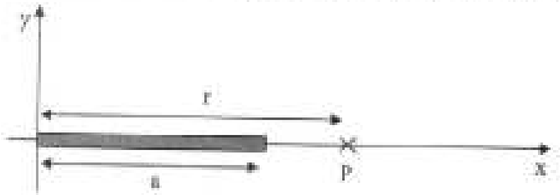
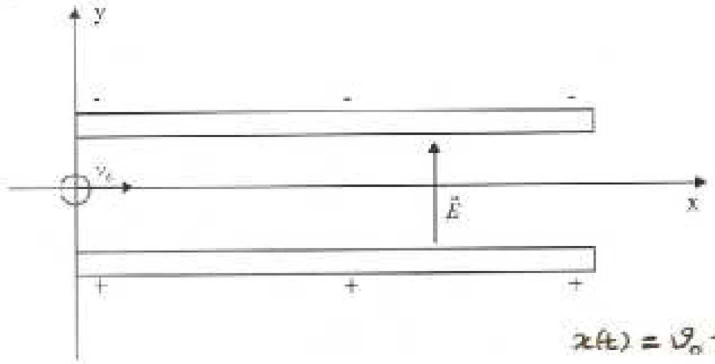
12- Positive charge Q is distrubuted uniformly along the x-axis from $x=0$ to $x=a$. Calculate the x and y components of the electric field produced by the charge distribution Q at the point P located on the positive x-axis at $x=r$, where $r>a$.



$$E_x = \int_0^a \frac{1}{4\pi\epsilon_0}\frac{\lambda\, dx}{(r-x)^2}, \quad \lambda = \frac{Q}{a}$$

$$E_x = \frac{\lambda}{4\pi\epsilon_0}(r-x)^{-1}\Big|_0^a = \frac{\lambda}{4\pi\epsilon_0}\left[\frac{1}{r-a} - \frac{1}{r}\right] \qquad E_x = \frac{\lambda}{4\pi\epsilon_0}\frac{a}{(r-a)(r)}$$

13- An electron is launched into a uniform electric field (magnitude E, direction shown in the figure below) with an initial horizontal velocity v_0. Initial coordinates of the electron are $x_0=0$ and $y_0=0$. Find out the equation describing the trajectory of the electron ($y=f(x)$, where f is a function of e, E, m, and v_0. The charge of an electron is $-e$.).

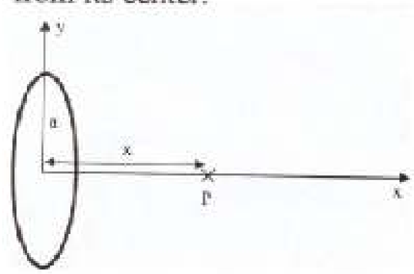


$$x(t) = v_0 t$$

$$y(t) = -\frac{1}{2}\frac{eE}{m}t^2 = -\frac{1}{2}\frac{eE}{m}\left(\frac{x}{v_0}\right)^2 \rightarrow$$

$$\boxed{y = -\frac{1}{2}\frac{eE}{m v_0^2}x^2}$$

14- A ring shaped conductor with radius a carries a total charge Q uniformly distributed around it. Find the electric field at a point P that lies on the axis of the ring at a distance x from its center.

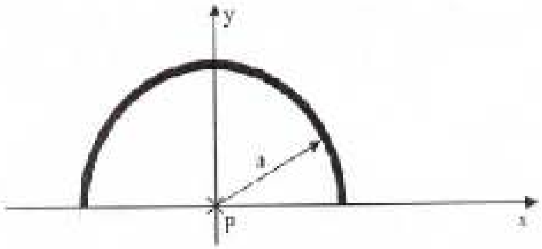


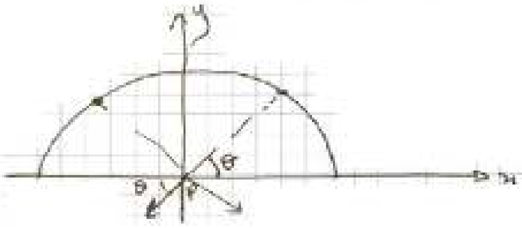
$$E_x = \frac{1}{4\pi\epsilon_0}\frac{Q}{(x^2+a^2)}\cdot\frac{x}{\sqrt{x^2+a^2}}$$

$$= \frac{Q}{4\pi\epsilon_0}\frac{x}{(x^2+a^2)^{3/2}},$$

along the +x direction

15- Positive charge Q is distributed uniformly around a semicircle of radius a. Find the electric field (magnitude and direction) at the center of curvature P.



