Unit 2. Non-LOS Propagation and Link Budget Analysis

EL-GY 6023: WIRELESS COMMUNICATIONS

PROF. SUNDEEP RANGAN

Learning Objectives

- Perform simple noise and interference calculations
- Define key communication requirements
  - BER, BLER, information rate, spectral efficiency, bandwidth
  - SNR: Energy per bit and energy per symbol
- Estimate rate based on simple models or from link curves
- Perform simple link budget calculations
- Qualitatively describe various propagation mechanisms in real world settings
- Compute reflected power from the radar equation
- Generate samples from a statistical path loss model
- Compute rate and SNR distributions using a statistical path loss model

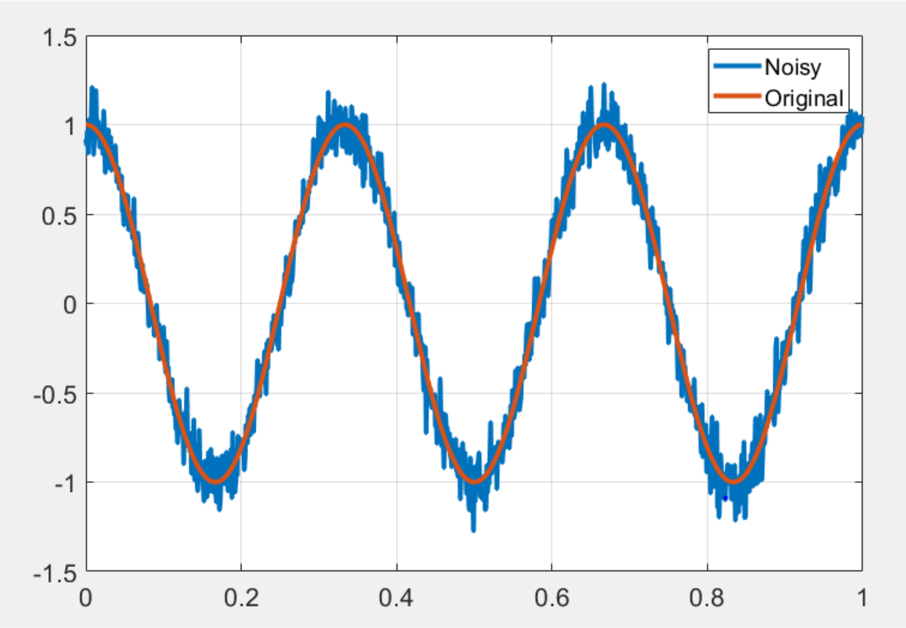
Outline

- Noise, Interference and SNR
- Communication Requirements and Link Budget Analysis
- Non-LOS Propagation
- Statistical Models for Path Loss
- Demo: Estimating Rates with a 3GPP model

What is Noise?

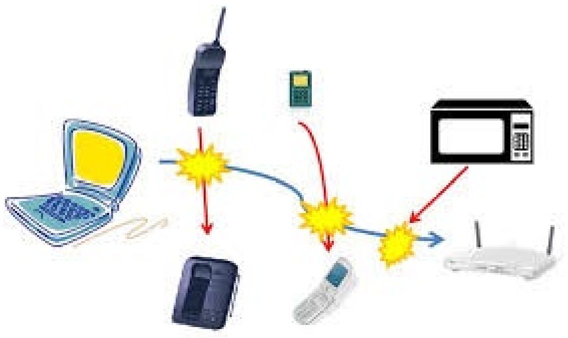
- Noise: Any unwanted component of the signal

- Key challenge in communication:
  - Estimate the transmitted signal in the presence of noise



Types of “Noise”

- Internal / thermal noise:
  - From imperfections in the receiver
  - Thermal noise: From random fluctuations of electrons
- Distortions:
  - Phase noise, quantization, channel estimation errors
  - Not really noise, but sometimes modeled as noise
- External Interference
  - Signals from other sources
  - In-band: Transmitters in the same frequency
    Ex: Multiple devices in a cellular band
  - Out-of-band: From leakage out of carrier
  - Some texts do not consider “interference” as noise



AWGN Noise Model

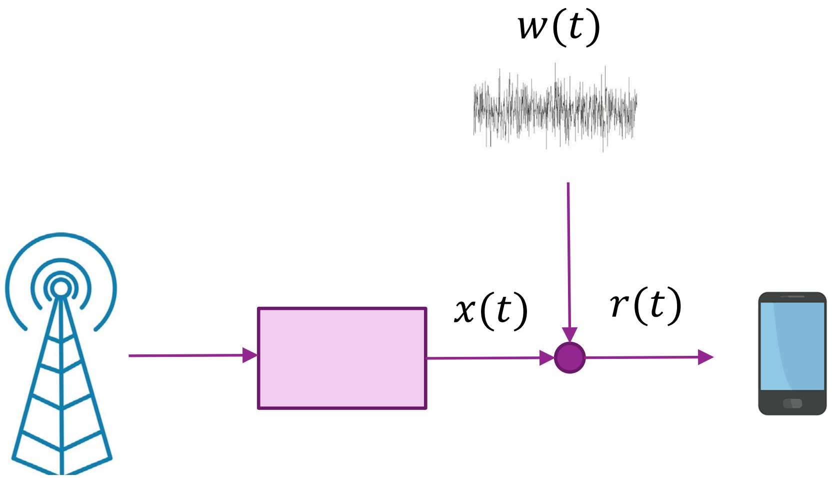
- Thermal noise is often modeled as Additive White Gaussian:

$$r(t) = x(t) + w(t)$$

  - $x(t)$: Desired component
  - $w(t)$: Noise
  - $r(t)$: Total received signal

- In real passband:
  - $w(t)$ is a real Gaussian process with PSD $\frac{N_0}{2}$

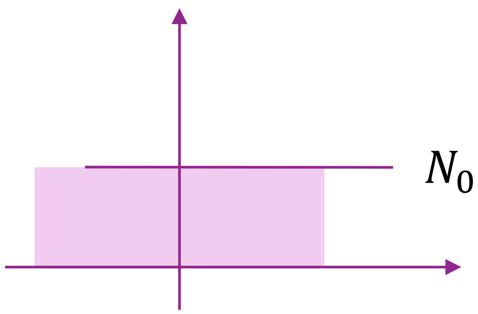
- In complex baseband:
  - $w(t)$ is a complex Gaussian process with PSD N_0



Thermal Noise Units

- Units for noise PSD: N_0 = W/Hz (in linear scale)

- Also, $N_0 = \text{Joules}$
  - Energy per degree of freedom
  - Equivalently, energy in any orthogonal sample



- Often written in dB scale: $N_0 = 10\log_{10}\left(\frac{N_0}{1\ \mathrm{mJ}}\right)$ [dBm/Hz]

Limits and Noise Figure

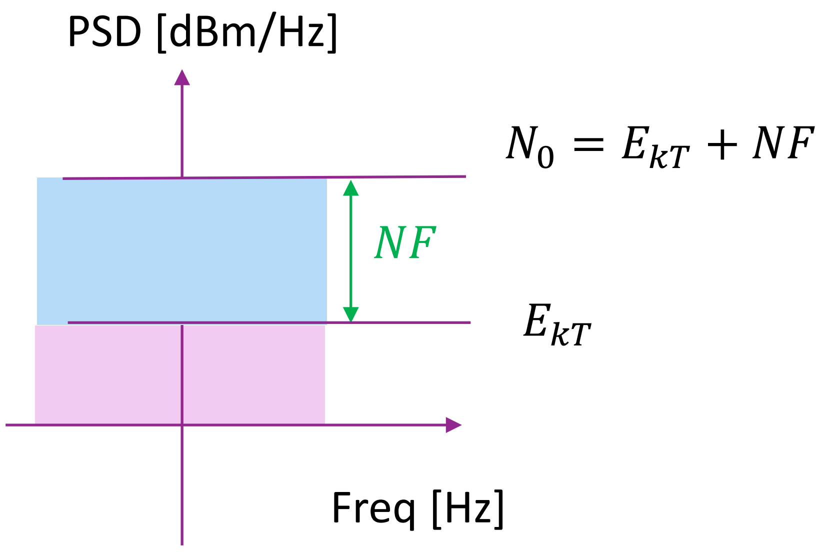
- Fundamental limit determined by statistical physics: $N_0 = kT$
  - k = Boltzman constant,
  - T = Temperature in Kelvin
  - At room temperature (T= 290K):

$$E_{kT} = 10\log_{10}(kT) = -174 \ \ \text{dBm/Hz}$$

- Practical systems see higher noise power

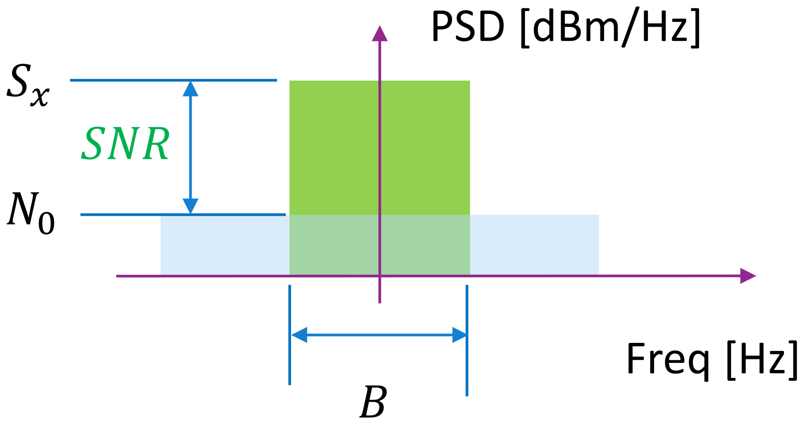
$$N_0 = 10\log_{10}(kT) + NF \ \ (\text{dBm/Hz})$$

  - Receiver imperfections
  - NF = Noise figure
  - Typical values are 2 to 9 dB in most wireless systems



Signal To Noise Ratio

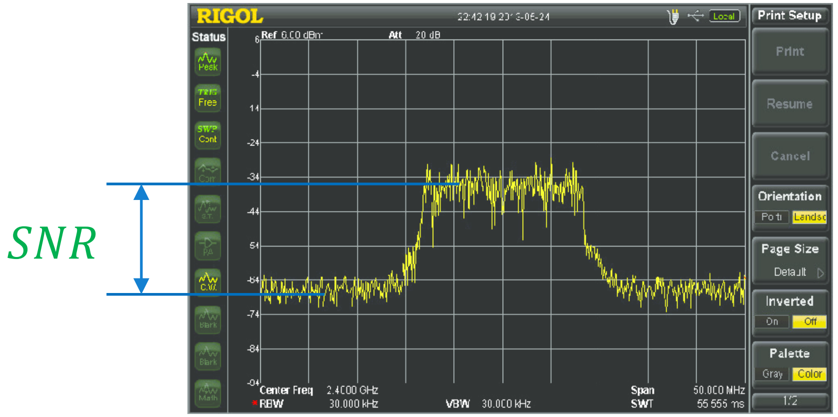
- Suppose: $r(t) = x(t) + w(t)$
  - $x(t)$ occupies bandwidth B with total RX power P_{rx}
  - Hence, PSD of $x(t) = S_x = \frac{P_{rx}}{B}$
  - $w(t)$ has PSD N_0



- Signal-to-Noise Ratio:

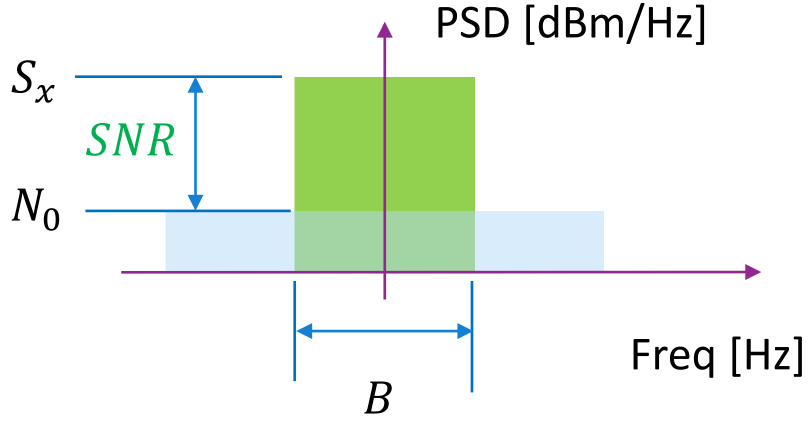
$$\boxed{SNR = \frac{S_x}{N_0} = \frac{P_{rx}}{BN_0}}$$

- Key performance metric:
  - Determines possible spectral efficiency
- Typically quoted in dB:
  - $SNR\,[\text{dB}] = P_{rx}[\text{dBm}] - N_0[\text{dBm/Hz}] - 10\log_{10}(B[\text{Hz}])$



Example: SNR Calculation

- Suppose that:
  - RX power = -80 dBm
  - Noise figure = 6 dB
  - Bandwidth = 20 MHz
- What is the SNR?
- Solution:
  - Thermal noise: $N_0 = E_{kT} + NF$
  - $$\begin{aligned} SNR &= P_{rx} - N_0 - 10\log_{10}(B) \\ &= P_{rx} - E_{kT} - NF - 10\log_{10}(B) \\ &= -80 - (-174) - 6 - 10\log_{10}(20(10)^6) \\ &= \text{15.0 dB} \end{aligned}$$



