1) A region in space contains a total positive charge Q that is distributed spherically such that the volume charge density $\rho(r)$ is given by:

$\rho(r) = 0$ for $r \leq R/2$
$\rho(r) = \alpha[1-(r/R)^2]$ for $R/2 \leq r \leq R$
$\rho(r) = 0$ for $r \geq R$

$dV = 4\pi r^2 dr$

Here α is a positive constant having units of C/m^3.

a) Determine α in terms of Q and R.
b) Using Gauss's law derive an expression for the magnitude of electric field as a function of r separately for all three regions. Express your answers in terms of the total charge Q.

(7 Mart 2009 1. Arasınavı)

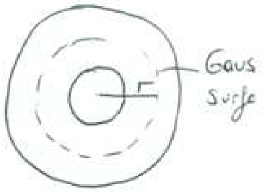
1) a) $Q = \int \rho\, dV = \int_{R/2}^{R} \alpha\left[1 - \frac{r^2}{R^2}\right] 4\pi r^2 dr \qquad Q = 4\pi\alpha\left[\int r^2 dr - \int \frac{r^4}{R^2} dr\right]$

$Q = 4\pi\alpha\left[\frac{r^3}{3}\Big|_{R/2}^{R} - \frac{r^5}{5R^2}\Big|_{R/2}^{R}\right] = 4\pi\alpha\left[\underset{5.32}{\frac{R^3}{3}} - \underset{5.4}{\frac{R^3}{3.8}} - \underset{3.32}{\frac{R^3}{5}} + \underset{3}{\frac{R^3}{5.32}}\right]$

$Q = \frac{4\pi\alpha R^3 47}{480} = \frac{47\pi\alpha R^3}{120} \Rightarrow \alpha = \frac{120 Q}{47\pi R^3}$

b) $r < \frac{R}{2} \qquad E = 0$ ①

$\frac{R}{2} \leq r \leq R \qquad \oint \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_0}$ ①



$q = \int_{R/2}^{r} \rho\, dV = 4\pi\alpha\left[\int_{R/2}^{r} r^2 dr - \int_{R/2}^{r} \frac{r^4}{R^2} dr\right] \qquad q = 4\pi\alpha\left[\frac{r^3}{3} - \frac{R^3}{3.8} - \frac{r^5}{5R^2} + \frac{R^3}{5.32}\right]$

$E\, 4\pi r^2 = \frac{4\pi\alpha}{\varepsilon_0}\left[\frac{r^3}{3} - \frac{R^3}{24} - \frac{r^5}{5R^2} + \frac{R^3}{160}\right]$

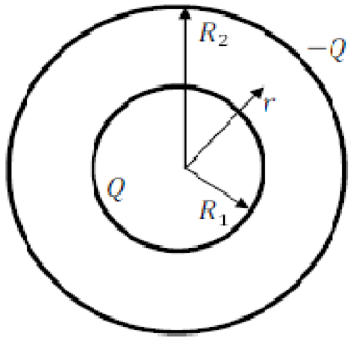
$E = \frac{\alpha}{\varepsilon_0}\left[\frac{r}{3} - \frac{R^3}{r^2} - \frac{r^3}{5R^2} + \frac{R^3}{160 r^2}\right]$

$E = \frac{120 Q}{47\varepsilon_0\pi}\left[\frac{r}{3R^3} - \frac{1}{r^2} - \frac{r^3}{5R^5} + \frac{1}{160 r^2}\right]$

$E = \frac{120 Q}{47\varepsilon_0\pi}\left[\frac{r}{3R^3} - \frac{159}{160 r^2} - \frac{r^3}{5R^5}\right]$

$r \geq R \qquad \oint \vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon_0} \qquad E = \frac{Q}{4\pi\varepsilon_0 r^2}$

2- An air-filled spherical capacitor is constructed with an inner shell of radius R_1 and an outer shell of radius R_2. If a charge of Q is placed on the inner conductor , and –Q on the outer conductor,



I) What is the electric field for region $r<R_1$?

a) $E = \frac{Q}{4\pi\varepsilon_0 r^2}$ **b) 0** c) $E = \frac{Q}{4\pi\varepsilon_0 r}$ d) $E = -\frac{Q^2}{4\pi\varepsilon_0 r^2}$

