27.22. **IDENTIFY:** For motion in an arc of a circle, $a = \frac{v^2}{R}$ and the net force is radially inward, toward the center of the circle.

SET UP: The direction of the force is shown in Figure 27.22. The mass of a proton is 1.67×10^{-27} kg.

EXECUTE: **(a)** $\vec{F}$ is opposite to the right-hand rule direction, so the charge is negative. $\vec{F} = m\vec{a}$ gives $|q|vB\sin\phi = m\frac{v^2}{R}$. $\phi = 90°$ and $v = \frac{|q|BR}{m} = \frac{3(1.60\times10^{-19}\text{ C})(0.250\text{ T})(0.475\text{ m})}{12(1.67\times10^{-27}\text{ kg})} = 2.84\times10^6\text{ m/s}$.

(b) $F_B = |q|vB\sin\phi = 3(1.60\times10^{-19}\text{ C})(2.84\times10^6\text{ m/s})(0.250\text{ T})\sin 90° = 3.41\times10^{-13}\text{ N}$.

$w = mg = 12(1.67\times10^{-27}\text{ kg})(9.80\text{ m/s}^2) = 1.96\times10^{-25}\text{ N}$. The magnetic force is much larger than the weight of the particle, so it is a very good approximation to neglect gravity.

EVALUATE: **(c)** The magnetic force is always perpendicular to the path and does no work. The particles move with constant speed.

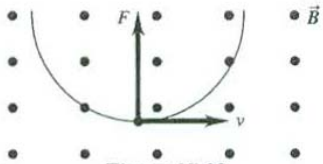


Figure 27.22

27.24. **IDENTIFY:** The magnetic force on the beam bends it through a quarter circle.

SET UP: The distance that particles in the beam travel is $s = R\theta$, and the radius of the quarter circle is $R = mv/qB$.

EXECUTE: Solving for R gives $R = s/\theta = s/(\pi/2) = 1.18\text{ cm}/(\pi/2) = 0.751\text{ cm}$. Solving for the magnetic field: $B = mv/qR = (1.67\times10^{-27}\text{ kg})(1200\text{ m/s})/[(1.60\times10^{-19}\text{ C})(0.00751\text{ m})] = 1.67\times10^{-3}\text{ T}$

EVALUATE: This field is about 10 times stronger than the Earth's magnetic field, but much weaker than many

27.68 **SET UP:** First find the current. The equivalent resistance across the battery is 30.0 Ω, so the total current is 4.00 A, half of which goes through the bar. Applying Newton's second law to the bar gives $\sum F = ma = F_B - mg = iLB - mg$.

EXECUTE: Solving for the acceleration gives

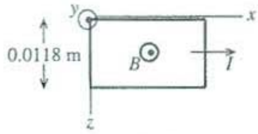
$$a = \frac{iLB - mg}{m} = \frac{(2.0\text{A})(1.50\text{ m})(1.60\text{ T}) - 3.00\text{ N}}{(3.00\text{ N}/9.80\text{ m/s}^2)} = 5.88\text{ m/s}^2.$$

The direction is upward.

EVALUATE: Once the bar is free of the conducting wires, its acceleration will become 9.8 m/s^2 downward since only gravity will be acting on it.

27.51. **IDENTIFY:** The drift velocity is related to the current density by Eq.(25.4). The electric field is determined by the requirement that the electric and magnetic forces on the current-carrying charges are equal in magnitude and opposite in direction.

(a) SET UP: The section of the silver ribbon is sketched in Figure 27.51a.



$J_x = n|q|v_d$

so $v_d = \dfrac{J_x}{n|q|}$

Figure 27.51a

EXECUTE: $J_x = \dfrac{I}{A} = \dfrac{I}{y_1 z_1} = \dfrac{120\text{ A}}{(0.23\times10^{-3}\text{ m})(0.0118\text{ m})} = 4.42\times10^{7}\text{ A/m}^2$

$$v_d = \frac{J_x}{n|q|} = \frac{4.42\times10^{7}\text{ A/m}^2}{(5.85\times10^{28}/\text{m}^3)(1.602\times10^{-19}\text{ C})} = 4.7\times10^{-3}\text{ m/s} = 4.7\text{ mm/s}$$

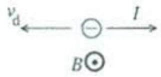
(b) magnitude of $\vec{E}$

$|q|E_z = |q|v_d B_y$

$E_z = v_d B_y = (4.7\times10^{-3}\text{ m/s})(0.95\text{ T}) = 4.5\times10^{-3}\text{ V/m}$

direction of $\vec{E}$

The drift velocity of the electrons is in the opposite direction to the current, as shown in Figure 27.51b.



