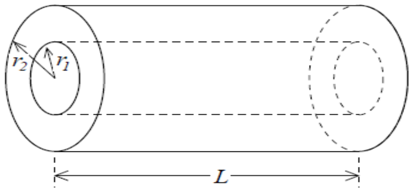
FIZ102E PHYSICS-II

PART I. Analytic Problems

1) A hollow cylindrical resistor with inner radius r_1 and outer radius r_2, and length L, is made of a material whose resistivity is ρ. Suppose current flows radially outward from the inner radius to the outer.



I) What is the current density $\vec{J}$?

a) $\frac{I}{2\pi rL}\hat{r}$ b) $\frac{I}{\pi r^2}\hat{r}$ c) $\frac{-Ir}{2\pi L}\hat{r}$ d) $\frac{I}{\pi L^2}\hat{r}$ e) $\frac{-I}{\pi r^2}\hat{r}$

II) What is the electric field vector $\vec{E}$?

a) $\frac{\rho I}{2\pi rL}\hat{r}$ b) $\frac{\rho I}{\pi r^2}\hat{r}$ c) $\frac{-\rho}{2\pi rL}\hat{r}$ d) $\frac{\rho}{\pi L^2}\hat{r}$ e) $\frac{2\rho I}{\pi r^2}\hat{r}$

III) What is the potential difference, V between the inner and outer cylinders?

a) $\frac{\rho I}{2\pi L}\ln\frac{r_2}{r_1}$ b) $\frac{\rho I}{L}\ln\frac{r_2}{r_1}$ c) $\frac{\rho I}{2L}\ln\frac{r_1}{r_2}$ d) $\frac{\rho I}{\pi L}\ln\frac{r_1}{r_2}$ e) $\frac{\rho I}{\pi r^2}\ln\frac{r_2}{r_1}$

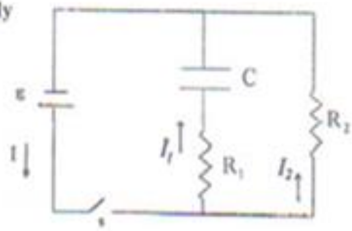
IV) Find the resistance R of the cylinder?

a) $\frac{\rho}{2\pi L}\ln\frac{r_2}{r_1}$ b) $\frac{\rho}{2\pi}\ln\frac{r_2}{r_1}$ c) $\frac{\rho}{2\pi L}\ln r_2$ d) $\frac{\rho}{\pi L}\ln r_1$ e) $\frac{\rho}{\pi L}\ln\frac{r_1}{r_2}$

V) What is the total power dissipated in the resistor?

a) $\frac{\rho I^2}{2\pi L}\ln\frac{r_2}{r_1}$ b) $\frac{\rho I^2}{2L}\ln\frac{r_1}{r_2}$ c) $\frac{\rho^2 I}{\pi L}\ln\frac{r_2}{r_1}$ d) $\frac{\rho I^2}{2\pi L}\ln(r_2-r_1)$ e) $\frac{\rho I^2}{2\pi L}(\frac{r_2}{r_1})$

2) The switch of the circuit shown in the figure is closed at time t=0. The capacitor is initially uncharged and the battery has no internal resistance.



I. What are the currents I_1 and I_2 at t=0?

a) $\frac{\varepsilon}{R_1}$ and $\frac{\varepsilon}{R_2}$ b) $\frac{\varepsilon}{2R_1}$ and $\frac{\varepsilon}{2R_2}$ c) $\frac{\varepsilon R_1}{R_2(R_1+R_2)}$ and $\frac{\varepsilon R_2}{R_1(R_1+R_2)}$

d) $\frac{2\varepsilon}{R_1}$ and $\frac{\varepsilon}{R_2}$ e) $\frac{\varepsilon}{2R_1}$ and $\frac{\varepsilon}{R_2}$

II. What are the charge on the capacitor and the current I_2 at $t\to\infty$?

a) εC and $\varepsilon/(R_1+R_2)$ b) εC and 0 c) εC and ε/R_2 d) $\varepsilon C/2$ and 0 e) $2\varepsilon C$ and ε/R_1

III. What is the charge on the capacitor as a function of time after the switch is closed?

a) $\varepsilon C(1-e^{-\frac{t}{(R_1+R_2)C}})$ b) $\varepsilon C(1-e^{-\frac{t}{R_1C}})$ c) $\varepsilon Ce^{-\frac{t}{((R_1+R_2)C)}}$ d) $\varepsilon C(1-e^{-\frac{t}{R_2C}})$ e) $\varepsilon Ce^{-\frac{t}{R_2C}}$

IV. When the capacitor is fully charged, the switch is opened. What is the current on the resistor R_2 immediately after the switch is opened?

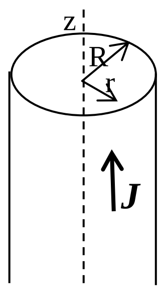
a) ε/R_2 b) $\varepsilon(R_1+R_2)/(R_1.R_2)$ c) $\varepsilon R_1/(R_2(R_1.R_2))$ d) $\varepsilon/(R_1+R_2)$ e) ε/R_1

V. What is the charge on the capacitor as a function of time after the switch is opened?

a) $\varepsilon Ce^{-\frac{t}{(R_1+R_2)C}}$ b) $\varepsilon Ce^{-\frac{t}{R_1C}}$ c) $\varepsilon C(1-e^{-\frac{t}{R_2C}})$ d) $\varepsilon C(1-e^{-\frac{t}{R_2C}})$ e) $\varepsilon C(1-e^{-\frac{t}{(R_1+R_2)C}})$

3) **<u>Questions 11-15</u> (Fall 2012, Final exam)**

A long, straight, solid cylinder, with radius R, is oriented with its axis in the z direction. It carries a current I whose current density is ***J***. The current density, although symmetric about the cylinder axis, is not constant, but varies according to $\vec{J}=\propto r\hat{k}$ **,** where α is a constant and r is the distance from the axis of the cylinder.



11) What is the total current I, flowing through the cylinder?

a) $2\pi\alpha R^3$ b) $2\pi\alpha R^3/3$ c) $\pi\alpha R^2/3$ d) $\pi\alpha R^3/2$ e) $2\alpha R^2/3$

12) What is the SI unit of α?

a) A^3/m^2 b) A^2/m c) A/m d) A/m^2 e) A/m^3

13) Find the magnitude and the direction of the magnetic field, B(r) in the region $r \leq R$

a) $\mu_0\alpha r^2/2R$, CW b) $\mu_0\alpha r^2/3$, CW c) $\mu_0\alpha r^2/3$, CCW d) $\mu_0\alpha r^2/3R^3$, CCW e) $\mu_0\alpha r/3R$, CW

14) Find the magnitude and the direction of the magnetic field, B(r) in the region $r \geq R$

a) $\mu_0 I/2\pi r$, CCW b) $\mu_0\alpha I/3r$, CW c) $\mu_0 Ir/2\pi$, CCW d) $\mu_0 I/2r$, CW e) $\mu_0 I/\pi r$, CCW

15) What is the total energy stored in the magnetic field of such a cylinder with length L in the region $r \leq R$?

a) $\mu_0\pi\alpha^2 L/5R^3$ b) $\mu_0\pi\alpha R^3 L/9$ c) $\mu_0\alpha^2 R^5/4$ d) $\mu_0\pi\alpha^2 R^6 L/54$ e) $\mu_0\pi\alpha R^2/4L$