II) What is the electric field for region $r>R_2$?

a) $E = \frac{Q}{4\pi\varepsilon_0 R_2^2}$ b) $E = -\frac{Q}{4\pi\varepsilon_0 R_2^2}$ c) $E = \frac{Q}{4\pi\varepsilon_0 r^2}$ **d) 0**

III) What is the electric field for region $R_1<r<R_2$?

a) 0 b) $E = \frac{Q}{4\pi\varepsilon_0 r^3}$ c) $E = \frac{-Q}{4\pi\varepsilon_0 r^2}$ **d)** $\boldsymbol{E = \frac{Q}{4\pi\varepsilon_0 r^2}}$

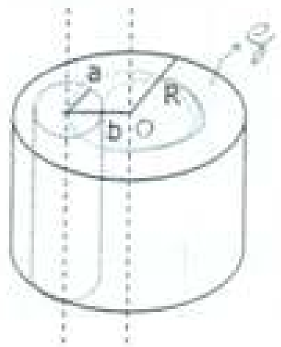
IV) What is the electric potential difference between the inner and outer conductor ?

a) $V(r) = \frac{Q}{4\pi\varepsilon_0 r}$ **b)** $\boldsymbol{V(r) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$ c) $V(r) = 0$ d) $V(r) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1^2} - \frac{1}{R_2^2}\right)$

V) What is the capacitance of the spherical capacitor ?

a) $C = \frac{Q}{4\pi\varepsilon_0}(R_2 - R_1)$ **b)** $\boldsymbol{C = \frac{Q}{4\pi\varepsilon_0}\left(\frac{R_1 R_2}{R_2 - R_1}\right)}$ c) $V(r) = 0$ d) $V(r) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1^2} - \frac{1}{R_2^2}\right)$

3) A very long, solid, insulating cylinder with radius R has a cylindrical hole with radius a bored along its entire length. The axis of the hole is a distance b from the axis of the cylinder, where $a+b<R$. The solid material of the cylinder has a uniform volume charge density $+\rho$. Find the electric field vector inside the hole. (Hint: You may want to consider the hole to be filled with both $+\rho$ and $-\rho$ and then use Gauss's law.)



3) $\oint \vec{E}\cdot d\vec{s} = \frac{Q_{ic}}{\varepsilon_0}$ $\quad E\, 2\pi r L = \frac{Q_{ic}}{\varepsilon_0}$

$$E \cdot 2\pi r L = \frac{\rho(\pi r^2 L)}{\varepsilon_0} \qquad \vec{E} = \frac{\rho(r)}{2\varepsilon_0}\hat{r}$$

$$\vec{E} = \vec{E}_+ + \vec{E}_- = \frac{\rho r}{2\varepsilon_0}\hat{r} - \frac{\rho r'}{2\varepsilon_0}\hat{r}'$$

$$\vec{E} = \frac{\rho}{2\varepsilon_0}(\vec{r} - \vec{r}') = \frac{\rho}{2\varepsilon_0}\vec{b}$$

$\rho = \frac{Q}{V} = \frac{Q}{\pi R^2 L}$

$\rho = \frac{Q'}{V'} = \frac{Q'}{\pi r^2 L}$

$\oint \vec{E}\, d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}$

$E\, 2\pi r L = \frac{\rho \pi r^2 L}{\varepsilon_0}$

$\vec{E} = \frac{\rho r'}{2\varepsilon_0}(\hat{r})$

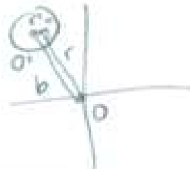
radius r $\quad E = \frac{\rho r}{2\varepsilon_0}$

$R < r \rightarrow \oint E\, da = \frac{Q_{ic}}{\varepsilon_0} \Rightarrow E \cdot 2\pi r L = \frac{\rho \pi r^2 L}{\varepsilon_0}$

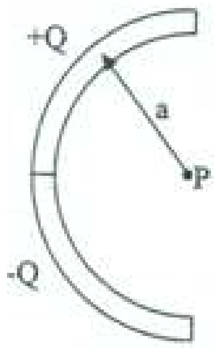
$E_h = \frac{\rho}{2\varepsilon_0}(b - r)$

