangle is directed **out of the page** at a point inside the rectangle. This is shown as $\vec{B_1}$ in the figure above. Another right-hand rule for finding the direction of the magnetic field due to the current loop is that when you curl your fingers around the loop following the current, your thumb points in the direction of the magnetic field. This gives the same result.

(c) Give the direction of the magnetic force on each segment of the rectangular loop due to the current in the long straight wire. Enter "zero" if there is no force.

The magnetic force on a short length dl of the rectangle is $\vec{F} = I_2\vec{dl} \times \vec{B_1}$, where $\vec{dl}$ points along the current, with $\vec{B_1}$ into the page. Curling your right fingers from any of the segments into the page will cause your thumb to point inward, toward the center of the rectangle, giving the directions shown in the table on the next page.

Segment	Magnetic Force Direction
Left	Right
Right	Left
Top	Down
Bottom	Up

(d) Calculate the magnitude of the total magnetic force on the rectangular loop due to the current in the long straight wire.

The forces on any segments of the top and bottom wire equidistant from current I_1 will cancel, so there is no net force in the vertical direction. (This can also be inferred from the symmetry.) The force on the left segment is to the right, with positive value

$$F_L = I_2 L B_1 = \frac{\mu_0 I_1 I_2 L}{2\pi R},$$

and the force on the left segment is to the left, with negative value

$$F_R = I_2 L B_1 = -\frac{\mu_0 I_1 I_2 L}{4\pi R},$$

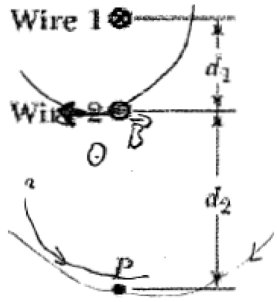
which is half the strength of F_L since this segment is twice as far from I_1. The net force is then

$$F = F_L + F_R = \frac{\mu_0 I_1 I_2 L}{4\pi R} = \frac{(10^{-7}\text{N/A}^2)(20\text{A})(12\text{A})(0.30\text{m})}{0.10\text{ m}} = 72.0\,\mu\text{N}.$$

(e) What is the direction of the force in part (d)?

As noted in the previous part, the net force is positive **to the right.**

8. In the Figure two very long straight wires are perpendicular to the page and separated by distance $d_1 = 0.75$ cm. Wire 1 carries a current of 6.5 A directed into the page.



a) What are the magnitude and direction (into or out of the page) of the current in wire 2 if the net magnetic field due to the two currents is zero at point P, located at distance d_2 = 1.50 cm from wire 2?

$$B_1 = \frac{\mu_0 i_1}{2\pi r_1} = B_2 = \frac{\mu_0 i_2}{2\pi r_2} \qquad r_2 = d_2 \qquad r_1 = d_1 + d_2$$

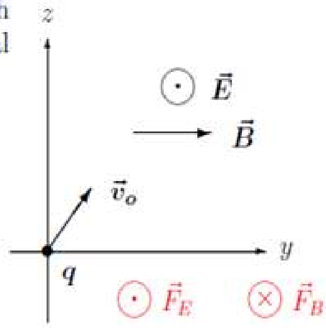
$$i_2 = \frac{r_2}{r_1} i_1 = 4.33\ \text{A}$$ out of page

b) What are the magnitude and direction of the magnetic force on wire 2 due to the current in wire 1?

$$F = i\vec{L}\times\vec{B} = iL\cdot B = L\cdot i_2\cdot\frac{\mu_0 i_1}{2\pi d_1}$$ downwards

$$\frac{A\cdot\frac{H}{m}\cdot A}{m} = \frac{A\,\frac{Tm}{A}\cdot A}{m} = TA = \frac{N}{Am}\cdot A = \frac{N}{m} \qquad \frac{F}{L} = 0{,}75\ \frac{mN}{m}$$

6. A positively charged particle, with charge $q = 2\,m\text{C}$, enters a region with a **uniform** electric field with $\vec{E} = (0.4\,\text{V/m})\,\hat{\imath}$ and a **uniform** magnetic field with $\vec{B} = (0.1\,\text{T})\,\hat{\jmath}$. The charged particle's initial velocity is $\vec{v}_o = (3\,\text{m/s})\,\hat{\jmath} + (4\,\text{m/s})\,\hat{k}$.



(a) What is the **electric force** on the charged particle as it enters the region? Present the answer as the magnitude and direction **OR** use the unit vector notation.

