Logic - Theorem Formalization With a Proof Assistant

CSE 495 - Final Presentation

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24.01.2024

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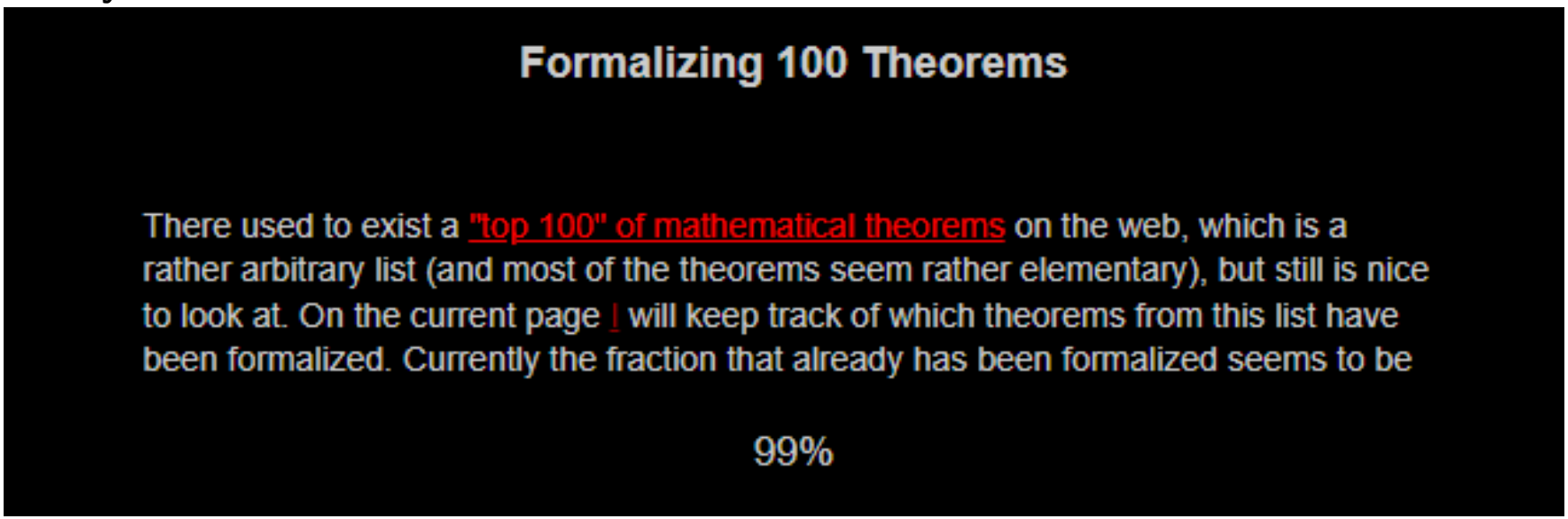
Definiton of Project

Proof assistant softwares are fairly new concept in computer science. LEAN is one of the most user friendly one. Like all others, it helps you manipulate the computer to check if the logic you follow make sense.

So in this project, I used LEAN theorem prover and formalized the Pick's Thoerem.

At a mathematics conference in July, 1999, Paul and Jack Abad presented their list of "The Hundred Greatest Theorems." Their ranking is based on the following criteria: "the place the theorem holds in the literature, the quality of the proof, and the unexpectedness of the result.

There is a person in Radboud University, Dr. Freek Wiedijk. He keeps track of these theorems and he has a website about how many of these theorem's has been formalized.



This is the list of theorem prove assistants that he checks.

Isabelle	89
HOL Light	87
Coq	79
Lean	76
Metamath	74
Mizar	69
nqthm/ACL2	45
ProofPower	43
PVS	26
Megalodon	12
Naproche	10
NuPRL/MetaPRL	8

About My Project

Pick's theorem is ranked 92nd in the top 100 math theorems list and has not yet been formalized by Lean. And hasn't been formalized by so many.

91. **The Triangle Inequality**
    Isabelle, Steven Obua: statement
    HOL Light, John Harrison: statement
    Coq, Frédérique Guilhot: statement
    Lean, Zhouhang Zhou: statement
    Metamath, Norman Megill: statement
    Mizar, Czeslaw Bylinski: statement
    ACL2, Ruben Gamboa
    ProofPower, Rob Arthan: statement
    PVS, NASA library, Ricky Butler & Cesar Munoz

92. **Pick's Theorem**
    HOL Light, John Harrison: statement

93. **The Birthday Problem**
    Isabelle, Lukas Bulwahn: statement
    HOL Light, John Harrison: statement
    Coq, Jean-Marie Madiot: statement
    Lean, Eric Rodriguez: statement
    Metamath, Mario Carneiro: statement
    Mizar, Cezary Kaliszyk: statement
    ACL2, David M. Russinoff
    ProofPower, Rob Arthan: statement

About My Project

There is a lean page called Missing theorems from this list. That's how I picked this theorem.

Missing theorems from Freek Wiedijk's list of 100 theorems

These theorems are not yet formalized in Lean. Here is the list of the formalized theorems.

- 92: Pick's Theorem
- 99: Buffon Needle Problem

Success Criteria

- Discrete Math, Logic, Proof Methods Knowledge
- Learning LEAN Software
- Formalize a theorem with the help of LEAN Proof Assistant.

References

- https://leanprover-community.github.io/100-missing.html
- https://www.cs.ru.nl/ freek/100/
- http://pirate.shu.edu/ kahlnath/Top100.html