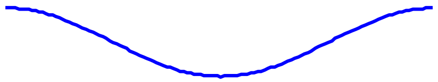
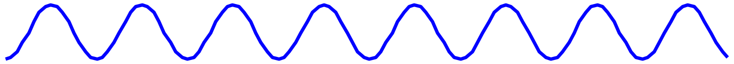
Low frequency



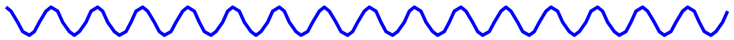
plain wall

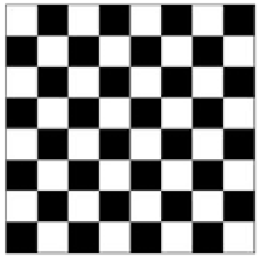
High frequency



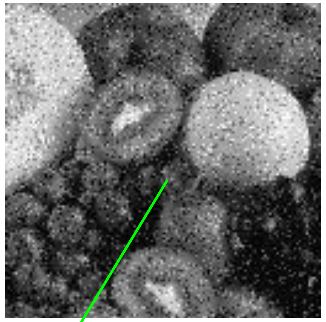
picket fence

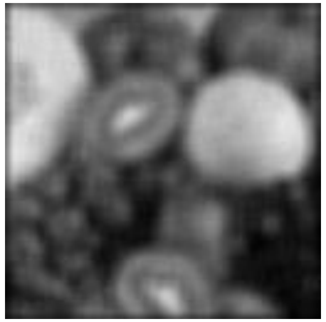
Very High frequency





checkerboard





Higher frequencies due to sharp image variations (e.g., edges, noise, etc.)

Any signal / function / curve / data can be represented as a **linear combination** of a set of **basic components**

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{2\pi n x}{L}\right) + b_n \sin\left(\frac{2\pi n x}{L}\right)\right]$$

–**Fourier components**: sinusoidal patterns

–**Fourier coefficients**: weighting factors assigned to the Fourier components

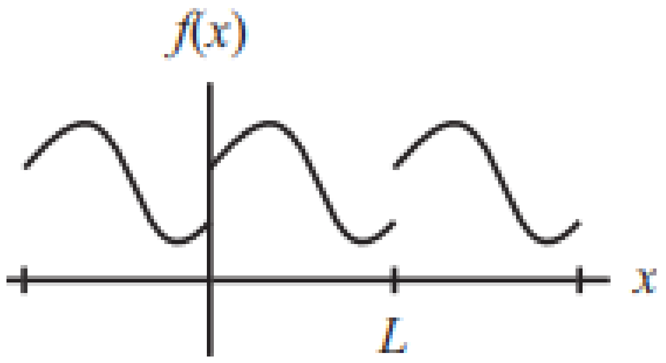
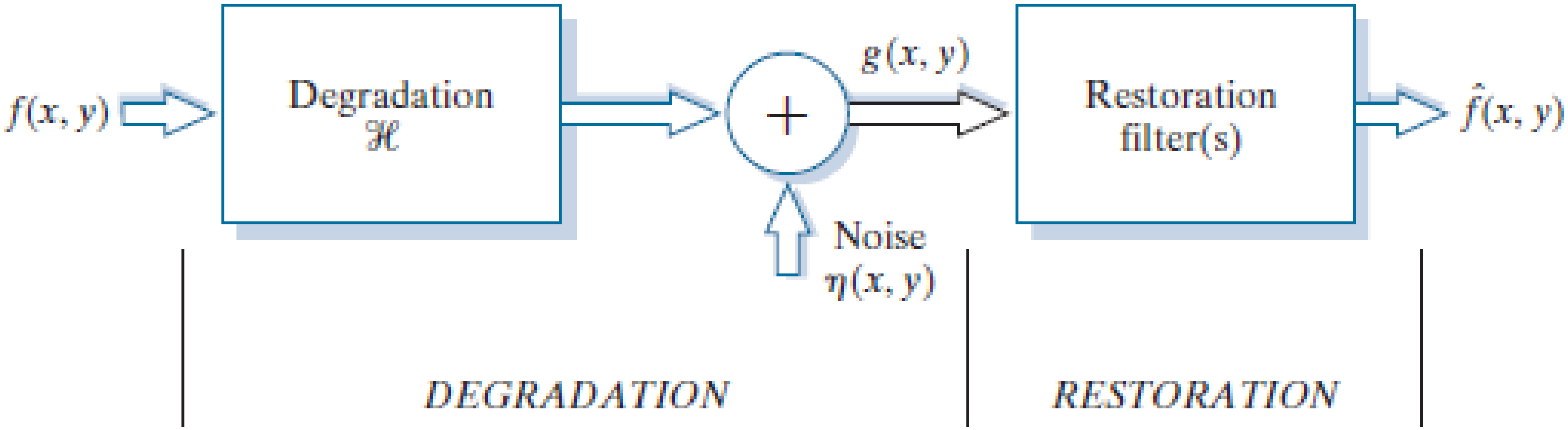


Image Restoration and Reconstruction

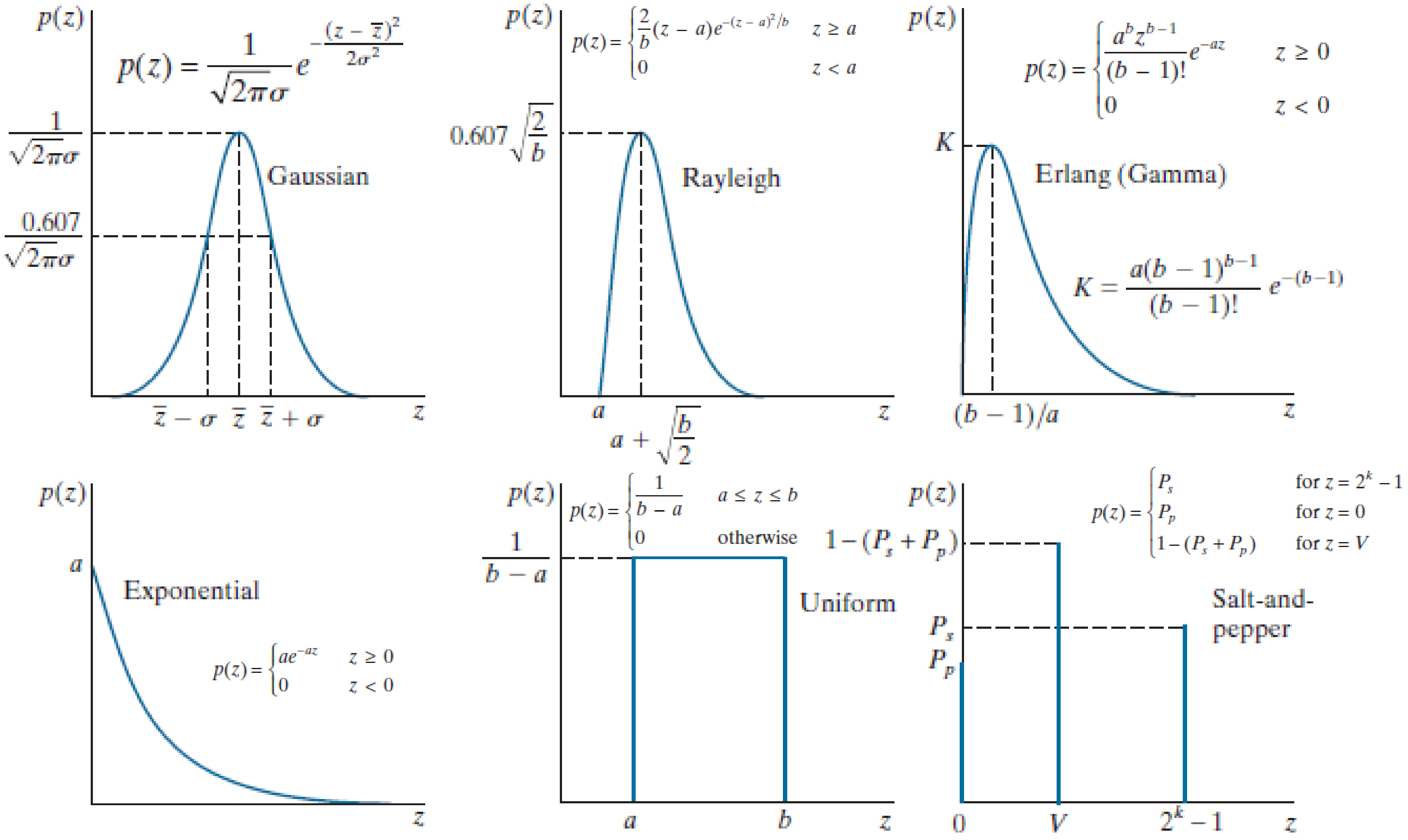
- A Model of the Image Degradation/Restoration process
- Noise Models
- Restoration in the Presence of Noise Only—Spatial Filtering
- Periodic Noise Reduction Using Frequency Domain Filtering
- Linear, Position-Invariant Degradations
- Estimating the Degradation Function
- Inverse Filtering
- Minimum Mean Square Error (Wiener) Filtering
- Constrained Least Squares Filtering
- Geometric Mean Filter

Upon completion of this chapter, readers should:

- Be familiar with the characteristics of various noise models used in image processing, and how to estimate from image data the parameters that define those models.
- Be familiar with linear, nonlinear, and adaptive spatial filters used to restore (denoise) images that have been degraded only by noise.
- Know how to apply notch filtering in the frequency domain for removing periodic noise in an image.
- Understand the foundation of linear, space invariant system concepts, and how they can be applied in formulating image restoration solutions in the frequency domain.
- Be familiar with direct inverse filtering and its limitations.
- Understand minimum mean-square-error (Wiener) filtering and its advantages over direct inverse filtering.
- Understand constrained, least-squares filtering.
- Be familiar with the fundamentals of image reconstruction from projections, and their application to computed tomography.