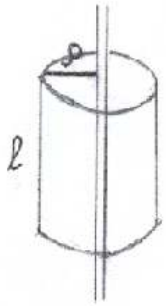


$$E_y = \int -\frac{1}{4\pi\epsilon_0}\frac{dQ}{a^2}\sin\theta \quad , \quad dQ = r\,d\theta\,\lambda$$

$$= \int -\frac{1}{4\pi\epsilon_0}\frac{r\lambda\,d\theta}{a^2}\sin\theta = -\frac{1}{4\pi\epsilon_0}\frac{r\lambda}{a^2}\int_0^\pi \sin\theta\,d\theta = -\frac{1}{4\pi\epsilon_0}\frac{r\lambda}{a^2}(-\cos\theta)\Big|_0^\pi = -\frac{1}{4\pi\epsilon_0}\frac{r\lambda}{a^2}2$$

$$\boxed{E_y = -\frac{1}{2\pi\epsilon_0}\frac{r\lambda}{a^2}}$$ along the $-y$ direction.

17- An infinite wire with uniform charge density ρ_1 (unit = Coulomb/m) is parallel to the x axis and crosses the point $\vec{r}'_0 = 5\,\hat{i} - 10\,\hat{j} + 3\,\hat{k}$. Derive the electric field $\vec{E}$ at $\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$.



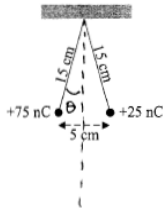
By symmetry, $\vec{E} = E(\rho)\hat{\rho}$.

Gauss' Law: $\int_{\text{cylinder}} \vec{E}\cdot d\vec{s} = E\int_{\text{side surface}} ds = E\,2\pi\rho l = \frac{\rho_1 l}{\epsilon_0}$.

$$E = \frac{\rho_1}{2\pi\epsilon_0\rho}.$$

$$\vec{E}(x,y,z) = \frac{\rho_1\hat{\rho}}{2\pi\epsilon_0\rho} = \frac{\rho_1\vec{\rho}}{2\pi\epsilon_0\rho^2} = \frac{\rho_1\left[(y+10)\hat{j} + (z-3)\hat{k}\right]}{2\pi\epsilon_0\left[(y+10)^2 + (z-3)^2\right]}$$

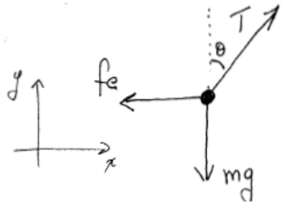
7- Two identical small spheres, having the same mass, and negligibly small radius, carry charges of 75 nC and 25 nC as shown in the figure. The spheres are attached to non-conducting, 15 cm long threads, of negligible mass. The far ends of the threads are secured to a pin in the ceiling and the spheres are allowed to hang freely as shown, coming to an equilibrium condition when they are 5.0 cm apart.



a) Calculate the magnitude of the force exerted by one sphere on the other when they are in equilibrium.

$$f_e = \frac{1}{4\pi\epsilon_0}\left(\frac{|Q_1 Q_2|}{R^2}\right) = 9\times10^9 \frac{N\cdot m^2}{C^2}\left[\frac{(75\times10^{-9}c)(25\times10^{-9}c)}{(.05m)^2}\right] = \underline{6.75\times10^{-3}\ \text{newton}}$$

b) In the space below, draw a free body diagram for either one of the spheres. Include all relevant forces.



$$\sin\theta = \frac{2.5\,cm}{15\,cm} \Rightarrow \theta = 9.6^\circ$$

c) Calculate the mass of either one of the spheres.

$$\Sigma f_x = -f_e + T\sin\theta = m\cancel{a_x}^{0}$$
$$\Sigma f_y = -mg + T\cos\theta = m\cancel{a_y}^{0}$$

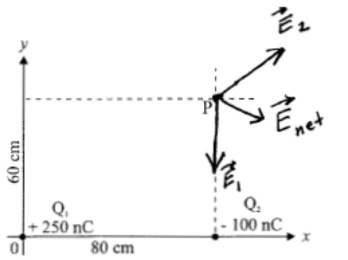
$a_x = a_y = 0$ since sphere is in equilibrium

$$T = \frac{f_e}{\sin\theta} \quad \text{then} \quad -mg + \left(\frac{f_e}{\sin\theta}\right)\cos\theta = 0 \quad g = \frac{f_e}{g\tan\theta} = \frac{6.75\times10^{-3}\,N}{(9.8\times10^{0}\,m/s^2)(\tan 9.6^\circ)} = 4.1\times10^{-3}\,kg = \underline{4.1\ \text{gram}}$$

