Question 1 – Exoplanet Characterization

In this question, you will estimate the mass and radius of a planet from its radial velocity and transit data.

A mysterious new (and fake!) planet, GJ 8999 b, has been detected orbiting the M dwarf GJ 8999. GJ 8999 is a *very* small star, with a mass of $0.2M_\odot$ and a radius of $0.2R_\odot$. (If you haven't seen those symbols before, $M_\odot$ and $R_\odot$ are the mass and radius of the Sun, respectively.)

The cunning astronomer you are, you have been measuring transit and radial velocity data of this star to figure out the planet's mass and radius of this planet, so you can publish a paper on the system! Let's characterize this planet now.

a) What is the inclination of GJ 8999 b?

Tr:
Gezegenin **transit yaptığı** (yani yıldızın önünden geçtiği) gözlemlenmiş.

Transit gözlemi yapılabiliyorsa, gezegenin yörüngesi gözlemcinin bakış doğrultusuna **neredeyse dik** olmalıdır.
Bu da yörünge eğikliğinin yaklaşık **90 derece** olduğunu gösterir.

En:
The planet has been observed to transit (i.e., pass in front of its star).
If a transit is observed, the planet's orbit must be nearly edge-on from our point of view.
This means the orbital inclination is approximately **90 degrees**.

b) New transit data from the Transiting Exoplanet Survey Satellite (TESS) has come in, and it very much looks like we have some exoplanet transits! A plot of the flux from the full 28-day observation period of TESS is shown here, as well as a plot that is zoomed into a single transit.

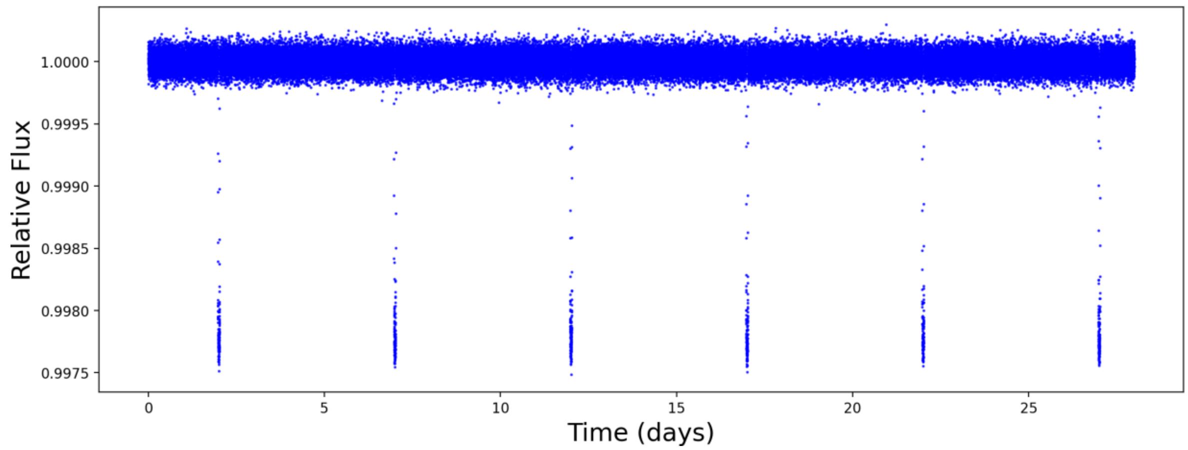

Figure 1: A plot of the flux of GJ 8999 over time over a 28-day period.

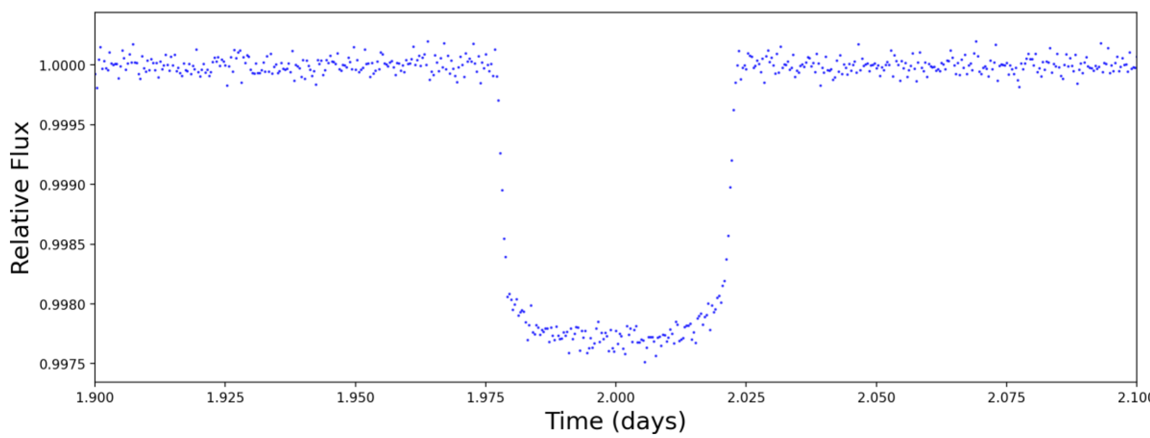

Figure 2: A plot of the flux of GJ 8999 over time, zoomed into a single exoplanet transit.