$\vec{v}\times\vec{B}\uparrow$

$\vec{F}_B = q\vec{v}\times\vec{B} = -e\vec{v}\times\vec{B}\downarrow$

Figure 27.51b

The directions of the electric and magnetic forces on an electron in the ribbon are shown in Figure 27.51c.

$\vec{F}_E$ must oppose $\vec{F}_B$ so $\vec{F}_E$ is in the $-z$-direction

Figure 27.51c

$\vec{F}_E = q\vec{E} = -e\vec{E}$ so $\vec{E}$ is opposite to the direction of $\vec{F}_E$ and thus $\vec{E}$ is in the $+z$-direction.

(c) The Hall emf is the potential difference between the two edges of the strip (at $z = 0$ and $z = z_1$) that results from the electric field calculated in part (b). $\mathcal{E}_{\text{Hall}} = Ez_1 = (4.5\times10^{-3}\text{ V/m})(0.0118\text{ m}) = 53\ \mu\text{V}$

EVALUATE: Even though the current is quite large the Hall emf is very small. Our calculated Hall emf is more than an order of magnitude larger than in Example 27.13. In this problem the magnetic field and current density are larger than in the example, and this leads to a larger Hall emf.

27.40. **IDENTIFY:** The magnetic force $\vec{F}_B$ must be upward and equal to mg. The direction of $\vec{F}_B$ is determined by the direction of I in the circuit.

SET UP: $F_B = IlB\sin\phi$, with $\phi = 90°$. $I = \dfrac{V}{R}$, where V is the battery voltage.

EXECUTE: **(a)** The forces are shown in Figure 27.40. The current I in the bar must be to the right to produce $\vec{F}_B$ upward. To produce current in this direction, point a must be the positive terminal of the battery.

(b) $F_B = mg$. $IlB = mg$. $m = \dfrac{IlB}{g} = \dfrac{VlB}{Rg} = \dfrac{(175\text{ V})(0.600\text{ m})(1.50\text{ T})}{(5.00\ \Omega)(9.80\text{ m/s}^2)} = 3.21\text{ kg}$.

EVALUATE: If the battery had opposite polarity, with point a as the negative terminal, then the current would be clockwise and the magnetic force would be downward.

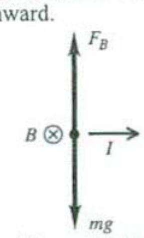


Figure 27.40

27.62. **IDENTIFY:** The net magnetic force on the wire is the vector sum of the force on the straight segment plus the force on the curved section. We must integrate to get the force on the curved section.

SET UP: $\sum F = F_{\text{straight, top}} + F_{\text{curved}} + F_{\text{straight, bottom}}$ and $F_{\text{straight, top}} = F_{\text{straight, bottom}} = iL_{\text{straight}}B$. $F_{\text{curved, }x} = \int_0^{\pi} iRB\sin\theta d\theta = 2iRB$

(the same as if it were a straight segment $2R$ long) and $F_y = 0$ due to symmetry. Therefore, $F = 2IL_{\text{straight}}B + 2IRB$

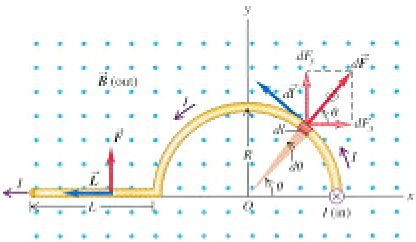
EXECUTE: Using $L_{\text{straight}} = 0.55$ m, $R = 0.95$ m, $I = 3.40$ A, and $B = 2.20$ T gives $F = 22$ N, to right.

EVALUATE: Notice that the curve has no effect on the force. In other words, the force is the same as if the wire were simply a straight wire 3.00 m long.

