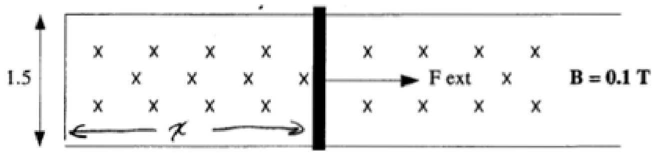
**3)** In the figure above a bar with resistance 0.2 Ω is sliding on frictionless perfectly conducting rails 1.5 meters apart, as shown. A constant external force of 2 N is applied in the direction shown. There is a constant magnetic field B=0.1 T into the page.




(a) What is the direction of the induced current around the circuit formed by the bar and rails? (circ

**CLOCKWISE** **COUNTERCLOCKWISE**

(b) When the bar reaches a constant velocity, what is the magnitude of the current?

when $V$ = constant, acceleration = 0, so $\Sigma(\text{forces}) = 0$,

$F_{ext} = F_{mag} = i l B \cancel{\sin\theta}$ $\quad i = \frac{F_{ext}}{l B} = \frac{2\,N}{(1.5m)(.1T)} = 13.3\,A$

(c) calculate the terminal velocity of the bar.

for a current $i$, an Emf $iR$ is required, the Emf depends on the velocity of the bar

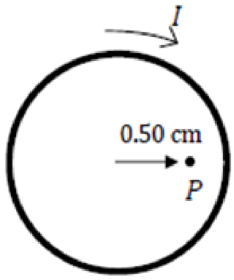
$$|\mathcal{E}| = \frac{d\Phi}{dt} = \frac{d}{dt}(BA) = B\frac{dA}{dt} = B\frac{d}{dt}\left[(1.5m)(x)\right]$$

$$|\mathcal{E}| = (1.5m)(B)\frac{dx}{dt} \quad \text{so } V = \frac{\mathcal{E}}{(1.5m)B} = \frac{iR}{(1.5m)B} = \frac{(13.3A)(.2\Omega)}{(1.5m)(.1T)}$$

velocity $\quad V = 17.3\ m/s$

**6)** The figure shows the cross-section of a 30 cm long solenoid with a cross-sectional area of 2.4 cm$^2$. The solenoid has 1600 turns of wire. If the time $t$ is given in Amperes, the current in the wire is

$I = (40.0\,t^2 + 1.80)$ Amperes.




(a) What is the magnetic flux through the cross section shown at $t = 0$? Is it into or out of the page, if the current circles clockwise about the solenoid

$$\Phi_m = AB = A\mu_0\frac{NI}{l} = (2.4\times10^{-4}\,\text{m}^2)\left(4\pi\times10^{-7}\frac{\text{Tm}}{\text{A}}\right)\frac{1600\,(1.80\,\text{A})}{0.30\,\text{m}} = 2.90\,\mu\text{Wb}.$$

(b) What is the inductance of the solenoid?

$$L = \frac{N\Phi_m}{I} = \frac{1600\,(2.90\,\mu\text{Wb})}{1.80\,\text{A}} = 2.58\,\text{mH}.$$

(c) What emf is generated in the solenoid at time t = 0?

$$\mathcal{E} = L\frac{dI}{dt} = 0$$

(d) What is the electric field at point $P$, a distance of 0.50 cm from the center? Give the magnitude and direction (up, down, left, right, into or out of the page).

The emf about a circle of radius $r = 0.50$ cm inside the solenoid is (with $t$ in seconds)

$$\oint \vec{E}\cdot d\vec{s} = 2\pi r\,E = \frac{d\Phi_m}{dt} = \frac{L}{N}\frac{dI}{dt} = \frac{(2.58\,\text{mH})(80\,t\,\text{A/s})}{1600} = (129\,t)\,\mu\text{V}.$$

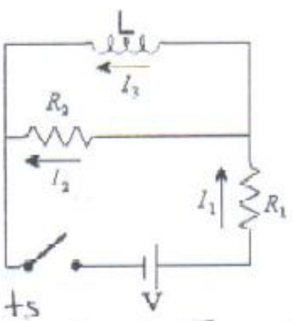
$$E = \frac{(129\,t)\,\mu\text{V}}{2\pi(0.0050\,\text{m})} = (\mathbf{4.11}\,\boldsymbol{t})\frac{\textbf{mV}}{\textbf{m}}.$$

The direction is such that a current following the emf would oppose the increase in flux into the page. Thus, the emf is counter-clockwise, to generate flux out of the page. At point P, the electric field is directed **upward** in the figure. At $t = 0, E$ vanishes. That answer would also be accepted.