What is the period of this exoplanet?

Tr:

Yıldızın parlaklığı genellikle 1.000 civarında seyrediyor. Grafiklerde yıldızın önünden gezegen geçtiğinde parlaklık ani şekilde düşüyor. Bu düşüşler **2.5, 7.5 ve 12.5** gün civarında görülmüş. Transitler arasındaki zaman farkı, yani gezegenin yörünge süresi, yaklaşık olarak:

P = 7.5 – 2.5 = 5 gün

Böylece GJ 8999 b'nin yörünge periyodu yaklaşık 5 gün olarak bulunuyor.

En:

The star's brightness stays around 1.000 normally. When the planet passes in front, the brightness drops sharply. These dips appear near **2.5, 7.5, and 12.5 days**. The time between two transits gives the orbital period, so:

P = 7.5 – 2.5 = 5 days

Therefore, the orbital period of GJ 8999 b is about 5 days.

c) What is the radius of this planet?

Tr:

Yıldızın parlaklığı transit öncesinde 1.0000, transit sırasında 0.9975'e düşüyor.
Buna göre transit derinliği:

$$\Delta = 1.0000 - 0.9975 = 0.0025$$

Transit derinliği, gezegen ile yıldız yarıçapının karesine eşittir:

$$(R_p / R_\bigstar)^2 = 0.0025 \Rightarrow R_p / R_\bigstar = \sqrt{0.0025} = 0.05$$

Yıldız yarıçapı $R_\bigstar = 0.2\ R_\odot$ olduğuna göre, gezegen yarıçapı:

$$R_p = 0.05 \times 0.2\ R_\odot = 0.01\ R_\odot$$

Güneş yarıçapı yaklaşık 109 Dünya yarıçapına eşit olduğundan,

$$R_p = 0.01 \times 109 = 1.1\ R_\oplus$$

Yani gezegenin yarıçapı yaklaşık **1.1 Dünya yarıçapı** kadardır.

En:

The star's brightness drops from 1.0000 before transit to 0.9975 during transit.
So, the transit depth is:

$$\Delta = 1.0000 - 0.9975 = 0.0025$$

This equals the square of the planet-to-star radius ratio:

$$(R_p / R_\bigstar)^2 = 0.0025 \Rightarrow R_p / R_\bigstar = \sqrt{0.0025} = 0.05$$

Given the star's radius is $0.2\ R_\odot$, the planet's radius is:

$$R_p = 0.05 \times 0.2\ R_\odot = 0.01\ R_\odot$$

Since the Sun's radius is about 109 times Earth's radius,

$$R_p = 0.01 \times 109 = 1.1\ R_\oplus$$

So, the planet's radius is about **1.1 times Earth's radius**.

d) Luckily for us, we have gotten some radial velocity data to figure out this planet's mass, too. This data, taken over a period of 30 days, measures the star's Doppler shift as it moves back and forth due to the planet's gravity.