Example 27.8 Magnetic force on a curved conductor

In Fig. 27.30 the magnetic field $\vec{B}$ is uniform and perpendicular to the plane of the figure, pointing out. The conductor has a straight segment with length L perpendicular to the plane of the figure on the right, with the current opposite to $\vec{B}$; followed by a semicircle with radius R; and finally another straight segment with length L parallel to the x-axis, as shown. The conductor carries a current I. Find the total magnetic force on these three segments of wire.

27.30 What is the total magnetic force on the conductor?



SOLUTION

IDENTIFY: Two of the three segments of wire are straight and the magnetic field is uniform, so we can find the force on these using the ideas of this section. We can analyze the curved segment by first dividing it into a large number of infinitesimal straight segments. We find the force on one such segment and then integrate to find the force on the curved segment as a whole.

SET UP: We find the force on the straight segments using Eq. (27.19) and the force on an infinitesimal part of the curved segment using Eq. (27.20). The total magnetic force on all three segments is the vector sum of the forces on each individual segment.

EXECUTE: Let's do the easy parts (the straight segments) first. There is *no* force on the segment on the right perpendicular to the plane of the figure because it is antiparallel to $\vec{B}$; $\vec{L} \times \vec{B} = 0$, or $\phi = 180°$ and $\sin\phi = 0$. For the straight segment on the left, $\vec{L}$ points to the left (in the direction of the current), perpendicular to $\vec{B}$. The force has magnitude $F = ILB$, and its direction is up (the $+y$-direction in the figure).

The fun part is the semicircle. The figure shows a segment $d\vec{l}$ with length $dl = R\,d\theta$, at angle θ. The direction of $d\vec{l} \times \vec{B}$ is radially outward from the center; make sure you can verify this direction. Because $d\vec{l}$ and $\vec{B}$ are perpendicular, the magnitude dF of the force on the segment $d\vec{l}$ is just $dF = I\,dl\,B$, so we have

$$dF = I(R\,d\theta)B$$

The components of the force $d\vec{F}$ on segment $d\vec{l}$ are

$$dF_x = IR\,d\theta\,B\cos\theta \qquad dF_y = IR\,d\theta\,B\sin\theta$$

To find the components of the total force, we integrate these expressions, letting θ vary from 0 to π to take in the whole semicircle. We find

$$F_x = IRB\int_0^{\pi} \cos\theta\,d\theta = 0$$

$$F_y = IRB\int_0^{\pi} \sin\theta\,d\theta = 2IRB$$

Finally, adding the forces on the straight and semicircular segments, we find the total force:

$$F_x = 0 \qquad F_y = IB(L + 2R)$$

or

$$\vec{F} = IB(L + 2R)\hat{j}$$

EVALUATE: We could have predicted from symmetry that the x-component of force on the semicircle would be zero. On the right half of the semicircle the x-component of the force is positive (to the right) and on the left half it is negative (to the left); the positive and negative contributions to the integral cancel.

Note that the net force on all three segments together is the same force that would be exerted if we replaced the semicircle with a straight segment along the x-axis. Do you see why?

27.71. **IDENTIFY:** $R = \dfrac{mv}{|q|B}$.

SET UP: After completing one semicircle the separation between the ions is the difference in the diameters of their paths, or $2(R_{13} - R_{12})$. A singly ionized ion has charge $+e$.

EXECUTE: **(a)** $B = \dfrac{mv}{|q|R} = \dfrac{(1.99\times10^{-26}\text{ kg})(8.50\times10^3\text{ m/s})}{(1.60\times10^{-19}\text{ C})(0.125\text{ m})} = 8.46\times10^{-3}$ T.

(b) The only difference between the two isotopes is their masses. $\dfrac{R}{m} = \dfrac{v}{|q|B} =$ constant and $\dfrac{R_{12}}{m_{12}} = \dfrac{R_{13}}{m_{13}}$.

$$R_{13} = R_{12}\left(\frac{m_{13}}{m_{12}}\right) = (12.5\text{ cm})\left(\frac{2.16\times10^{-26}\text{ kg}}{1.99\times10^{-26}\text{ kg}}\right) = 13.6\text{ cm}.$$ The diameter is 27.2 cm.