$\boxed{\vec{E} = \frac{\rho r}{2\varepsilon_0}(\hat{r})}, \quad \hat{r} = \frac{\vec{r}}{r}$

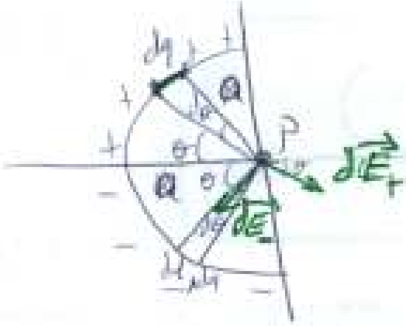
$\vec{E} = \frac{\rho \vec{r}}{2\varepsilon_0}$



6) A thin glass rod is bent into a semicircle of radius a. A charge $+Q$ is uniformly distributed along the upper half, and a charge $-Q$ is uniformly distributed along the lower half, as shown in the figure. Find the magnitude and the direction of the electric field at point P, the center of the semicircle.



6)



$dq = \lambda \cdot dl$

$|\lambda_+| = |\lambda_-| = \lambda$

$\lambda = \frac{Q}{\left(\frac{2\pi a}{4}\right)} = \frac{2Q}{\pi a}$

$\left(k = \frac{1}{4\pi\epsilon_0}\right)$

At point P:

$d\vec{E}_P = d\vec{E}_+ + d\vec{E}_-$

$d\vec{E}_P = (dE_{1x} + dE_{2x})\hat{i} + (dE_{1y} + dE_{+y})\hat{j}$

$= -2dE_+ \sin\theta\, \hat{j}$

$dl = a \cdot d\theta \Rightarrow dq = \lambda a\, d\theta$

$dE_+ = k\frac{dq}{a^2} = \frac{k\lambda a\, d\theta}{a^2} = k\frac{\lambda\, d\theta}{a}$

$dE_- = dE_+$ (due to the symm

$dE_{+x} = dE_+\cos\theta$, $dE_{+y} = -dE_+\sin\theta$

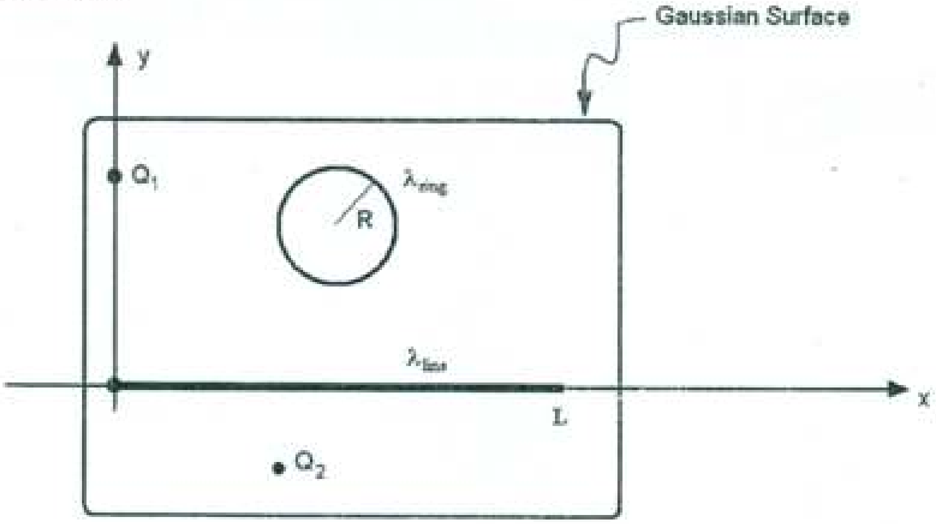
$dE_{-x} = -dE_{+x}$, $dE_{-y} = dE_{+y}$

$$\Rightarrow \int dE_P = E_P = \int[-2dE_+ \sin\theta] = \int_0^{\pi/2}\left[-\frac{2k\lambda\, d\theta}{a}\sin\theta\right]$$