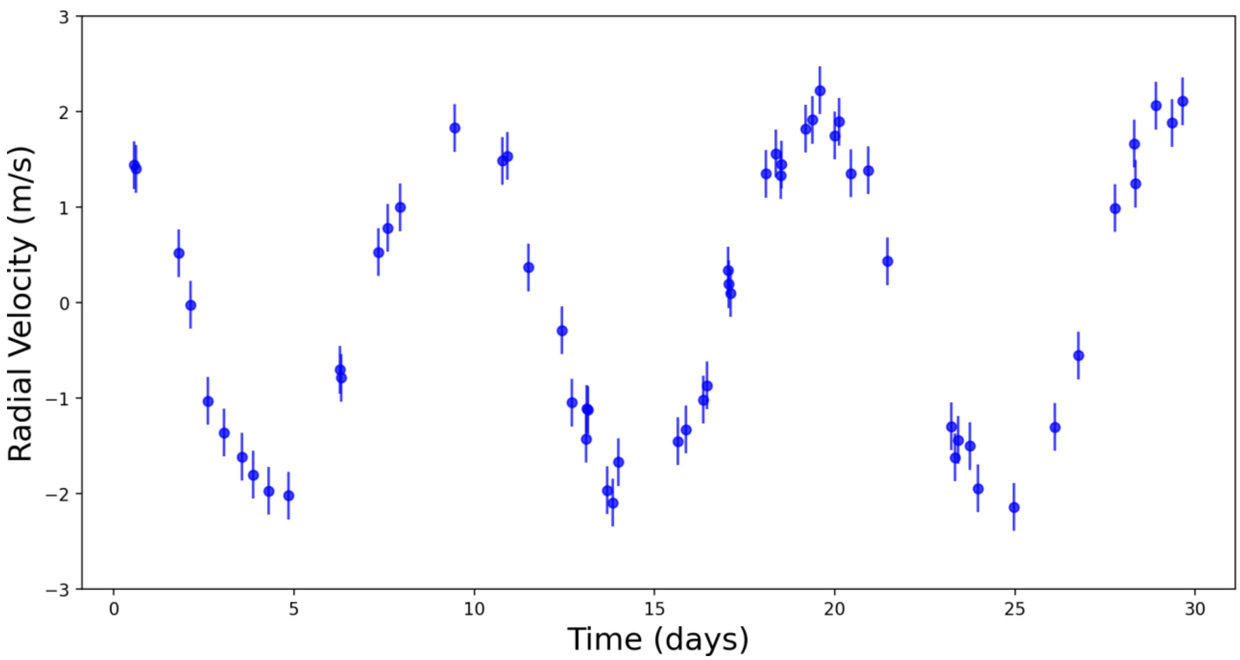

Figure 3: A plot of the radial velocity of GJ 8999 over time.

What is the semi-amplitude K of this planetary signal?

Tr:

Radial velocity grafiğinde yıldızın hızındaki değişim gezegenin etkisiyle oluyor.
Grafikte hız en yüksek **+2.3 m/s**, en düşük **−2.3 m/s.**
Yarı-amplitüd KKK, bu iki hız farkının yarısıdır:

K = (2.3 − (−2.3)) ÷ 2 = 4.6 ÷ 2 = 2.3 m/s

Yani **KKK yaklaşık 2.3 metre/saniye**.

En:

On the radial velocity graph, the star's speed changes due to the planet's pull.
The highest speed is about +2.3 m/s, the lowest is −2.3 m/s.
The semi-amplitude KKK is half the difference between these:

K = (2.3 − (−2.3)) ÷ 2 = 4.6 ÷ 2 = 2.3 m/s

So, **KKK is about 2.3 meters per second**

e) What is the mass of this planet?

Tr:

Gezegenin kütlesini (Mp) hesaplamak için radial velocity denklemini kullanıyoruz:

$K = (Mp \times \sin(i)) \times [(2\pi G) / (P \times M^{*2})]^{(1/3)}$

Buradan Mp şöyle bulunur:

$Mp = K \div [\sin(i) \times ((2\pi G) / (P \times M^{*2}))^{(1/3)}]$

Verilen değerler:

$K = 2.3$ m/s
$P = 5$ gün $= 432{,}000$ saniye
$M^* = 0.2 \times 1.989 \times 10^{30}$ kg
$i = 90°$, bu yüzden $\sin(i) = 1$
$G = 6.67430 \times 10^{-11}\ m^3/kg/s^2$

Gezegen kütlesi (Mp) için formül:

$Mp = (K \times M^{*(2/3)} \times P^{(1/3)}) \div ((2\pi G)^{(1/3)} \times \sin(i))$

1. $M^{*(2/3)}$ yaklaşık 5.42×10^{19}
2. $P^{(1/3)}$ yaklaşık 75.6
3. $(2\pi G)^{(1/3)}$ yaklaşık 7.49×10^{-4}

$Mp = (2.3 \times 5.42 \times 10^{19} \times 75.6) \div (7.49 \times 10^{-4}) \approx 1.26 \times 10^{25}$ kg

Dünya kütlesi yaklaşık 5.972×10^{24} kg olduğundan, gezegenin kütlesi Dünya'nın yaklaşık **2.1 katıdır.**

En:

To calculate the planet’s mass (Mp), we use the radial velocity equation:

$K = (Mp \times \sin(i)) \times [(2\pi G) / (P \times M^{*2})]^{(1/3)}$

Solving for Mp:

$Mp = K \div [\sin(i) \times ((2\pi G) / (P \times M^{*2}))^{(1/3)}]$

Given values:

$K = 2.3$ m/s
$P = 5$ days $= 432{,}000$ seconds
$M^* = 0.2 \times 1.989 \times 10^{30}$ kg
$i = 90°$, so $\sin(i) = 1$
$G = 6.67430 \times 10^{-11}\ m^3/kg/s^2$

Formula for Mp:

$Mp = (K \times M^{*(2/3)} \times P^{(1/3)}) \div ((2\pi G)^{(1/3)} \times \sin(i))$

1. $M_*^{(2/3)} \approx 5.42 \times 10^{19}$
2. $P^{(1/3)} \approx 75.6$
3. $(2\pi G)^{(1/3)} \approx 7.49 \times 10^{-4}$

$M_p = (2.3 \times 5.42 \times 10^{19} \times 75.6) \div (7.49 \times 10^{-4}) \approx 1.26 \times 10^{25}$ kg

Earth's mass is about 5.972×10^{24} kg, so the planet's mass is roughly **2.1 times Earth's mass**.

f) So, now that we've found the mass and radius of our planet, let's try to figure out what it's made of!

The following plot shows (very rough) 'mass-radius curves' of rocky exoplanets of different compositions. A planet lying on a given curve has a mass and radius consistent with being made of the corresponding composition.

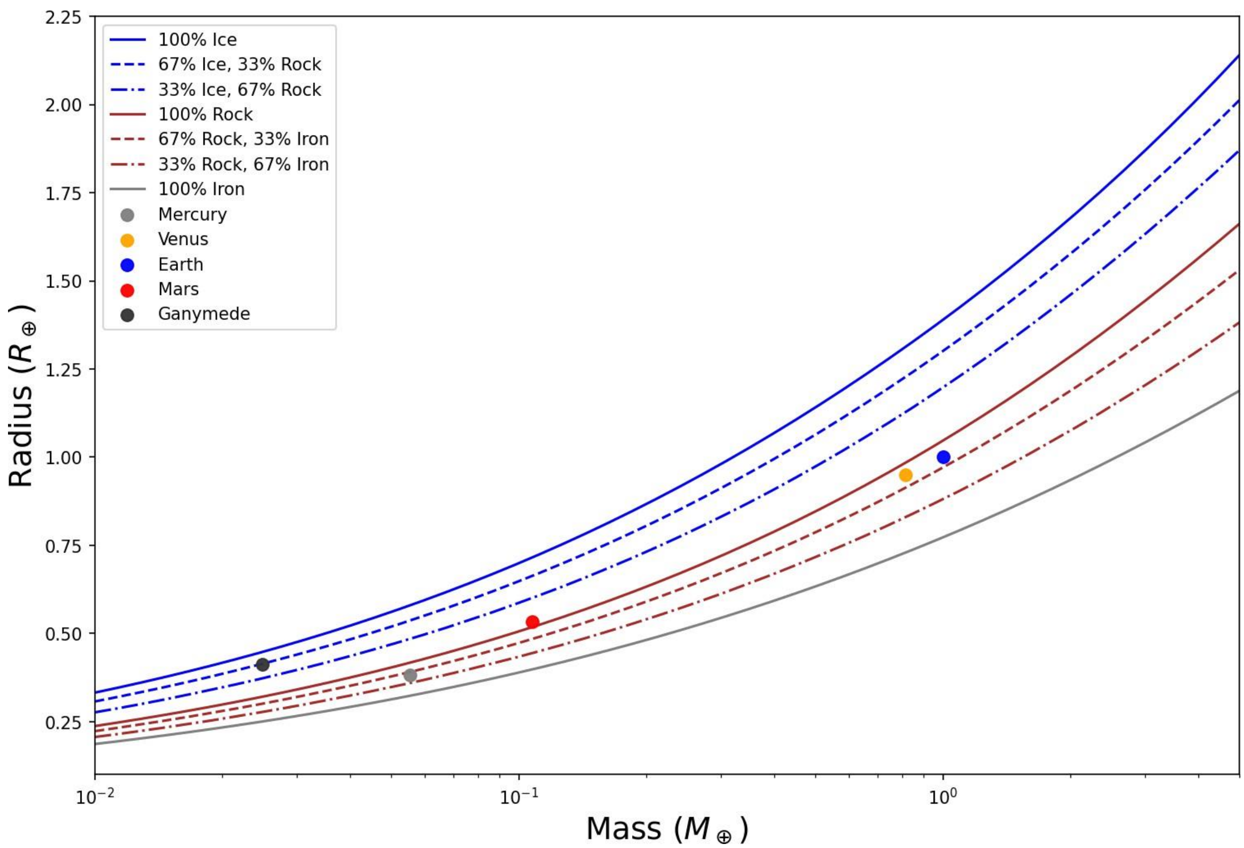

Figure 4: A plot showing the mass-radius curves for different exoplanet compositions.