Sampled Data Systems

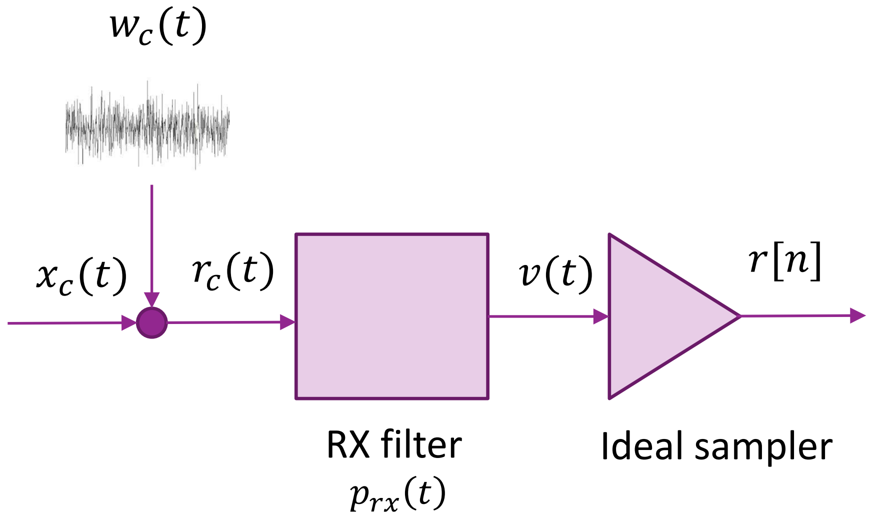
- Most receiver process sampled signals
- RX continuous-time complex baseband signal:

$$r_c(t) = x_c(t) + w_c(t)$$

- Assume typical digitally sampling model:
  - Filtering: $v(t) = p_{rx}(t) * r_c(t)$
  - Ideal sampler: $r[n] = v(nT)$
- Then, in discrete-time:

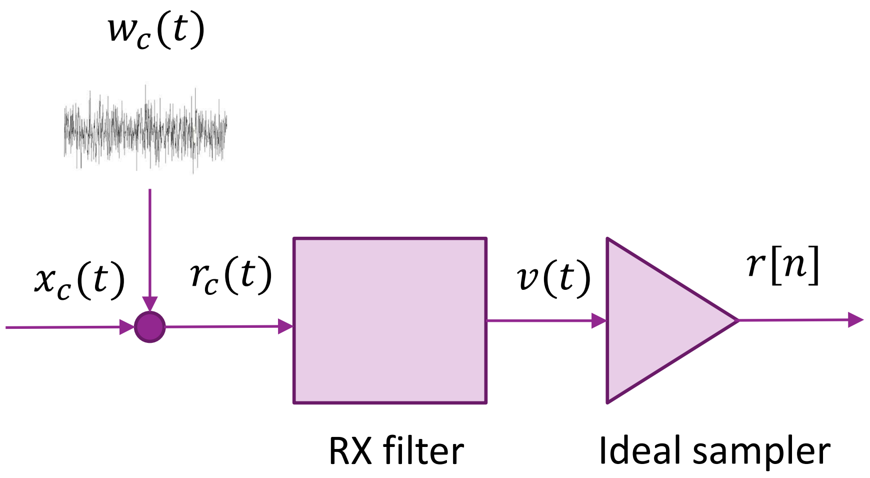
$$r[n] = x[n] + w[n]$$

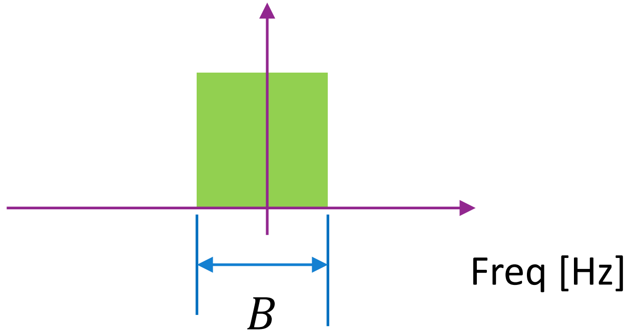
  - Signal component: $x[n]$ = sampled from $x_c(t)$
  - Noise component: $w[n]$ = sampled from $w_c(t)$



SNR in Sampled Data Systems

- Continuous-time: $r_c(t) = x_c(t) + w_c(t)$
- Assume an ideal low-pass filter:
  - $p(t) = \frac{1}{\sqrt{T}} sinc\left(\frac{t}{T}\right)$
  - Bandwidth $B = \frac{1}{T}$
- If $r_c(t)$ is band-limited to $|f| \le \frac{B}{2}$ then:
  - Energy per sample is $E_s = P_{rx}T$
- If $w_c(t)$ is AWGN with PSD N_0 then:
  - $w[n] \sim CN(0, N_0)$
  - N_0 = noise energy per sample
- Resulting SNR: $\frac{E_s}{N_0} = \frac{P_{rx}T}{N_0} = \frac{P_{rx}}{BN_0}$



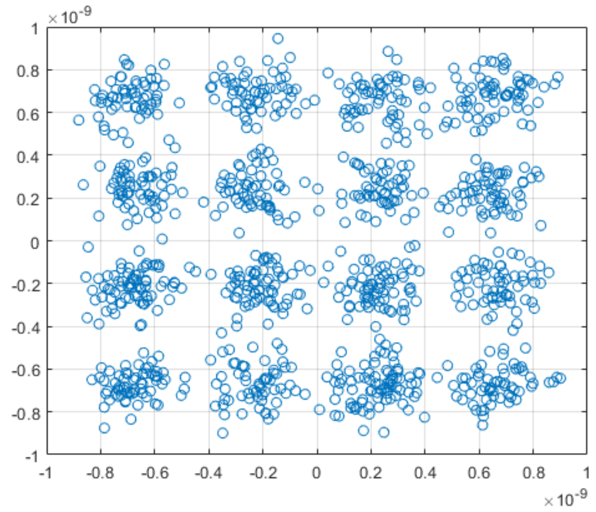


Simulating Noise in MATLAB

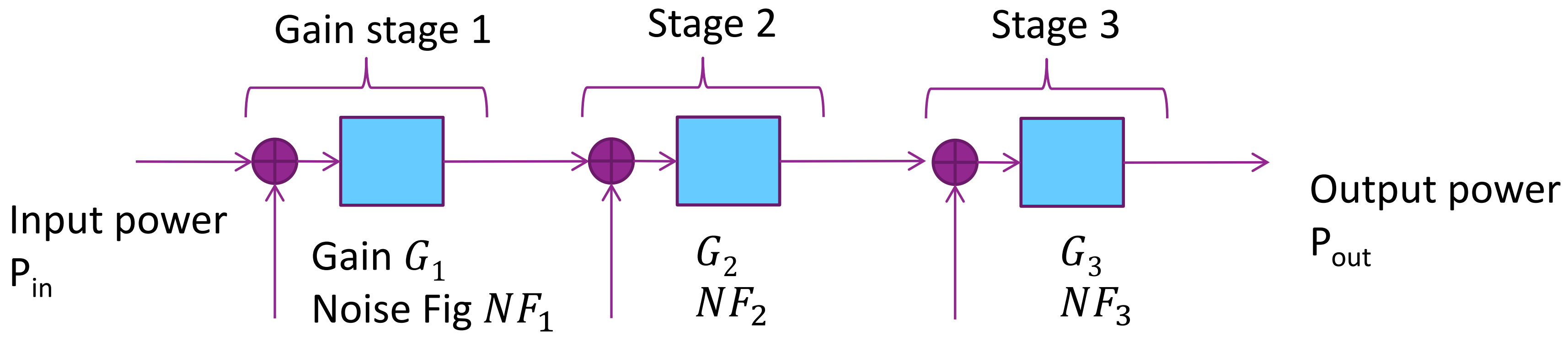
```
modRate = 4;    % num bits per symbol (4=16 QAM)
M = 2^modRate;  % QAM order
nsym = 1024;    % num symbols

% Generate data
nbits = nsym*modRate;
bits = randi([0,1],nbits,1);
sym = qammod(bits,M,'InputType','bit','UnitAveragePower',true);
```

```
% Energy per sample in dBJ.
% Note we have to subtract 30 since MATLAB uses W not mW
Es = Prx - 30 - 10*log10(fsamp);

% Rescale signal
s = 10.^(0.05*Es)*sym / sqrt(mean(abs(sym).^2));

% Create a thermal noise object
tn = comm.ThermalNoise('NoiseMethod', 'Noise figure', ...
    'NoiseFigure', NF, 'Add290KAntennaNoise', true);

% Add the noise
r = tn.step(s);

% Plot the constellation
plot(real(r), imag(r), 'o');
grid();
```

- Simulating noise is easy in MATLAB
- Example in demo:
  - Generate 16-QAM symbols
  - Scale for the RX power
  - Add thermal noise

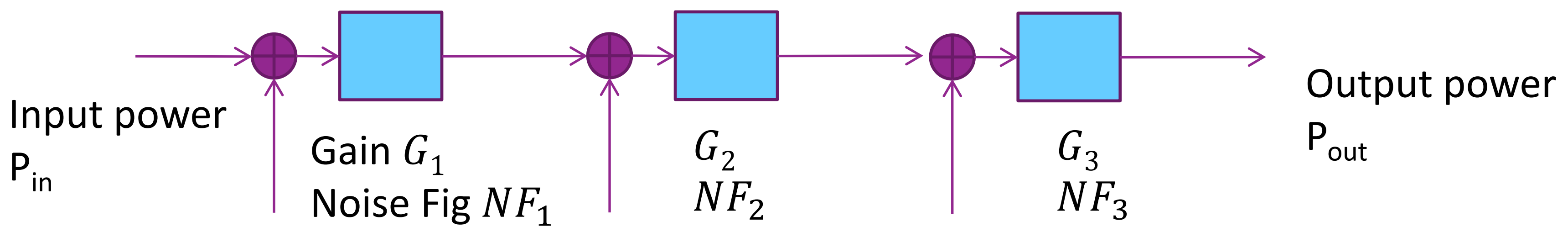


Cascade of Elements



- Most receivers are built with multiple stages
  - Ex: LNA, Mixer, …
- Each stage has a gain and noise figure
- Some stages (typically amplifiers) add noise with a noise figure

NF for Cascade of Elements



- Total gain and noise figure:

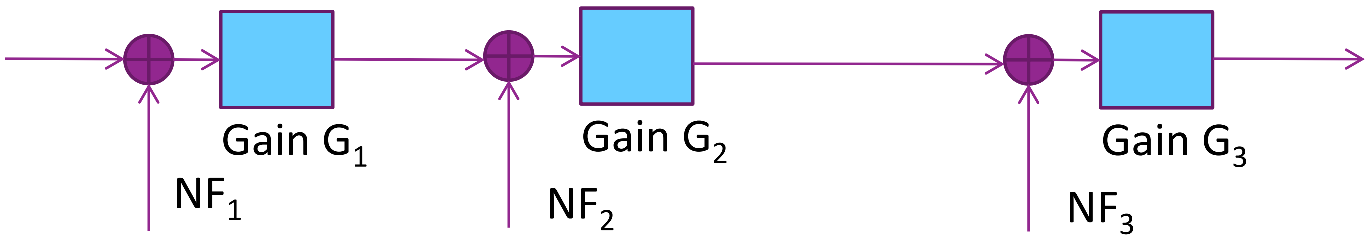
$$G_{tot} = G_1 G_2 G_3, \qquad NF_{tot} = NF_1 + \frac{NF_2 - 1}{G_1} + \frac{NF_3 - 1}{G_1 G_2}$$

  - NF formula arises since each stage adds $(NF_i - 1)kT$ noise

- Consequence: Most designs start with a low noise amplifier (LNA)
  - Has low noise figure NF_1 and high gain G_1
  - Suppresses noise figure of later stages $NF_2, NF_3, \ldots$

Example Problem

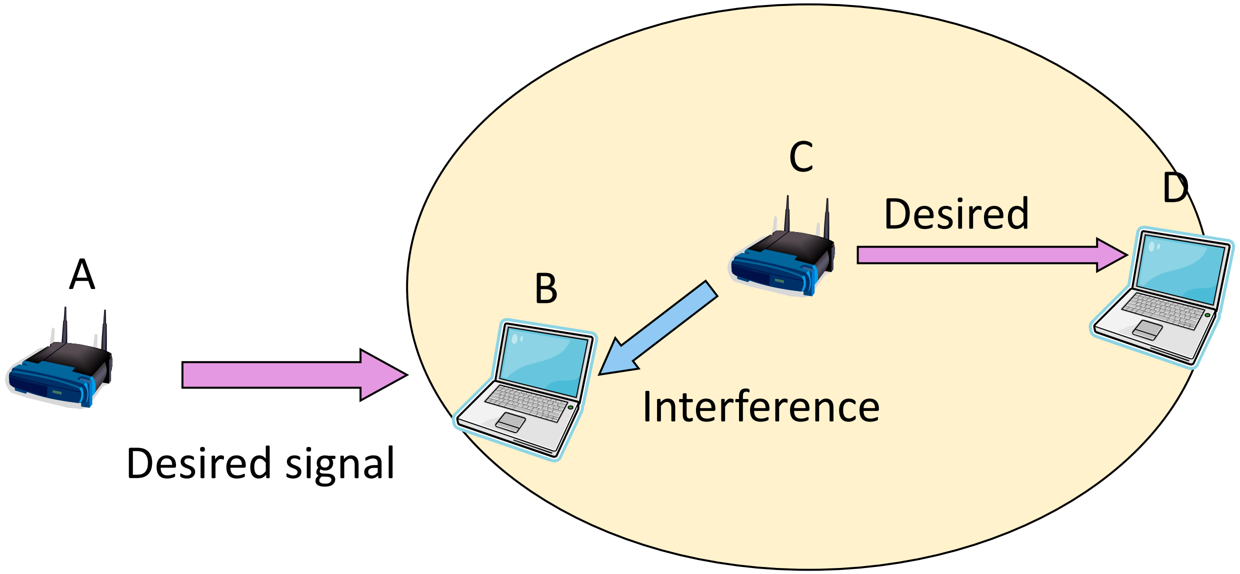
Molisch 3.1



- What is total NF and gain?
- Answer: Compute G and NF (in linear units) for each stage:
  - Attenuator: $NF_1 = 1$, [Does not add noise], $G_1 = 10^{-0.1(1.5)} = 0.707$ [Note sign]
  - LNA: $NF_2 = 10^{0.1(4)} = 2.51$, $G_2 = 10^{0.1(10)} = 10$
  - Mixer: $NF_3 = 10^{0.1(1)} = 1.25$, $G_3 = 1$ (unit gain)
  - Total gain: $G = -1.5 + 10 + 0 = 8.5$ dB
  - Noise figure $NF_{tot} = NF_1 + \frac{NF_2 - 1}{G_1} + \frac{NF_3 - 1}{G_1 G_2} = 1 + \frac{2.51 - 1}{0.707} + \frac{1.25 - 1}{(0.707)(10)} = 3.15 \approx 5.0$ dB