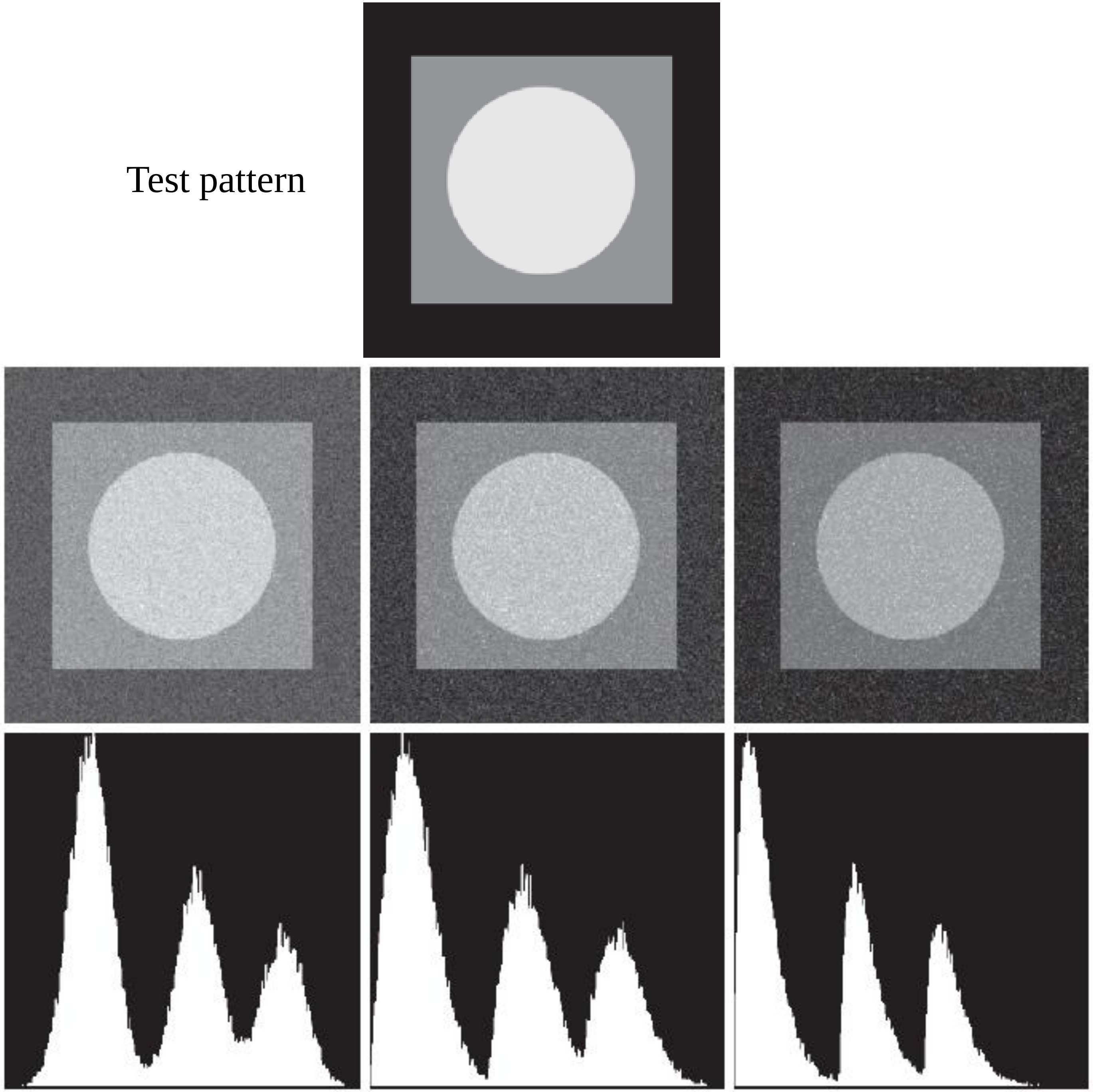


A model of the image degradation / restoration process.

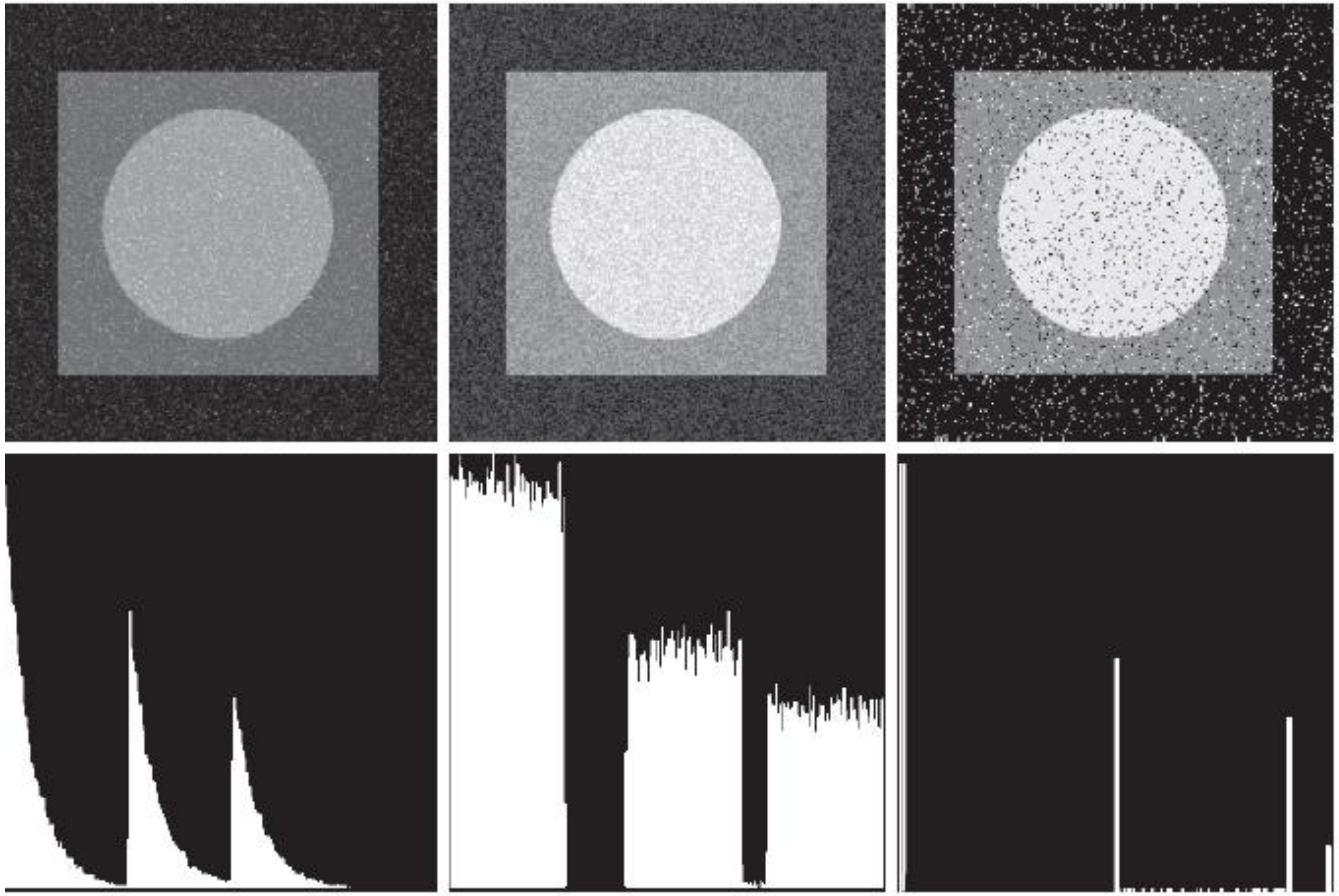


SOME IMPORTANT NOISE PROBABILITY DENSITY FUNCTIONS

Test pattern

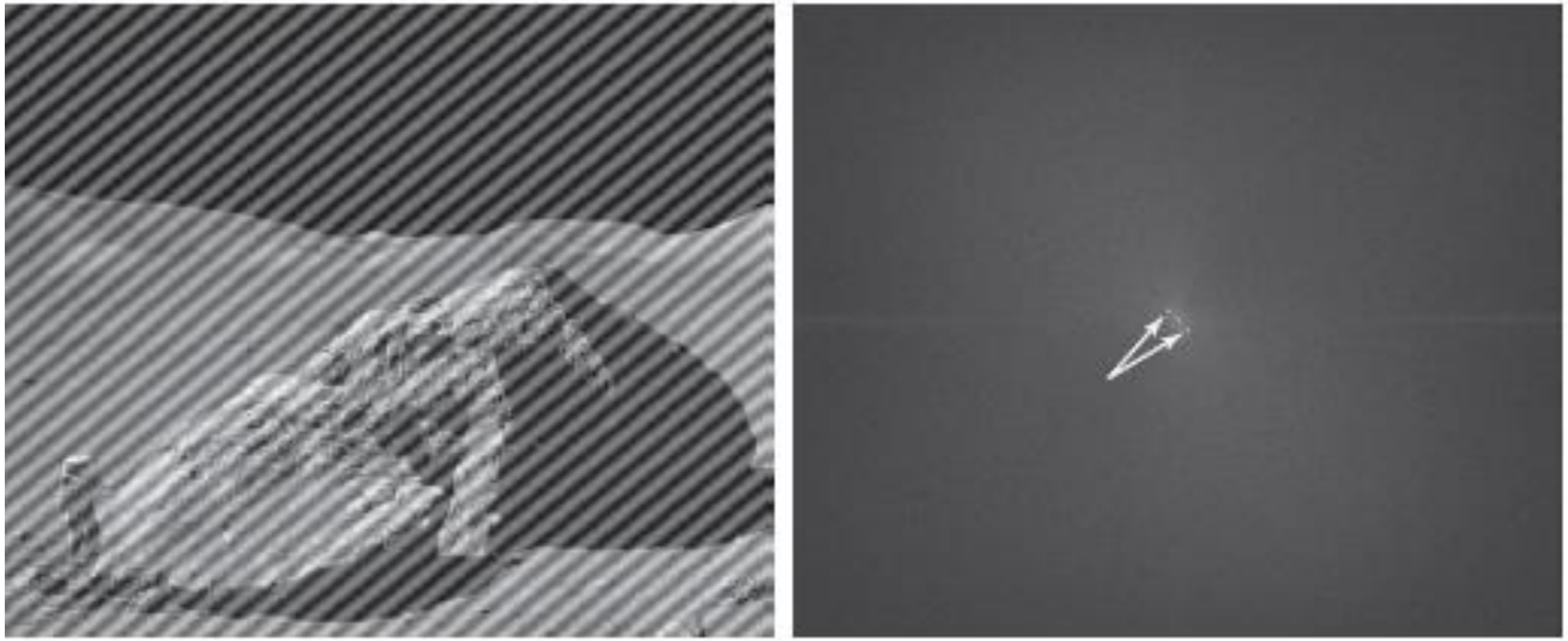


Gaussian, Rayleigh, Erlanga noise



Exponential Uniform Salt-and-Pepper Noise

PERIODIC NOISE



(a) Image corrupted by additive sinusoidal noise.
(b) Spectrum showing two conjugate impulses caused by the sine wave.
(Original image courtesy of NASA.)