8- Two point charges are fixed in place in an xy coordinate system as shown in the figure. There is no charge located at the point P.

a) Sketch, at point P on the figure, three vectors, representing the electric field due to Q_1 alone, due to Q_2 alone, and the net field due to both.

b) Calculate the net electric field vector (expressed in unit vector form) at P due to the two charges.



$$E_1 = k\frac{Q_1}{R_1^2} = \left(9\times10^9 \frac{N\cdot m^2}{C^2}\right)\frac{100\times10^{-9}C}{(.6m)^2}$$

E_1 in $-y$ direction so $\vec{E}_1 = 2500(-\hat{j})\ V/m$

$$E_2 = k\frac{Q_2}{R_2^2} = \left(9\times10^9 \frac{N\cdot m^2}{C^2}\right)\frac{(250\times10^{-9}c)}{(.6m)^2+(.8m)^2} = 2250\ V/m$$

E_1 in $.8\hat{i} + .6\hat{j}$ direction, so $\vec{E}_2 = 1800\hat{i} + 1350\hat{j}\ V/m$

$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 = 1800\hat{i} - 1150\hat{j}\ V/m$$

c) Calculate the net electric potential at P due to the two charges

$$V_P = V_1 + V_2 = k\frac{Q_1}{R_1} + k\frac{Q_2}{R_2} \qquad \text{scalar}$$

$$V_P = \left(9\times10^9\frac{N\cdot m^2}{C^2}\right)\left\{\left[\frac{-100\times10^{-9}C}{.6m}\right] + \left[\frac{+250\times10^{-9}C}{\sqrt{(.8m)^2+(.6m)^2}}\right]\right\} = +750\,V = (-1500V) + (+2250V)$$

d) Calculate the amount of work that the **electric field** would do on a +5.0 nC charge brought to point P from infinitely far away.

$$W = -q\,\Delta V = -q\,V_P = -(5\times10^{-9}c)(750V - 0V) = -3.8\times10^{-6}\,J$$

21- Consider a system of two positive point charges of magnitude q on the y axis at coordinates $(0, a)$ and $(0, -a)$, as shown in Fig.3. Find the electric potential at a point $P(x, y)$ and evaluate the electric field using the knowledge of the potential field.

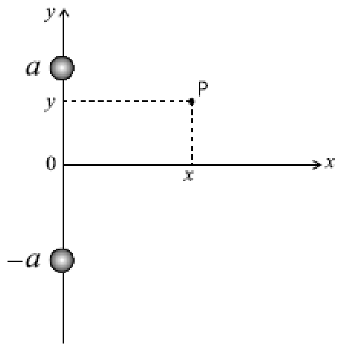


FIG. 3: A system of two point charges of magnitude q.

Solution The electric potential at (x, y) is given by the superposition

$$V(x,y) = kq\left(\frac{1}{\sqrt{x^2+(y-a)^2}} + \frac{1}{\sqrt{x^2+(y+a)^2}}\right)$$

from which we can evaluate $E_x = -dV/dx$ to be

$$E_x(x,y) = kqx\left(\frac{1}{(x^2+(y-a)^2)^{3/2}} + \frac{1}{(x^2+(y+a)^2)^{3/2}}\right).$$

Similarly $E_y = -dV/dy$ becomes,

$$E_y(x,y) = kq\left(\frac{y-a}{(x^2+(y-a)^2)^{3/2}} + \frac{y+a}{(x^2+(y+a)^2)^{3/2}}\right).$$

22- Consider a system of two positive point charges of magnitude q on the y axis at coordinates $(0, a)$ and $(0, -a)$, as shown in Fig.4. Calculate the electric field at a point $P(x, 0)$ and determine the electric potential from the knowledge of the electric field.

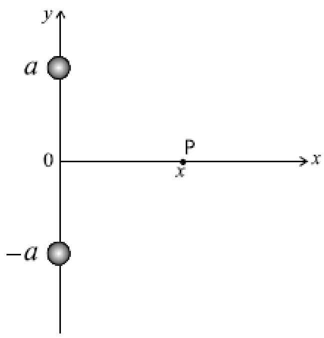


FIG. 4: A system of two point charges of magnitude q.