Interference

- Signals from other transmitters using same band and same time.
- Fundamental to the broadcast nature of the wireless medium.
- Adds to total noise seen at receiver
- Example:
  - A transmits to B (desired signal)
  - C transmits to D (desired signal)
  - But B gets interference from A



Interference Calculations

- Remember: Noise and interference powers add in linear scale (not in dB)!
- Example:
  - NF = 4 dB, B = 20 MHz
  - Interference power = -95 dBm, RX signal power = -80 dBm
  - Find the SNR=signal to noise and SINR = signal to interference + noise
- Solution:
  - Thermal noise density $N_0 = -174 + 4 = -170$ dBm/Hz
  - Noise power $P_N = N_0 B$. In dBm: $P_N = -170 + 10\log_{10}(2(10)^7) = -97$ dBm
  - SNR = -80 – (-97) = 17 dB
  - Noise + interference power, $P_{NI} = 10^{-9.7} + 10^{-9.5} = 5.2(10)^{-9} = -92.9$ dBm
  - Note: You add in linear scale first before converting to dB!!
  - SINR = -80 – (-92.9) = 12.9 dB

In-Class Problem

- Cascade of two elements
  - LNA
  - Mixer
- Simulation:
  - Transmit symbols
  - Add noise in each stage
  - Measure resulting SNR

Problem 1: Adding Noise and Measuring SNR

In this problem, we will simulate a simple cascade of two receiver elements:

- An LNA with gain `GLna` and noise figure `NFLna`
- A mixer with unity gain and noise figure `NFmix`

The received signal has power `Prx` and bandwidth `B`.

```
Prx = -80;      % RX power in dBm
NFLna = 6;      % Noise figure in the LNA in dB
GLna = 15;      % LNA gain in dB
NFmix = 13;     % mixer noise figure
B = 18e6;       % Bandwidth in Hz
```

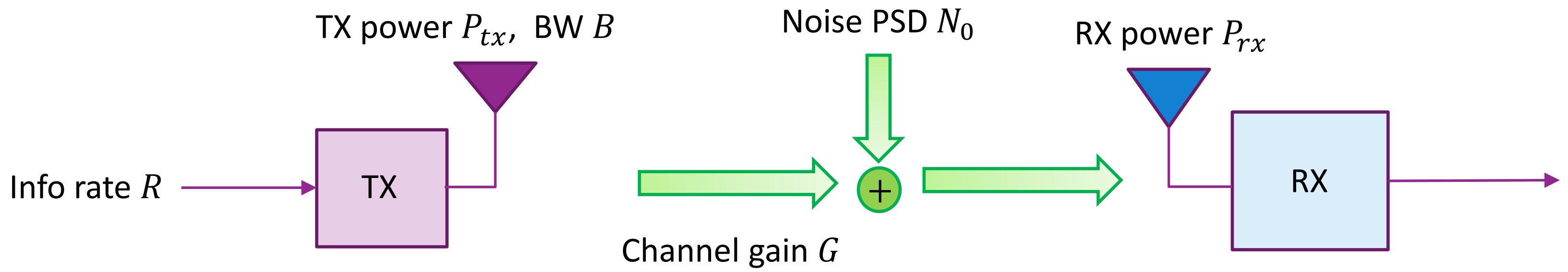
Compute and print the effective noise figure.

```
% TODO
%    NFeff = ...
```

Outline

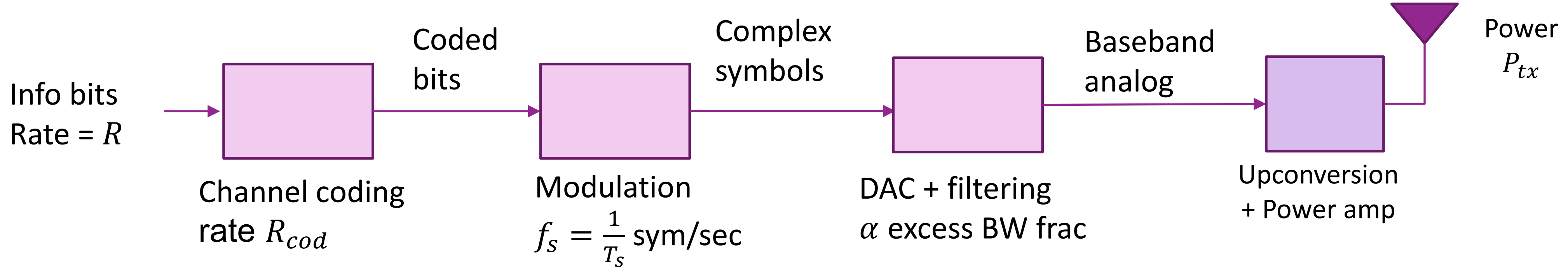
- ❑Noise and Interference
- ❑Communication Requirements and Link Budget Analysis
- ❑Non-LOS Propagation
- ❑Statistical Models for Path Loss
- ❑Demo: Estimating Rates with a 3GPP model

Communication Requirements



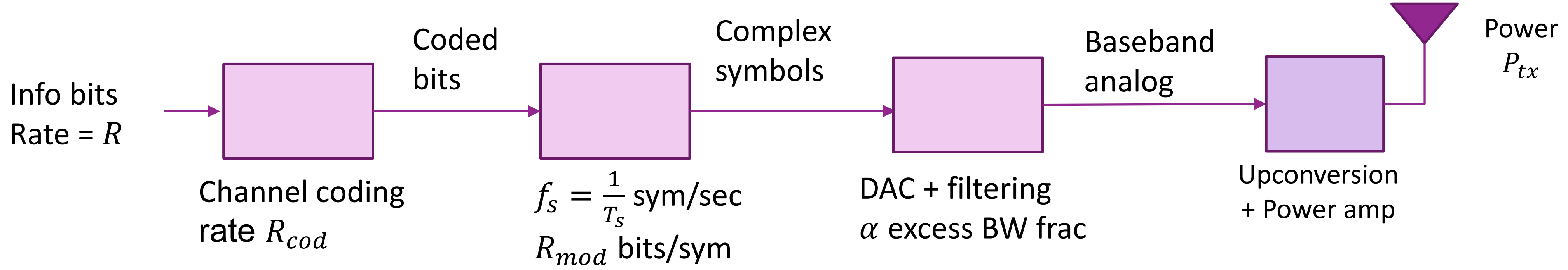
- Basic tradeoff in all communication systems:
  - Information rate: Amount of information we try to send
  - Bandwidth: Spectrum the signal occupies
  - Reliability: The probability it is received correctly
  - Channel quality: Typically measured by the signal to noise ratio

Review: Typical Transmitter Steps



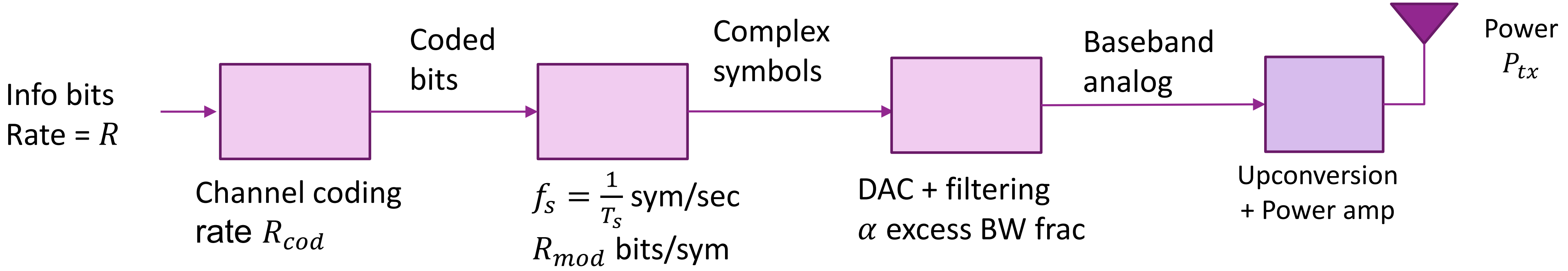
- Communication systems typically use four stages
  - Channel coding: ex. Convolutional, turbo, LDPC
  - Modulation: QPSK, 16-QAM, 64-QAM, …
  - Pulse shaping or transmit filtering
  - Upconversion
- Determines modulation + coding scheme (MCS)

Information Rate and Bandwidth



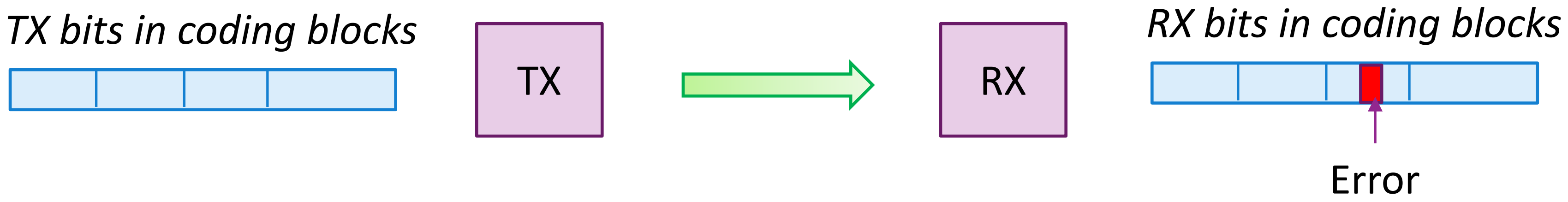
- Information rate: $R = R_{cod} R_{mod} f_s$ [bits/sec]
- Signal bandwidth: f_s [Hz]
- Occupied bandwidth: $B = (1+\alpha) f_s$ [Hz]
- Spectral efficiency $\frac{R}{B}$ (Units are bps/Hz or bits/DOF)

Example Calculation



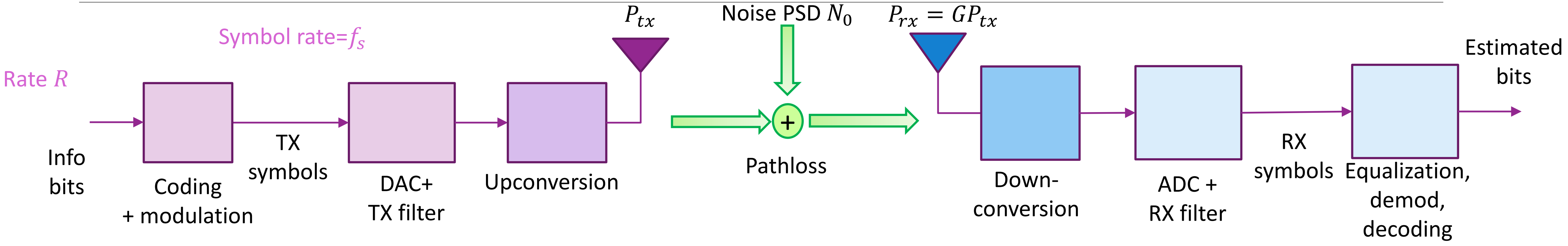
- Ex: A system transmits with rate ½ coding, 16-QAM, 10 Msym/s and excess BW 10%
  - The modulation and code rate are $R_{cod} = \frac{1}{2}$; $R_{mod} = 4$ (16-QAM)
  - Sample rate $f_s = 10$ Msym/s
  - Excess bandwidth $\alpha = 0.1$
  - Hence information rate is $R = R_{cod}R_{mod}f_s = (0.5)(4)(10) = 20$ Mbps
  - Occupied bandwidth = $B = (1+\alpha)f_s = (1.1)(10) = 11$ MHz

Reliability: BER and BLER



- Most communication system TX data into blocks
  - Framing may be used for coding, MAC layer transport blocks or IP layer
  - Range in size from 100s to > 10000 bits per blocks
- Measure reliability by either:
  - BER: Bit error rate = fraction of bits in error (useful when there is no blocks)
  - BLER: Block error rate = fraction of blocks that have at least one error
- Errors typically need to be corrected at some higher layer
  - Retransmissions / ARQ at MAC or transport layer (A lot more on this later!)
  - Corrected by application (e.g. voice masking in audio)
  - Acceptable level for BER and BLER depend on many MAC and application layer factors

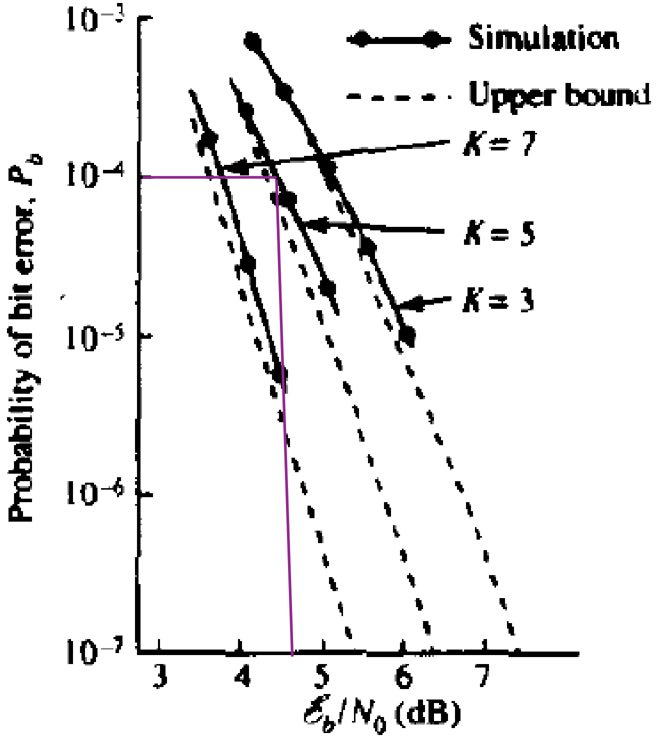
SNR and Power Relations



- Channel quality is measured by SNR = signal to noise ratio
  - SNR per symbol: $\frac{E_s}{N_0} = \frac{P_{rx}T_s}{N_0}$
  - SNR per bit $\frac{E_b}{N_0} = \frac{P_{rx}}{N_0 R}$ (pronounced "ebb-no")
- Requirement will depend on the MCS used and the quality of the receiver