When an image is degraded only by additive noise

$$g(x,y) = f(x,y) + \eta(x,y)$$

$$G(u,v) = F(u,v) + N(u,v)$$

MEAN FILTERS

Arithmetic Mean Filter

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(r,c)\in S_{xy}} g(r,c)$$

Geometric Mean Filter

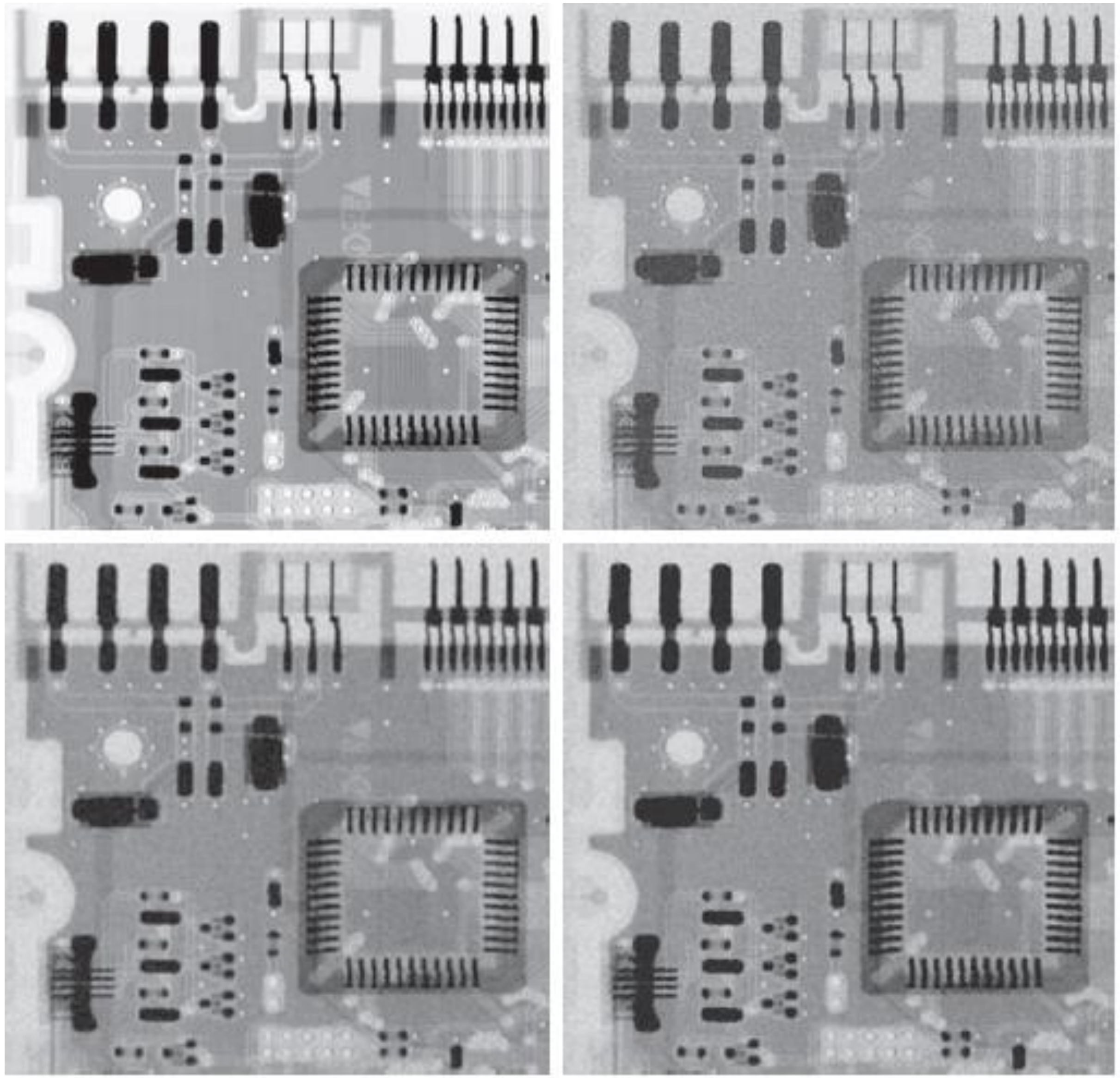
$$\hat{f}(x,y) = \left[\prod_{(r,c)\in S_{xy}} g(r,c) \right]^{\frac{1}{mn}}$$

Harmonic Mean Filter

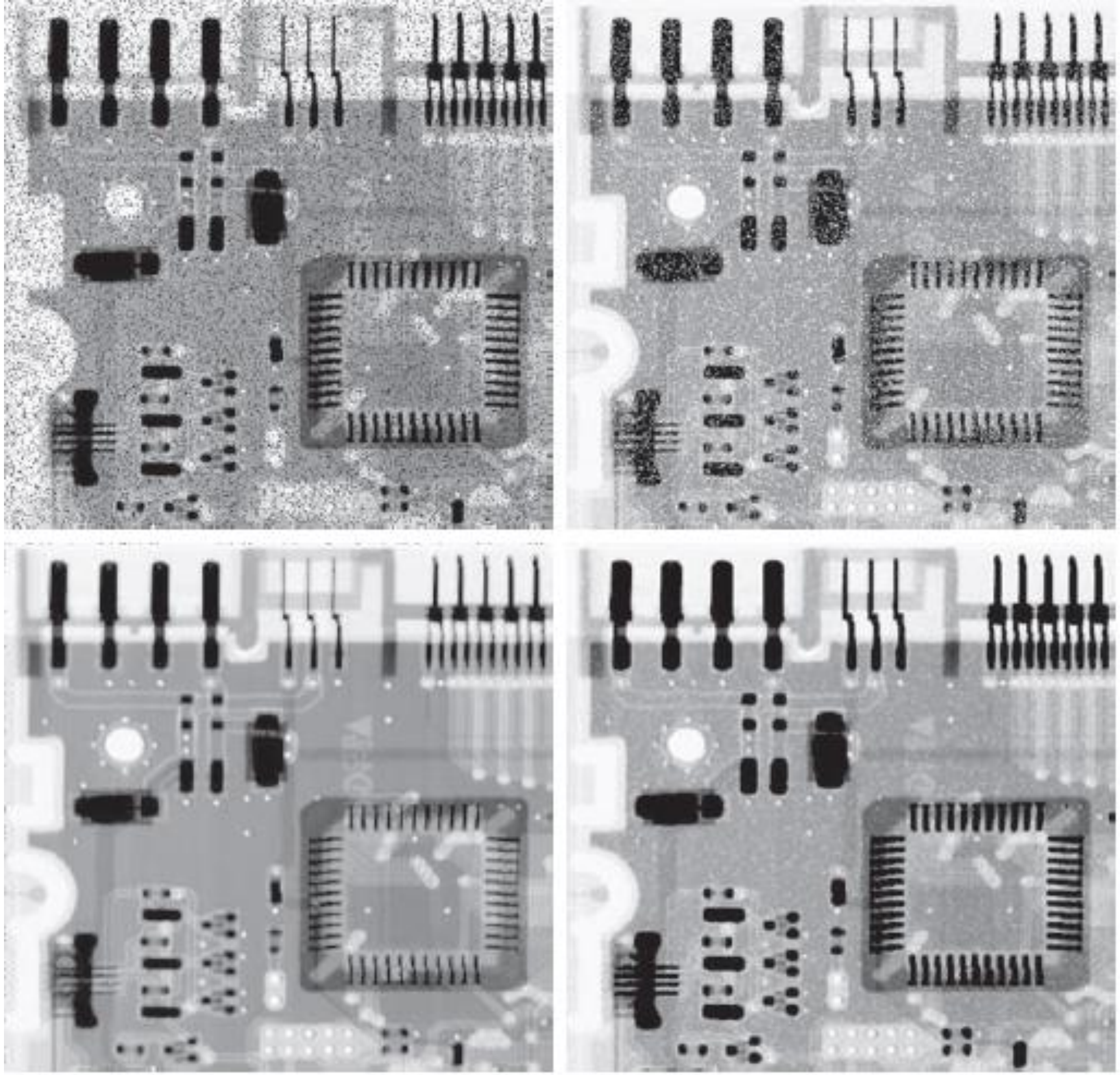
$$\hat{f}(x,y) = \frac{mn}{\sum_{(r,c)\in S_{xy}} \frac{1}{g(r,c)}}$$

Contraharmonic Mean Filter

$$\hat{f}(x,y) = \frac{\sum_{(r,c)\in S_{xy}} g(r,c)^{Q+1}}{\sum_{(r,c)\in S_{xy}} g(r,c)^{Q}}$$



(a) X-ray image of circuit board.
(b) Image corrupted by additive Gaussian noise.
(c) Result of filtering with an arithmetic mean filter of size 3 x 3.
(d) Result of filtering with a geometric mean filter of the same size
(Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)



(a) Image corrupted by pepper noise with a probability of 0.1.
(b) Image corrupted by salt noise with the same probability.
(c) Result of filtering (a) with a 3 x 3 contraharmonic Filter $Q = 1\ 5$
(d) Result of filtering (b) with $Q = -1\ 5$.