QUESTIONS 11–15

11) total current, $I = \int \vec{J}\cdot d\vec{A} = \int J.dA$, area in dr at $r = dA = 2\pi r\,dr$

current in dr at $r = J.2\pi r.dr = \alpha r.2\pi r\,dr = \alpha 2\pi r^2 dr$

$\Rightarrow \underline{I} = \int_0^R \alpha 2\pi r^2 dr = \underline{\underline{\frac{2\pi}{3}\alpha R^3}}$

12) $\alpha = \frac{3I}{2\pi R^3}$ [A/m^3 is correct units]

13) $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}$, $I_{encl} = \int \vec{J}\cdot d\vec{A} = \int_0^r J.dA = \frac{2\pi}{3}\alpha r^3$

$2\pi r.B = \mu_0 \frac{2\pi}{3}\alpha r^3 \rightarrow \boxed{B = \frac{\mu_0\alpha r^2}{3}}$ $r \leq R$ (CCW)

14) $\oint \vec{B}\cdot d\vec{l} = \mu_0 I \Rightarrow 2\pi r B = \mu_0 I$, $\boxed{B = \frac{\mu_0 I}{2\pi r}}$ $r \geq R$ (CCW)

15) Energy stored in the volume of a cylindrical shell of radii r and r+dr with length L;

$dU = u.dV$, where u is energy density and dV is the volume of the cylindrical shell

$u = \frac{1}{2\mu_0}B^2$ and $dV = 2\pi r\,dr.L \Rightarrow dU = \frac{1}{2\mu_0}B^2 \cdot 2\pi r\,dr.L$

4) A long coaxial cable in Figure 3 consists of a long cylindrical inner conductor with radius a and an outer hollow cylindrical conductor with inner and outer radii b, c respectively. The wire carries opposite currents I through its inner and outer conductors with densities $J_{in}=\alpha r$ and $J_{out}=\beta r$ respectively, where r is the radial distance from the central axis and α, β are some proportionality constants.

16-What is the value of the constant β ?

a) $\dfrac{I}{\pi(c^3-b^3)}$ b) $\dfrac{I}{2\pi(c^3-b^3)}$ c) $\dfrac{2I}{\pi(a^3-c^3)}$ d) $\dfrac{I}{\pi(b^3-a^3)}$ e) $\dfrac{3I}{2\pi(c^3-b^3)}$

17-What is the magnetic field inside the inner conductor at a radial distance r ($0\le r<a$)?

a) $\dfrac{\alpha r \mu_0 a^2}{2(a-r)}$ b) $\mu_0 \alpha r^2-\mu_0 I$ c) $\mu_0 \alpha r^2-\mu_0 \beta(b^2-r^2)$ d) $\dfrac{1}{3}\mu_0 \alpha r^2$ e) $\dfrac{\alpha r \mu_0 a^2}{2\pi a}$

18-What is the magnetic field inside the outer conductor at a radial distance r ($b<r<c$)?

a) $\mu_0(\dfrac{1}{3}\alpha r^2-\dfrac{I}{2\pi r})$ b) $\mu_0 \dfrac{I}{2\pi(r-b)}$ c) $\mu_0 \beta r^2-\dfrac{\mu_0 I}{2\pi b}$ d) $\mu_0(\dfrac{I}{2\pi r}-\dfrac{2}{3}\alpha r^2)$ e) $\mu_0(\dfrac{I}{2\pi r}-\dfrac{1}{3r}\beta(r^3-b^3))$

19-What is the magnetic field outside the coaxial cable at a radial distance r ($r>c$)?

a) Zero b) $\dfrac{\mu_0}{2\pi r}(\alpha a^2+\beta(c^2-b^2))$ c) $\mu_0 \beta(c^2-b^2)-\dfrac{\mu_0 I}{2\pi a}$ d) $\mu_0 \dfrac{I}{\pi r}$ e) $\dfrac{\mu_0}{2r}(\alpha a^2-\beta(c^2-b^2))$

20-What is the magnetic flux per unit length through the space between the conductors (a<r<b)?

a) zero b) $\dfrac{\mu_0 I}{2\pi} ln\dfrac{b}{a}$ c) $\mu_0 I ln\dfrac{b}{a}$ d) $\dfrac{1}{2}\mu_0 \alpha\beta(b^2-a^2)ln\dfrac{b}{a}$ e) $\dfrac{1}{2}\mu_0 \alpha(b^2-a^2)ln\dfrac{b}{a}$

5) The figure below shows the cross section of a long conductor of a type called a coaxial cable. The radius of the inner solid cylinder is a, and the outer cylindrical shell has inner radius b and outer radius c, as shown in the figure below. The conductors carry equal but opposite currents I, with the current in the inner conductor flowing out of the page. The currents are uniformly distributed over the cross-sectional area in each case. The coordinate r measures the distance from the axis of the cylinders. Express your answers in terms of I, a, b, c and possibly other constants.

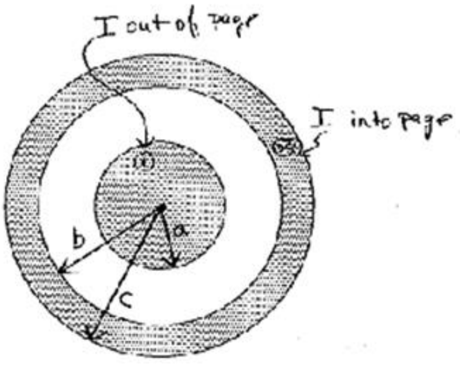
(a) Determine the magnitude of the current density $|\vec{J}_{inner}|$ in the region $r<a$. Determine the magnitude of the current density $|\vec{J}_{outer}|$ in the region $b<r<c$.

(b) Determine the magnetic field $\vec{B}(r)$ in the range $r<a$, being sure to indicate the direction of $\vec{B}$.

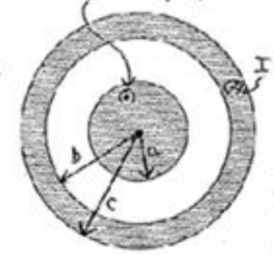
(c) Determine $\vec{B}(r)$ in the region $a<r<b$.

(d) Determine $\vec{B}(r)$ in the region $b<r<c$.

(e) Determine $\vec{B}(r)$ in the region $r>c$.



5)



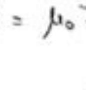
(a) I uniformly distributed over area

$|\vec{J}_{inner}| = \dfrac{I}{\pi a^2}$

$|\vec{J}_{outer}| = \dfrac{I}{\pi(c^2-b^2)}$ since area of outer cylindrical shell $= \pi(c^2-b^2)$

(b) Because of the symmetry of an infinite wire, we can use Ampere's Law

$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl.}$



$\vec{B}$ forms concentric loops, ccw in this case.

$\int \vec{B}\cdot d\vec{l} = \int B\,dl = 2\pi r B = \mu_0 \int_0^r J_{inner} \underbrace{dA}_{2\pi r\,dr}$

$(\vec{B} \parallel d\vec{l})$

$2\pi r|\vec{B}| = \dfrac{\mu_0 I}{\pi a^2}\displaystyle\int_0^r 2\pi r' dr'$ Only the current enclosed in $0 \to r$ contributes.