Example Problem

- A system uses:
  - Convolutional code shown ($K = 5$) and $R_{cod} = \frac{1}{2}$, 16-QAM
  - Symbol rate $f_s = 100$ Msym/s. No excess bandwidth
  - Transmit power: 23 dBm
  - RX noise PSD: -170 dBm/Hz (including NF)
- For a BER of 10^{-4} find the maximum path loss
- Solution:
  - From graph we need $\gamma_b = \frac{E_b}{N_0} \approx 4.5$ dB
  - Information rate is $R = R_{cod} R_{mod} f_s = (0.5)(4)(100) = 200$ Mbps
  - $\gamma_b = \frac{E_b}{N_0} = \frac{P_{rx}}{N_0 f_s}$.
  - In dB scale: $P_{rx} = \gamma_b + 10\log_{10}(R) + N_0 = 4.5 + 3 + 80 - 170 = -82.5$ dBm
  - Hence, max path loss is $L = P_{tx} - P_{rx} = 23 - (-82.5) = 105.5$ dB



From Proakis

Shannon Capacity

- Capacity = max rate achievable given bandwidth and SNR
  - Rate optimized over all possible MCSs and communication schemes.
- Given by classic Shannon formula

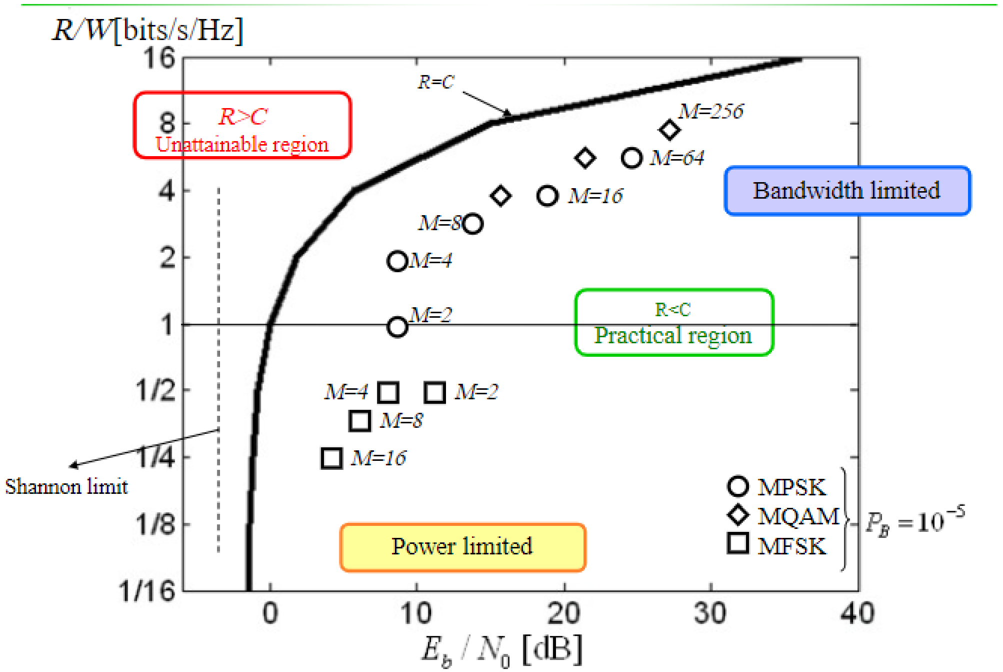
$$C = B\log_2(1+\gamma), \qquad \gamma = SNR = \frac{P_{rx}}{N_0 B}$$

- Capacity relates theoretical rate to two key parameters:
  - B = bandwidth in Hz
  - SNR in linear scale (not dB!!!)
- Max spectral efficiency $\frac{C}{B}$ = max bps/Hz
- Mathematical result from classic paper in 1948.

Bandwidth and Power-Limited Regions

- Shannon formula:

$$C = B\log_2(1+\gamma) = B\log_2\left(1+\frac{P_{rx}}{N_0 B}\right)$$



- Power-Limited region:
  - As $B \to \infty, C \to \log_2(e)\frac{P_{rx}}{N_0}$
  - Rate linearly increases with power / SNR $\frac{P_{rx}}{N_0}$
- Bandwidth-limited region:
  - For large SNR, $C \approx B\log_2(\gamma)$
  - Linearly increases in bandwidth,
  - Logarithmically increase in SNR.
  - Increasing SNR has little practical value

Actual Rate vs. Shannon Capacity

- Theoretical Shannon capacity cannot be achieved
  - Needs infinite computation and delay.
- Practical modems achieve a rate below Shannon limit.
- Useful model:

$$R = (1 - \Delta)B \min\{\log_2(1 + \beta\gamma), \rho_{max}\}$$

  - Δ=fraction overhead
  - β=loss factor, usually quoted in dB
  - Often say "system is β dB below capacity"
  - $\gamma_{\mathrm{eff}} = \beta\gamma$ can be thought of as an "effective" SNR
- Usually loss from capacity is at least 3dB
  - Often higher depending on receiver complexity and other factors

Rate vs. Capacity Example

- What is the maximum rate for a system:
  - 20% overhead, 10 MHz bandwidth, SNR=12 dB
  - Operates at 3dB below Shannon capacity
- Answer:
  - Effective SNR: $\gamma_{\text{eff}} = 12 - 3 = 9$ dB. In linear scale $\gamma_{\text{eff}} = 8$ (since 9=3(3))
  - Therefore,

$$R = (1 - \Delta)B \log_2(1 + \beta\gamma) = (0.8)(10) \log_2(1 + 8) = 25.3 \text{ Mbps}$$

  - Note the final units are in Mbps

Example Link Budget

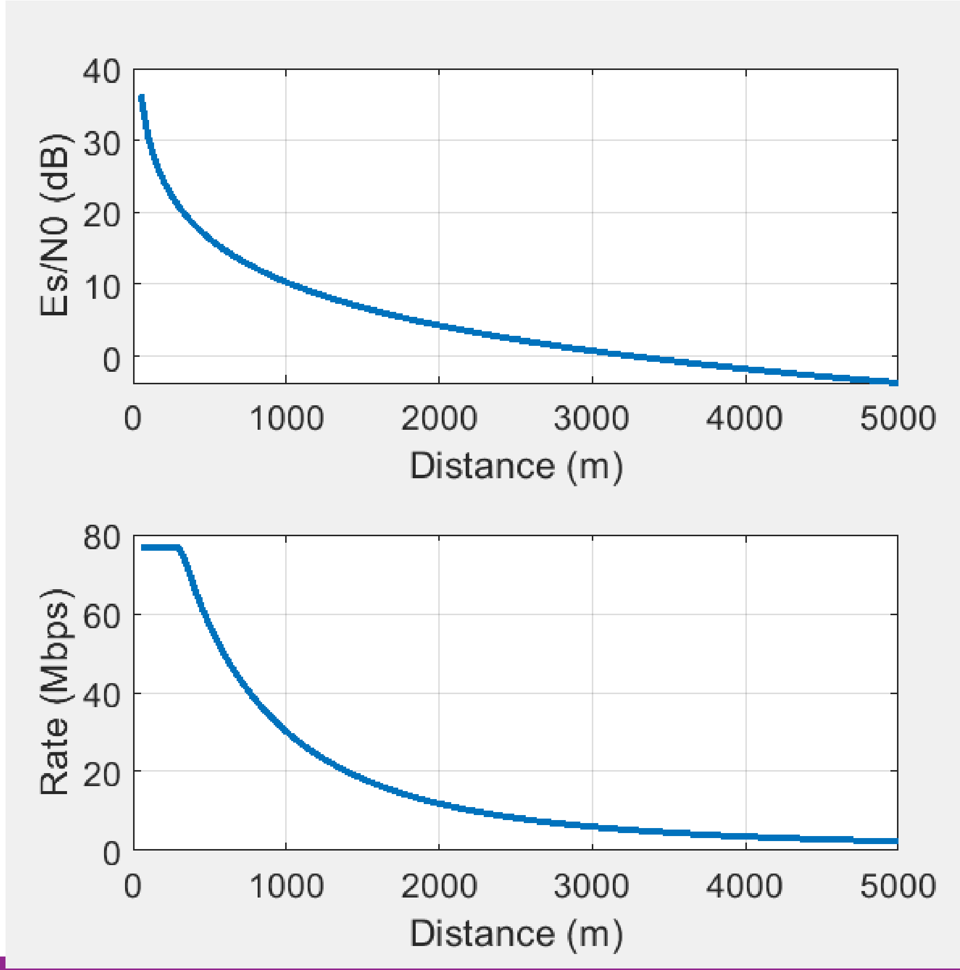
Item	Value	Remarks
Transmit power (dBm)	23.0	200 mW transmitter
Distanced based path loss (dB)	90.0	Will depend on propagation model
Shadowing (dB)	20.0	Will depend on obstructions
Receive power (dBm)	-87.0	TX power - path loss - shadowing
Bandwidth (MHz)	20.0	BW of 802.11 signal
Noise figure	5.0	Will depend of implementation of receiver
Noise power (dBm)	-96.0	-174 +10log(BW) + NF
SNR (dB)	9.0	RX pow - Noise pow

- Link budget: Measures final SNR as a function of TX power and all impairments

Example: Rate in Free Space

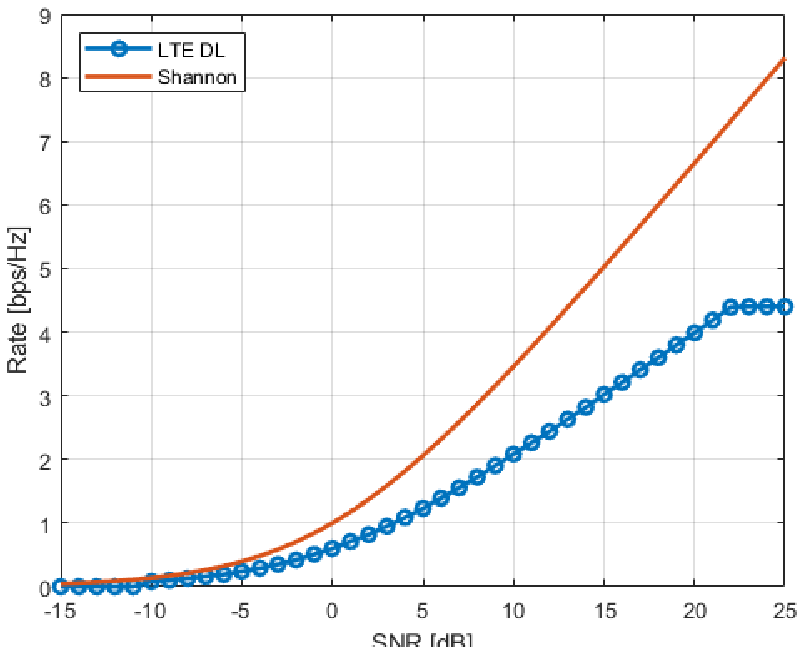
- Parameters similar to small cell transmission
- Can get data rate of 5 km!
- Since we assume free-space
  - Will be much worse in reality

```
% Parameters
B = 20e6;    % bandwidth
fc = 2.3e9;  % carrier
NF = 6;      % noise figure
snrLoss = 6; % loss from Shannon capacity
maxSE = 4.8;
bwLoss = 0.2;
Ptx = 15;  % Power in dBm
dist = linspace(50,5000,100)';  % distance

% Compute the FS path loss
vp = physconst('lightspeed');  % speed of light
lambda = vp/fc;   % wavelength
pl = fspl(dist, lambda);

% Compute SNR
kT = -174;
EsN0 = Ptx - pl - kT - NF - 10*log10(B);

% Compute rate
snrEff = 10.^(0.1*(EsN0-snrLoss));
rateMbps = B*(1-bwLoss)*min(log2(1 + snrEff), maxSE)/1e6;
```



In-Class Problem

- Fit the Rate vs. SNR for 3GPP LTE



Problem 2. Estimating the SNR Requirements for LTE

In this problem, we will see how the performance of a commercial LTE system comparse to the Shannon theory. of the system in various scenarios. You can find any of the 3GPP standards by googling, e.g. "3GPP 36.942". E SNR for the uplink and downlink.