(c) The separation is $2(R_{13} - R_{12}) = 2(13.6\text{ cm} - 12.5\text{ cm}) = 2.2$ cm. This distance can be easily observed.

EVALUATE: Decreasing the magnetic field increases the separation between the two isotopes at the detector.

27.75. **IDENTIFY:** For the loop to be in equilibrium the net torque on it must be zero. Use Eq.(27.26) to calculate the torque due to the magnetic field and use Eq.(10.3) for the torque due to the gravity force.

SET UP: See Figure 27.75a.

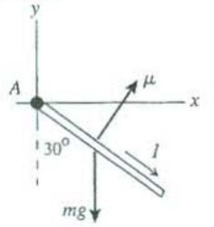


Figure 27.75a

Use $\sum \tau_A = 0$, where point A is at the origin.

EXECUTE: See Figure 27.75b.

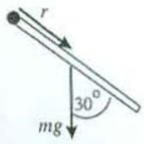


Figure 27.75b

$\tau_{mg} = mgr\sin\phi = mg(0.400 \text{ m})\sin 30.0°$

The torque is clockwise; $\vec{\tau}_{mg}$ is directed into the paper.

For the loop to be in equilibrium the torque due to $\vec{B}$ must be counterclockwise (opposite to $\vec{\tau}_{mg}$) and it must be that $\tau_B = \tau_{mg}$. See Figure 27.75c.

Then continue and find B=0.024T

27.77. **IDENTIFY:** Use Eq.(27.20) to calculate the force and then the torque on each small section of the rod and integrate to find the total magnetic torque. At equilibrium the torques from the spring force and from the magnetic force cancel. The spring force depends on the amount x the spring is stretched and then $U = \frac{1}{2}kx^2$ gives the energy stored in the spring.

(a) SET UP:

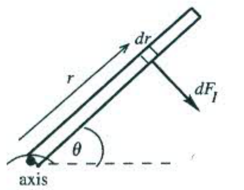


Figure 27.77

Divide the rod into infinitesimal sections of length dr, as shown in Figure 27.77.

EXECUTE: The magnetic force on this section is $dF_I = IBdr$ and is perpendicular to the rod. The torque $d\tau$ due to the force on this section is $d\tau = rdF_I = IBr\,dr$. The total torque is $\int d\tau = IB\int_0^l r\,dr = \frac{1}{2}Il^2B = 0.0442 \text{ N}\cdot\text{m}$, clockwise.

(b) SET UP: F_I produces a clockwise torque so the spring force must produce a counterclockwise torque. The spring force must be to the left; the spring is stretched.

EXECUTE: Find x, the amount the spring is stretched:

$\sum \tau = 0$, axis at hinge, counterclockwise torques positive

$(kx)l\sin 53° - \frac{1}{2}Il^2B = 0$

$$x = \frac{IlB}{2k\sin 53.0°} = \frac{(6.50 \text{ A})(0.200 \text{ m})(0.340 \text{ T})}{2(4.80 \text{ N/m})\sin 53.0°} = 0.05765 \text{ m}$$

$U = \frac{1}{2}kx^2 = 7.98\times10^{-3} \text{ J}$

27.84. **IDENTIFY** and **SET UP:** Follow the procedures specified in the problem.

EXECUTE: **(a)** $d\vec{l} = dl\hat{t}$, where $\hat{t}$ is a unit vector in the tangential direction. $d\vec{l} = Rd\theta\left[-\sin\theta\hat{i} + \cos\theta\hat{j}\right]$. Note that this implies that when $\theta = 0$, the line element points in the $+y$-direction, and when the angle is $90°$, the line element points in the $-x$-direction. This is in agreement with the diagram.

$d\vec{F} = Id\vec{l}\times\vec{B} = IRd\theta\left[-\sin\theta\hat{i} + \cos\theta\hat{j}\right]\times(B_x\hat{i}) = IB_xRd\theta[-\cos\theta\hat{k}]$.

(b) $\vec{F} = \int_0^{2\pi} -\cos\theta IB_xR\, d\theta\hat{k} = -IB_xR\int_0^{2\pi}\cos\theta d\theta\hat{k} = 0$.