$2\pi r|\vec{B}| = \dfrac{\mu_0 I}{a^2}\cdot\dfrac{2r^2}{2}$ $\boxed{|\vec{B}| = \dfrac{\mu_0 I r}{2\pi a^2}}$ for $\underline{r<a}$ Direction is Counter-clockwise loops.

(OR)  Amperian loop

$I_{encl} = J_{inner}\cdot A = \dfrac{I}{\pi a^2}\cdot \pi r^2$

$\boxed{I_{encl} = I\cdot\left(\dfrac{r}{a}\right)^2}$

$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}$

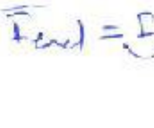
$\Downarrow$

$2\pi r\cdot B = I\cdot\left(\dfrac{r}{a}\right)^2$

$\Rightarrow \boxed{B = I\cdot\dfrac{r}{2\pi a^2}}$ $\underline{r<a}$

(c) $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} \Rightarrow \boxed{B(r) = \dfrac{\mu_0 I}{2\pi r}}$ $(I_{encl} = I)$.


Amperian loop for $\underline{a<r<b}$

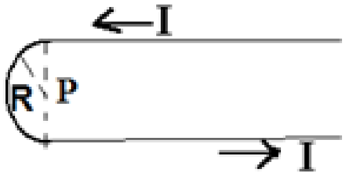
(d)  Amperian loop? for $\underline{b<r<c}$ $\Rightarrow$ $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}$, $I_{encl} = I - J_{outer}\,A$

$I_{encl} = I - \dfrac{I}{\pi(c^2-b^2)}\cdot\pi(r^2-c^2) = I\left[1-\dfrac{r^2-b^2}{c^2-b^2}\right]$

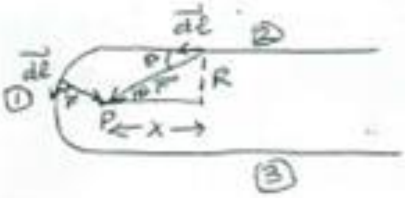
$\Rightarrow 2\pi r B = \mu_0 I_{encl} \rightarrow B = \dfrac{I}{2\pi r}\left(1-\dfrac{r^2-c^2}{c^2-b^2}\right]$

(e) $I_{encl} = I - I = 0 \Rightarrow B = 0$ $\underline{r>c}$

6) Consider the wire shown in the figure. Calculate the magnetic field at point P, the conter of the half-circle of the radius R around which the wire turns, as a function of R and the current I carried by the wire.



6)) We divide the wire into three pieces.



For the first piece

$$B_1 = \frac{\mu_0 I}{4\pi}\int \frac{d\ell\, r \sin 90^\circ}{r^3}$$

$$B_1 = \frac{\mu_0 I}{4\pi R^2}\int d\ell = \frac{\mu_0 I}{4\pi R^2}(\pi R) \quad \boxed{B_1 = \frac{\mu_0 I}{4R}}$$

For the second piece:

$B_2 = \frac{\mu_0 I}{4\pi}\int \frac{d\ell\, r\sin\theta}{r^3}$ where $r^2 = x^2 + R^2$;

$\sin\theta = \frac{R}{r}$ and $d\ell = -dx$. Hence, $B_2 = \frac{\mu_0 I R}{4\pi}\int_{\infty}^{0} \frac{(-dx)}{(x^2+R^2)^{3/2}}$

$$B_2 = \frac{\mu_0 I R}{4\pi}\int_0^{\infty} \frac{dx}{(x^2+R^2)^{3/2}} = \frac{\mu_0 I R}{4\pi}\left[\frac{1}{R^2}\frac{x}{\sqrt{x^2+R^2}}\right]_0^{\infty}$$

$$= \frac{\mu_0 I R}{4\pi R^2}\left[1-0\right] \quad \boxed{B_2 = \frac{\mu_0 I}{4\pi R}}$$

From symmetry

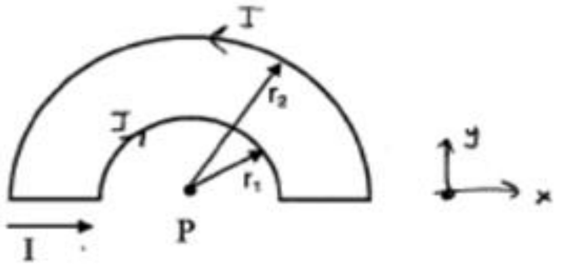
$$B_3 = B_2 = \frac{\mu_0 I}{4\pi R}.$$

Hence $B = B_1 + B_2 + B_3$

$$\boxed{B = \mu_0 I\left(\frac{1}{4R} + \frac{1}{2\pi R}\right)} \odot \vec{B}$$

$\vec{B}$ is out of th paper.

(a) A wire loop is bent into the shape shown below, with two semicircles of radii r_1 and r_2, and two straight edges of length d. If a current I flows around the loop in the direction shown, what is the magnetic field $\vec{B}$ at the center of curvature (point P)?

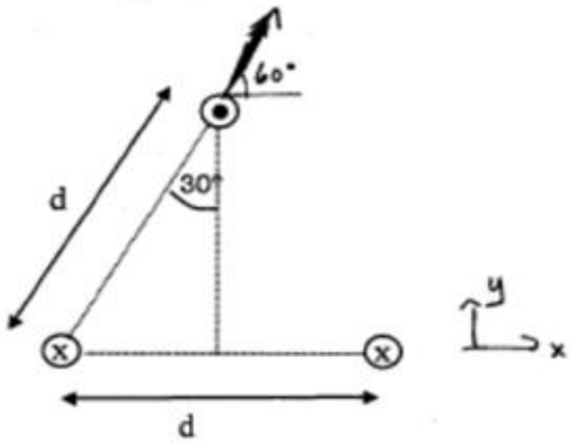


let $\hat{z}$ be out of page →

$$B_{arc} = \frac{\mu_0 I}{4\pi r}\theta \qquad \vec{B}_{NET} = \left(\frac{\mu_0 I \pi}{4\pi r_1} - \frac{\mu_0 I \pi}{4\pi r_2}\right)(-\hat{z}) = \frac{\mu_0 I}{4}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)(-\hat{z}) \quad \text{(into page)}$$

(b) Three long wires, each with linear mass density μ, carry equal currents in the directions shown. The lower two wires are a distance d apart and are attached to a table.

1. What is the net force (magnitude and direction) on the upper wire?
2. What current I will allow the upper wire to levitate so as to form an equilateral triangle with the lower wires as shown?



1. $$\vec{F}_{net} = \vec{F}_{13} + \vec{F}_{23}$$

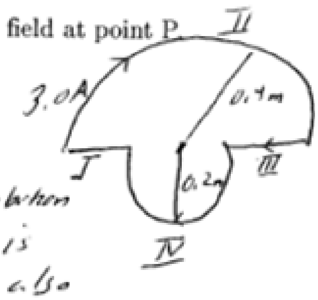
$$= \frac{\mu_0 I^2 l}{2\pi d}(\cos 60\hat{x} + \sin 60\hat{y}) + \frac{\mu_0 I^2 l}{2\pi d}(-\cos 60\hat{x} + \sin 60\hat{y})$$

$$\boxed{\frac{\vec{F}_{net}}{l} = \frac{\mu_0 I^2}{\pi d}\frac{\sqrt{3}}{2}\hat{y}} \qquad \text{(x component vanishes by symmetry)}$$

2. levitate if $|\vec{F}_{net\,mag}| = mg$ mass density $\mu = m/l$

$$\frac{\mu_0 I^2 \sqrt{3}}{2\pi d}\cancel{l} = \mu g \cancel{l} \qquad \boxed{I = \sqrt{\frac{\mu g\, 2\pi d}{\sqrt{3}\mu_0}}}$$

8) Find the magnitude and direction of the magnetic field at point P.