ORDER-STATISTIC FILTERS

Median Filter

$$\hat{f}(x,y) = \underset{(r,c)\in S_{xy}}{\text{median}}\{g(r,c)\}$$

Max and Min Filters

$$\hat{f}(x,y) = \max_{(r,c)\in S_{xy}}\{g(r,c)\}$$

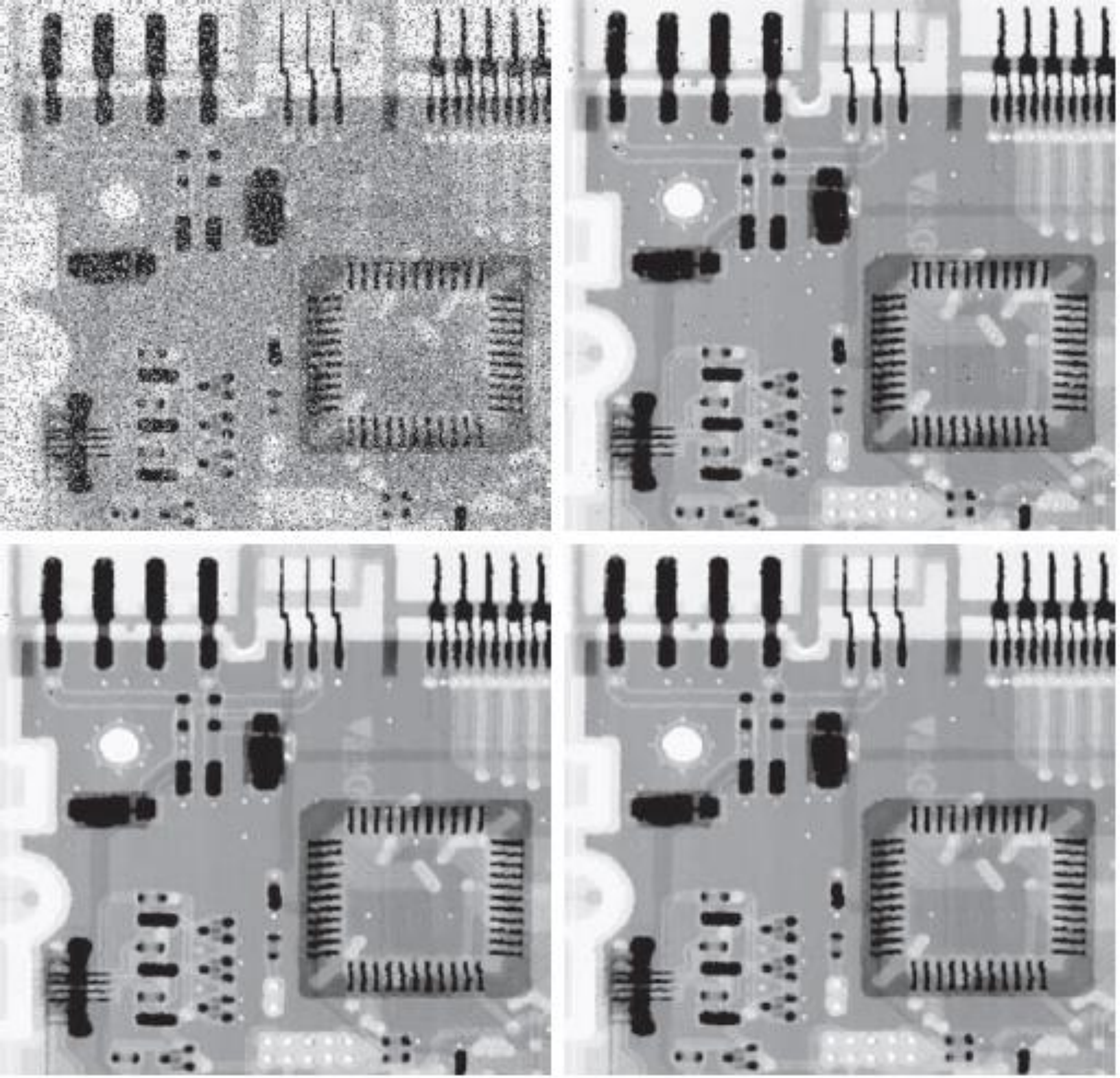
$$\hat{f}(x,y) = \min_{(r,c)\in S_{xy}}\{g(r,c)\}$$

Midpoint Filter

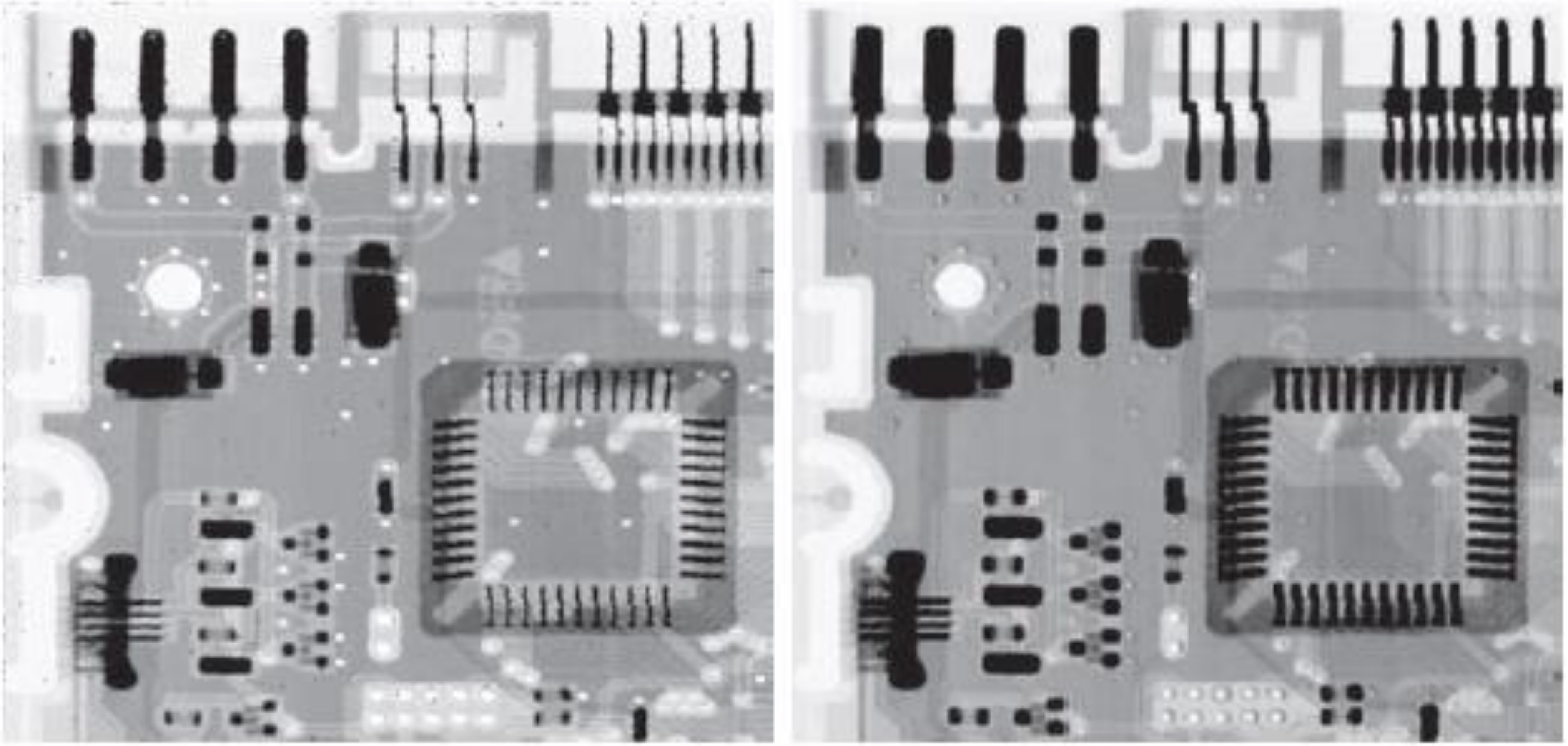
$$\hat{f}(x,y) = \frac{1}{2}\left[\max_{(r,c)\in S_{xy}}\{g(r,c)\} + \min_{(r,c)\in S_{xy}}\{g(r,c)\}\right]$$

Alpha-Trimmed Mean Filter

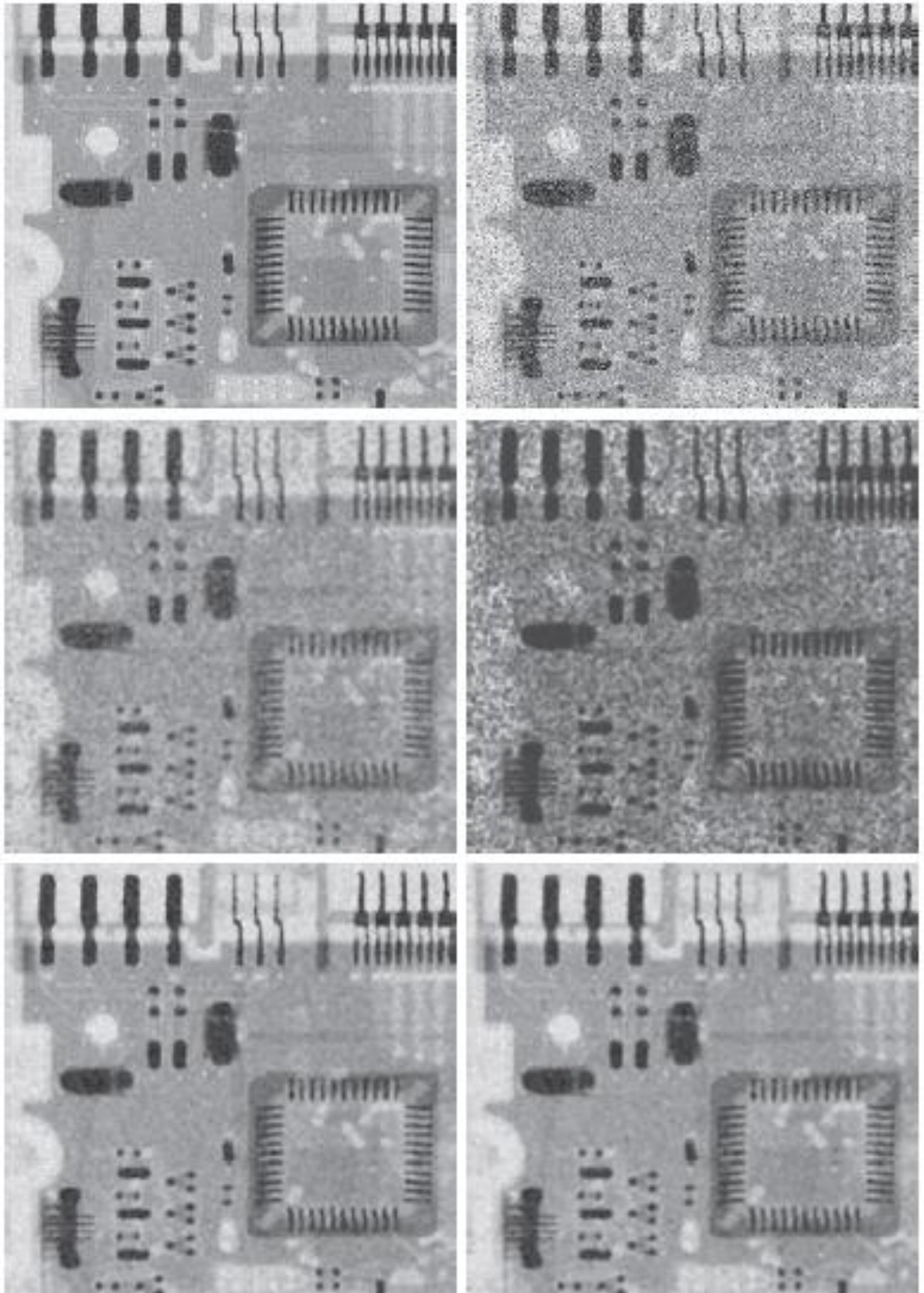
$$\hat{f}(x,y) = \frac{1}{mn-d}\sum_{(r,c)\in S_{xy}} g_R(r,c)$$