The five rocky planets (plus Ganymede) are all shown on the plot as well. For example, Earth lies very near the '67% rock, 33% iron' curve, and Earth's composition IS indeed about 67% rock and 33% iron.

With this in mind, what is the composition of GJ 8999 b?

Tr:

Bu gezegenin kütle-yarıçap grafiğindeki yeri, onun ne tür bir yapıya sahip olduğunu anlamamıza yardımcı oluyor. Yaklaşık 2.1 Dünya kütlesi ve 1.1 Dünya yarıçapı ile GJ 8999 b, kayalık gezegenlere çok benziyor.

Bildiğimiz gibi, 1 Güneş kütlesi yaklaşık 333,000 Dünya kütlesine eşit. Buna göre bu gezegenin kütlesi yaklaşık 6.3×10^{-6} Güneş kütlesi.

Bu durum bize gezegenin çoğunlukla kayadan oluştuğunu, belki az miktarda buz da içerdiğini gösteriyor. Çünkü bu kadar yoğun bir gezegen buz ağırlıklı olamaz, ama demir açısından da çok zengin değil.

Yani kısaca, GJ 8999 b muhtemelen bizim Güneş Sistemi'ndeki Dünya gibi kayalık bir gezegen.

En:

The planet's position on the mass-radius diagram helps us understand what it might be made of. With about 2.1 Earth masses and 1.1 Earth radii, GJ 8999 b looks a lot like a rocky planet.

As we know, 1 solar mass is roughly equal to 333,000 Earth masses. So, this planet's mass is about 6.3×10^{-6} solar masses.

This tells us the planet is mostly made of rock, maybe with a little ice. It's too dense to be mostly ice, but not dense enough to be very iron-rich.

In short, GJ 8999 b is probably a rocky planet, similar to Earth and other terrestrial planets in our Solar System.

Detailed Explanations for: Exoplanet Detection Methods

Question 1a: Inclination of GJ 8999 b

If a planet transits its star from our point of view, this means its orbital inclination must be nearly 90°.
A transit can only be observed when the planet passes directly in front of the star, partially blocking its light.
This occurs when the orbital plane is almost edge-on relative to the observer.

Hence, we conclude:
Answer: Inclination $i \approx 90°$

Question 1b: Orbital Period

From the TESS light curve, transit dips are observed at days 2.5, 7.5, and 12.5. These dips in brightness indicate that the planet passes in front of its host star at regular intervals.

The orbital period P is calculated using the time between two successive transits:
P = 7.5 days – 2.5 days = 5 days

Answer: Orbital Period ≈ 5 days

Question 1c: Radius of the Planet

The depth of the transit (Δ) indicates how much light is blocked by the planet and is given by:
$\Delta = (R_p / R_\star)^2$

From the graph, the flux drops from 1.0000 to 0.9975:
$\Delta = 1.0000 - 0.9975 = 0.0025 \Rightarrow (R_p / R_\star)^2 = 0.0025 \Rightarrow R_p / R_\star = \sqrt{0.0025} \approx 0.05$

The star's radius $R_\star$ is 0.2 R■, so the planet's radius is:
$R_p = 0.05 \times 0.2$ R■ = 0.01 R■

Given R■ ≈ 109 $R_\oplus$, then:
$R_p = 0.01 \times 109 \approx 1.1\ R_\oplus$

Answer: Radius ≈ 1.1 Earth radii

Question 1d: Radial Velocity Semi-Amplitude (K)

The radial velocity graph shows that the star's motion varies between +2.3 m/s and –2.3 m/s due to the gravitational pull of the planet.

Semi-amplitude is half the peak-to-peak value:
$K = (2.3 - (-2.3)) / 2 = 4.6 / 2 = 2.3$ m/s

Answer: Semi-amplitude K ≈ 2.3 m/s

Question 1e: Mass of the Planet

Using the radial velocity formula:
$K = (M_p \times \sin(i)) \times [(2\pi G) / (P \times M^{*2})]^{(1/3)}$

Solving for Mp:
$M_p = (K \times M^{*(2/3)} \times P^{(1/3)}) / ((2\pi G)^{(1/3)} \times \sin(i))$

Given:
K = 2.3 m/s
P = 5 days = 432,000 s
$M^* = 0.2 \times 1.989 \times 10^{3}$■ kg = 3.978×10^{2}■ kg
i = 90°, so sin(i) = 1