$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\, d\vec{l}\times\hat{r}}{r^2}$$

On segments I & III, there is no contribution since $d\vec{l} \parallel \hat{r}$. On segment II, $d\vec{l}\times\hat{r}$ is into the page. On segment IV, $d\vec{l}\times\hat{r}$ is also into the page.

$$dB = \frac{\mu_0}{4\pi}\left[\frac{I\underbrace{\pi r_1}_{\int dl}}{r_1^2} + \frac{I\underbrace{\pi r_2}_{\int dl}}{r_2^2}\right] = \frac{\mu_0}{4} I\left[\frac{1}{r_1}+\frac{1}{r_2}\right]$$

$$= \frac{4\pi\times10^{-7}}{4}(3.0A)\left[\frac{1}{0.2}+\frac{1}{0.4}\right] = 7.07\times10^{-6}\,T \quad \text{(direction into the page)}$$

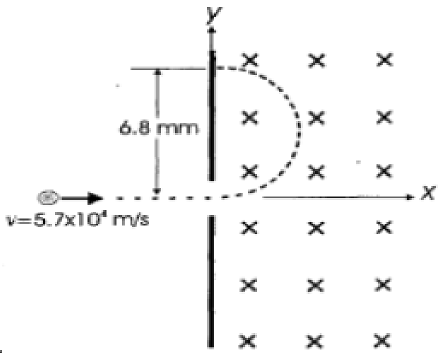
9) A uniform magnetic field, directed into the page as shown, exists to the right of a barrier. An ion with a mass of 2.5×10^{-26} kg and a speed of 5.7×10^{4} m/s enters this region of uniform magnetic field through an aperture at the origin as shown. The ion moves in a counterclockwise direction around a semicircular path that intersects the barrier 6.8 mm above the aperture as shown.

(a) Is the ion positively charged or negatively charged?

(b) What is the centripetal acceleration of the ion when it is in the region of uniform magnetic field?

(c) Assuming that the **magnitude** of the charge on the ion is e (1.6×10^{-19} C), what is the magnitude of the magnetic field?

(d) A negative ion having the same speed and magnitude of charge but a mass of 1.75×10^{-26} kg is projected through the aperture and into the region of magnetic field. Where will its path intersect the barrier?



(b) $$\frac{v^2}{r} = a = \frac{(5.7\times10^4\ m/s)^2}{\left(\frac{0.0068\ m}{2}\right)} = 9.56\times10^{11}\ m/s^2$$

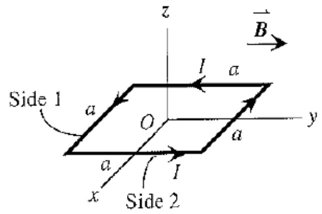
(c) $$a = \frac{F}{m} \Rightarrow \frac{v^2}{r} = \frac{qvB}{m} \Rightarrow B = \frac{ma}{qv} = \frac{(2.5\times10^{-26}kg)(9.56\times10^{11}m/s^2)}{(1.6\times10^{-19}C)(5.7\times10^4 m/s)}$$

$B = 2.62$ T

(d) $$r = \frac{mv}{qB} = \frac{(1.75\times10^{-26}kg)(5.7\times10^4 m/s)}{(1.6\times10^{-19}C)(2.62\,T)} = 2.38\times10^{-3}\,m = 2.38\ mm$$

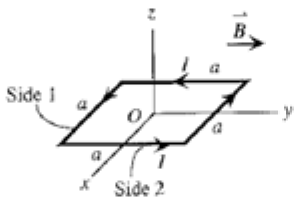
IT INTERSECTS THE BARRIER AT $y = -2r = -4.76$ mm.

10) A square loop of wire with side "a" lies in the xy plane. The loop carries a current I as shown in the figure. There is a



uniform magnetic field $\vec{B} = B_0\hat{j}$ along the +y axis.
1. What is the magnitude and direction of the forces acting on the side 1 and side 2 of the loop?
a. $\vec{F_1} = -IaB_0\hat{k}$, $\vec{F_2} = IaB_0\hat{j}$ b. $\vec{F_1} = 0$, $\vec{F_2} = IaB_0\hat{k}$ **c.** $\boldsymbol{\vec{F_1} = IaB_0\hat{k}}$ **,** $\boldsymbol{\vec{F_2} = 0}$
d. $\vec{F_1} = -IaB_0\hat{\imath}$, $\vec{F_2} = -IaB_0\hat{k}$
2. What is the magnitude and direction of the magnetic dipole?
a. $\vec{\mu} = -Ia^2\hat{\imath}$ **b.** $\boldsymbol{\vec{\mu} = Ia^2\hat{k}}$ c. $\vec{\mu} = Ia^2\hat{\jmath}$ d. $\vec{\mu} = -Ia^2\hat{k}$
3. What is the magnitude and direction of the torque acting on the loop for the position shown in the figure?
a. $\boldsymbol{\vec{\tau} = -Ia^2B_0\,\hat{\imath}}$ b. $\vec{\tau} = -Ia^2B_0\,\hat{\jmath}$ c. $\vec{\tau} = 2Ia^2B_0\,\hat{k}$ d. $\vec{\tau} = -Ia^2B_0\,\hat{k}$
4. How much work must be done by an external agent to turn the loop from the initial position shown in the figure to a final position so that the magnetic dipole moment will be along the y-axis.
a. $W = Ia^2B_0^2$ b. $W = -2Ia^2B_0$ c. $W = -\frac{1}{2}Ia^2B_0$ **d. W=** $\boldsymbol{Ia^2B_0}$
5. Suppose the loop is removed, that means ignore the loop. A proton of charge +e moves with initial velocity $\vec{v} = v_0\,\hat{\imath}$ along the +x axis into the uniform magnetic field $\vec{B} = B_0\,\hat{\jmath}$. What is the magnitude and direction of the electric field to be applied in the region so that the proton moves without deflection.
a. $\boldsymbol{\vec{E} = -v_0B_0\hat{k}}$ b. $\vec{E} = -v_0B_0\hat{\imath}$ c. $\vec{E} = v_0B_0\hat{k}$ d. $\vec{E} = v_0B_0\hat{\imath}$

A square loop of wire with side a lies in the xy plane. The loop carries a current I as shown in the figure. There is a uniform magnetic field $\vec{B} = B_0\hat{j}$ along the +y axis. (See the figure. Answer the questions in terms of the parameters given in the questions, I, a, B_0, v_0, express the vector quantities in unit vector notation.)



(a) (6 pts) Find the magnitude and direction of the forces acting on the side 1 and side 2 of the loop. Side 1 and side 2 are shown in the figure.

Side 1 : $\vec{F}_1 = I[a\hat{i} \times B_0\hat{j}]$

$F_1 = IaB_0 k$ (N)

Side 2 : $\vec{F}_2 = 0$

(b) (6 pts) Find the magnitude and direction of the torque acting on the loop for the position shown in the figure. ....