(a) Image corrupted by saltand- pepper noise with probabilities $P_s = P_p = 0.1$
(b) Result of one pass with a median filter of size 3x3.
(c) Result of processing (b) with this filter.
(d) Result of processing (c) with the same filter.



(a) Result of filtering with a max filter of size 3x 3.
(b) Result of filtering with a min filter of the same size.



(a) Image corrupted by additive uniform noise.

(b) Image additionally corrupted by additive salt-and pepper noise.

(c)-(f) Image (b) filtered with a 5 x 5:

(c) Arithmetic mean filter;

(d) Geometric mean filter;

(e) median filter;

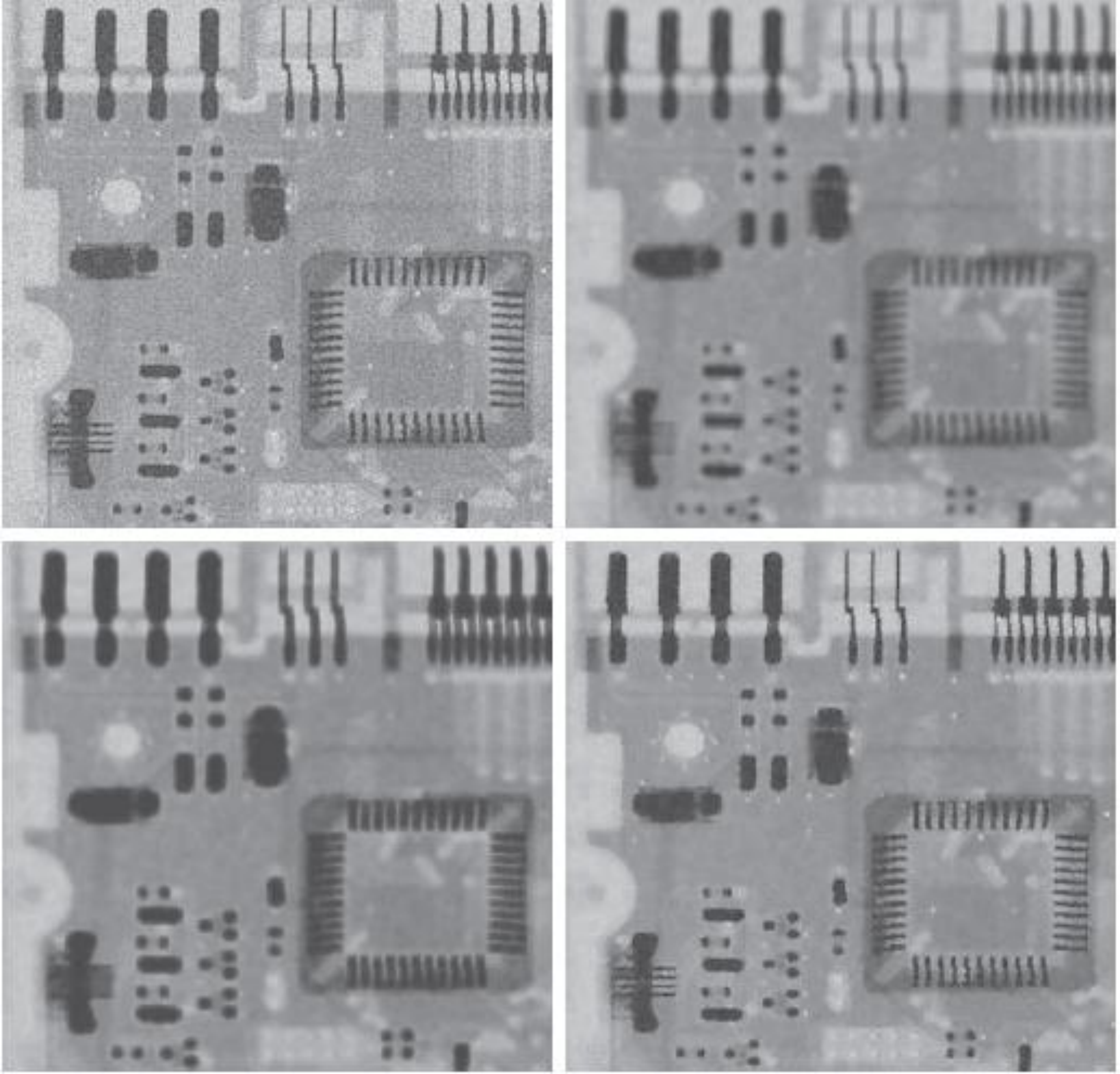
(f) alpha-trimmed mean filter, with $d = 6$.

Adaptive, Local Noise Reduction Filter

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_{S_{xy}}^2}\left[g(x,y) - \bar{z}_{S_{xy}}\right]$$

1. If σ_η^2 is zero, the filter should return simply the value of g at (x, y). This is the trivial, zero-noise case in which g is equal to f at (x, y).
2. If the local variance $\sigma_{S_{xy}}^2$ is high relative to σ_η^2, the filter should return a value close to g at (x, y). A high local variance typically is associated with edges, and these should be preserved.
3. If the two variances are equal, we want the filter to return the arithmetic mean value of the pixels in *Sxy*. This condition occurs when the local area has the same properties as the overall image, and local noise is to be reduced by averaging.

adaptive filters are capable of performance superior to that of the filters discussed thus far.



(a) Image corrupted by additive Gaussian noise of zero mean and a variance of 1000.
(b) Result of arithmetic mean filtering.
(c) Result of geometric mean filtering.
(d) Result of adaptive noise reduction filtering.
All filters used were of size 7 x 7.

Adaptive Median Filter

The adaptive median-filtering algorithm uses two processing levels, denoted level *A* and level *B*, at each point (x, y) :

Level A: If $z_{min} < z_{med} < z_{max}$, go to Level B
Else, increase the size of S_{xy}
If $S_{xy} \leq S_{max}$, repeat level A
Else, output z_{med}.

Level B: If $z_{min} < z_{xy} < z_{max}$, output z_{xy}
Else output z_{med}.

z_{min} = minimum intensity value in S_{xy}

z_{max} = maximum intensity value in S_{xy}

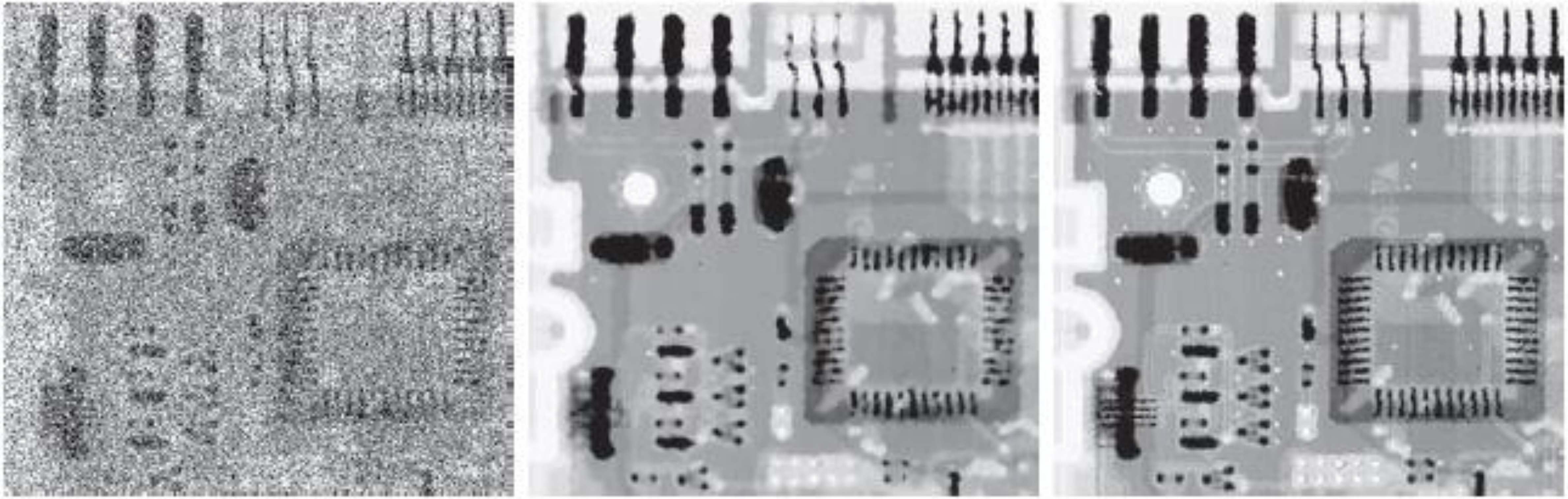
z_{med} = median of intensity values in S_{xy}

z_{xy} = intensity at coordinates (x, y)

S_{max} = maximum allowed size of S_{xy}

Adaptive Median Filter produces a slightly less blurred result, but can fail to detect salt (pepper) noise embedded in a constant background having the same value as pepper (salt) noise.

Image denoising using adaptive median filtering.



a b c

FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_s = P_p = 0.25$. (b) Result of filtering with a 7×7 median filter. (c) Result of adaptive median filtering with $S_{max} = 7$.

