DISCRETE MATHEMATICS MIDTERM EXAM

90 minutes. April 1, 2011

Id	Fullname	Signature

Q1	Q2	Q3	Q4	Q5	Q6	Total
/20	/15	/15	/15	/20	/15	/100

1. Let $P(x)$: "x is a clear explanation", $Q(x)$: "x is satisfactory", and $R(x)$: "x is an excuse". Suppose that the universe of discourse for x is the set of all English text. Express each of the following statements using these predicates:

   (a) All clear explanations are satisfactory.

   (b) Some excuses are unsatisfactory.

   (c) Some excuses are not clear explanations.

   (d) Does (c) follow from (a) and (b)? If not, is there a correct conclusion? Explain your answer.

2. If $A \triangle B = A$ what can be said about the sets A and B? Explain. *Note*: $A \triangle B = (A \cup B) - (A \cap B)$

3. *(ANSWER THIS QUESTION ON THE FRONT SIDE OF THE SECOND PAPER).*
   How many positive integers n less than 6000 satisfy $gcd(n, 6000) = 1$?

4. What is wrong with the following "proof" that $a^n = 1$ for all $n \in \mathbb{N}$ whenever $a \neq 0$:

$$\begin{aligned} BASIS\,STEP &: \; a^0 = 1 \\ INDUCTIVE\,STEP &: \; a^{n+1} = \frac{a^n \cdot a^n}{a^{n-1}} = \frac{1 \cdot 1}{1} = 1 \end{aligned}$$

5. Let $m \in \mathbb{Z}^+$where m is an odd integer. Prove that there exists a positive integer n such that m divides $2^n - 1$. (*Hint*: You can use the pigeonhole principle.)

6. *(ANSWER THIS QUESTION ON THE BACK SIDE OF THE SECOND PAPER).* Let $f : A \rightarrow B$ be an onto function. Define the relation R on A such that aRb if $f(a) = f(b)$. Is R an equivalence relation?