The unit vector notation: $\vec{F}_E = q\vec{E} = (2\times10^{-3}\,\text{C})(0.4\,\text{V}/m)\hat{\imath} = (0.8\times10^{-3}\,\text{N})\hat{\imath}$

The magnitude of the electric force is: $F_E = qE = (2\times10^{-3}\,\text{C})(0.4\,\text{V}/m) = 0.8\,m\text{N}$

The direction of $\vec{F}_E$ is the same as the direction of $\vec{E}$, which is in the positive x direction.

(b) What is the **magnetic force** on the charged particle as it enters the region? Present the answer as the magnitude and direction **OR** use the unit vector notation.

The unit vector notation: $\vec{F}_B = q\vec{v}\times\vec{B} = (2\times10^{-3}\,\text{C})(3\hat{\jmath}+4\hat{k})(\text{m/s})\times(0.1\,\text{T})\hat{\jmath} = -(0.8\times10^{-3}\,\text{N})\hat{\imath}$
because $\hat{\jmath}\times\hat{\jmath} = 0$, and $\hat{k}\times\hat{\jmath} = -\hat{\imath}$

The magnitude of the magnetic force is: $F_B = qvB\sin\theta = qv_zB = (2\times10^{-3}\,\text{C})(4\,\text{m/s})(0.1\,\text{T}) = 0.8\times10^{-3}\,\text{N}$

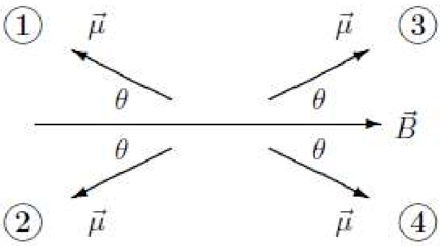
The direction of $\vec{F}_B$ is into the page which is in negative x direction.

(c) What should be the **magnitude** of the **magnetic field** for the charged particle to go through this region **undeflected**?

In order for a charged particle to move undeflected through an electric and magnetic filed these fields have to be crossed (perpendicular to each other), the electric and the magnetic forces on the particle have to have the same magnitude, and the directions of the two forces have to be opposite. In such a case the net force acting on the particle is zero and it moves undeflected (only the nonzero net force can change the velocity of the particle!)

Inspecting our results in parts (a) and (b) we can see that all the conditions just stated are met, therefore our charged particle is moving undeflected through the space with the two filds given in the problem.

4. The figure shows four orientations, at angle θ, of a magnetic dipole $\vec{\mu}$ in a magnetic field $\vec{B}$.



(a) Rank the orientations according to the **magnitude** of the **torque** on the dipole, **greatest** first. (Circle the right answer).

[(i)] All tie (ii) $|\vec{\tau}_1| > |\vec{\tau}_2| = |\vec{\tau}_3| > |\vec{\tau}_4|$ (iii) $|\vec{\tau}_1| > |\vec{\tau}_2| > |\vec{\tau}_3| > |\vec{\tau}_4|$ (iv) $|\vec{\tau}_1| = |\vec{\tau}_2| > |\vec{\tau}_3| = |\vec{\tau}_4|$

The magnitude of a torque: $|\vec{\tau}| = |\vec{\mu}||\vec{B}||sin\theta|$

Also: $\sin\theta = \sin(\pi-\theta) = -\sin(\pi+\theta) = -\sin(-\theta)$ which have all the same magnitude.

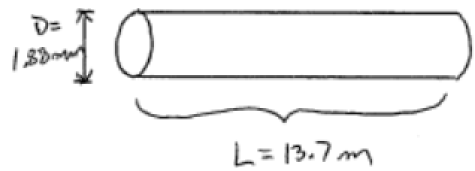
(b) Rank the orientations according to the **potential energy** of the dipole, **greatest** first. (Circle the right answer).

(i) All tie (ii) $U_1 = U_3 > U_2 = U_4$ [(iii)] $U_1 = U_2 > U_3 = U_4$

The potential energy of a dipole: $U = -|\vec{\mu}||\vec{B}|cos\theta$

Also: $\cos\theta = -\cos(\pi-\theta) = -\cos(\pi+\theta) = \cos(-\theta)$

17. A gold wire of length 13.7 cm and diameter 0.188 mm is going to be used to construct a thermocouple. Gold has a temperature coefficient of resistivity of 3.40×10^{-3} °C^{-1}. (a) At 20.0°C, gold has a resistivity of 2.44×10^{-8} Ω·m, what is the resistance of this wire at this temperature? (b) After constructing our thermocouple, we place it into a liquid of unknown temperature and apply a voltage across the leads of the termocouple of 0.224 V. If we measure a current of 0.884 A going through this thermocouple, what is the temperature of this liquid in °C?