**7)** The switch in the circuit below has been open for a long, long time. Determine the currents $I_1$, $I_2$, $I_3$ in the resistors and in the self-inductor at the moment

**a)** the switch is closed,
**b)** a long time after the switch is closed

The internal resistance of the battery is negligibly small. Express your answers ONLY in terms of $V$, $R_1$, $R_2$ and $L$.



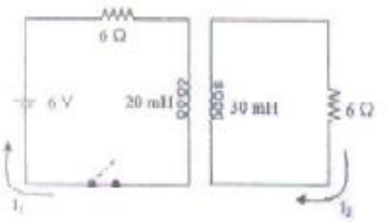

a) Before the switch is closed, all currents are zero. Immediately after the switch is closed, $I_3$ will still be zero, for the inductor prevents instantaneous changes in $I_3$. That leaves us with

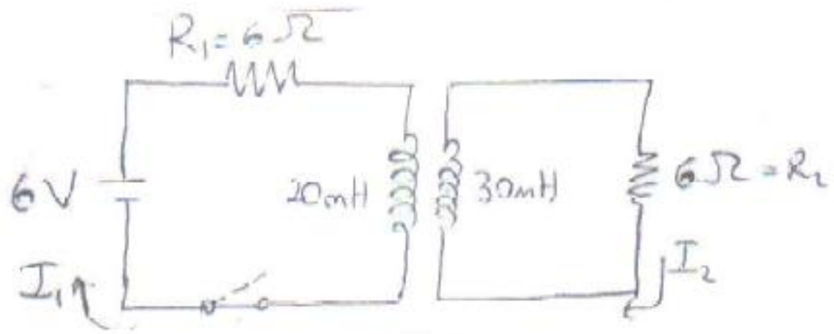
$V = R_1 I_1 + R_2 I_2$ and $I_1 = I_2$ because $I_3 = 0$

So $I_1 = I_2 = V/(R_1+R_2)$

b) The ohmic resistance of self inductor is zero. So if we wait a long time, until the self inductor is no longer opposing changes in $I_3$, we have a wire with zero resistance in parallel with $R_2$. Thus $I_2 = 0$ and $I_1 = I_3$. So $I_1 = I_3 = V/R_1$

**8)** The two identical coils in the circuit are placed close to each other and their mutual inductance is 0.7 mH. Suppose that the switch has been closed for a long time and is then opened at t=0. Calculate the current in the circuit at t = 10 ms.




Before the switch is oppened $I_1 = \frac{E}{R_1} = \frac{6V}{6\Omega} = 1A$ $\quad I_2 = 0$

The flux through $L_2$ is

$\Phi_{B21} = MI_1 = (0.7 mH)(1A) = 0.7\, mWb$

When the switch is opened the induced emf in $L_2$ wants to maintain this flux at t=0 the initial current in $L_2$ is

$\left(L = \frac{N\Phi_B}{I}\right)$ $I_{20} = \frac{\Phi_{B21}}{L_2} = \frac{(0.7\, mWb)}{30 mH} = 0,023\, A = 23\, mA$

This current reduces exponentially $I_2 = I_{20}\, e^{-\frac{R_2 t}{L_2}} = (0,023 A)\, e^{-\frac{(6\Omega)(18ms)}{30mH}}$

$= 0,63\, mA$

9) A very large cylindrical capacitor of radius $R$ and plate separation $d$ is being charged slowly with constant current $I$. As the capacitor charges, the electric field between the plates increases with time. Take $r$ to be the radial distance from the axis of the capacitor, as shown in the figure below.

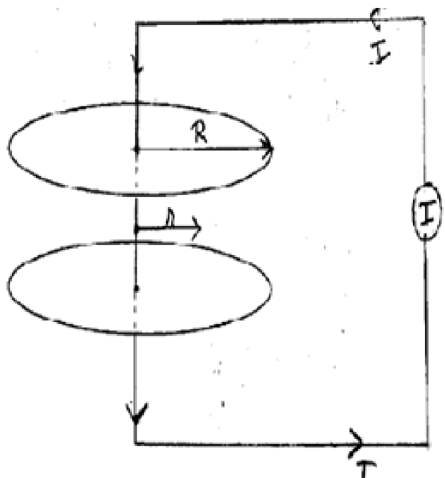
(a) At a particular point in time, the surface charge density on the plates is $\sigma$. What is the electric field $\vec{E}$ between the plates at that time?