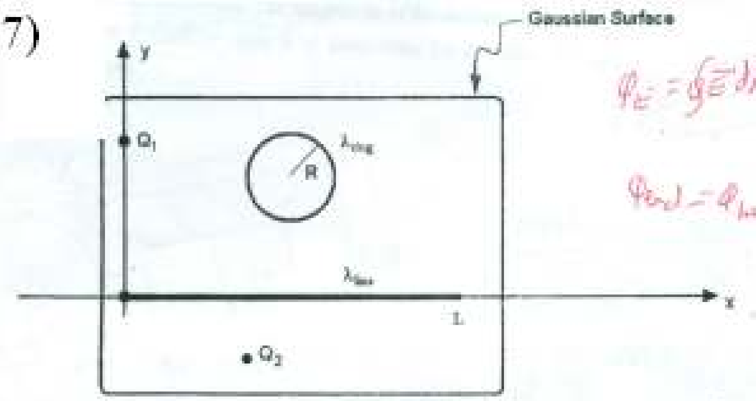
$$= -\frac{2k\lambda}{a}\int_0^{\pi/2}\sin\theta\, d\theta = +\frac{2k\lambda}{a}$$

$$\lambda = \frac{2Q}{\pi a} \Rightarrow E_P = +\frac{4kQ}{\pi a^2} \Rightarrow \boxed{\vec{E}_P = -\frac{4kQ}{a^2\pi}\hat{j}}$$

7) The volume enclosed by the Gaussian surface in the Figure includes point charges $Q_1=5Q$ and $Q_2=-2Q$, the ring of charge distribution with a uniformly distributed linear charge density $\lambda_{ring}=-7Q/L$, and a line of charge on the x-axis with a non-uniformly distributed linear charge density $\lambda_{line}=(3Q/L)(1-(x/L))$. The length of the line of charge is L and the radius of the ring is $R=L/6$. Calculate the flux of electric vector field, Φ, through the Gaussian surface.



7)



$\Phi_E = \oint \vec{E}\cdot d\vec{A} = \frac{q_{encl}}{\varepsilon_0}$

$q_{encl} = q_{line} + Q_1 + Q_2 + Q_{ring}$

$Q_{line} = \int_0^L \lambda_{line}\,dx = \int_0^L \left(\frac{3q}{L}\right)\left(1-\frac{x}{L}\right)dx = 3q\int_0^1 (1-u)\,du \quad \left(u = x/L,\ du = \frac{dx}{L}\right)$

$= 3q\left(u - u^2/2\,\Big|_0^1\right) = \frac{3q}{2}$

$q_{ring} = \lambda_{ring}\cdot 2\pi R = -\frac{7q}{L}\cdot\left(2\pi\frac{L}{6}\right) = -\frac{7q\pi}{3}$

enclosed charge: $\Sigma q = q_1 + q_2 + q_{line} + q_{ring} = 5q - 2q + \frac{3q}{2} - \frac{7\pi q}{3}$
$= q\left(9/2 - 7\pi/3\right)$

Φ_E = flux through the surface $\Rightarrow \boxed{\Phi_E = \frac{\Sigma q}{\varepsilon_0} = \frac{q(9/2 - 7\pi/3)}{\varepsilon_0}}$

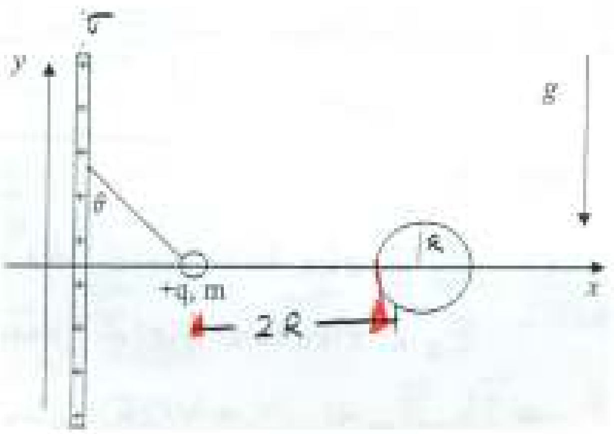
9) A positive point charge of q with mass m is connected with a neutral non-conducting wire, of length R, to an infinitesimally thin but large, two dimensional, positively charged, insulator sheet as shown from the side on the figure. The sheet has a uniform charge density of σ. In this system there is also a positively charged spherical insulator of radius R, with a non uniform charge density (with a radial charge distribution) of $\rho = Ar/R$. The sphere is fixed in its place. The distance between the surface of the sphere and the point charge is $2R$. If the point charge is stable as shown in the figure, while making an angle of θ with the charged sheet:

a) Calculate the electric field due to the charged, two dimensional, sheet at the position of the point charge. (Perform the calculation, do not just write the final result!)

