

Logic - Theorem formalization with a proof assistant

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What is logic?

Logic is the study of how to reason and make sense of information in a clear and systematic way. It helps us determine what's true or false, and it provides a set of rules for drawing conclusions based on evidence and facts

What is theorem?

A theorem is a statement or proposition in mathematics or logic that has been proven to be true based on a set of axioms and previously established theorems.

What is proposition?

A proposition is a declarative sentence (that is, a sentence that declares a fact) that is either true or false, but not both

What is axiom?

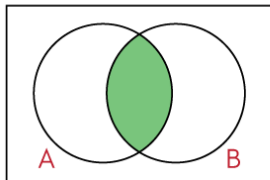
An axiom is a rule or statement that is generally accepted to be true without proof and axioms do not require proof because they are self-evident or accepted as a starting point.

What is proof?

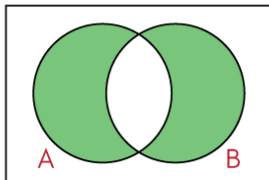
A proof is a way to show that something is true or correct. It's like providing evidence or a clear explanation that leaves no doubt about the truth of a statement. In mathematics, proofs are used to make sure that mathematical facts and ideas are absolutely correct and can be trusted.

Boolean Operations

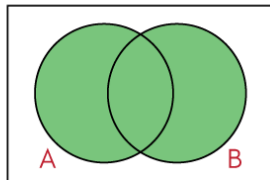
AND



XOR



OR



A logic example

“You can access the Internet from campus only if you are a computer science major or you are not a freshman.”

In particular, we let a , c , and f represent;

a : “You can access the Internet from campus,”

c : “You are a computer science major,”

f : “You are a freshman,” respectively.

$$a \rightarrow (c \vee \neg f)$$

Proof Methods

Exhaustive Proof and Proof by Cases

Existence Proofs

Uniqueness Proofs

Exhaustive Proof and Proof by Cases

An exhaustive proof shows that a statement is true by considering and demonstrating all possible cases.

It leaves no room for uncertainty, as it covers every situation or condition.

It's like checking all the boxes to make sure nothing is left out.

Proof by cases breaks down a statement into separate cases or situations.

It shows that if the statement is true for each individual case, it is true overall.

Existence Proof

An existence proof demonstrates that something satisfying a particular condition or property does indeed exist. It confirms that there is at least one example that meets the criteria.

Uniqueness Proof

A uniqueness proof demonstrates that there is only one thing that satisfies a certain condition or property.
It shows that no other options exist.

Source

Rosen - Discrete Mathematics and its Applications 7th edition

Codingame.com

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