$\alpha = 3.40 \times 10^{-3}\ {}^{\circ}C^{-1}$

$\rho = 2.44 \times 10^{-8}\ \Omega\, m, \quad T = 20.0^{\circ}C$

$$A = \frac{\pi D^2}{4} = \frac{\pi}{4}\left(1.88 \times 10^{-4} m\right)^2 = 2.78 \times 10^{-8} m^2$$

a) $$R = \rho \frac{L}{A} = \left(2.44 \times 10^{-8}\ \Omega m\right) \frac{0.137\ m}{2.78 \times 10^{-8}\ m^2}$$

$$= \boxed{0.120\ \Omega} = R_0 \quad (\text{for problem below})$$

at $T_0 = 20.0^{\circ}C$

b) $$R = \frac{\Delta V}{I} = \frac{0.224\ V}{0.884\ A} = 0.253\ \Omega$$

$$R = R_0\left[1 + \alpha(T - T_0)\right] = R_0 + \alpha R_0 (T - T_0)$$

$$\alpha R_0 (T - T_0) = R - R_0, \quad T - T_0 = \frac{R - R_0}{\alpha R_0}$$

$$T = T_0 + \frac{R - R_0}{\alpha R_0} = 20.0^{\circ}C + \frac{0.253\ \Omega - 0.120\ \Omega}{\left(3.40 \times 10^{-3}\ {}^{\circ}C^{-1}\right)(0.120\ \Omega)}$$

$$= 20.0^{\circ}C + 3\underline{2}6^{\circ}C = \boxed{346^{\circ}C}$$

5) A copper wire is 50 cm long and has a circular cross section with a diameter 0.8mm.

a) Calculate the resistance of the wire. The resistivity of copper is $1.7\times10^{-8}\ \Omega\text{m}$.

$$R = \rho\frac{L}{A} = 1.7\times10^{-8}\,\Omega\cdot m \cdot \frac{0.5m}{\pi\,(0.4mm)^2} = 0.017\,\Omega$$
$$= 1.7\times10^{-2}\,\Omega$$

b) A 5.0V potential difference is applied between the ends of the wire. Calculate the current.

$$I = \frac{\Delta V}{R} = \frac{5.0V}{0.017\,\Omega} = 296A$$

c) For the situation in part b), calculate the power dissipated in the wire.

$$P = \Delta V\cdot I = 1,480\,W \approx 1.5\,kW$$

Problem 4

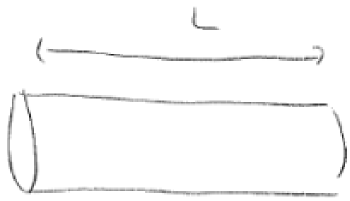
Consider a cylindrical resistor of length L, radius a, and resistivity ρ. The cylinder is positioned with its axis along the z-axis. A current of magnitude I flows through the resistor in the positive-z direction. Assume that the current is distributed uniformly within the cylinder.

(a) Find the resistance R of the cylinder and the power P dissipated in it.

(b) Derive expressions for the electric field within the cylinder and for the magnetic field both inside and outside of the cylinder. Be sure to specify both the magnitude and direction of the fields. In determining the magnetic field, approximate the length of the cylinder as infinite.

($\mu/4\pi = 4\pi/c$)

(a) $\boxed{R=\rho\frac{L}{\pi a^2}}$ $\quad P=I^2R=\boxed{\frac{I^2\rho L}{\pi a^2}}$



(b) Voltage across cylinder = IR

$\bar{E}=V/L = IR/L = \boxed{\frac{I\rho}{\pi a^2}\hat{z}}$

For magnetic field, use Ampere's Law: $\oint B\cdot d\ell = \frac{4\pi}{c} I_{enclosed}$

$\odot\hat{z}$

$r<a$

$B\cdot 2\pi r = \frac{4\pi}{c}(\pi r^2 J)$ $\qquad \frac{I}{\pi a^2}$

$B\cdot 2\pi r = \frac{4\pi}{c}\pi r^2\frac{I}{\pi a^2}$

$\bar{B} = \frac{4\pi^2 r^2 I}{2\pi^2 r c a^2} = \boxed{\frac{-2I}{ca^2} r\hat{\theta}}$

$r\geq a$: $B\cdot 2\pi r = \frac{4\pi}{c} I$ $\quad \boxed{\bar{B} = \frac{-2I}{cr}\hat{\theta}}$

4) a) A wire 2.5 m long carries a current of 8 A and makes an angle of 25° with a uniform magnetic field of magnitude $B = 0.7$ T. Calculate the magnetic force on the wire. Explain clearly your answer.