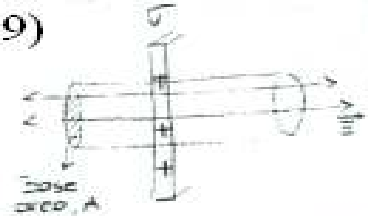
b) Calculate the electric field due to the charged sphere at the position of the point charge. (Perform the calculation, do not just write the final result!)

c) What is the value of σ in terms of q, R, m and θ ?

(Gravitational acceleration is g and $A = 36q/\pi R^3$) (*Hint: Use the Gauss Law.*)



9)

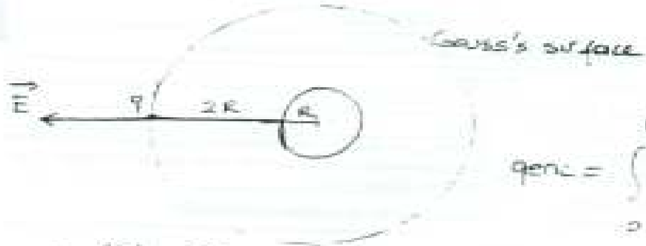


$\epsilon_0 \oint \vec{E}\cdot d\vec{A} = q_{enc}$

cylinder

$q_{enc} = \sigma A$ $\qquad \epsilon_0 E A \times 2 = \sigma A$

$\boxed{\vec{E}_{at\,q} = \frac{\sigma}{2\epsilon_0}\,\hat{i}}$ ($\vec{E}$ due to sheet)



sphere

$q_{enc} = \int_0^R \rho \cdot dV = \int_0^R A\frac{r}{R}\, 4\pi r^2 dr = \frac{A}{R}\, 4\pi \left.\frac{r^4}{4}\right|_0^R$

$\epsilon_0 \oint \vec{E}\, d\vec{A} = q_{enc}$ $\qquad q_{enc} = \frac{36\, q}{\pi R^3}\, \frac{4\pi}{R}\, \frac{R^4}{4} = 36q$

$\epsilon_0 E\, 4\pi r^2 = 36q$

$\epsilon_0 E\, 4\pi (3R)^2 = 36q \rightarrow \boxed{\vec{E}_{at\,q} = \frac{1}{\epsilon_0 \pi}\, \frac{q}{R^2}\,(-\hat{i})}$ (due to

ee body diagram for q

E_1 = electric field due to sheet
E_2 = electric field due to sphere

$T_y + mg = 0 \quad \ldots (a)$
$T_x + qE_1 + qE_2 = 0 \quad \ldots (b)$

$\rightarrow T\cos\theta = mg \longrightarrow T = \frac{mg}{\cos\theta} \rightarrow -T\sin\theta - qE_2 + qE_1 = 0$

$mg\tan\theta + q\,\frac{1}{\epsilon_0 \pi}\,\frac{q}{R^2} = q\,\frac{\sigma}{2\epsilon_0}$ $\qquad \sigma = 2mg\,\frac{\epsilon_0}{q}\tan\theta + \frac{2}{\pi}\,\frac{q}{R^2}$

10) A very long, uniform line of charge with positive linear charge density $+\lambda$ lies along the x-axis. An identical line of charge lies along the y-axis.

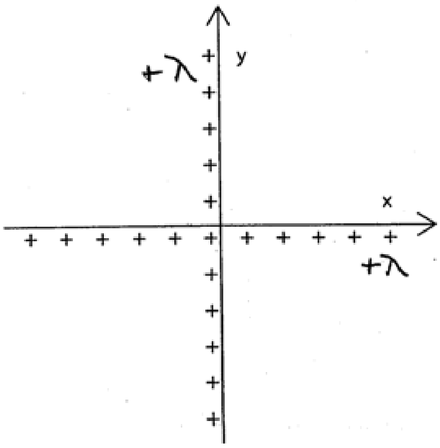
(a) Determine the electric field $\vec{E}(x, y)$ for all points in the $x-y$ plane.

(b) Determine the change in electrostatic potential ΔV between the points $x = a, y = a$ and $x = a, y = 3a$.