Table A.2 Look-Up-Table of UL and DL Throughput vs. SNIR for Baseline E-UTRA Coexistence Studies

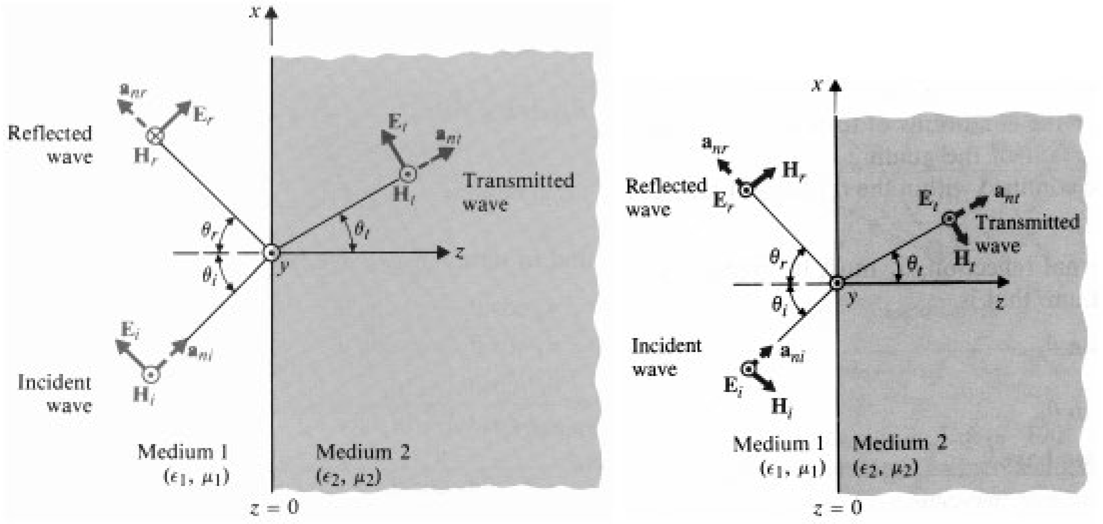
SNIR	Throughput				SNIR	Throughput			
	bps/Hz		kbps per 375kHz RB			bps/Hz		kbps per 375kHz RB	
dB	DL	UL	DL	UL	dB	DL	UL	DL	UL
-15	0	0	0	0	6	1.39	0.93	521	347
-14	0	0	0	0	7	1.55	1.04	582	388
-13	0	0	0	0	8	1.72	1.15	646	430
-12	0	0	0	0	9	1.90	1.26	711	474
-11	0	0	0	0	10	2.08	1.38	778	519
-10	0.08	0.06	31	21	11	2.26	1.51	847	565
-9	0.10	0.07	38	26	12	2.44	1.63	917	611
-8	0.13	0.08	48	32	13	2.63	1.76	988	658
-7	0.16	0.10	59	39	14	2.82	1.88	1059	706
-6	0.19	0.13	73	48	15	3.02	2.00	1131	750
-5	0.24	0.16	89	59	16	3.21	2.00	1204	750
-4	0.29	0.19	109	73	17	3.41	2.00	1277	750
-3	0.35	0.23	132	88	18	3.60	2.00	1350	750
-2	0.42	0.28	159	106	19	3.80	2.00	1424	750
-1	0.51	0.34	190	127	20	3.99	2.00	1498	750
0	0.60	0.40	225	150	21	4.19	2.00	1572	750
1	0.71	0.47	265	176	22	4.39	2.00	1646	750
2	0.82	0.55	308	206	23	4.40	2.00	1650	750
3	0.95	0.63	356	237	24	4.40	2.00	1650	750
4	1.09	0.72	408	272	25	4.40	2.00	1650	750
5	1.23	0.82	463	309					

Outline

- ❑Noise and Interference
- ❑Communication Requirements and Link Budget Analysis
- ❑Non-LOS Propagation
- ❑Statistical Models for Path Loss
- ❑Demo: Estimating Rates with a 3GPP model

Reflections and Refractions

Sean Hum, [Lecture Notes](), U of Toronto



(a) Parallel polarization (b) Perpendicular polarization

$$\Gamma_{\parallel} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$T_{\parallel} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

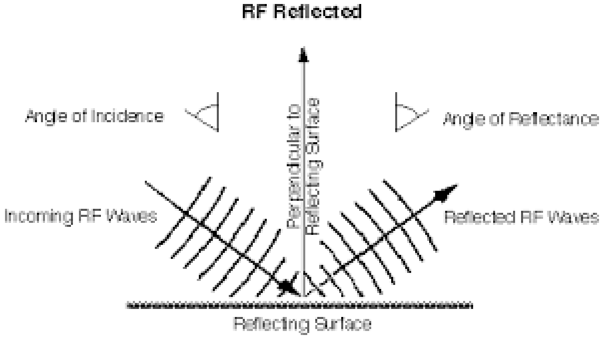
$$\Gamma_{\perp} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

$$T_{\perp} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

- Occur at any dielectric interface
  - Change in characteristic impedance η
- Consider two separate polarizations
  - Parallel and perpendicular
- Reflected components
  - $\theta_r = \theta_i$: Reflects in opposite angle
- Refracted / transmission component:
  - $\sqrt{\mu_1 \epsilon_1} \sin\theta_i = \sqrt{\mu_2 \epsilon_2} \sin\theta_t$
  - May be no components
- Complex gains on each component
  - Γ, T
- Polarization may in general change

Metal Reflectors

Sean Hum, Lecture Notes, U of Toronto



$$\Gamma_{\parallel} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$
$$T_{\parallel} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$\Gamma_{\perp} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$
$$T_{\perp} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

- Special case of a perfect conductor:
  - $\eta_2 = 0$
- All signal is reflected back:
  - $\Gamma_{||} = \Gamma_{\perp} = -1$
- No transmission component:
  - $T_{||} = T_{\perp} = 0$
- Polarization direction is unchanged

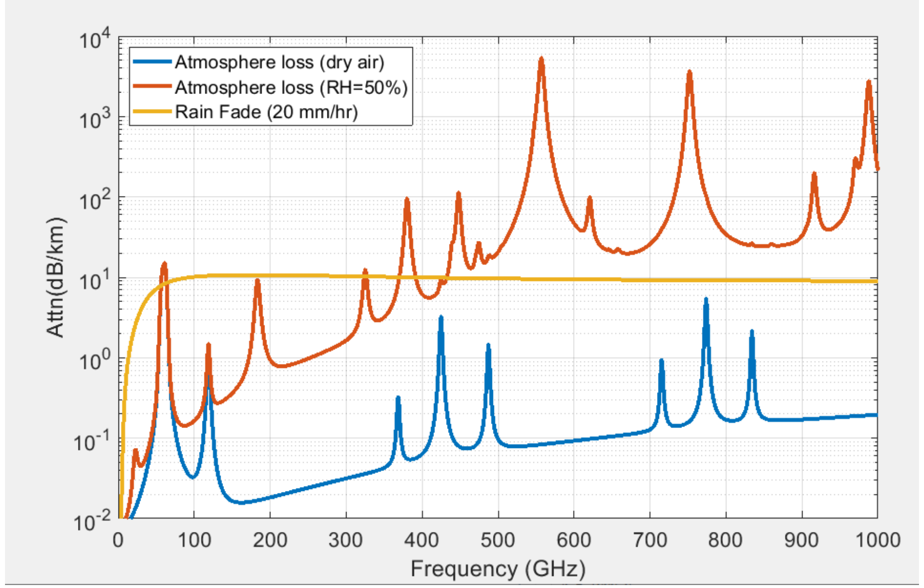
Transmission Through Typical Materials

Building Material	2.4 GHz Attenuation
Solid Wood Door1. 75"	6 dB
Steel Fire/Exit Door 1.75"	13 dB
Steel Fire/Exit Door 2.5"	19 dB
Brick 3.5"	6 dB
Concrete Wall 18"	18 dB
Glass Divider 0.5"	12 dB
Interior Solid Wall 5"	14 dB
Marble 2"	6 dB
Exterior Double Pane Coated Glass 1"	13 dB
Exterior Single Pane Window 0.5"	7 dB

- Radio waves can transmit through materials, but with attenuation
- Source: City of Cumberland, Maryland WiFi study

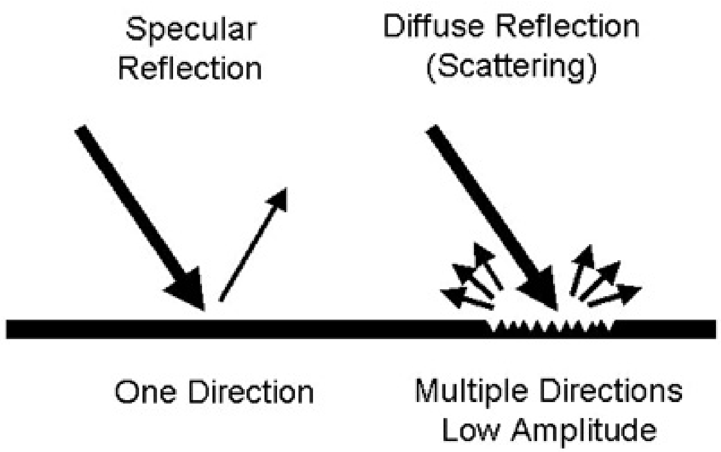
Attenuation Models in MATLAB

- MATLAB Phased Array Toolbox has many models atmospheric attenuation
  - Commands for free space path loss, and attenuation for fog, gas and rain
  - Based on well-studied measurements
- Note: Water absorption is particularly important to model for mmWave!




```
% Frequencies to test
freq = linspace(1,1000,1000)'*1e9;
range = 1000;  % Compute attenuation at 1 km
rr = 20;       % Rain rate

T = 15;       % temperature in C
P = 101300.0;  % atmospheric pressure
Wsat = 4.8;  % vapor density at saturation (g/m^3)
RH = 0.5;    % relative humidity
W = RH*Wsat;  % vapor density

% Compute attenuations
attn_dry = gaspl(range,freq,T,P,0)';
attn_humid = gaspl(range,freq,T,P,W)';
attn_rain = rainpl(range, freq, rr)';
```

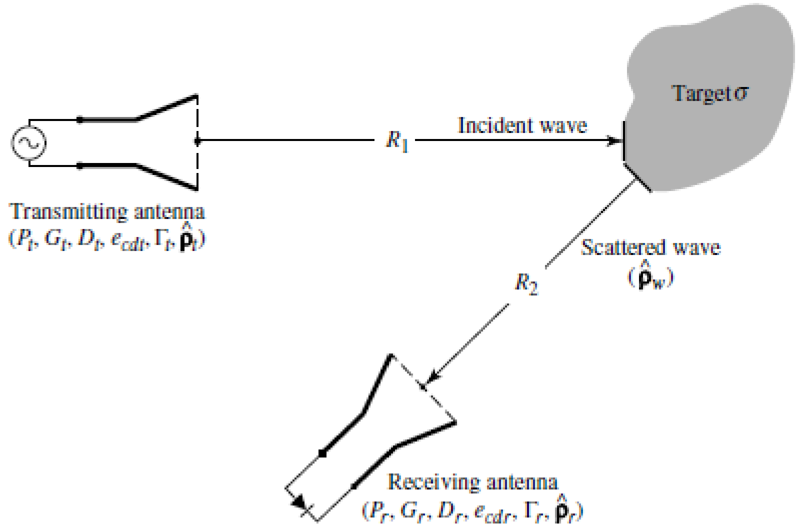
Specular Reflections & Scattering



- Due to surface roughness, reflected radio waves can be scattered in many directions.
- Amount of power loss in specular component related to height of surface irregularities.
- Texts provides probabilistic models to estimate power loss based on random height variations.

Radar Cross Section

- Intuition:
  - Let W_i = incident radiation density
  - Reflect total power = σW_i
  - σ = effective area captured for reflection
  - Scattered density is: $W_s = \frac{\sigma W_i}{4\pi R_2^2}$
- Define radar cross section:
  - $\sigma = \lim_{R_2 \to \infty} \left[4\pi R_2^2 \frac{W_s}{W_i} \right]$
- Defines effective area of receiving target
- Generally depends on angle

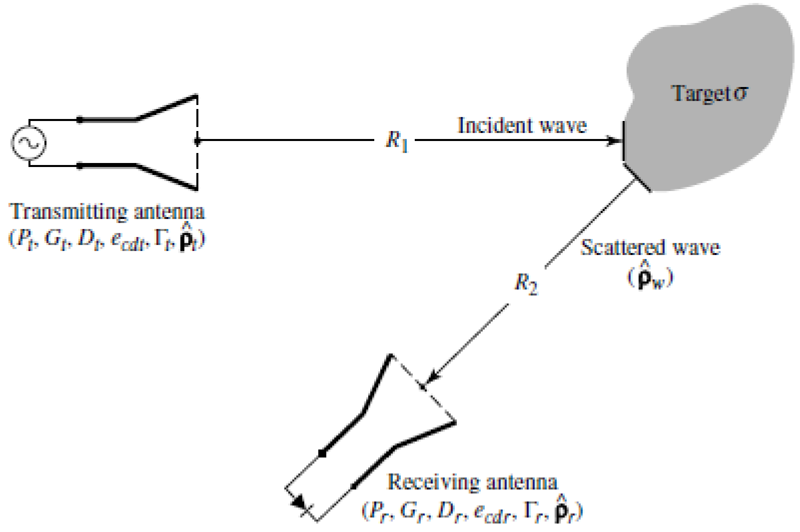


Radar Equation

- Radar Equation: Ratio of RX to TX power

$$\frac{P_r}{P_t} = \frac{\sigma D_r D_t}{4\pi}\left(\frac{\lambda}{4\pi R_1 R_2}\right)^2$$

  - Proof: Next slide



- Provides basic equation for radar link budgets
- Key points:
  - Power decays as $\frac{1}{R^4}$
  - Much faster than LOS free-space $\frac{1}{R^2}$
  - Remember directivity and RCS depend on angle

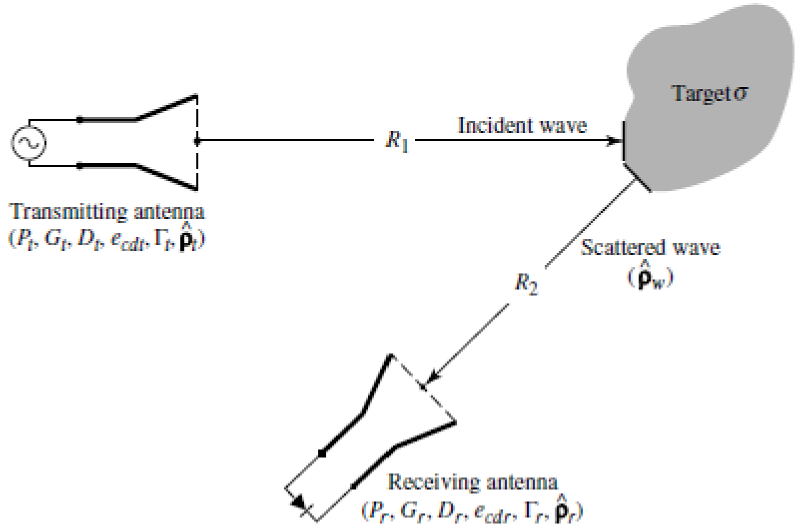
Proof of the Radar Equation

- Incident density: $W_i = \frac{P_t D_t}{4\pi R_1^2}$
- Reflected density:

$$W_s = \frac{\sigma W_i}{4\pi R_2^2} = \frac{\sigma P_t D_t}{(4\pi R_1 R_2)^2}$$