Solution Electric field at a point $(x, 0)$ is calculated directly as

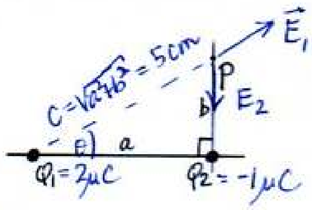
$$\vec{E}(x) = k\frac{2qx}{(x^2+a^2)^{3/2}}\hat{x}.$$

Using the electric field we evaluate the potential at the same point relative to a reference point at infinity through

$$\begin{aligned} V(x) &= \int_x^\infty E(x)dx = 2kq\int_x^\infty \frac{xdx}{(x^2+a^2)^{3/2}}, \\ &= 2kq\frac{1}{\sqrt{x^2+a^2}}. \end{aligned}$$

In evaluating the integral the change of variable $u = x^2 + a^2$ may be used.

5. In the diagram shown, the charges $Q_1 = 2.00\ \mu C$ and $Q_2 = -1.00\ \mu C$ are fixed in place, and $a = 4.00$ cm, $b = 3.00$ cm.



a) (6) Determine the x component of the electric field produced at point P.

Only Q_1 produces an x-component at P.

$$E_1 = \frac{kQ_1}{r^2} = \frac{(8.99\times10^9 \frac{Nm^2}{C^2})(2\mu C)}{(0.05m)^2} = 7.192\ MN/C$$

$$E_x = E_1\cos\theta = E_1\cdot\frac{a}{c} = 7.192\frac{MN}{C}\cdot\frac{4}{5} = \boxed{5.75\ MN/C}$$

b) (6) Determine the y component of the electric field produced at point P.

$$E_{1y} = E_1\sin\theta = E_1\frac{b}{c} = 7.192\frac{MN}{C}\cdot\frac{3}{5} = 4.315\ MN/C$$

$$E_{2y} = E_2 = \frac{kQ_2}{r^2} = \frac{kQ_2}{b^2} = \frac{(8.99\times10^9\frac{Nm^2}{C^2})(-1\mu C)}{(0.03m)^2} = -9.989\ MN/C$$

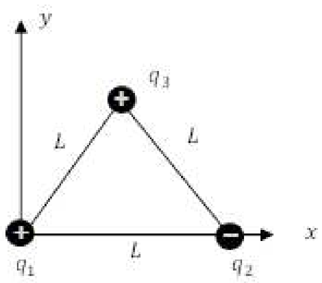
$$\text{Total } E_y = E_{1y}+E_{2y} = (4.315-9.989)\frac{MN}{C} = \boxed{-5.67\ MN/C}$$

c) (4) If a third charge, $q = 0.500\ \mu C$ is placed at P, what is the electric force that acts on it? Give the result in components, like $\vec{F} = (F_x, F_y)$.

$$\vec{F} = q\vec{E} = 0.500\mu C \times \left(5.75\frac{MN}{C}, -5.67\frac{MN}{C}\right) \qquad (\mu\times M = 1)$$

$$\vec{F} = (F_x, F_y) = \boxed{(2.88\ N, -2.84\ N)}$$

4. Three charges $q_1 = +4.00\ \mu C, q_2 = -6.00\ \mu C,\ q_3 = +8.00\mu C$ are placed at the corners of an equilateral triangle as shown, with edge $L = 0.650$ m.



(a) Calculate the electric field at the position of charge q_1 due to q_2 and q_3. Give its magnitude and direction relative to the $+x$ axis.

The electric field due to the charge q_3 is $\vec{E}_1 = \frac{k_e q_3}{L^2}(-\hat{\imath}\cos 60^o - \hat{\jmath}\sin 60^o)$

and the electric field due to the charge q_2 is $\vec{E}_2 = -\frac{kq_2}{L^2}\hat{\imath}$.

Combining them gives a total electric field

$$\vec{E} = -\frac{9\times10^9}{(0.650)^2}\left\{\hat{\imath}\left(-6.0+\frac{8.0}{2}\right)+\hat{\jmath}\,\frac{8.0\sqrt{3}}{2}\right\}\frac{\mu\text{N}}{\text{C}} = (42.6\,\hat{\imath} - 147.6\,\hat{\jmath})\frac{\text{kN}}{\text{C}}.$$

$$E = \sqrt{E_x^2 + E_y^2} = \mathbf{153.6\,\frac{kN}{C}}, \quad \theta = \text{atan}\frac{E_y}{E_x} = \mathbf{-\,73.9^o}.$$