PERIODIC NOISE REDUCTION USING FREQUENCY DOMAIN FILTERING

a b
c d

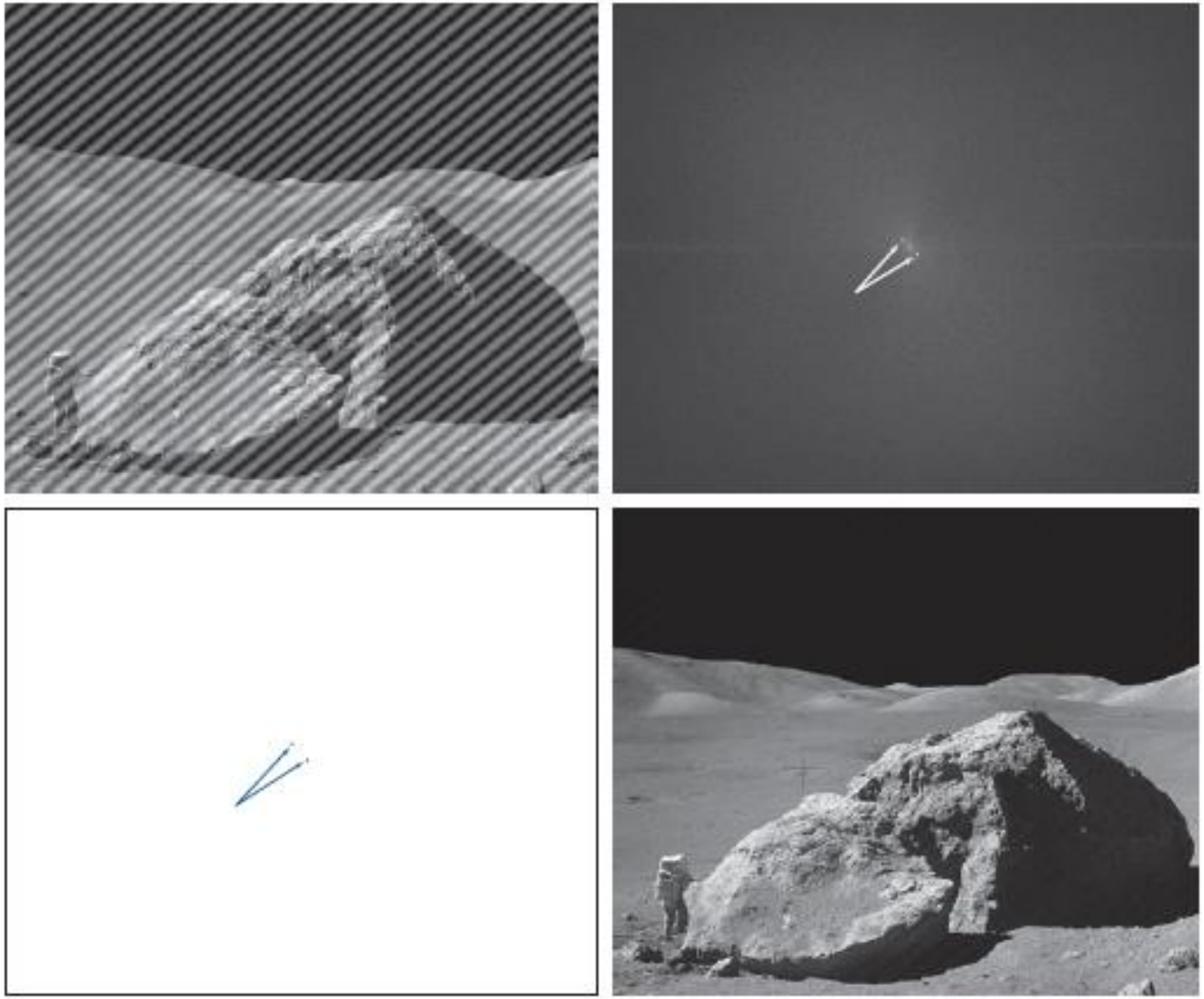


FIGURE 5.16 (a) Image corrupted by sinusoidal interference. (b) Spectrum showing the bursts of energy caused by the interference. (The bursts were enlarged for display purposes.) (c) Notch filter (the radius of the circles is 2 pixels) used to eliminate the energy bursts. (The thin borders are not part of the data.) (d) Result of notch reject filtering. (Original image courtesy of NASA.)

LINEAR, POSITION-INVARIANT DEGRADATIONS

$$g(x,y) = \mathcal{H}\left[f(x,y)\right] + \eta(x,y)$$

let us assume that $\eta(x,y) = 0$ so that $g(x,y) = \mathcal{H}\left[f(x,y)\right]$.

Linearity

$$\mathcal{H}\left[af_1(x,y) + bf_2(x,y)\right] = a\mathcal{H}\left[f_1(x,y)\right] + b\mathcal{H}\left[f_2(x,y)\right]$$

Scale (position) invarient .

$$\mathcal{H}\left[f(x-\alpha, y-\beta)\right] = g(x-\alpha, y-\beta)$$

ESTIMATION BY IMAGE OBSERVATION

Spatial Domain : Convolution

$$g(x,y) = (h \star f)(x,y) + \eta(x,y)$$

Fourier Domain : Multplication

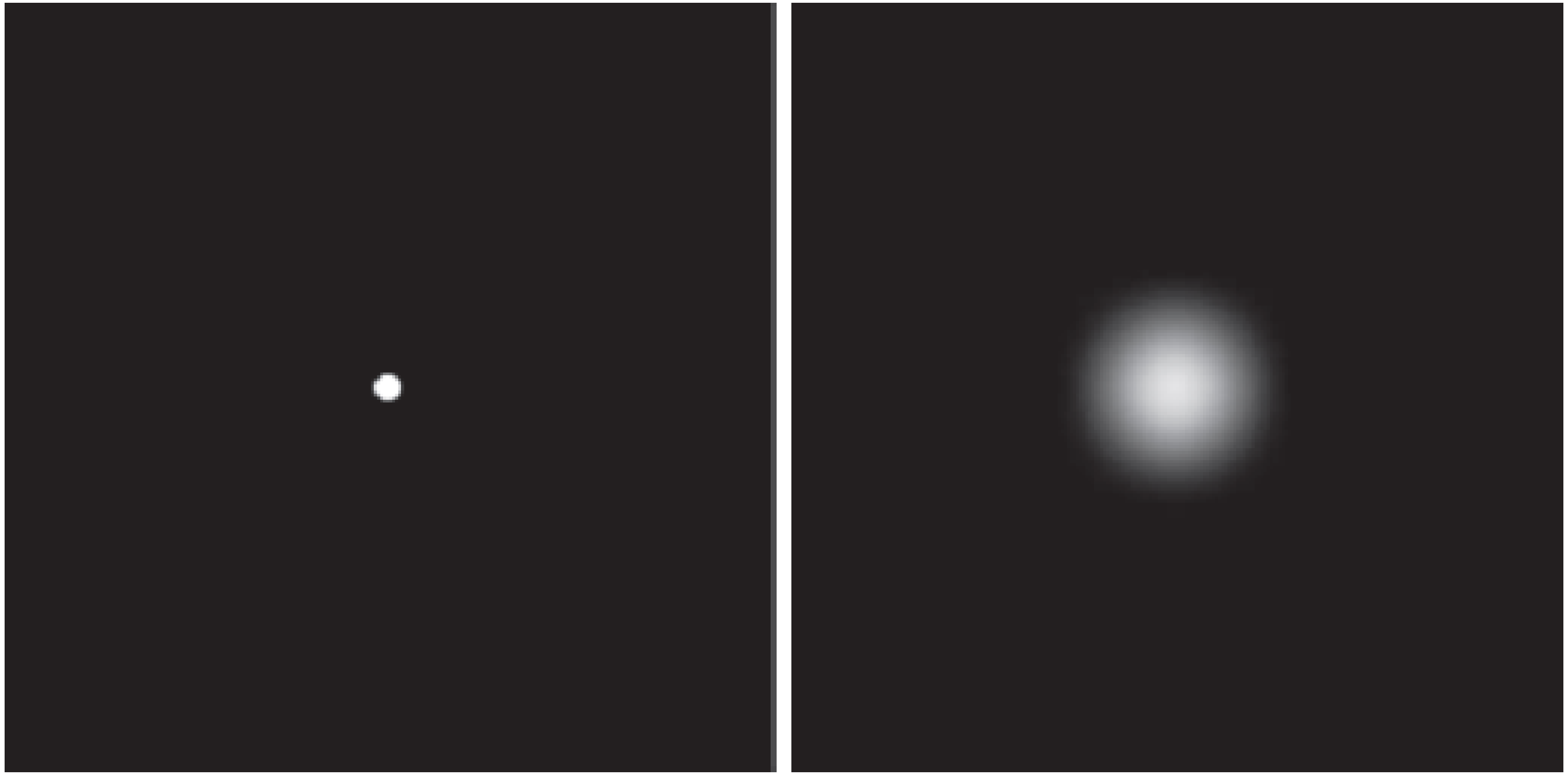
$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

Assume that : No noise

$$H_s(u,v) = \frac{G_s(u,v)}{\hat{F}_s(u,v)}$$

a b

FIGURE 5.24 Estimating a degradation by impulse characterization. (a) An impulse of light (shown magnified). (b) Imaged (degraded) impulse.