- Received power $P_r = A_r W_s$
- Using antenna-directivity relation: $A_r = \frac{D_r \lambda^2}{4\pi}$
- Therefore:

$$P_r = \frac{\sigma \lambda^2 D_r D_t}{4\pi (4\pi R_1 R_2)^2} P_t$$



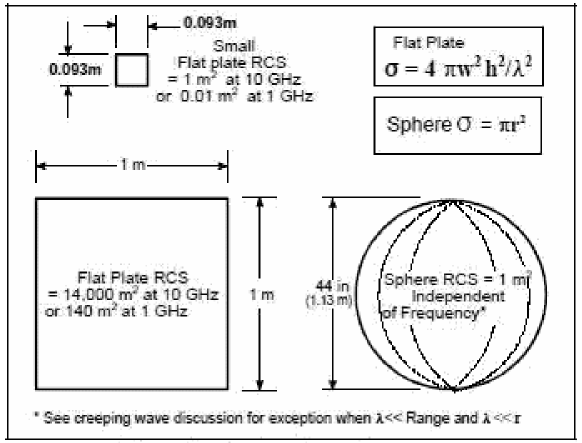
Typical RCS Values

TABLE 2.2 RCS of Some Typical Targets

	Typical RCSs [22]	
Object	**RCS (m^2)**	**RCS (dBsm)**
Pickup truck	200	23
Automobile	100	20
Jumbo jet airliner	100	20
Large bomber *or* commercial jet	40	16
Cabin cruiser boat	10	10
Large fighter aircraft	6	7.78
Small fighter aircraft *or* four-passenger jet	2	3
Adult male	1	0
Conventional winged missile	0.5	−3
Bird	0.01	−20
Insect	0.00001	−50
Advanced tactical fighter	0.000001	−60

From Balanis

- Values are often quoted in dBsm
  - $\sigma[\text{dBsm}] = 10\log_{10}\left(\frac{\sigma}{1\ m^2}\right)$
- For small perfect electric conductor (PEC)

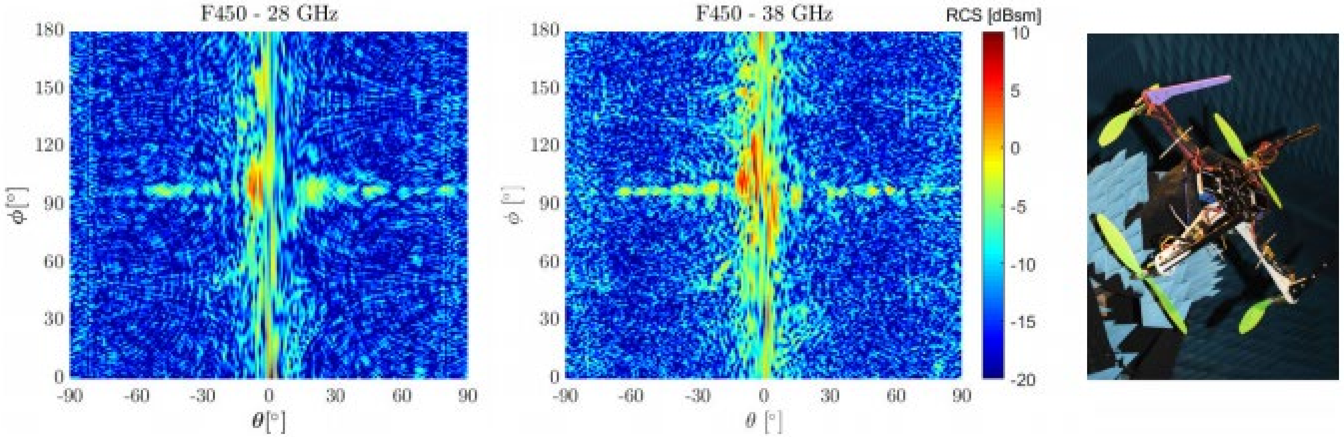


From RFCafe

Note formulas only true valid in optical range:

- Range $\gg \lambda$
- Area $\gg \lambda^2$

Example: RCS Measurements of a UAV



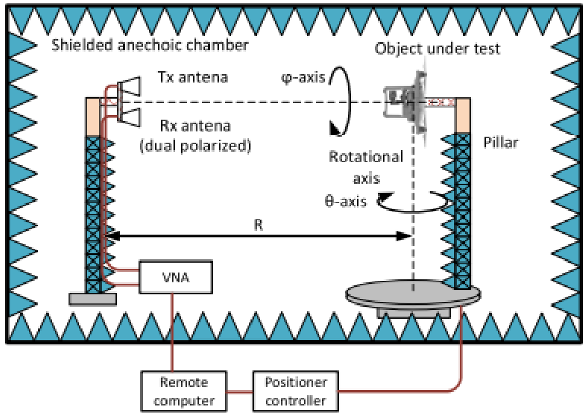


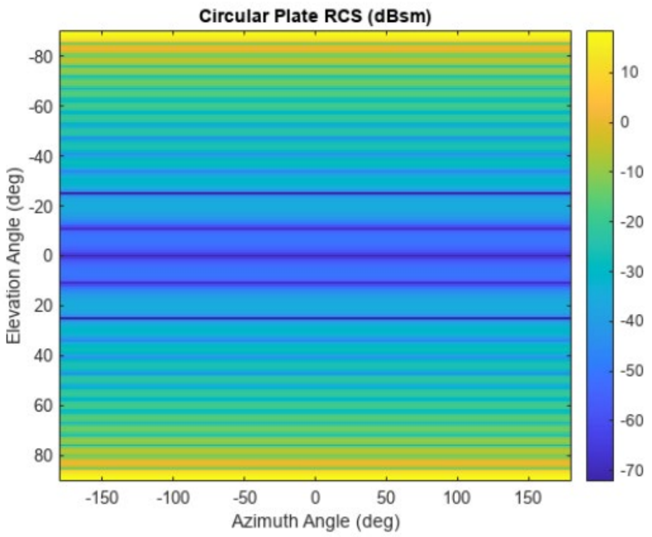
FIGURE 1. **The schematic view of the measurement setup.**

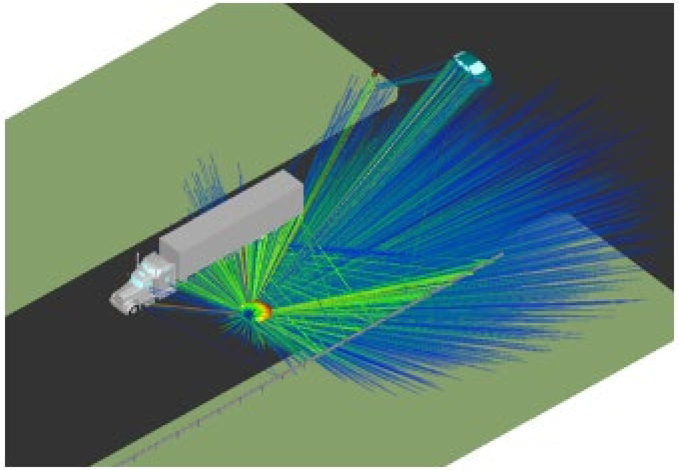
- Measurements of RCS of commercial UAVs
  - Use an anechoic chamber (removes reflections)
- Notice RCS is much larger when facing the drone
- Very small when seeing drone from side

From Semkin, Vasilii, et al. "Analyzing Radar Cross Section signatures of diverse drone models at mmWave frequencies." *IEEE Access* (2020).

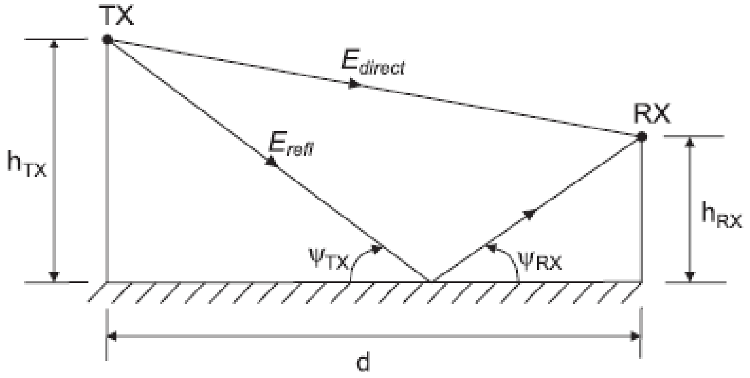
RCS Simulation Tools

- MATLAB’s RADAR toolbox
  - Has analytic formulae for simple scattering objects
  - Ex: Disc, cylinder, plate, …
- Remcomm Wavefarer
  - Performs detailed scattering analysis
  - Solves current integrals on arbitrary surface
  - Ex: automotive RADAR, indoor scenes
- You can use these in your project





Propagation Loss with Reflections



$$\frac{P_r}{P_t} = G_1 G_2 \frac{h_{tx}^2 h_{rx}^2}{d^4}$$

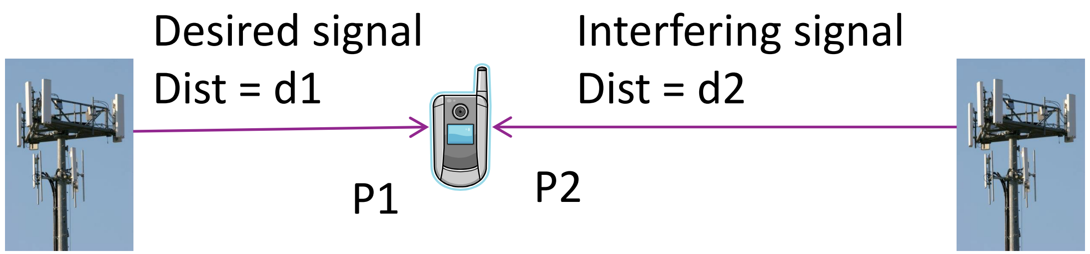
- In free space, we saw power density decays as d^2
- Due to ground reflections, power decays faster than d^2
- For d large, decays as d^α, α= path loss exponent.
  - α has been observed from 1.5 to 5.5, but usu. btw 3 and 5.
- For single ground reflection, can show α=4.
  - Based on reflected wave canceling direct wave
  - See www.wiley.com/go/molisch Appendix 4-A

Path Loss Exponent, Coverage & Interference

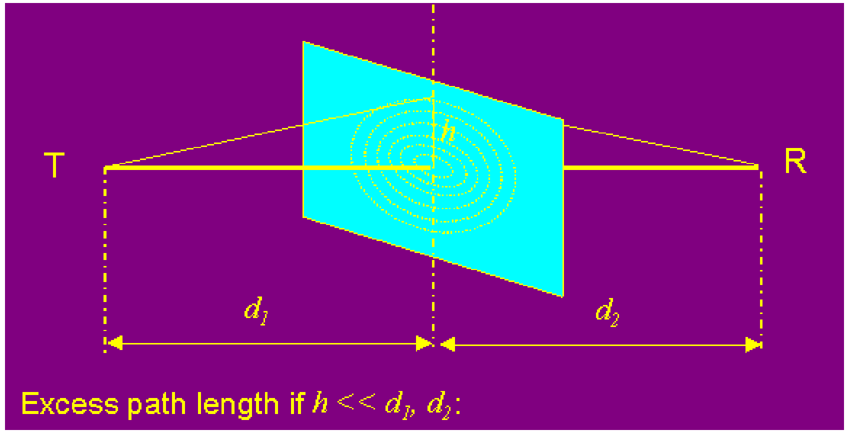
- For coverage-limited systems, low α is good
  - Power decays slower => signals have greater range

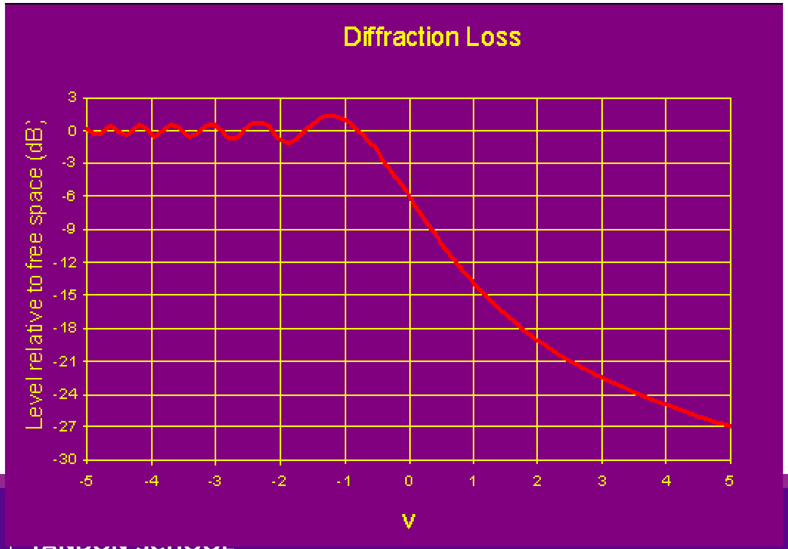
- For interference-limited systems, high α is good
  - Ex: If thermal noise is negligible:

$$SINR = \frac{P_1}{P_{noise} + P_2} \approx \frac{P_1}{P_2} = \left(\frac{d_2}{d_1}\right)^{\alpha}$$



Diffraction





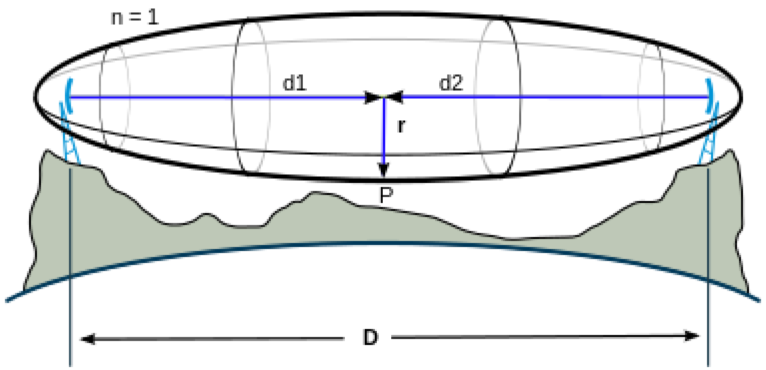
- Interfering objects (IOs) do not result in sharp shadows.
  - Due to wave nature of EM radiation
  - Simple ray model is not correct.
- Waves diffract at IO boundaries,
  - Intensity after IO can be stronger in parts than with no IO!
- Example: Knife-edge diffraction:
  - Diffraction around semi-infinite barrier
  - Loss dictated by normalized excess path length