(c) $d\vec{\tau} = \vec{r}\times d\vec{F} = R(\cos\theta\hat{i} + \sin\theta\hat{j})\times(IB_xR\, d\theta[-\cos\theta\hat{k}]) = -R^2IB_xd\theta(\sin\theta\cos\theta\hat{i} - \cos^2\theta\hat{j})$

(d) $\vec{\tau} = \int d\vec{\tau} = -R^2IB_x\left(\int_0^{2\pi}\sin\theta\cos\theta d\theta\hat{i} - \int_0^{2\pi}\cos^2\theta d\theta\hat{j}\right) = IR^2B_x\left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right)_0^{2\pi}\hat{j}$. $\vec{\tau} = IR^2B_x\pi\hat{j} = I\pi R^2B_x\hat{j} = IA\hat{k}\times B_x\hat{i}$

and $\vec{\tau} = \vec{\mu}\times\vec{B}$.

EVALUATE: Section 27.7 of the textbook derived $\vec{\tau} = \vec{\mu}\times\vec{B}$ for the case of a rectangular coil. This problem shows that the same result also applies to a circular coil.

27.1. **IDENTIFY** and **SET UP:** Apply Eq.(27.2) to calculate $\vec{F}$. Use the cross products of unit vectors from Section 1.10.

EXECUTE: $\vec{v} = (+4.19\times10^4 \text{ m/s})\hat{i} + (-3.85\times10^4 \text{ m/s})\hat{j}$

(a) $\vec{B} = (1.40 \text{ T})\hat{i}$

$\vec{F} = q\vec{v}\times\vec{B} = (-1.24\times10^{-8} \text{ C})(1.40 \text{ T})\left[(4.19\times10^4 \text{ m/s})\hat{i}\times\hat{i} - (3.85\times10^4 \text{ m/s})\hat{j}\times\hat{i}\right]$

$\hat{i}\times\hat{i} = 0, \hat{j}\times\hat{i} = -\hat{k}$

$\vec{F} = (-1.24\times10^{-8} \text{ C})(1.40 \text{ T})(-3.85\times10^4 \text{ m/s})(-\hat{k}) = (-6.68\times10^{-4} \text{ N})\hat{k}$

EVALUATE: The directions of $\vec{v}$ and $\vec{B}$ are shown in Figure 27.1a.

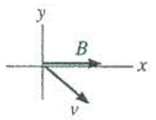


Figure 27.1a

The right-hand rule gives that $\vec{v}\times\vec{B}$ is directed out of the paper (+z-direction). The charge is negative so $\vec{F}$ is opposite to $\vec{v}\times\vec{B}$;

$\vec{F}$ is in the $-z$- direction. This agrees with the direction calculated with unit vectors.

(b) **EXECUTE:** $\vec{B} = (1.40 \text{ T})\hat{k}$

$\vec{F} = q\vec{v}\times\vec{B} = (-1.24\times10^{-8} \text{ C})(1.40 \text{ T})\left[(+4.19\times10^4 \text{ m/s})\hat{i}\times\hat{k} - (3.85\times10^4 \text{ m/s})\hat{j}\times\hat{k}\right]$

$\hat{i}\times\hat{k} = -\hat{j}, \hat{j}\times\hat{k} = \hat{i}$

$\vec{F} = (-7.27\times10^{-4} \text{ N})(-\hat{j}) + (6.68\times10^{-4} \text{ N})\hat{i} = \left[(6.68\times10^{-4} \text{ N})\hat{i} + (7.27\times10^{-4} \text{ N})\hat{j}\right]$

EVALUATE: The directions of $\vec{v}$ and $\vec{B}$ are shown in Figure 27.1b.

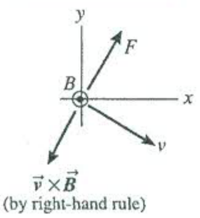


Figure 27.1b

The direction of $\vec{F}$ is opposite to $\vec{v}\times\vec{B}$ since q is negative. The direction of $\vec{F}$ computed from the right-hand rule agrees qualitatively with the direction calculated with unit vectors.