$\vec{\tau} = \vec{\mu} \times \vec{B}$ $\vec{\tau} = (Ia^2 k)\times(B_0 j)$ $\vec{\tau} = -Ia^2B_0 i$ (N-m)

(c) (7 pts) How much work must be done by an external agent to turn the loop from the initial position shown in the figure to a final position so that the magnetic dipole moment will be along the $-y$ axis.

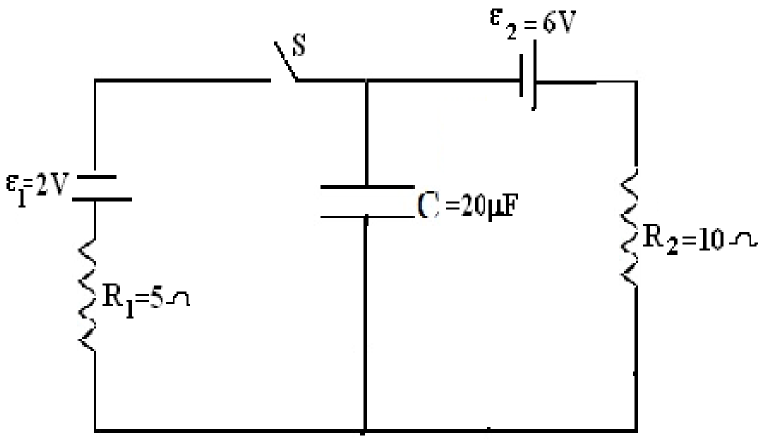
$U = -\vec{\mu}\cdot\vec{B}$ $U_i = 0$ $U_f = -[(-Ia^2 j)\cdot(B_0 j)] = Ia^2B_0$ (J)

$W = U_f - U_i \Rightarrow$ $W = Ia^2B_0$ (J)

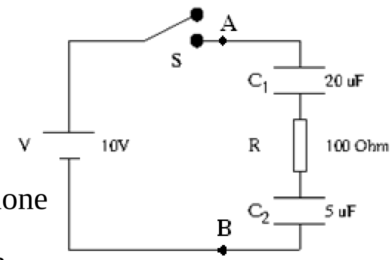
(d) (6 pts) Suppose the loop is removed, that means ignore the loop. A proton of charge $+e$ moves with initial velocity $\vec{v} = v_0\vec{i}$ along the $+x$ axis into the uniform magnetic field $\vec{B} = B_0\vec{j}$ stated in part (a). Find the magnitude and direction of the electric field to be applied in the region so that the proton moves without deflection.

$\vec{F} = e[\vec{E} + (v_0 i)\times(B_0 j)] = 0 \Rightarrow \vec{E} = -v_0B_0 k$ (V/m)

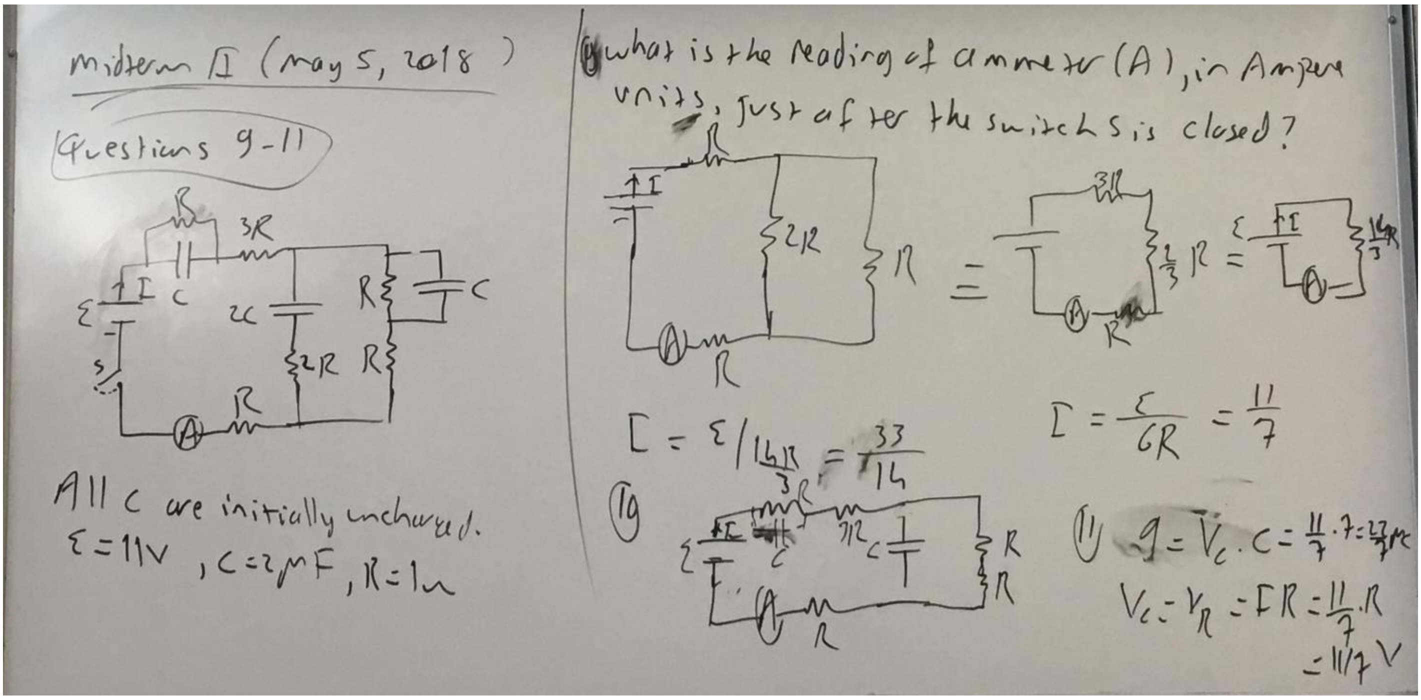
11) Consider the circuit shown in the figure below.
a) Find q on C when S has been open for a long time.
b) Now S is closed, find the currents in R_1 and R_2 immediately after S is closed.
c) A long time after S is closed, what are the resistor currents and capacitor charge?



12) Consider the circuit in the figure. The switch S is closed at t=0 and the capacitors are initially uncharged.



I) What is the time constant of the circuit?
a) 0.4ms b) 0.8 ms c)2.5 ms d) 1.25 ms e) none
II) What is the value of the current at t=0.8 ms?
a) 2A b) e^{-1} A c) 1A **d) (1/10)e^{-2} A** e) 0.5 A
III) What is the current right after the switch is closed?
a) 2 A **b) 0.1 A** c) 0.5 A d) e^{-2} A e) e^{-1} A
IV) After the equilibrium established, switch S is opened and A and B points are connected. What is the current rigth at the moment of the connection is established?
a) 0.1 A b) 2 A c) 0.5 A d) e^{-2} A e) e^{-1} A
V) Waiting long time after the A and B points are connected, what will be the charge in C_1 ?
a) 10 μC b) 40 μC **c) 0** d) 5μC e) 35 μC



Questions 11–15

11) τ = time constant $= R_{eq} \cdot C_{eq}$, $R_{eq} = R = 100\,\Omega$

$C_{eq} = C_1^{-1} + C_2^{-1} = \frac{20}{5} = 4\,\mu F$

$\Rightarrow \tau = (100)(4\times10^{-6} F) = \underline{0.4 ms}$

12) $t = 0.8ms = 2(0.4ms) = 2\tau$, $I = \frac{dq}{dt} = \frac{q_0}{RC_{eq}} e^{-t/\tau} = I_0 e^{-t/\tau}$

$\Rightarrow I = I_0 e^{-2\tau/\tau} = I_0 e^{-2} = \frac{q_0}{C_{eq}} \cdot \frac{1}{R} e^{-2} = \frac{V}{R} e^{-2}$

$\Rightarrow \boxed{I = \frac{1}{10} e^{-2} (A)}$

13) at $t=0 \Rightarrow I = I_0 = \frac{V}{R} = \frac{10V}{100} = \underline{0.1A}$ (right after the switch is closed)

14) Now, C_{eq} acts instantaneously like an emf of 10V.