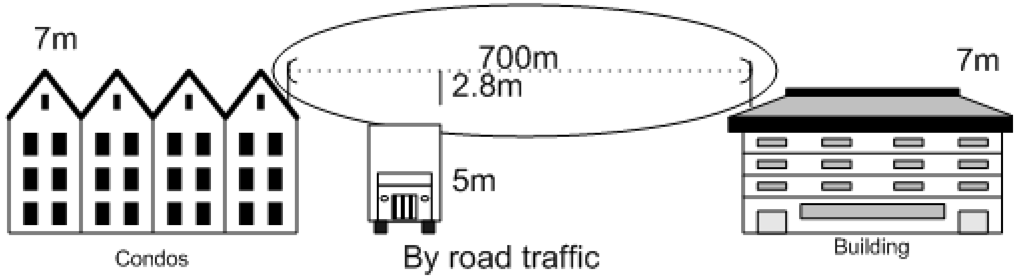
$$\nu = h\sqrt{\frac{2(d_1+d_2)}{\lambda d_1 d_2}}$$

http://www.mike-willis.com/Tutorial/diffraction.htm

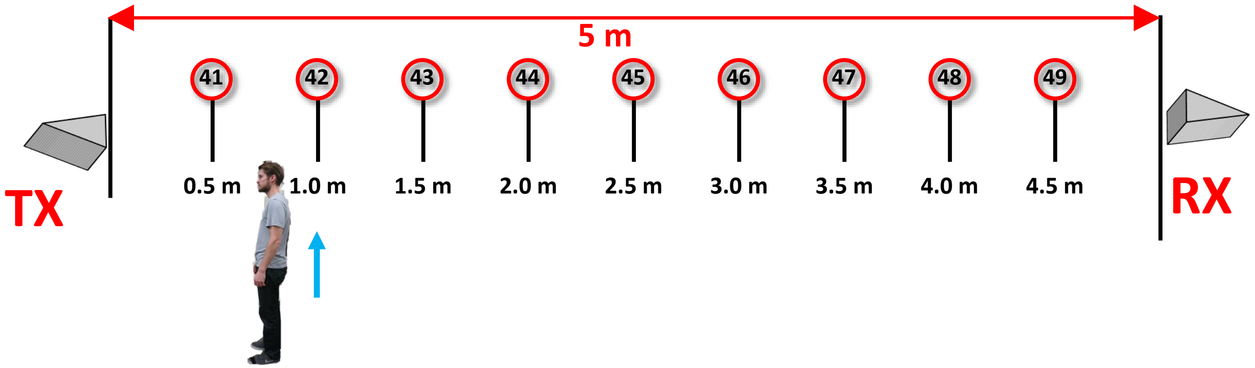
Fresnel Zone and Diffraction

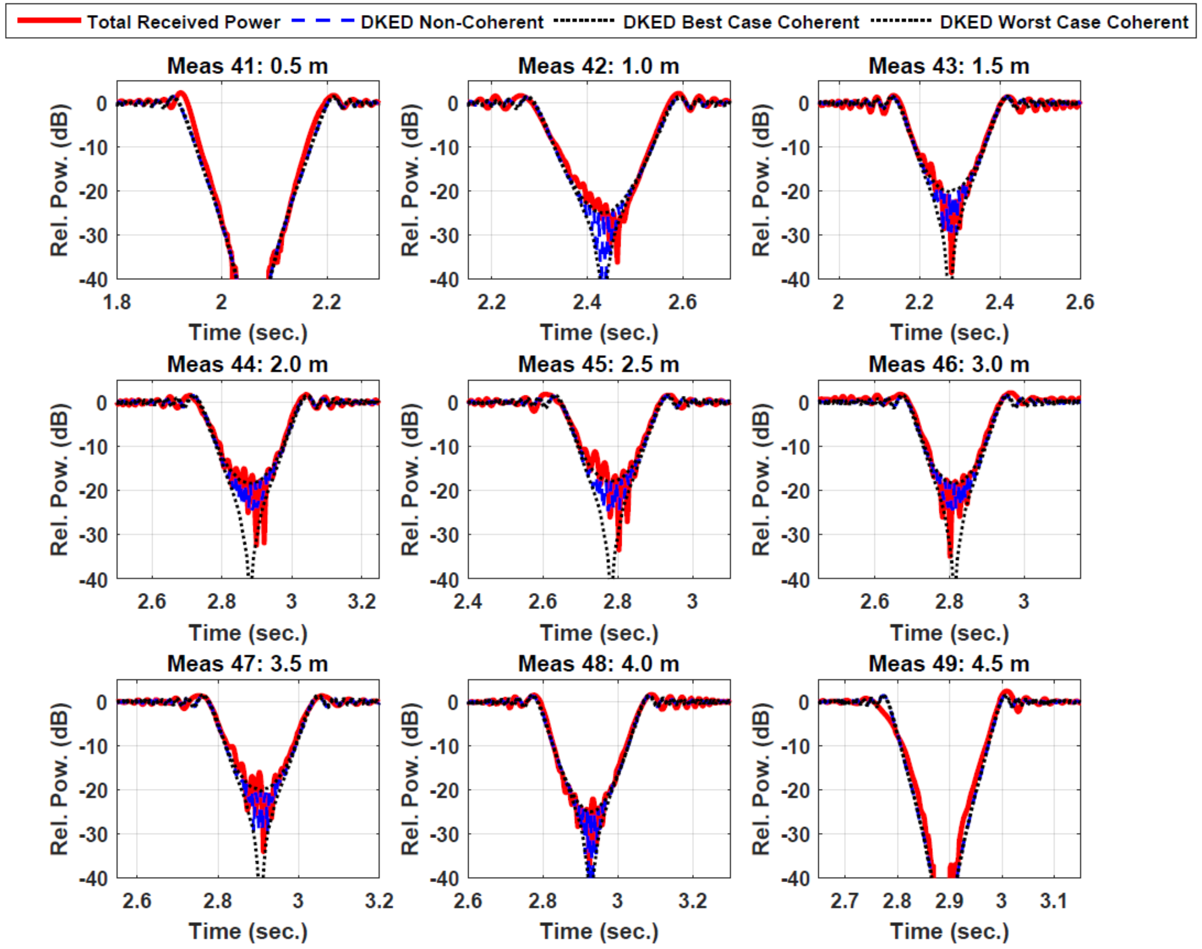
- Frensel Zone n: Set of points where:
  - Reflected distance = Direct distance $\frac{n\lambda}{2}$
  - Points lie on an ellipse
  - Distance from direct path is: $r_n = \sqrt{\frac{n\lambda d_1 d_2}{d_1+d_2}}$
- Physical meaning:
  - Objects in Frensel region will impact propagation
- Example: Diffraction related
  - Amount of diffraction $\nu = h/r_2$
- Rule of thumb: For unobstructed propagation:
  - Keep 60% of first Frensel zone free





Human Blocking at 73GHz Example

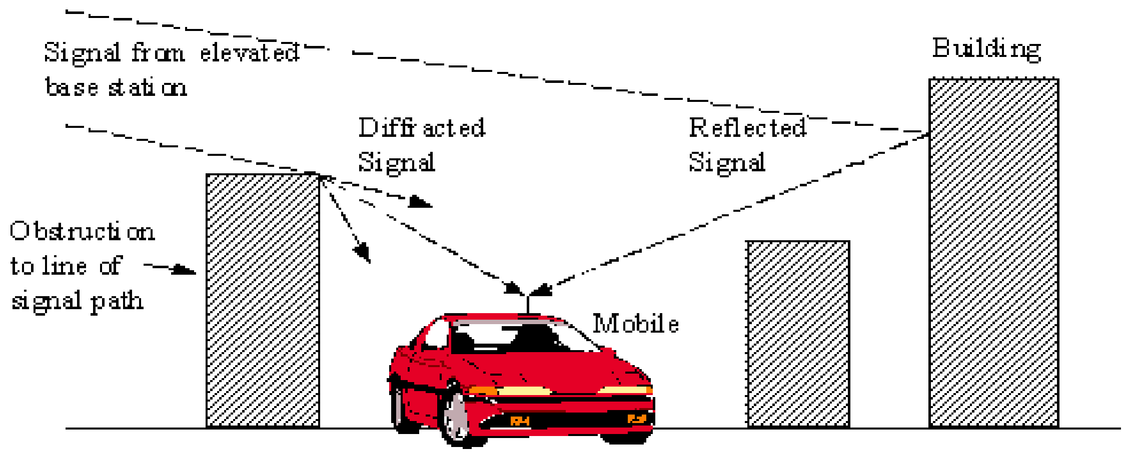


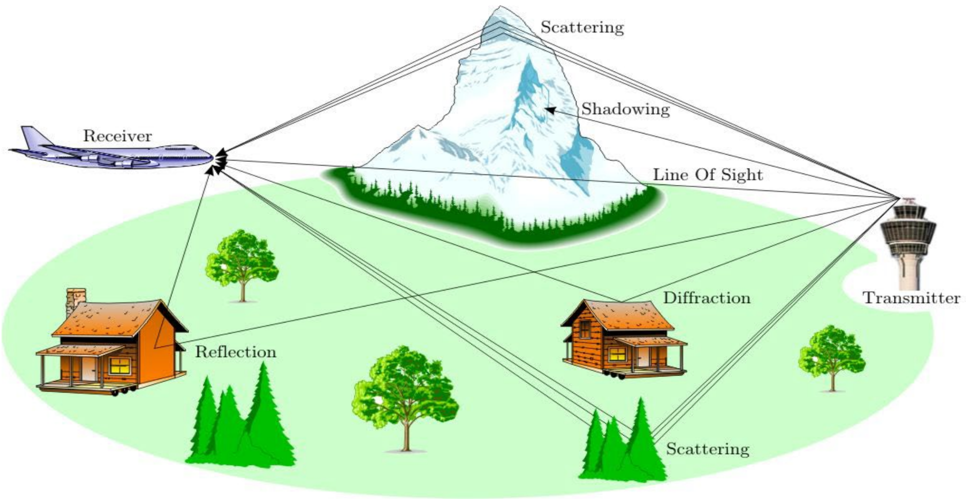


- Between 25 and 40 dB blockage
  - Depends on distance
- Total power predicted by double knife edge diffraction
- Piecewise linear model used by 3GPP

G. R. MacCartney, Jr., S. Deng, S. Sun, and T. S. Rappaport, "73 GHz Millimeter-Wave Human Blockage and Dynamic Measurements," IEEE VTC 2016