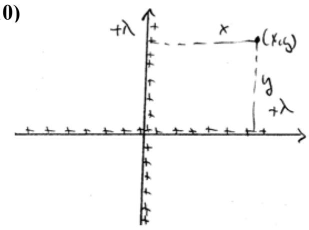
(c) Determine ΔV between the points $x = a, y = a$ and $x = 3a, y = a$.

(d) How much work must be done to move a small negative charge $-q$ from the point $x = 3a, y = 3a$ to the point $x = a, y = a$?

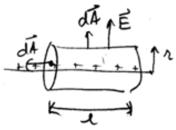
(e) For a very long linear charge distribution, we do not define the zero of electrostatic potential to be an infinity. Why not?



10)



(a) By Gauss's law, the field due to a long line of charge is



$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$ $\qquad 2\pi r l E = \frac{\lambda l}{\varepsilon_0}$

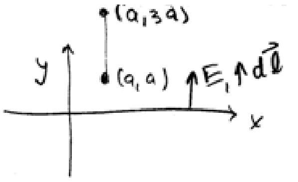
$E = \frac{\lambda}{2\pi\varepsilon_0 r}$ — direction is radially outward from the wire. r measures the distance from the wire.

The line of charge on the x-axis produces a field

$\vec{E}_1 = \frac{\lambda}{2\pi\varepsilon_0 y}\hat{y}$ - correct for all values of y since $\vec{E}_1$ changes direction when y changes direction.

The field due to the line of charge on the y-axis is $\vec{E}_2 = \dfrac{\lambda}{2\pi\varepsilon_0 x}\hat{\imath}$

The total field is the sum $\vec{E} = \dfrac{\lambda}{2\pi\varepsilon_0 x}\hat{\imath} + \dfrac{\lambda}{2\pi\varepsilon_0 y}\hat{\jmath}$

(b) 

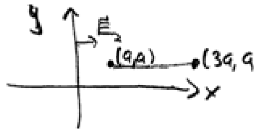
In moving from (a,a) to $(a,3a)$, we are moving perpendicular to $\vec{E}_2$, so it does not contribute to ΔV

$$\Delta V = -\int \vec{E}\cdot d\vec{l} \qquad d\vec{l} = \vec{d}_y$$

$$\Delta V = -\int_a^{3a} \frac{\lambda}{2\pi\varepsilon_0 y}\,dy = \frac{-\lambda}{2\pi\varepsilon_0}\ln\left(\frac{3a}{a}\right)$$

$$\boxed{\Delta V = -\frac{\lambda}{2\pi\varepsilon_0}\ln 3}$$

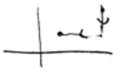
The potential is decreasing, as expected since we are moving in the direction of $\vec{E}_1$

(c) 

Now we are moving perpendicular to $\vec{E}_1$, so only $\vec{E}_2$ contributes.

$$\Delta V = -\int_a^{3a} \frac{\lambda}{2\pi\varepsilon_0 x}\,dx = \boxed{\frac{-\lambda}{2\pi\varepsilon_0}\ln 3 = \Delta V}$$

(d) In going from $(3a,3a)$ to (a,a) we can consider the path $(3a,3a) \to (3a,a) \to (a,a)$. Then



$$\Delta V = \Delta V_1 + \Delta V_2 = \frac{+\lambda}{2\pi\varepsilon_0}\ln 3 + \frac{\lambda}{2\pi\varepsilon_0}\ln 3$$

The sign changes because we are moving in the opposite direction as was considered in (b) & (c). Then $W = -q\Delta V = -\dfrac{q\lambda}{\pi\varepsilon_0}\ln 3$

This is the work done by an external agent. The field does positive work.