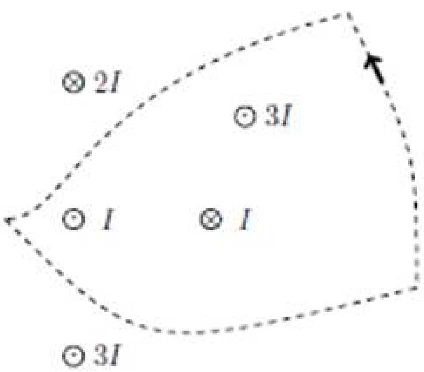
b) A circular coil of 150 turns has a radius of 3 cm. Calculate the current that results in a magnetic dipole moment of $5A.m^2$.
Explain clearly your answer.

a) $F = BiL\sin\theta = 5.92N$

b) $\mu = NiA \quad \Rightarrow \quad i = \frac{5A.m^2}{150\pi(0.03m)^2} = 11.8A$

5) a) A square loop of wire of edge length a carries current i. Calculate the magnitude of the magnetic field at the center of the loop.
Explain clearly your answer.

b) Evaluate $\oint \vec{B}\cdot d\vec{s}$ for the path shown in figure. Explain clearly your answer.

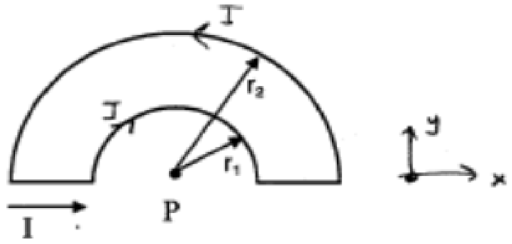


a) $B = \frac{\mu_0 I\sqrt{2}}{2\pi a}\cdot 4 = \frac{2\sqrt{2}\mu_0 I}{\pi a}$

b) $\oint \vec{B}\times\vec{ds} = 3\mu_0 I$

Problem 3:

(a) A wire loop is bent into the shape shown below, with two semicircles of radii r_1 and r_2, and two straight edges of length d. If a current I flows around the loop in the direction shown, what is the magnetic field $\vec{B}$ at the center of curvature (point P)?

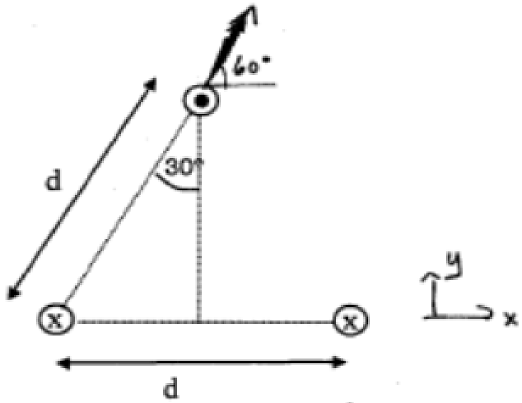


let $\hat{z}$ be out of page →

$B_{arc} = \frac{\mu_0 I}{4\pi r}\theta$ $\vec{B}_{net} = \left(\frac{\mu_0 I \pi}{4\pi r_1} - \frac{\mu_0 I \pi}{4\pi r_2}\right)(-\hat{z}) = \frac{\mu_0 I}{4}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)(-\hat{z})$ (into page)

(b) Three long wires, each with linear mass density μ, carry equal currents in the directions shown. The lower two wires are a distance d apart and are attached to a table.

1. What is the net force (magnitude and direction) on the upper wire?
2. What current I will allow the upper wire to levitate so as to form an equilateral triangle with the lower wires as shown?