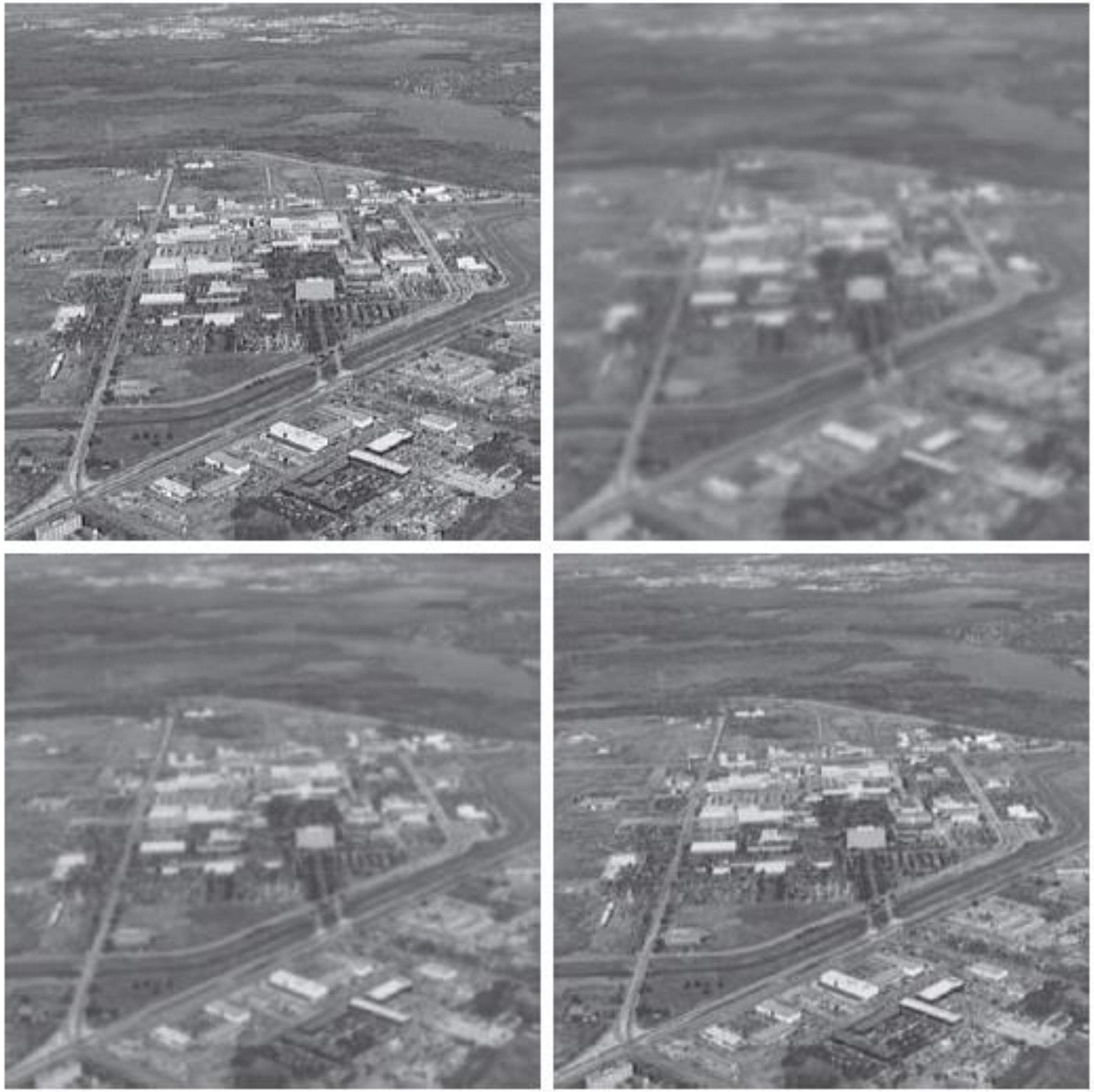


$$H(u,v) = \frac{G(u,v)}{A}$$

ESTIMATION BY MODELING

a b
c d

FIGURE 5.25 Modeling turbulence. (a) No visible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. All images are of size 480×480 pixels. (Original image courtesy of NASA.)

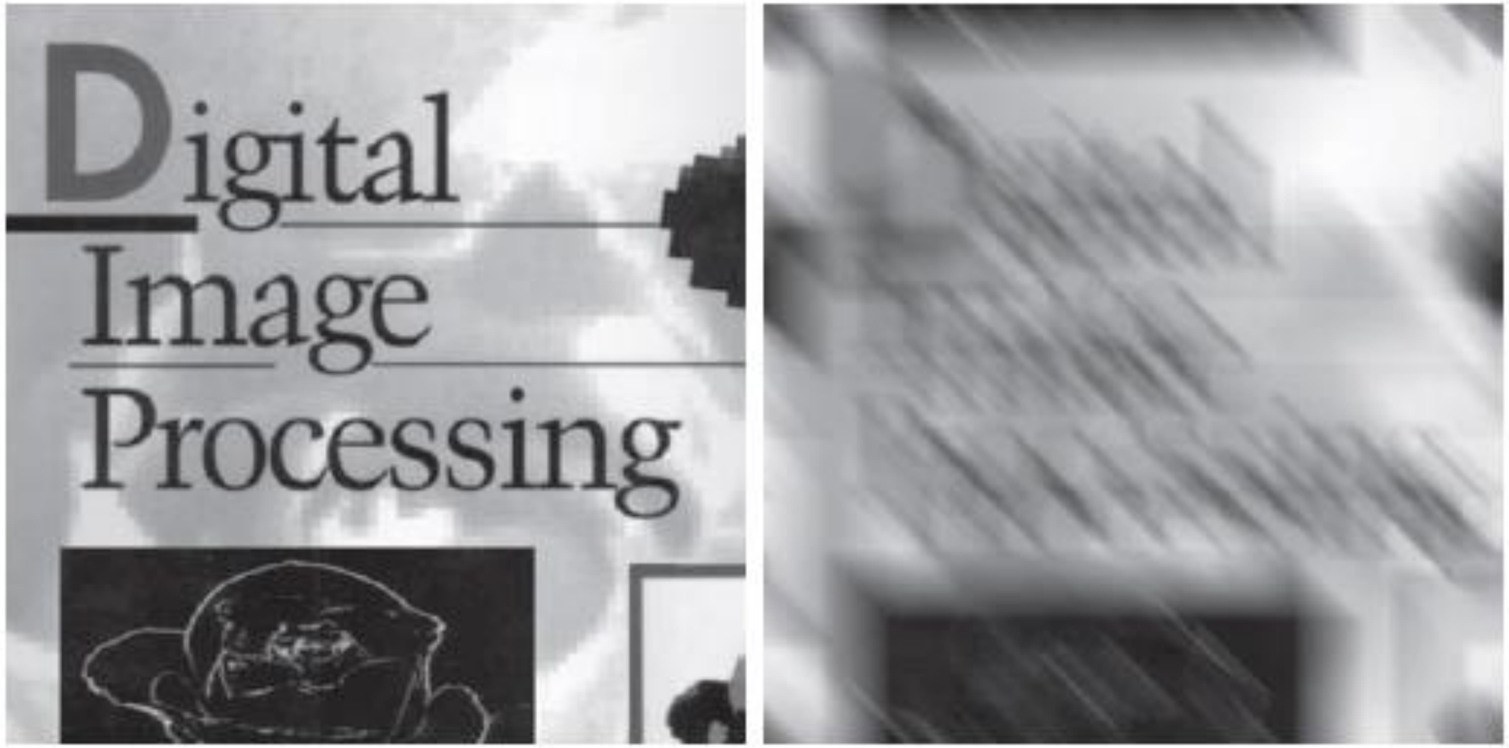


$$H(u,v) = e^{-k(u^2 + v^2)^{5/6}}$$

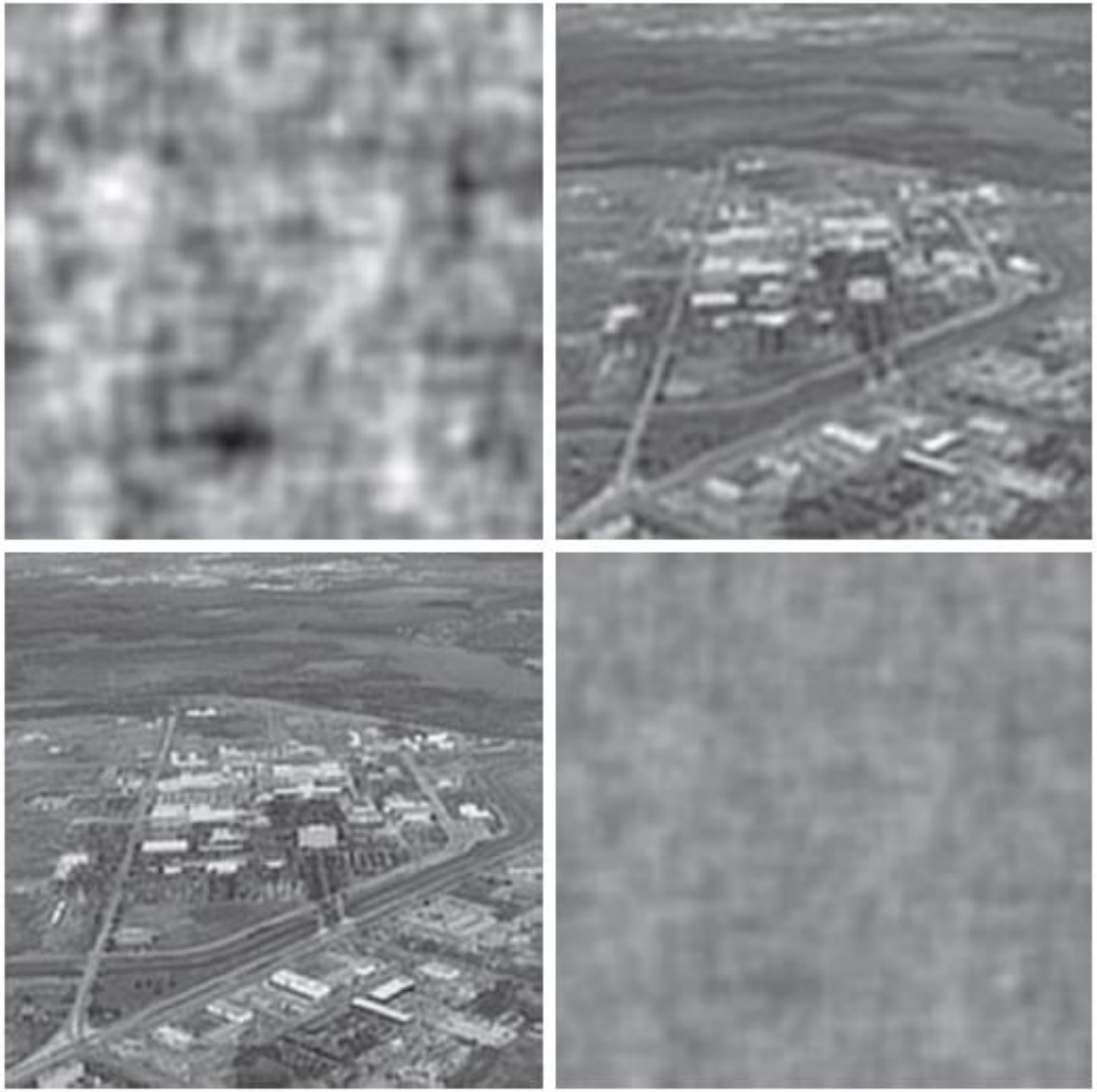
MOTION BLUR

a b

FIGURE 5.26 (a) Original image. (b) Result of blurring using the function in Eq. (5-77) with $a = b = 0.1$ and $T = 1$.