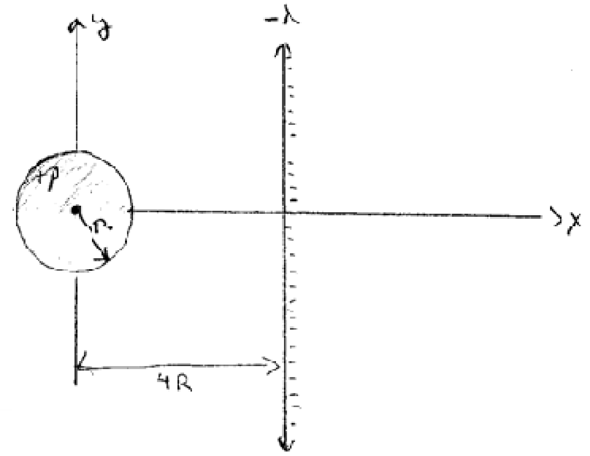
(e) Unlike a spherical charge distribution, for a line of charge $E \sim 1/r$. Then

$$\Delta V = -\int_{r_0}^{\infty} \vec{E}\cdot d\vec{r} = -\int_{r_0}^{\infty} \frac{\lambda}{2\pi\varepsilon_0 r}\,dr = \frac{\lambda}{2\pi\varepsilon_0}\ln(r)\Big|_{r_0}^{\infty} \to \infty$$

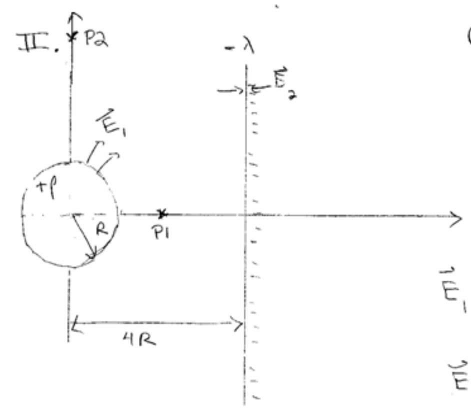
The integral defining ΔV diverges if we take the limit $\to \infty$. So instead we have to select finite limits to define ΔV.

11) An insulating sphere of radius R is centered at the origin. It carries a positive uniform volume charge density ρ. In addition, a very long, thin insulating rod runs parallel to the y-axis at $x = 4R$. The rod carries a negative uniform linear charge density $-\lambda$.
Express your answers in terms of ρ, R, λ, and possibly other constants.

(a) Determine the electric field $\vec{E}$ at the point $x = 2R, y = 0$.
(b) Determine the electric field $\vec{E}$ at the point $x = 0, y = 3R$.
(c) Determine the contribution to the x-component of the electric field, E_x, due to the rod only, as a function of position x on the x-axis.
(d) Determine the contribution to the x-component of the electric field, E_x, due to the sphere only, as a function of position x on the x-axis.
(e) Determine the electric flux Φ_E through a cube of side $\frac{1}{3}R$ centered at $x = 0, y = 2R$.



11)



(a) Find $\vec E$ at $x=2R, y=0$ (P1)

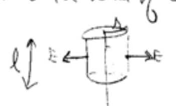
$\vec E_1$ due to sphere: $\oint \vec E \cdot d\vec A = \frac{Q_{enc}}{\varepsilon_0}$ all the charge on the sphere is enclosed.

$4\pi r^2 E_1 = \frac{Q_{TOT}}{\varepsilon_0}$ $\quad Q_{TOT} = \frac{4}{3}\pi\rho R^3$

$\vec E_1 = \frac{Q_{TOT}}{4\pi\varepsilon_0 r^2} = \frac{\rho R^3}{3\varepsilon_0 r^2}\hat r$ where $r = 2R$ at point P1 and $\hat r = \hat\imath$ at P1

$\vec E_1(P1) = \frac{\rho R}{12\varepsilon_0}\hat\imath$ – due to sphere. +3

$\vec E_2$ due to the line of charge:



$\oint \vec E \cdot d\vec A = \frac{Q_{encl}}{\varepsilon_0}$ $\quad E_2 = \frac{-\lambda}{2\pi\varepsilon_0 r}\hat r$

$2\pi r l E = \frac{-\lambda l}{\varepsilon_0}$ $\quad$ (radially inward)

At point P1, $\hat r = -\hat\imath$ and $r = 2R$

Note at P1 $\vec E_2$ is in the +x direction.