1. $\vec{F}_{net} = \vec{F}_{13} + \vec{F}_{23}$

$= \frac{\mu_0 I^2 l}{2\pi d}(\cos 60\,\hat{x} + \sin 60\,\hat{y}) + \frac{\mu_0 I^2 l}{2\pi d}(-\cos 60\,\hat{x} + \sin 60\,\hat{y})$

$$\boxed{\frac{\vec{F}_{net}}{l} = \frac{\mu_0 I^2}{\pi d}\frac{\sqrt{3}}{2}\,\hat{y}}$$

(x component vanishes by Symmetry)

2. levitate if $|\vec{F}_{net\,mg}| = mg$ mass density $\mu = m/l$

$\frac{\mu_0 I^2 \sqrt{3}}{2\pi d}\cancel{l} = \mu g \cancel{l}$ $$\boxed{I = \sqrt{\frac{\mu g\, 2\pi d}{\sqrt{3}\mu_0}}}$$

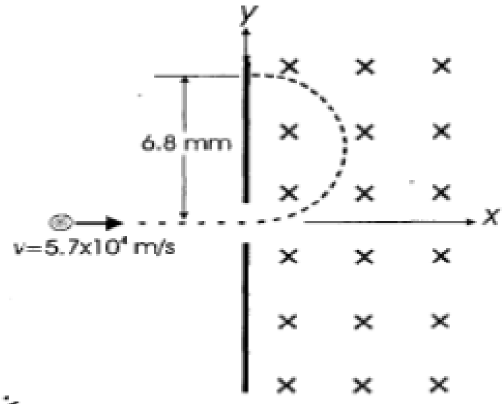
[10] A uniform magnetic field, directed into the page as shown, exists to the right of a barrier. An ion with a mass of 2.5×10^{-26} kg and a speed of 5.7×10^{4} m/s enters this region of uniform magnetic field through an aperture at the origin as shown. The ion moves in a counterclockwise direction around a semicircular path that intersects the barrier 6.8 mm above the aperture as shown.

(a) Is the ion positively charged or negatively charged?

(b) What is the centripetal acceleration of the ion when it is in the region of uniform magnetic field?

(c) Assuming that the **magnitude** of the charge on the ion is e (1.6×10^{-19} C), what is the magnitude of the magnetic field?

(d) A negative ion having the same speed and magnitude of charge but a mass of 1.75×10^{-26} kg is projected through the aperture and into the region of magnetic field. Where will its path intersect the barrier?



(b) $\frac{v^2}{r} = a = \frac{(5.7\times10^4\ m/s)^2}{\left(\frac{0.0068\ m}{2}\right)} = 9.56\times10^{11}\ m/s^2$

(c) $a = \frac{F}{m} \Rightarrow \frac{v^2}{r} = \frac{qvB}{m} \Rightarrow B = \frac{ma}{qv} = \frac{(2.5\times10^{-26}kg)(9.56\times10^{11}m/s^2)}{(1.6\times10^{-19}C)(5.7\times10^4 m/s)}$

$B = 2.62\ T$

(d) $r = \frac{mv}{qB} = \frac{(1.75\times10^{-26}kg)(5.7\times10^4 m/s)}{(1.6\times10^{-19}C)(2.62T)} = 2.38\times10^{-3}m = 2.38\ mm$

IT INTERSECTS THE BARRIER AT $y = -2r = -4.76$ mm.

8. A long straight solid cylinder of radius a, with its axis along the z direction, carries a non-uniform current density $\vec{J}(r) = J(r)\hat{k}$. The non-uniform current density is

$$J(r) = \frac{2I_0}{\pi a^2}(1-(r/a)^2),\ \ r \le a \quad \text{and} \quad J(r) = 0,\ \ r > a.$$

(a) What is the total current enclosed by the cylinder ?

(b) What is the B field outside the cylinder ?

$J(r) = \frac{\text{current}}{\text{area}}$ $\quad A = \pi r^2$, $dA = 2\pi r\,dr$

$I(r) = \int_0^a J(r)\,dA = \int_0^a \frac{2I_0}{\pi a^2}(1 - r^2/a^2)\,2\pi r\,dr$

$= \frac{2I_0}{\pi a^2}\int_0^a 2\pi\,(r - r^3/a^2)\,dr$

$$= \frac{2I_0(2\pi)}{\pi a^2}\left[\frac{r^2}{2} - \frac{r^4}{4a^2}\right]_0^a = \frac{4I_0}{a^2}\left[\frac{a^2}{2} - \frac{a^4}{4a^2}\right]$$