$\Rightarrow \boxed{I = \frac{10V}{R} = 0.1 A}$

③

15) Both capacitors will be fully discharged through the resistor R.

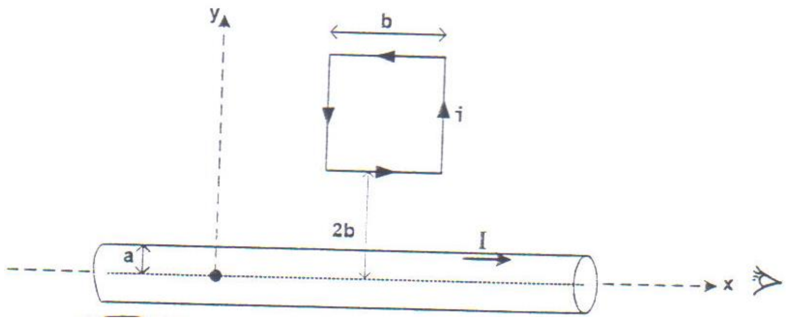
$\Rightarrow \boxed{q_1 = 0}$

13-

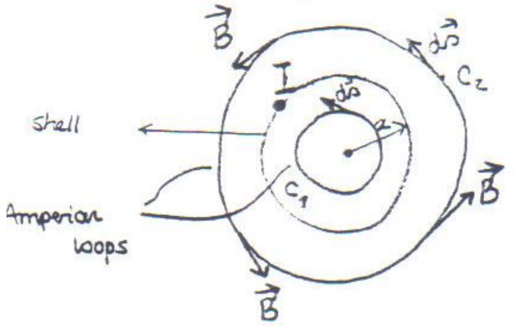
7) Consider a long conducting cylindrical *shell* of radius a, lying along the x-axis as shown in the figure. The current on the shell is I.

(i) Find the magnetic field vector $\vec{B}(r)$ in the regions $r < a$ and $r > a$ where r is the distance measured from the axis of the cylinder. (You must show Amperian loops and directions on them clearly.)

(ii) Now suppose that a square loop of each side b carrying a current i lies in the xy-plane with horizontal sides parallel to this cylindrical shell as in the figure. The distance between the cylindrical shell and the nearest side of the current loop is $2b$. Find the magnitude and direction of the net force exerted on the loop by the magnetic field of the cylindrical shell. (In calculating the force on each side, neglect the effect of other sides of the square and consider only the force acted by the cylindrical shell.)



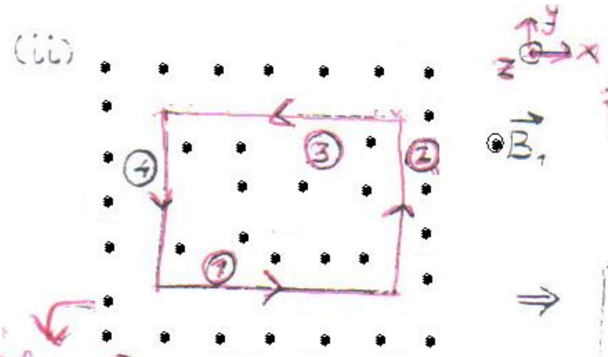
(i) View from the x-axis



Inside: $\vec{B} = 0$ - $i_{enc}(C_1) = 0$!

Outside: $\underbrace{\oint_{C_2} \vec{B} \cdot d\vec{s}}_{B.2\pi r} = \underbrace{\mu_0 i_{enc}(C_2)}_{I}$

(ii)



nonuniform B produced by the shell

$$\vec{F}_1 = i\vec{l}_1 \times \vec{B}_1 = (i b \hat{x}) \times \left(\frac{\mu_0 I}{2\pi 2b} \hat{z}\right)$$

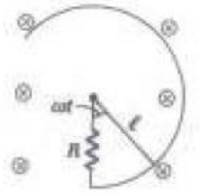
$$\Rightarrow \boxed{\vec{F}_1 = \frac{\mu_0 i I}{4\pi} (-\hat{y})}$$

$$\vec{F}_3 = i\vec{l}_3 \times \vec{B}_3 = i(-b\hat{x}) \times \left(\frac{\mu_0 I}{2\pi 3b} \hat{z}\right) \Rightarrow \boxed{\vec{F}_3 = \frac{\mu_0 i I}{6\pi} \hat{y}}$$

$\boxed{\vec{F}_2 + \vec{F}_4 = 0}$ magnitudes are the same but their directions are opposite; this is obvious from the symmetry.

$$\Rightarrow \vec{F}_{net} = \vec{F}_1 + \ldots + \vec{F}_4 \quad \boxed{\vec{F}_{net} = -\left(\frac{\mu_0 i I}{12\pi}\right)\hat{y}}$$

14) A conducting rod of length $\ell = 120$ mm is pivoted at one end as the other end slides on a circular conductor perpendicular to a uniform magnetic field $B = 400$ mT, as shown in Fig. 28-23. The rod rotates counterclockwise with constant angular speed $\omega = 370$ rad/s. Assume that all of the $R = 1200\text{-}\Omega$ resistance of the circuit is contained in the resistance symbol in the figure. *(a)* Determine an expression for the induced current in the circuit in terms of ℓ, B, ω, and R. *(b)* Evaluate the current using the values above. *(c)* What is the sense of the induced current? *(d)* Evaluate the magnitude of the magnetic torque on the rotating rod about an axis parallel to **B** and through the pivot point. How can the rod rotate with constant angular speed?



a) $\mathcal{E} = -N\dfrac{d\Phi_m}{dt} = -\dfrac{d}{dt}(\vec{B}\cdot\vec{A}) = -\dfrac{d}{dt}(BA\cos 180^\circ)$

$= \dfrac{d}{dt}(BA)$

The area A shown for $\theta = \omega t$, $A = \frac{1}{2}(\omega t)\ell^2$

$\mathcal{E} = \dfrac{d}{dt}\left[B\left(\frac{1}{2}\omega t \ell^2\right)\right] = \frac{1}{2}B\ell^2\omega$

$I = \dfrac{\mathcal{E}}{R} = \dfrac{\frac{1}{2}B\ell^2\omega}{R} = \dfrac{B\ell^2\omega}{2R}$

b) $I = 888\,\mu A$

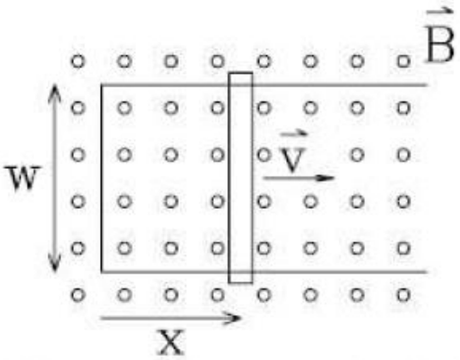
c) By Faraday's law, the induced current flows counterclockwise

d) For an element dr, the force $dF = (I\,dr)B$

Torque $= (I\,dr)Br$

$\tau = \int_0^\ell (I\,dr)Br = IB\int_0^\ell r\,dr = IB\dfrac{\ell^2}{2} = 2.55\,\mu N.m$

15) Consider the arrangement shown in the diagram: a conducting bar slides along a U-shaped conducting rail of width w under the action of an applied force which maintains its constant velocity $\vec{v} = v\hat{i}$ (i.e. to the right). There is a uniform magnetic field $\vec{B}$ directed perpendicular to, and out of, the page. Express all of your quantitative answers in terms of some or all of w, x, v, and/or B **only**.