(b) What is the potential energy of the set of three charges (relative to a situation where they are widely separated)?

$$U = k_e\left(\frac{q_1q_2}{r}+\frac{q_1q_3}{r}+\frac{q_2q_3}{r}\right) = \frac{9\times10^9}{0.650}(-24+32-48)\times10^{-12}\text{J} = \mathbf{-0.554\,J}.$$

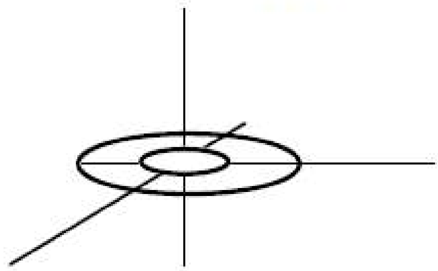
(c) Suppose charge q_1 is free to move but the other charges are held fixed, and has a mass $m_1 = 1.20$ g. Find its speed when far away from the other charges, assuming it is released at rest from the origin.

The electric potential at the location of q_1 is $U_1 = k_e\left(\frac{q_1q_2}{L}+\frac{q_1q_3}{L}\right) = \frac{9\times10^9}{0.650}(8\times10^{-12})\text{ J} = 0.111\text{ J}.$

Energy conservation implies that

$$\frac{1}{2}mv_f^2 = U_1, \qquad v_f = \sqrt{\frac{2(0.111\text{ J})}{1.2\times10^{-3}\text{kg}}} = \mathbf{13.6\,\frac{m}{s}}.$$

1. Consider a ring of charge that has an inner radius a and an outer radius b. It has a charge density σ



a) Derive an expression for the potential along its axis.

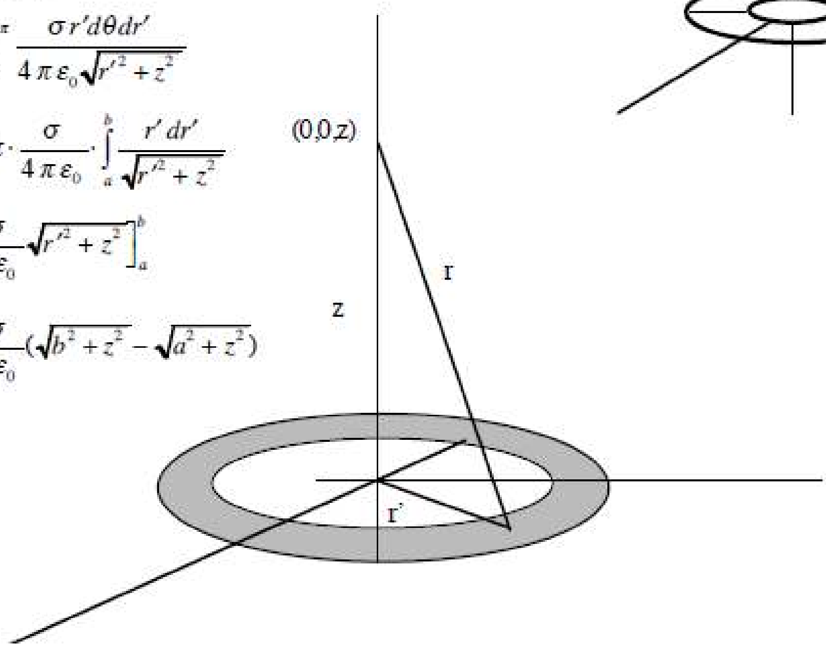
$$V = \int\frac{dq}{4\pi\varepsilon_0 r} \qquad dq = \sigma r' d\theta dr' \qquad r = \sqrt{r'^2 + z^2}$$

$$V = \int_{a,0}^{b,2\pi}\frac{\sigma r' d\theta dr'}{4\pi\varepsilon_0\sqrt{r'^2+z^2}}$$