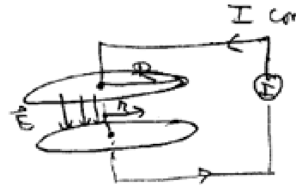
(b) Determine the time rate of change of the electric field between the plates, $\frac{dE}{dt}$, in terms of $I$, $R$, and other constants.

(c) Near the center of the plates, the electric field is constant in space. Determine the magnetic field $\vec{B}(r)$ between the plates for $r < R$ in terms of $I$, $R$ and other constants. Be sure to indicate both direction and magnitude of the field.

(d) Neglecting fringe effects around the edge of the capacitor plates, determine the magnetic field $\vec{B}(r)$ for $r = R$ and for $r > R$.

(e) Sketch $B(r)$ for all r.




(a) $|E| = \frac{\sigma}{\varepsilon_0}$, direction is vertically downward.

(b) $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} = \frac{Q}{\pi R^2 \varepsilon_0}$

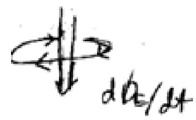
$$\frac{dE}{dt} = \frac{dQ/dt}{A\varepsilon_0} = \frac{I}{\pi R^2 \varepsilon_0} \qquad \boxed{\frac{dE}{dt} = \frac{I}{\pi R^2 \varepsilon_0}}$$

(c) The displacement current is (extension of Ampere's law)

$\oint \vec{B}\cdot d\vec{l} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$ for $r<R$, $\Phi_E = \pi r^2 E$

$\frac{d\Phi_E}{dt} = \pi r^2 \frac{dE}{dt} = \frac{\cancel{\pi} r^2 I}{\cancel{\pi} R^2 \varepsilon_0}$ $\oint \vec{B}\cdot d\vec{l} = 2\pi r B$ $\vec{B}$ forms concentric loops about $\frac{d\Phi_E}{dt}$

Direction is cw as viewed from the top (+) plate.




$2\pi r B = \frac{\mu_0 \cancel{\varepsilon_0} I r^2}{\cancel{\varepsilon_0} R^2}$ $\boxed{|\vec{B}| = \frac{\mu_0 I r}{2\pi R^2}}$ direction cw loops as viewed from above $r<R$

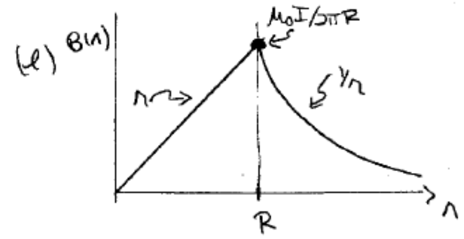
(d) for $r=R$ and $r>R$ $\Phi_E = \pi R^2 E$ (E extends only to R)

$\frac{d\Phi_E}{dt} = \pi R^2 \frac{dE}{dt}$ $\oint \vec{B}\cdot d\vec{l} = 2\pi r B = \mu_0 \cancel{\varepsilon_0} \left(\frac{\pi R^2 I}{\pi R^2 \cancel{\varepsilon_0}}\right) = \mu_0 I$

$\boxed{|\vec{B}| = \frac{\mu_0 I}{2\pi r}}$ Direction again cw as viewed from the top

$r \geq R$

(e)



**10)** A red laser beam with a wavelength of 700 nm shines on a dark target which absorbs the beam's energy. The beam has a radius of 1.00 mm and power is absorbed in the target at a rate of 150 mW.

**(a)** What is the frequency of the laser light (in Hz = cycles per second)?

$$f = \frac{c}{\lambda} = \frac{3 \times 10^{8}\,\mathrm{m/s}}{700 \times 10^{-9}\,\mathrm{m}} = \mathbf{4.29 \times 10^{14}\,Hz}.$$

**(b)** What is the amplitude of the electric field in the laser beam?

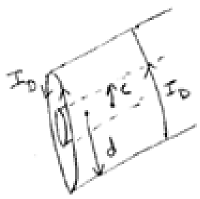
$$S_{\max} = \frac{E^2}{\mu_0 c} = 2S_{avg} = \frac{2P_{avg}}{A} = \frac{2(0.150\ \mathrm{W})}{\pi(0.001\mathrm{m})^2} = \mathbf{9.55 \times 10^{4}\,\frac{W}{m^2}}.$$

$$E = \sqrt{\mu_0 c S_{\max}} = \sqrt{(4\pi \times 10^{-7})(3 \times 10^{8})(9.55 \times 10^{4})}\,\frac{V}{m} = \mathbf{6000\,\frac{V}{m}}.$$

