

1-a)

$$T(n) = 1$$

$$T(n) = T(n-1) + 1$$

$$T(n-1) = T(n-2) + 1$$

$$T(n) = T(n-2) + \frac{1+1}{n \text{ times}}$$

$$T(n) = \underbrace{T(0)}_1 + n = \underbrace{\theta(n)}$$

1-b) $T(n) = T(n/2) + T(n/2) + 1$

Master theorem

$$= 2T(n/2) + 1$$

$$n^{\log_b a} = n^{\log_2 2} = n$$

$$n < 1 \text{ so } \theta(n^{\log_2 2}) = \theta(n)$$

Those algorithms almost the same. They have equal time complexity. b has two more variable than a but in today's conditions that won't be a problem. So I would choose "a", because it is easier to understand.

2)

4) Algorithm finds the distance between every point and others.

int x1, x2;

int min = 9999 9999

for i=0 to n-1

for j=i+1 to n-1

if (distance(set[i], set[j]) < min)

min = distance(set[i], set[j])

x1 = i

x2 = j

end if

end for

end for

return x1 and x2

end procedure

$$\longrightarrow \theta(n)$$

$$\longrightarrow \theta(n-i-1)$$

$$\left. \begin{array}{l} \theta(n) \\ \theta(n-i-1) \end{array} \right\} \theta(1)$$

$$n-1 + n-2 + n-3 + \dots + 1 + 0$$

$$\frac{(n-1)(n-2)}{2} = n^2$$

Time Complexity is $\theta(n^2)$

2)

```
coefs = [3, 4, -2] # coefficients a_n , a_{n-1} ...

def calculate(x_0):
    total = 0
    n = len(coefs)
    for a in range(n):
        total = total + (coefs[a] * pow(x_0, n - a - 1))
    return total

print(calculate(2))
```

Handwritten notes in red:

- $\rightarrow n \text{ times}$ (pointing to the `for` loop)
- $\rightarrow O(n)$ (pointing to the `pow` function)
- $= n \cdot O(1)$
- $= O(n^2)$
- $\text{pow} \rightarrow O(n - a - 1) = O(n)$

Yes, there is a way to do that faster with Horner's method. For example

$x^3 + 5x^2 + 2x - 1$. This polynomial can be evaluated as $((x + 5)x + 2)x - 1$. That means writing the polynomial as coefficients of x^n and repeatedly multiplying with x .

That way the time complexity will be $O(n)$

3)

```
def count_words(word_, first_, last_):  
    count = 0  
    for x in range(len(word_)):  $\rightarrow n$  times  
        if word[x] == first_:  $\rightarrow n - x - 1$  times  
            for y in range(x + 1, len(word_)):  $\rightarrow \theta(1)$   
                if word[y] == last_:  $n \cdot (n - x - 1) \cdot \theta(1)$   
                    count = count + 1  $= \theta(n^2)$   
    return count  
  
word = "arabamavan"  
print("for word: " + word + " first: a , last: r => " + str(count_words(word_, "a", "r")))  
  
word = "TXZXXJZWXX"  
print("for word: " + word + " first: X , last: Z => " + str(count_words(word_, "X", "Z")))
```

5)

```

1 regions = ["A", "B", "C", "D", "E", "F", "G"]
2 profits = [3, -5, 2, 11, -8, 9, -5]
3
4
5 def cluster_a():
6     current = 0
7     max = -9999999
8     curr_path = []
9     max_path = []
10    for x in range(len(regions)):
11        current = profits[x]
12        curr_path.clear()
13        curr_path.append(regions[x])
14        for y in range(x + 1, len(regions)):
15            current = current + profits[y]
16            curr_path.append(regions[y])
17            if current > max:
18                max_path = curr_path.copy()
19                max = current
20
21    print(max_path)
22
23
24    print("5-a")
25    cluster_a()

```

Handwritten annotations for `cluster_a()`:

- `for x in range(len(regions)):` → n times
- `curr_path.clear()` → $O(1)$
- `curr_path.append(regions[x])` → $O(1)$
- `for y in range(x + 1, len(regions)):` → $n - x - 1$ times
- `curr_path.append(regions[y])` → $O(1)$
- `max_path = curr_path.copy()` → $O(n)$

Overall complexity: $n \cdot (n - x - 1) \cdot n = O(n^3)$

```

def maxClusterConnected(l, m, r): # left middle right
    temp = 0
    left_total = -9999999
    right_total = -9999999

    for x in range(m, l - 1, -1):
        temp = temp + profits[x]
        if temp > left_total:
            left_total = temp

    temp = 0
    for x in range(m + 1, r + 1):
        temp = temp + profits[x]
        if temp > right_total:
            right_total = temp

    return max(left_total + right_total, left_total, right_total) # return the biggest one among them

```

Handwritten annotations for `maxClusterConnected`:

- `def maxClusterConnected(l, m, r):` → $O(n)$
- `for x in range(m, l - 1, -1):` → $n/2$ times
- `for x in range(m + 1, r + 1):` → $n/2$ times

Overall complexity: $\frac{n}{2} + \frac{n}{2} = n$

```

def cluster_b(l, r):
    if l == r:
        return profits[l]

    m = (l + r) // 2

    return max(cluster_b(l, m), cluster_b(m + 1, r), maxClusterConnected(l, m, r))

```

Handwritten annotations for `cluster_b`:

- Recurrence relation: $T(n) = 2T(n/2) + O(n)$
- Master theorem: $n^{\log_2 2} = n^{\log_2 2} = n \rightarrow \text{case 2}$
- Overall complexity: $O(n \log_2 n)$

Final line of code: `print("\nPart 5-b: ", cluster_b(0, len(profits) - 1))`

Handwritten note at the bottom: $O(n^{\log_2 2} \log n) = O(n \log n)$