INVERSE FILTERING



a b
c d

FIGURE 5.27 Restoring Fig. 5.25(b) using Eq. (5-78). (a) Result of using the full filter. (b) Result with H cut off outside a radius of 40. (c) Result with H cut off outside a radius of 70. (d) Result with H cut off outside a radius of 85.

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

$$H(u,v) = e^{-k\left[(u + M/2)^2 + (v - N/2)^2\right]^{5/6}}$$

MINIMUM MEAN SQUARE ERROR (WIENER) FILTERING

$$e^2 = E\left\{(f - \hat{f})^2\right\}$$

FREQUENCY DOMAIN

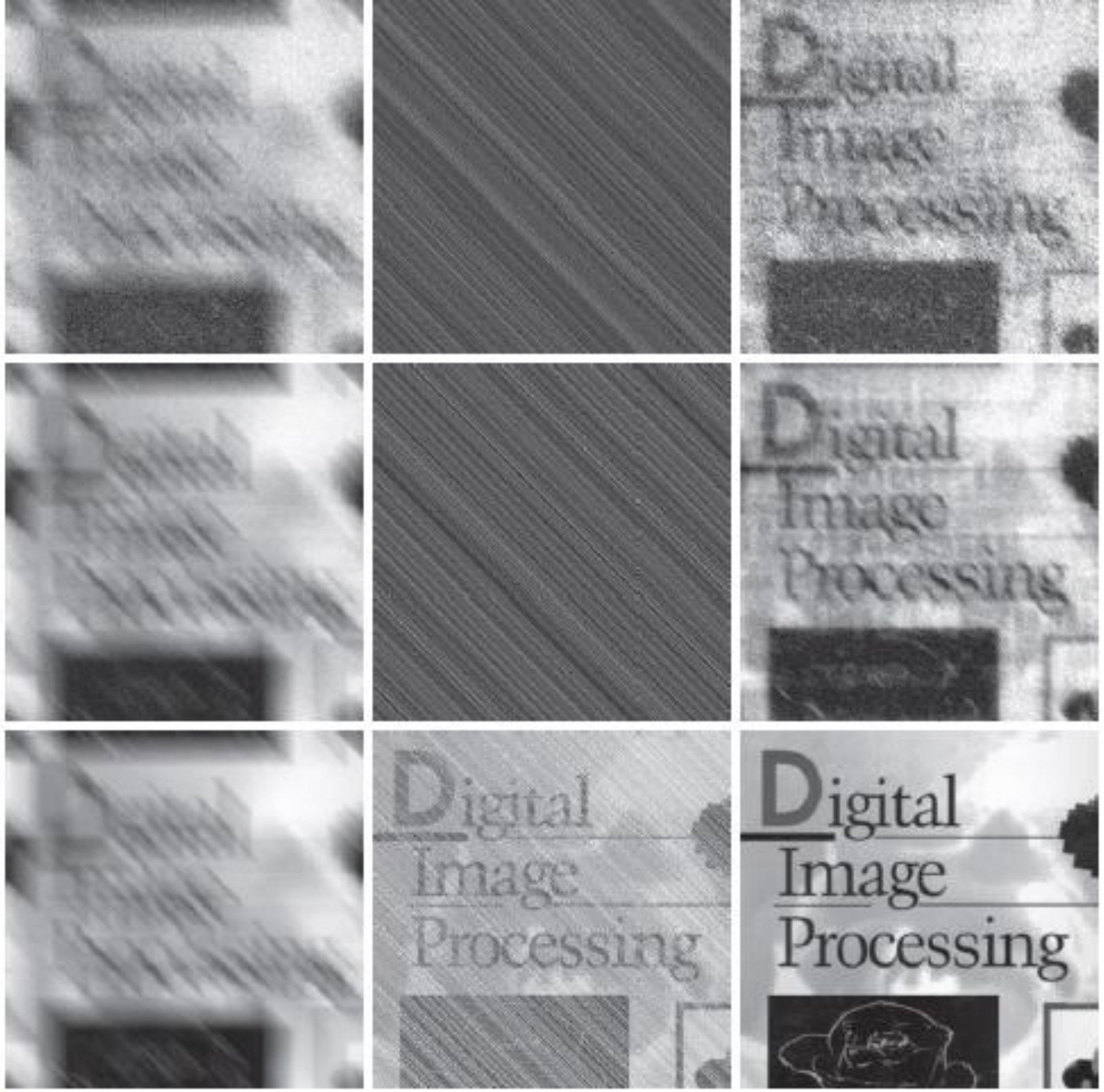
$$\begin{aligned}\hat{F}(u,v) &= \left[\frac{H^*(u,v)S_f(u,v)}{S_f(u,v)|H(u,v)|^2 + S_\eta(u,v)}\right]G(u,v)\\ &= \left[\frac{H^*(u,v)}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right]G(u,v)\\ &= \left[\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right]G(u,v)\end{aligned}$$

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2 + K}\right]G(u,v)$$

1. $\hat{F}(u,v)$ = Fourier transform of the estimate of the undegraded image.
2. $G(u,v)$ = Fourier transform of the degraded image.
3. $H(u,v)$ = degradation transfer function (Fourier transform of the spatial degradation).
4. $H^*(u,v)$ = complex conjugate of $H(u,v)$.
5. $|H(u,v)|^2 = H^*(u,v)H(u,v)$.
6. $S_\eta(u,v) = |N(u,v)|^2$ = power spectrum of the noise [see Eq. (4-89)]†
7. $S_f(u,v) = |F(u,v)|^2$ = power spectrum of the undegraded image.

a b c

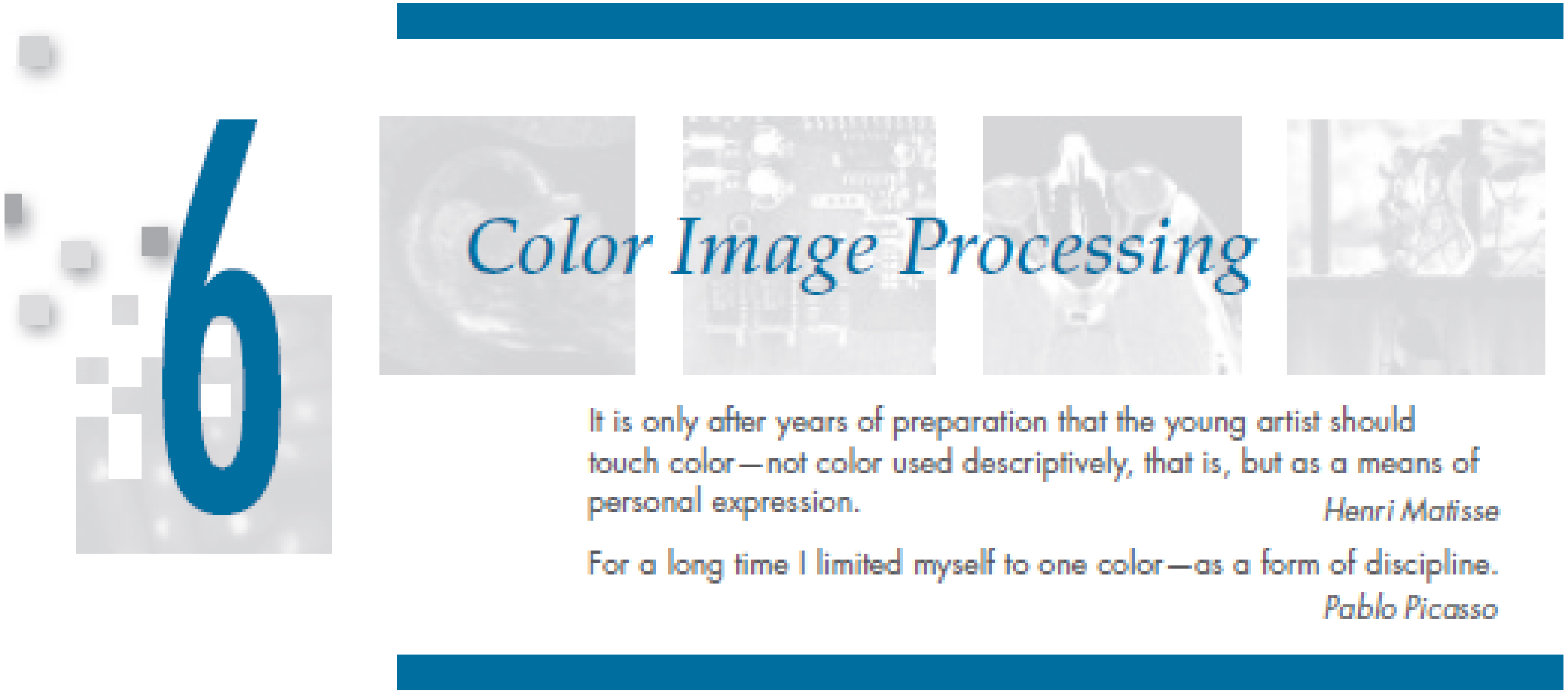
FIGURE 5.28 Comparison of inverse and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.



a b c
d e f
g h i

FIGURE 5.29 (a) 8-bit image corrupted by motion blur and additive noise. (b) Result of inverse filtering. (c) Result of Wiener filtering. (d)–(f) Same sequence, but with noise variance one order of magnitude less. (g)–(i) Same sequence, but noise variance reduced by five orders of magnitude from (a). Note in (h) how the deblurred image is quite visible through a "curtain" of noise.

Next Course



6

Color Image Processing

It is only after years of preparation that the young artist should touch color—not color used descriptively, that is, but as a means of personal expression.

Henri Matisse

For a long time I limited myself to one color—as a form of discipline.

Pablo Picasso