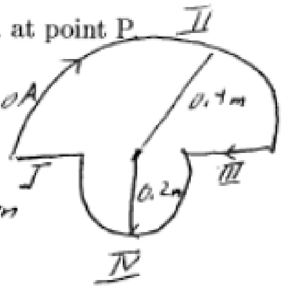
$$= \frac{4I_0}{a^2}\left[\frac{a^2}{2} - \frac{a^2}{4}\right] = \frac{4I_0}{a^2}\,\frac{a^2}{4} = I_0$$

$$\oint \vec{B}_3 \cdot d\vec{l} = B\,2\pi r = \mu_0 I_{enc}$$

$$B\,2\pi r = \mu_0 I_0$$

$$\boxed{B = \frac{\mu_0 I_0}{2\pi r}}\ \text{outside.}$$

9. Find the magnitude and direction of the magnetic field at point P.



$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2}$$

On segments I & III, there is no contribution since $d\vec{l} \parallel \hat{r}$. On segment II, $d\vec{l}\times\hat{r}$ is into the page. On segment IV, $d\vec{l}\times\hat{r}$ is also into the page.

$$dB = \frac{\mu_0}{4\pi}\left[\frac{I\overbrace{\pi r_1}^{\int dl}}{r_1^2} + \frac{I\overbrace{\pi r_2}^{\int dl}}{r_2^2}\right] = \frac{\mu_0 I}{4}\left[\frac{1}{r_1} + \frac{1}{r_2}\right]$$

$$= \frac{4\pi\times10^{-7}}{4}(3.0A)\left[\frac{1}{0.2} + \frac{1}{0.4}\right] = 7.07\times10^{-6}\,T \ \text{(direction into the page)}$$

Questions 21-25

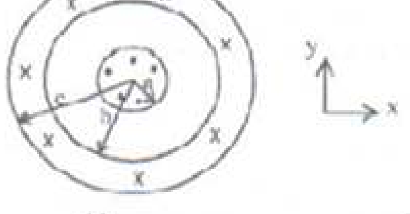
The figure below shows a cross section of a long cylindrical conductor of a type called coaxial cable. Its radii (a, b=3a, c=4a) are shown in the figure. Equal but opposite currents I_0 exist in the two conductors. The current density $\vec{J}$ is symmetric around the central axis of the cable but varies as a function of radial distance r from this axis according to the following relationship given in terms of the constants A and D:

$\vec{J} = Ar^2\hat{k}$ for $r \le a$

$\vec{J} = 0$ for $a \le r \le b$

$\vec{J} = -\frac{D}{r}\hat{k}$ for $b \le r \le c$

$\vec{J} = 0$ for $r \ge c$



21 . What is the constant A ?

a) $\frac{4I_0}{\pi a^4}$ b) $\frac{2I_0}{\pi a^4}$ c) $\frac{5I_0}{\pi a^4}$ d) $\frac{4I_0}{\pi a^3}$ e) $\frac{I_0}{\pi a^3}$

22 . What is the constant D?

a) $\frac{I_0}{\pi a}$ b) $\frac{2I_0}{\pi a^2}$ c) $\frac{I_0}{7\pi a^4}$ d) $\frac{I_0}{2\pi a}$ e) $\frac{3I_0}{7\pi a^2}$

23 . What is the magnitude of the magnetic field at $r = a/2$?

a) $\frac{\mu_0 A a^4}{64}$ b) $\frac{\mu_0 A a^3}{64}$ c) $\frac{\mu_0 A a^3}{32}$ d) $\frac{\mu_0 A a^4}{32}$ e) $\frac{\mu_0 A a^2}{64}$

24 . What is the magnitude of the magnetic field in the region where $a \le r \le b$?

a) $\frac{\mu_0 I_0}{\pi}$ b) $\frac{\mu_0 I_0}{\pi(b-a)}$ c) $\frac{I_0}{\pi r}$ d) $\frac{\mu_0 I_0}{2\pi a}$ e) $\frac{\mu_0 I_0}{2\pi r}$

25 . What is the magnitude of the magnetic field at $r = 3.5\,a$?

$\frac{\mu_0}{7}(\frac{I_0}{\pi a} - D)$ b) $\frac{\mu_0}{7}(\frac{I_0}{2\pi a} - D)$ c) $\frac{\mu_0}{6}(\frac{I_0}{\pi a} - 2D)$ d) $\frac{\mu_0}{6}(\frac{I_0}{2\pi a} - D)$ e) $\frac{\mu_0}{7}(\frac{I_0}{\pi a} - 2D)$