1. When the bar is at position x, what is the magnetic flux through the closed circuit.

   Bwx

2. What is the magnitude of the emf around the circuit?

   Bwv

3. The induced current will produce a contribution to the magnetic field. In which direction will this field be (inside the rectangular loop)?

   Into the page.

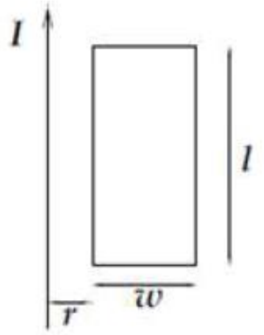
4. In which direction (up or down) through the conducting bar will the induced current flow?

   Down.

5. What is the direction of the magnetic force on the conducting bar?

   To the left.

16) A loop of wire in the shape of a rectangle of width w and length l, and a long straight wir carrying a current I are in a plane as shown. [Note: express all of your final answers in term of l, w, r (defined in the figure), I_2 (defined below), numerical constants (such as π), an constants that appear in Maxwell's equations.]



1. What is the magnitude and direction of the magnetic field at the exact center of the loop?

   $\frac{\mu_0 I}{2\pi(r+w/2)}$ into the page

2. What is the magnetic flux through the loop due to the current I? [Hint: You can't just multiply by an area; do an integral.]

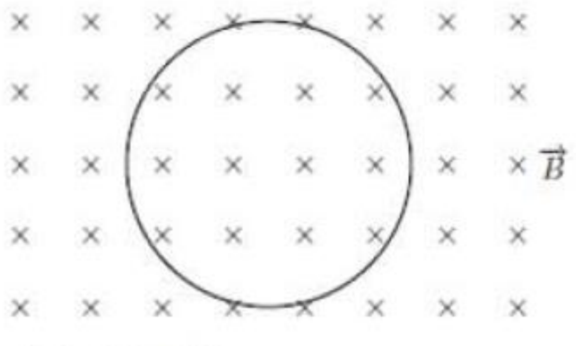
   $\frac{\mu_0 I l}{2\pi} \ln\left(1+\frac{w}{r}\right)$

3. If a current I_2 runs clockwise around the loop, what is the net force acting on the loop?

   $\frac{\mu_0 I I_2 l}{2\pi}\left(\frac{1}{r}-\frac{1}{r+w}\right)$ to the left

17) A circular loop of radius 2 cm is placed in a uniform magnetic field as seen in figure. The magnetic field is changed uniformly from 0.2 T to 0.8 T in a time interval of 1 s, beginning at $t = 0$.

a) Find the magnitude of magnetic field $t = 0$, $t = 0.5$ and $t = 1$ s.
b) What emf is induced in the loop at $t = 0.5$ s?
c) Find the direction of induced current in the loop. Explain clearly your answer.



a) $B = 0.2 + 0.6t$
$\Rightarrow \quad B(0) = 0.2\text{T},\ B(0.5) = 0.5\text{T},\ B(1) = 0.8\text{T}.$

b) $\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(BA) = -A\frac{dB}{dt} \quad = -\pi(2\times10^{-2}m)^2\,0.6T/s = -7.54\times10^{-4}\text{V}$

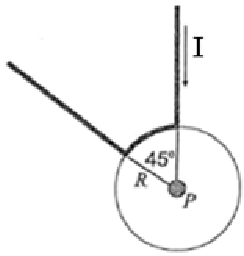
c) The magnetic field is into the page and increasing. By Lenz's law, induced current must decrease it. So the direction of the induced current is counter clock-wise.

FIZ102E PHYSICS-II

PART II. *Multiple Choice*

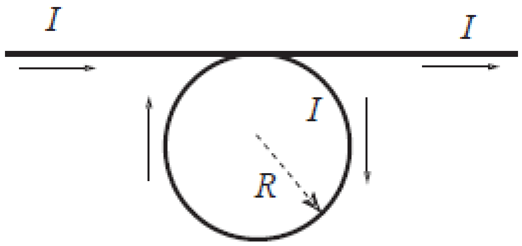
1) In the figure shown, a conductor consisting of a circular arch of 45^0 with radius R and two wires attached to this arc going radially outward (and effectively infinite in length) has a current I passing through it. What is the magnitude of the magnetic field at point P, the center of the arc?

a) $(\mu_0 I)/(16R)$ b) $(\mu_0 I)/(8R)$ c) 0 d) $(\mu_0 I)/(2R)$ e) $(\mu_0 I)/(4\pi R)$



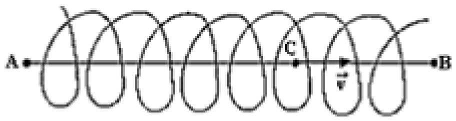
2) An infinitely long insulated wire carrying a current *I* is bent into the shape shown (straight line plus circle of radius *R* with the currents in the direction shown). The magnitude of the field *B* at the center of the circle is:

a) $\frac{\mu_0 I}{2R}(1+\pi)$ b) $\frac{\mu_0 I}{2\pi R}$ c) $\frac{\mu_0 I}{2R}$ **d) $\boldsymbol{\frac{\mu_0 I}{2\pi R}(1+\pi)}$** e) $\frac{\mu_0 I \pi}{2R}$



3) A solenoid carries a current I. An electron is injected with velocity **v** along the axis AB of the solenoid. When the electron is at C, it experiences a force that is

a) zero b) not zero and along AB c) not zero and along BA d) not zero and perpendicular to the page e) none of these is correct



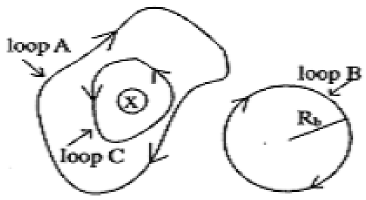
4) Two parallel conductors each of 0.50 m length, separated by 5.0×10^{-3} m and carrying 3.0 A in opposite directions, will experience what type and magnitude of mutual force? (magnetic permeability in empty space $\mu_0 = 4\pi \times 10^{-7}$ T.m/A).

a) repulsive, 0.6×10^{-4} N b) attractive, 0.06×10^{-4} N **c) repulsive, 1.8×10^{-4} N**
d) attractive, 1.8×10^{-4} N e) repulsive, 3.6×10^{-4} N

5) Consider a very long wire carrying a steady state current I going into the page as indicated in the figure below. Three oriented loops are also shown in the figure below. Which statement is correct?

