ITU Computer and Informatics Faculty
BLG 202E Numerical Methods in CE
2018 - 2019 Spring
Homework 2

Due 28.03.2019 23:00

- The solutions must be written on **white paper.** The written MATLAB codes should be included in the submitted report.
- Only **one** page should be used for each answer.
- Write your name and number at the top of each page.
- No late submissions will be accepted.
- In Case of Cheating and Plagiarism Strong **disciplinary action will be taken**.
- For any questions about the Homework 2, contact Beyza EKEN (beyzaeken@itu.edu.tr).

Submissions: Please submit your report through Ninova e-Learning System. Another way of submission will not be accepted.

Questions:

1. Write a MATLAB program to find all the roots of a given, twice continuously differentiable, function $f \in C^2[a, b]$.

   Your program should first probe the function $f(x)$ on the given interval to find out where it changes sign. (Thus, the program has, in addition to f itself, four other input arguments: *a*, *b*, the number $nprobe$ of equidistant values between *a* and *b* at which f is probed, and a tolerance tol.)

   For each subinterval $[a_i, b_i]$ over which the function changes sign, your program should then find a root as follows. Use either Newton's method or the secant method to find the root, monitoring decrease in $|f(x_k)|$. If an iterate is reached for which there is no sufficient decrease (e.g., if $|f(x_k)| \geq 0.5|f(x_{k-1})|$), then revert back to $[a_i, b_i]$, apply three bisection steps and restart the Newton or secant method. The ith root is deemed "found" as x_k if both

   $$|x_k - x_{k-1}| < tol(1 + |x_k|) \text{ and } |f(x_k)| < tol$$

   hold.

   a. Verify your program by finding the two roots of the function

   $$f(x) = 2cosh(x/4) - x\ ,$$

   starting your search with $[a, b] = [0,10]$ and $nprobe = 10$.

   b. Find all the roots of the function

$$f(x) = f(x) = \begin{cases} \dfrac{\sin(x)}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$$

in the interval $[-10,10]$ for $tol = 10^{-7}$.

2. Consider finding the root of a given nonlinear function $f(x)$ known to exist in a given interval $[a, b]$, using one of the following three methods: *bisection, Newton,* and *secant*. For each of the following instances, one of these methods has a distinct advantage over the other two. Match problems and methods and justify briefly.
    a. $f(x) = x - 1$ on the interval $[0,2.5]$.
    b. $f(x)$ is given in Figure 1 on [0, 4].
    c. $f(x) \in C^5[0.1,0.2]$, the derivatives of f are all bounded in magnitude by 1, and $f'(x)$ is hard to specify explicitly or evaluate.

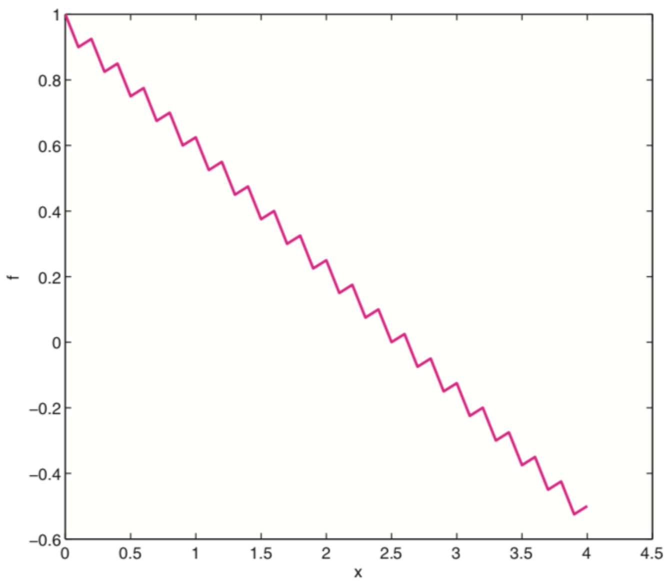

Figure 1: Graph of an anonymous function

3. Write a MATLAB function that solves tridiagonal systems of equations of size *n*. Assume that no pivoting is needed, but do not assume that the tridiagonal matrix *A* is symmetric. Your program should expect as input four vectors of size n (or $n - 1$): one right-hand-side $\boldsymbol{b}$ and the three nonzero diagonals of A. It should calculate and return $\boldsymbol{x} = A^{-1}\boldsymbol{b}$ using a Gaussian elimination variant that requires $O(n)$ flops and consumes no additional space as a function of n (i.e., in total $5n$ storage locations are required). Try your program on the matrix defined by $n = 10$, $a_{i-1,i} = a_{i+1,i} = -i$, and $a_{i,i} = 3i$ for all i such that the relevant indices fall in the range 1 to n. Invent a right-hand-side vector $\boldsymbol{b}$.

4. Let

$$A = \begin{pmatrix} 5 & 6 & 7 & 8 \\ 0 & 4 & 3 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & -2 \end{pmatrix}$$

a. The matrix A can be decomposed using partial pivoting as $PA = LU$, where U is upper triangular, L is unit lower triangular, and P is a permutation matrix. Find the 4×4 matrices U, L, and P.

a. Given the right-hand-side vector $\boldsymbol{b} = (26, 9, 1, -3)^T$, find $\boldsymbol{x}$ that satisfies $Ax = \boldsymbol{b}$**.** (Show your method, do not just guess.)