$\vec E_2(P_1) = \frac{-\lambda}{2\pi\varepsilon_0(2R)}(-\hat\imath) = \frac{+\lambda}{4\pi\varepsilon_0 R}\hat\imath$ +4

The total field is the superposition of these two contributions:

$\vec E = \vec E_1 + \vec E_2$ $\quad \boxed{\vec E(P1) = \left(\frac{\rho R}{12\varepsilon_0} + \frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat\imath}$

(b) P2 ⇒ $x=0$; $y=3R$ the same forms hold for E_1 & E_2

$\vec{E}_1 = \frac{\rho R^3}{3\varepsilon_0 r^2}\hat{r}$ at P2 $\hat{r}=\hat{j}$ and $r = 3R$

$\vec{E}_1(P2) = \frac{\rho R^3}{3\varepsilon_0 9R^2}\hat{j} = \frac{\rho R}{27\varepsilon_0}\hat{j}$ +3 Note $\vec{E}_1$ is in the +y direction.

$\vec{E}_2 = \frac{-\lambda}{2\pi\varepsilon_0 r}\hat{r}$ $\hat{r}$ is still $-\hat{\imath}$, $\vec{E}_2$ is $\perp$ to the line of charge.

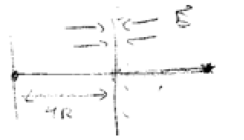
$r = 4R$ - the perpendicular distance from P2 to the line +4

$\vec{E}_2 = \frac{-\lambda}{2\pi\varepsilon_0(4R)}(-\hat{\imath})$ $\vec{E} = \vec{E}_1 + \vec{E}_2$

$$\boxed{\vec{E}(P2) = \frac{+\lambda}{8\pi\varepsilon_0 R}\hat{\imath} + \frac{\rho R}{27\varepsilon_0}\hat{j}}$$

(c) $\vec{E}$ due to rod only on x- axis

$\vec{E}_2 = \frac{-\lambda}{2\pi\varepsilon_0 r}\hat{r}$



⇒ $\vec{E} = E_x$ only, no other components

$E_2(\text{on x-axis}) = \frac{-\lambda}{2\pi\varepsilon_0|x-4R|}\hat{r}$

$$\boxed{E_{2x}(\text{on x-axis}) = \frac{-\lambda}{2\pi\varepsilon_0(x-4R)}\hat{\imath}}$$

Correct for all values of x - note change of sign as x passes through 4R

(d) $\vec{E}$ due to sphere only, on x-axis for $|x|>R$, the expression from (a) is correct.

$\vec{E} = \frac{\rho R^3}{3\varepsilon_0 r^2}\hat{r}$ on the x-axis, $\hat{r}=\hat{\imath}$ and $r=x$ for $x<0$, $\vec{E}$ changes direction, $\hat{r}=-\hat{\imath}$

$$\boxed{\vec{E}_{1x} = \frac{\rho R^3}{3\varepsilon_0 x^2}\hat{\imath} \quad \text{for } x > R} \qquad \boxed{E_{1x} = \frac{-\rho R^3}{3\varepsilon_0 x^2}\hat{\imath} \quad \text{for } x < -R}$$

for $|x| < R$, we have to consider the fact that only part of the charge is enclosed: $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0} = \frac{\rho(4/3\pi r^3)}{\varepsilon_0}$

$$\cancel{4\pi} r^2 E_r = \frac{\cancel{4\pi}\rho r^3}{3\varepsilon_0}$$

for $x>0$ $\hat{r} = \hat{\imath}$, $r = x$

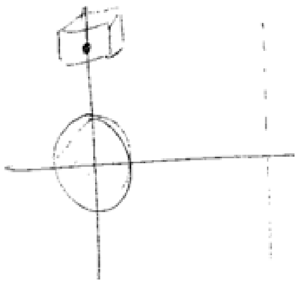
$$E_r = \frac{\rho r^3}{3\varepsilon_0 r^2} = \frac{\rho r}{3\varepsilon_0}\hat{r}$$

for $x<0$ $\hat{r} = -\hat{\imath}$

Then we have $\boxed{\vec{E}_{1x} = \frac{\rho x}{3\varepsilon_0}\hat{\imath} \quad \text{for } |x| < R}$

Correct for $x>0$ and $x<0$, note that x changes sign

(2)



Φ_E through cube at $x=0, y=2R$

side of cube $= R/3$.

By Gauss's law, $\oint \vec{E}\cdot d\vec{A} = \Phi_E = \frac{Q_{encl}}{\varepsilon_0}$

But $Q_{encl} = 0$, so

$$\boxed{\Phi_E = 0}$$