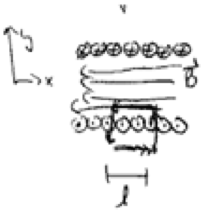
**(c)** What is the amplitude of the magnetic field in the laser beam?

$$B = \frac{E}{c} = \frac{6000}{3 \times 10^{8}}\,\mathrm{T} = \mathbf{2.00 \times 10^{-5}\,T}\ (= 0.20\ \mathrm{Gauss}).$$

**13)**



(a) first consider only Solenoid D, carrying current $I_D$. A slice through it looks like:



By symmetry $\vec{B}$ inside is constant & outside $\vec{B} \sim 0$. Use a small rectangle as an Amperian loop, as shown.

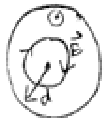
$$\oint \vec{B} \cdot d\vec{l} = Bl + \underbrace{0 + 0}_{\vec{B} \perp d\vec{l}\text{ along the vertical segments}} + \overset{B=0\text{ outside}}{0} = \mu_0 I \underbrace{nl}_{\text{number of turns enclosed}}$$

$Bl = \mu_0 I n l$

$\boxed{\vec{B} = \mu_0 I n\ (-\hat{i})}$ in to the left in the sketch.

(b) The current in solenoid D is increasing at the rate $\frac{dI_D}{dt}$. To determine $\vec{E}$ due to the changing $\vec{B}$, use Faraday's law.

$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$

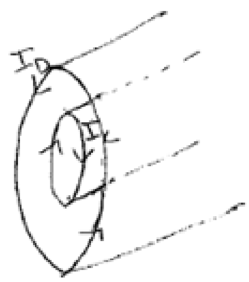


$\vec{B}$ out of page, and increasing upward.

$\vec{E}$ forms concentric loops around $\vec{B}$. In $r < d$, $2\pi r|\vec{E}| = \frac{d\Phi_B}{dt}$

$\Phi_B = \pi r^2 B = \pi r^2 \mu_0 n I$ $\quad 2\pi r|\vec{E}| = \frac{d\Phi_B}{dt} = \pi r^2 \mu_0 n\, dI_D/dt$

The direction will be so as to oppose the change in $\Phi_B$, so current will flow in the clockwise direction in solenoid C.



$|\vec{E}| = \frac{r\mu_0 n}{2}\frac{dI_D}{dt}$ $(r<d)$

The direction is given by Lenz's Law $\Rightarrow$ the direction of $\vec{E}$ is such as to produce a current opposing the change in $\Phi_B$. Since $\Phi_B$ is increasing upward, $\vec{E}$ will form clockwise loops to oppose the change in $\Phi_B$

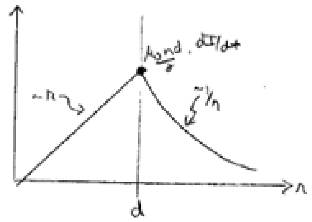
$$\boxed{\vec{E} = \frac{\mu_0 n r}{2}\cdot\frac{dI_D}{dt} \quad \text{clockwise loops as viewed from left} \quad r<d}$$

For $r>d$, $\Phi_B = \pi d^2 B$, since $B=0$ for $r>d$. Then

$$\oint \vec{E}\cdot d\vec{l} = 2\pi r|\vec{E}| = \pi d^2\frac{dB}{dt} = \pi d^2(\mu_0 n)\frac{dI_D}{dt}$$

$\underline{\underline{r>d}}$ $|\vec{E}| = \frac{\mu_0 n d^2}{2r}\frac{dI_D}{dt}$, direction CW loops as viewed from left

(c)




(d) $\mathcal{E} = -\frac{d\Phi_B}{dt}$ $\quad \Phi_B = \pi c^2 B$ for a single turn of wire in Solenoid C. Since $c<d$, Solenoid C is completely within the region of uniform field.

$\frac{d\Phi_B}{dt} = \pi c^2\frac{dB}{dt} = \pi c^2\mu_0 n\frac{dI_D}{dt}$ But Solenoid C has a total of $N = nL$ turns, each of which sees this $\frac{d\Phi_B}{dt}$. Then

$|\mathcal{E}| = N\frac{d\Phi_B}{dt} = nL(\pi c^2\mu_0 n)\frac{dI_D}{dt}$ $\quad \boxed{|\mathcal{E}| = \mu_0 n^2\pi c^2 L\frac{dI_D}{dt}}$