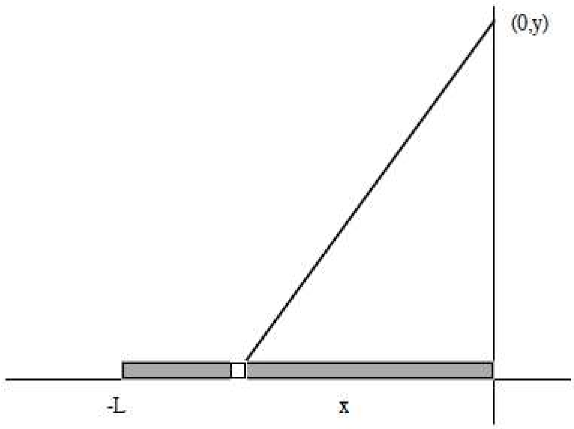
$$= 2\pi\cdot\frac{\sigma}{4\pi\varepsilon_0}\cdot\int_a^b\frac{r'dr'}{\sqrt{r'^2+z^2}}$$

$$= \frac{\sigma}{2\varepsilon_0}\sqrt{r'^2+z^2}\Big]_a^b$$

$$= \frac{\sigma}{2\varepsilon_0}(\sqrt{b^2+z^2} - \sqrt{a^2+z^2})$$



2. Consider the charged rod below. Assume that it is uniformly charged with charge per unit length λ.



a) Write an expression for the electric potential due to a small charge dq at the point p?

$$dV = \frac{dq}{4\pi\varepsilon_0 r} = \frac{dq}{4\pi\varepsilon_0\sqrt{x^2+y^2}}$$

b) Write an expression for the dq and the r in terms of x and the distance y.

$$dq = \lambda dx \qquad r = \sqrt{x^2+y^2}$$

c) What is the potential at the point indicated? You may need the integral

$$\int_{-L}^{0} \frac{dx}{\sqrt{x^2+y^2}} = \ln[y] - \ln[-L + \sqrt{L^2+y^2}]$$

$$V = \int_{-L}^{0} \frac{\lambda\, dx}{4\pi\varepsilon_0\sqrt{x^2+y^2}} = \frac{\lambda}{4\pi\varepsilon_0}\int_{-L}^{0}\frac{dx}{\sqrt{x^2+y^2}} = \frac{\lambda}{4\pi\varepsilon_0}\cdot(\ln[y] - \ln[-L + \sqrt{L^2+y^2}])$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\ln\frac{y}{-L+\sqrt{L^2+y^2}}$$

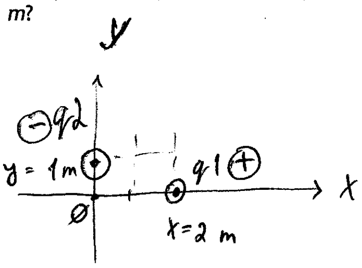
P3. A point charge $q1$=4.00 nC is located on the x-axis at x=2.00 m, and a second point charge $q2$=−6.00 nC is on the y-axis at y=1.00 m.

a) What is the total electric flux due to these two point charges through a spherical surface centered at the origin and with radius $r1$=0.5 m?

b) What is the total electric flux due to these two point charges through a spherical surface centered at the origin and with radius $r2$=1.5 m?

c) What is the total electric flux due to these two point charges through a spherical surface centered at the origin and with radius $r3$=2.5 m?

1). draw it!



2). solve it using Gauss's Law.

a). No charge enclosed $\rightarrow \Phi_E = \frac{Q_{encl}}{\varepsilon_0} = \varnothing$.

b). $\Phi_E = \frac{q_2}{\varepsilon_0} = \frac{-6\ nC}{\varepsilon_0} = \frac{-6\cdot 10^{-9}\ C}{8.85\cdot 10^{-12}\ C^2/Nm^2} = -678\ \frac{Nm^2}{C}$

c). $\Phi_E = \frac{q_1+q_2}{\varepsilon_0} = \frac{(+4-6)\cdot 10^{-9}\ C}{8.85\cdot 10^{-12}\ C^2/Nm^2} = -226\ \frac{Nm^2}{C}$

P4. The electric potential at points in a space is given by function $V=2x^2-3y^2+5z^3$. What is the magnitude and direction of the electric field at the point (3, 2, -1)?

1). $V(x,y,z)$ is given

2) $\boxed{\vec{E} = -\vec{\nabla} V}$

3). $\frac{\partial V}{\partial x} = 4 \cdot x$

$\frac{\partial V}{\partial y} = -6 \cdot y$

$\frac{\partial V}{\partial z} = 15 \cdot z^2$

$\vec{E}(3,2,-1) = -4 \cdot (3) \cdot \hat{x} + 6 (2) \hat{y} - 15 \cdot (-1)^2 \cdot \hat{z}$

$\vec{E}(3,2,-1) = -12 \cdot \hat{x} + 12 \cdot \hat{y} - 15 \cdot \hat{z}$

$\|\vec{E}\| = \sqrt{12^2 + 12^2 + 15^2} = 22.6 \ \frac{N}{C}$