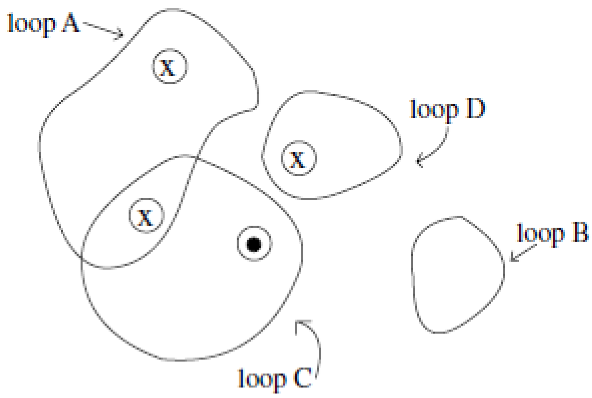
a) $\oint_B \vec{B}.\vec{dl} = \mu_0 I$b) $\oint_A \vec{B}.\vec{dl} = -\mu_0 I$ c) The magnetic field at every point around loop B is $\frac{\mu_0 I}{2\pi R_b}$

d) $\boldsymbol{\oint_A \vec{B}.\vec{dl} > \oint_B \vec{B}.\vec{dl} > \oint_C \vec{B}.\vec{dl}}$ e) $\oint_A \vec{B}.\vec{dl} = 0$



6) Consider four equal currents going into or out of the page as indicated in the figure below. Rank the line integral of the magnetic field $\oint \vec{B} \cdot d\vec{l}$ (from greatest to least) taken in the clockwise direction.

(a) $A = B = C = D$
(b) $A > C > B > D$
(c) $D > B > C > A$
(d) $B = C > D > A$
(e) $A > D > C = B$

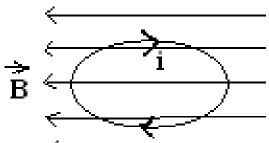


7) Consider a solenoid with radius R and length L ($R \ll L$). The magnetic field at the center of the solenoid is B_0. A second solenoid is constructed that has twice the radius, twice the length, and carries twice the current as the original solenoid, but has the same number of turns per meter. The magnetic field at the center of the second solenoid is

(A) $B_0/4$. (B) $B_0/2$. (C) B_0. (D) $2B_0$. (E) $4B_0$. $B = \mu_0 n i$

8) A circular coil with 200 turns, an area of 30 cm^2, carying a current of 40mA in clockwise sense, lies on a horizontal plane. The coil is in a horizontally directed uniform magnetic field of 20T, as shown in the figure. Find the magnitude of the torque on the coil.

a) 480 N.m b) 4.8 N.m c) 48 N.m d) 0.48 N.m e) 0.048 N.m



will result with a larger radius.

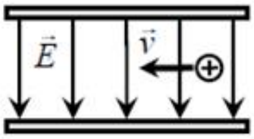
5) Torque on the dipole:

$\vec{\tau} = \vec{\mu} \times \vec{B}$, $\vec{\mu} = N \cdot i \cdot A\,\hat{n} \Rightarrow \mu = N \cdot i \cdot A = (200)(40\times10^{-3}A)(30\times10^{-4}m^2)$

since $\vec{B} \perp \vec{\mu}$, $|\vec{\tau}| = \mu \cdot B \cdot \sin 90°$ $\boxed{\vec{\mu} = -(24\times10^{-3} A.m^2)\hat{i}}$

$|\vec{\tau}| = \tau = (24\times10^{-3} A.m^2)(20T) = 0.48\,Nm$

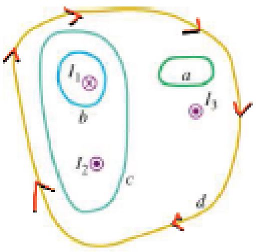
9) The figure at the right shows a positively-charged particle traveling through a velocity selector in which a uniform electric field is directed downward. What direction of magnetic field must be applied so that the particle travels in a straight line?



a. Upward, opposite to the direction of the electric field vector.
b. Downward, in the same direction as the electric field vector.
c. Out of the page.
d. Into the page.
e. It is not possible to use a magnetic field to make the particle travel in a straight line.

Rationale: *The electric force on a positive charge is parallel to the electric field, so it is downward. For the particle to travel in a straight line the magnetic force, $\vec{F} = q\vec{v}\times\vec{B}$, has to be upward. Then the right-hand rule [fingers along the first vector, $\vec{v}$, thumb along the cross-product direction (upward), and then the fingers curl out of the page] gives $\vec{B}$ out of the page.*

10) What is the line integral $\int \vec{B}\cdot d\vec{l}$ clockwise around the path d shown in the figure?

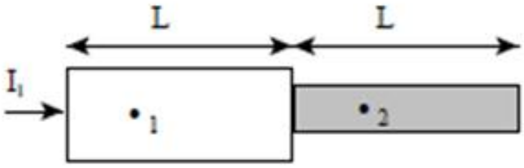


a) $\mu_0(I_1 + I_2 + I_3)$ b) $\mu_0(I_1 - I_2 - I_3)$ c) $\mu_0(I_2 - I_1 + I_3)$

d) $\mu_0(I_1 - I_2)$ e) 0

$\int \vec{B}.d\vec{l} = \mu_0 \times (\text{current enclosed})$
$= \mu_0 \times (I_1 - I_2 - I_3)$

11) A resistor which consists of two segments of equal lengths. The radius of segment 1 is twice as big as the radius of segment 2 . Their resistivities are the same. If the current in segment 1 is I_1, how big is the current in segment 2, I_2?



A) $I_2=¼I_1$ X B) $I_2=I_1$ C) $I_2=2\,I_1$ D) $I_2=4\,I_1$

12) Kirchoff's junction rule is an expression of:

A) Ohm's Law X B) Conservation of charge C) Conservation of energy D) Coulomb's Law

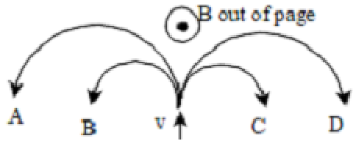
Kirchoff's loop rule is an expression of:

A) Ohm's Law B) Conservation of charge X C) Conservation of energy D) Coulomb's Law

13) A copper wire and an aluminum wire both have the same dimensions (same length, same cross-sectional area). Which one statement is true?

B) Both wires have the same resistance and the same resistivity.
X C) The wires have different resistances and different resistivities.
D) The wires have the same resistance but different resistivities.
E) The wires have different resistances but the same resistivity.

14) Four particles moving at constant speed v are entering a region of a constant magnetic field which points out of the page. Their masses and charges are, respectively: (M, +Q), (M, -Q), (2M, +Q) and (2M, -Q). Which is the trajectory that belongs to the particle with mass 2M and positive charge?



a. A b. B c. C d. D

15) Identify whether each statement below about magnetism is true or false.

a. Effect on charges: (One is true, one is false.)
   $\boxed{T}$ F A magnetic field exerts a net force only on moving charged particles.
   T $\boxed{F}$ A magnetic field exerts a net force on any electrically charged particle.

b. Positive and negative charges: (One is true, one is false.)
   T $\boxed{F}$ A magnetic field has the same effect on positive and negative charges.
   $\boxed{T}$ F A magnetic field has opposite effects on positive and negative charges.

c. Field lines: (One is true, one is false.)
   T $\boxed{F}$ Magnetic field lines begin at north poles and end at south poles.
   $\boxed{T}$ F Magnetic field lines run in continuous closed loops, with no beginning or end.

d. Magnetic poles: (One is true, one is false.)
   T $\boxed{F}$ Some particles are magnetic north poles, and others are magnetic south poles.
   $\boxed{T}$ F Magnetic north and south poles can never be separated from each other.