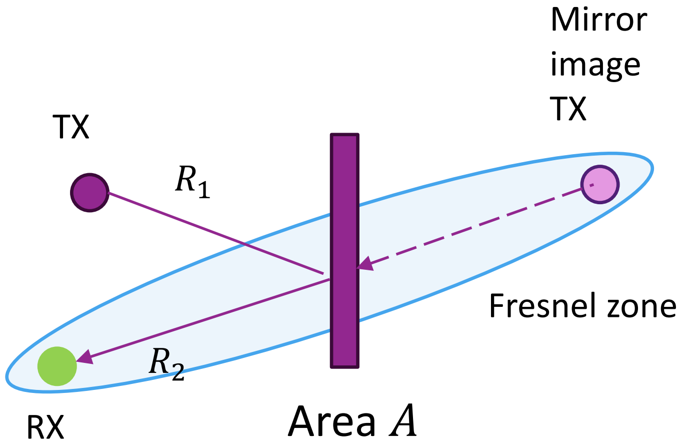
Radio Waves Have Many Paths



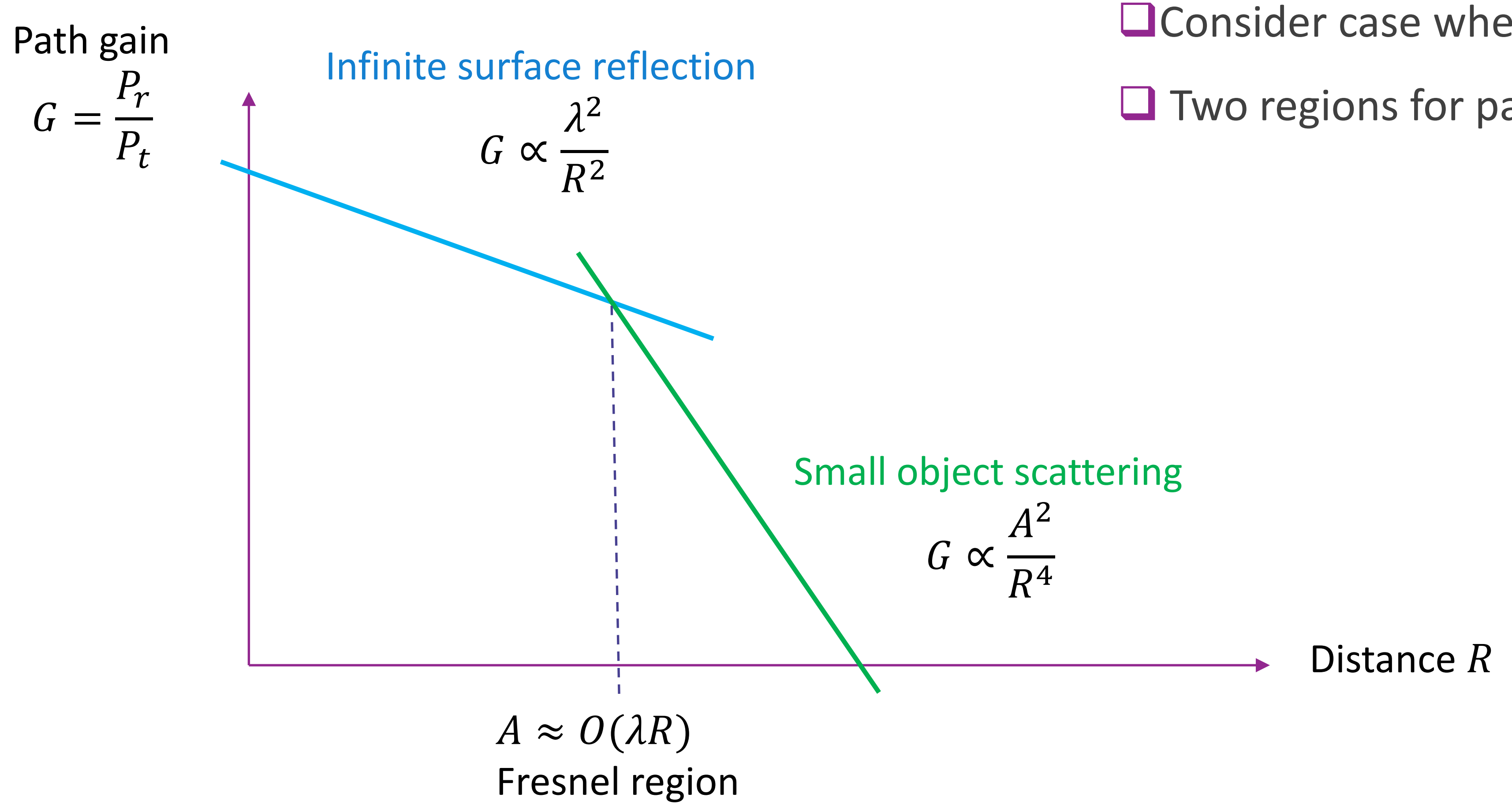


Fresnel Zone: Scattering vs. Reflection

- Consider scattering from a PEC plate: $\sigma = \frac{4\pi A^2}{\lambda^2}$
- Path loss from radar equation:
  - $L_{scat} = \frac{\sigma D_r D_t}{4\pi}\left(\frac{\lambda}{4\pi R_1 R_2}\right)^2 = A^2 D_r D_t \left(\frac{1}{4\pi R_1 R_2}\right)^2$
- Path loss from an infinite surface reflection:
  - Reflection appears as direct transmission from mirror image
  - $L_{ref} = D_r D_t \left(\frac{\lambda}{4\pi(R_1+R_2)}\right)^2$ by Friis' law
- Then $L_{scat} = L_{ref}$ when: $\lambda A = \frac{R_1 R_2}{R_1+R_2} \Rightarrow A = \ r_1^2$
- Conclusion: Object becomes infinite surface reflection when:
  - Area $\gg$ Fresnel zone



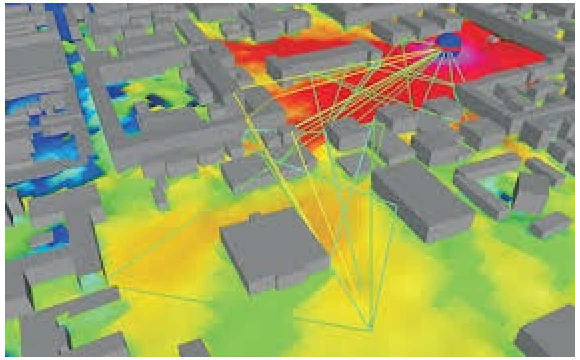
Scattering vs. Reflection

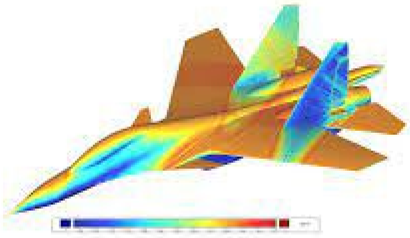


- Consider case when $R_1 = R_2 = R$
- Two regions for path gain

Ray Tracing vs. Radar Simulation

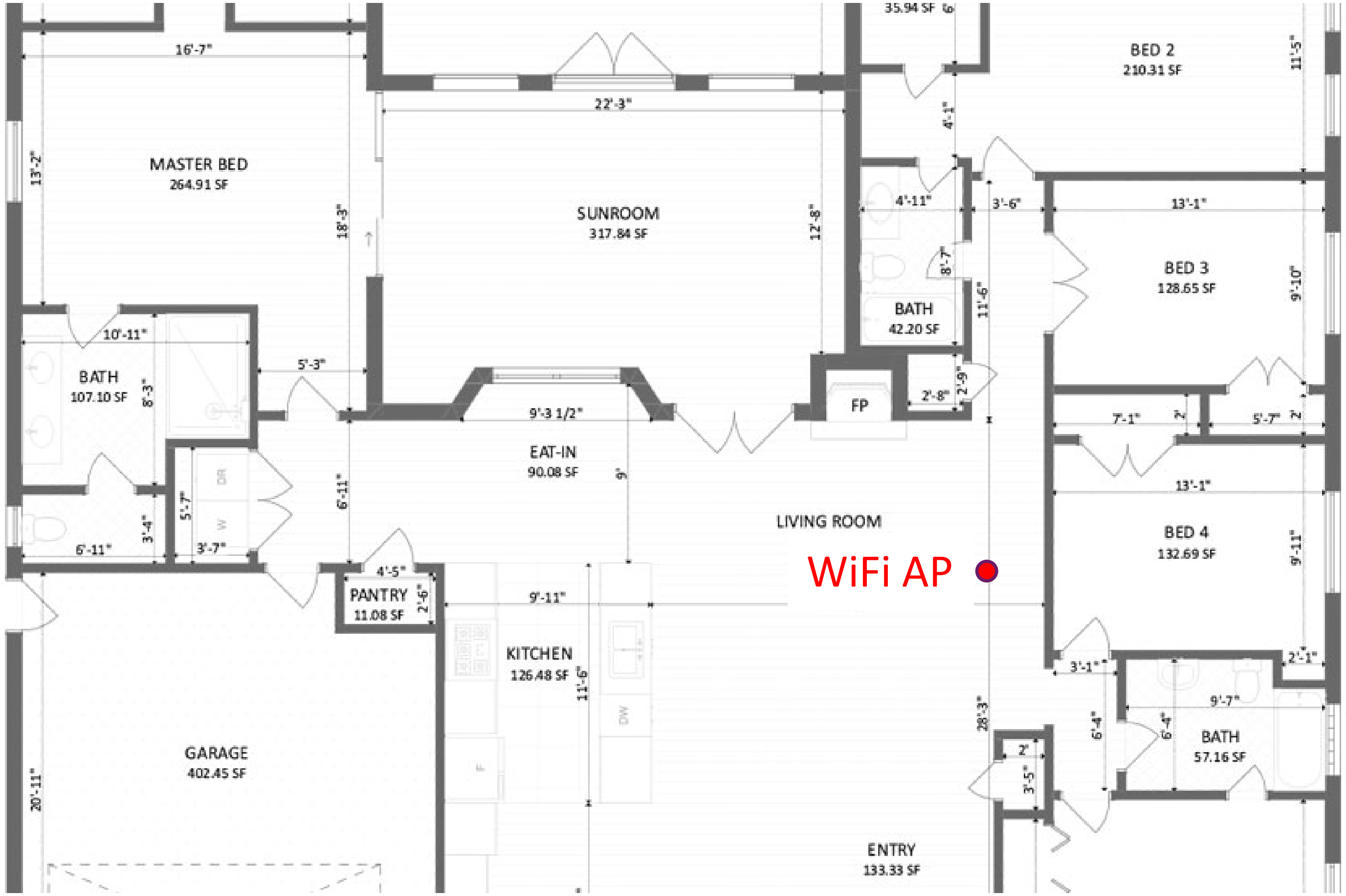
- Propagation simulation increasingly important
  - Particularly for training ML-based methods
- Ray Tracing
  - Use infinite surface reflections
    + approximation of diffraction
  - Computationally fast, but less accurate
- Radar simulation
  - Uses exact computation of surface interactions
  - More exact but higher computational demand
- Still much to study in comparing methods
- Fresnel region provides some guidance





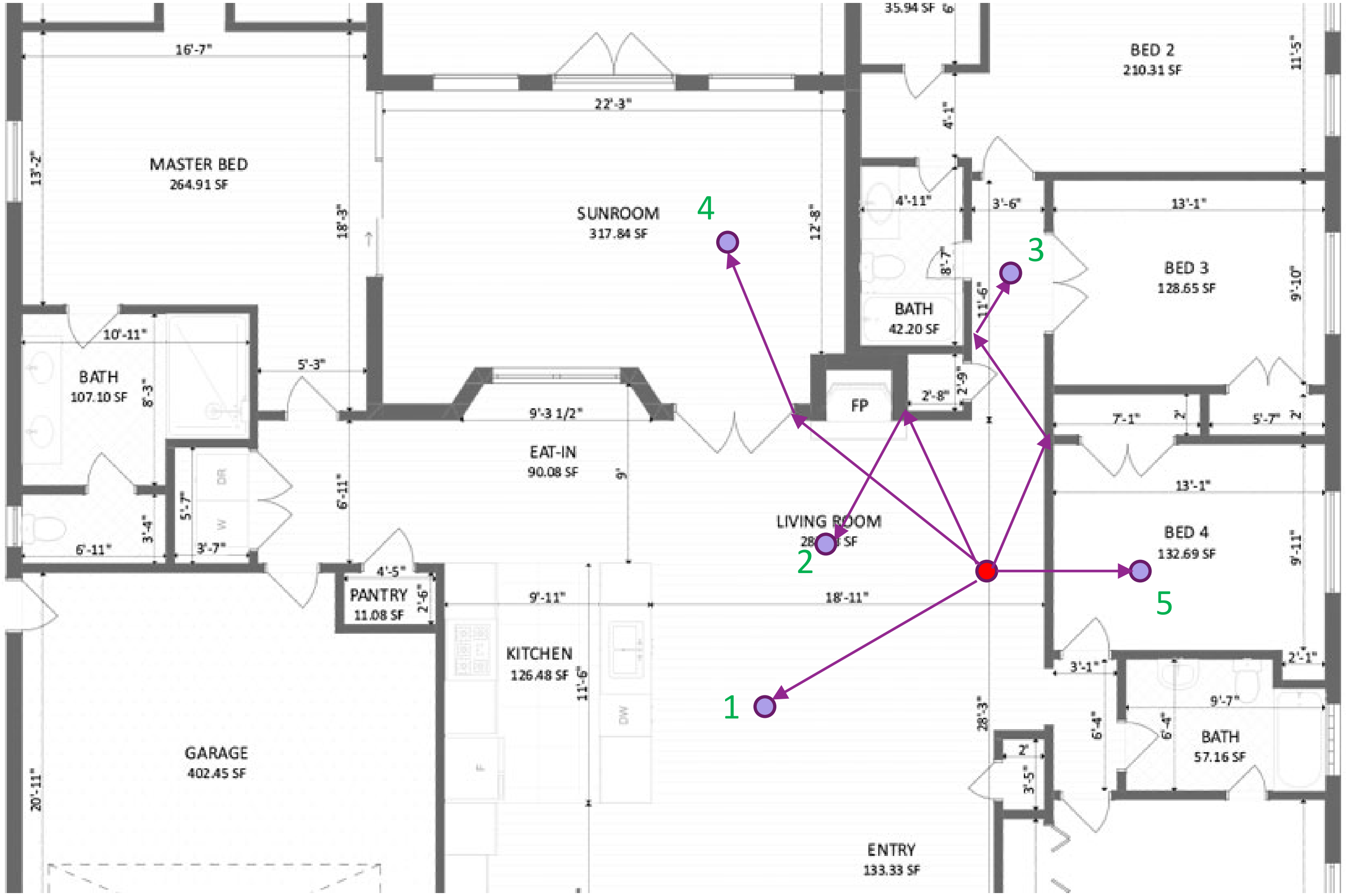
In-Class Exercise

- A WiFi access point is located as marked.
- Find locations where each of the following types of paths could be received. Also, draw the path:
  - Direct LOS
  - Reflected path one bounce
  - Reflected path two bounces
  - Diffracted path
  - Transmission through wall or window



In-Class Exercise Solution

- A WiFi access point is located as marked.
- Find locations where each of the following types of paths could be received. Also, draw the path:
  1. Direct LOS
  2. Reflected path one bounce
  3. Reflected path two bounces
  4. Diffracted path
  5. Transmission through wall or window

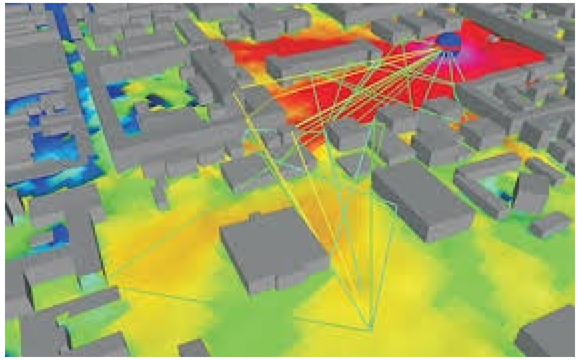


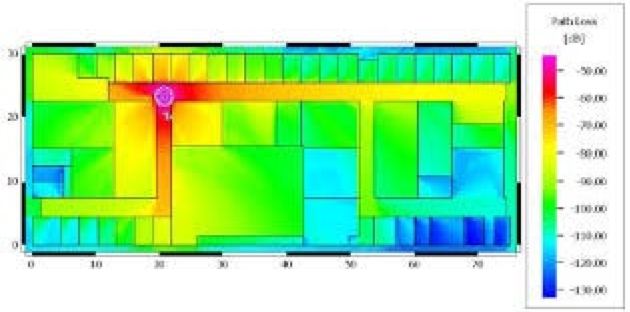
Outline

- ❑Noise and Interference
- ❑Communication Requirements and Link Budget Analysis
- ❑Non-LOS Propagation
- ❑Statistical Models for Path Loss
- ❑Demo: Estimating Rates with a 3GPP model

Real Path Loss

- Path loss is a complex function of environment
- Varies with distance, obstacles, reflections …
- Site specific path loss can be predicted with ray tracing

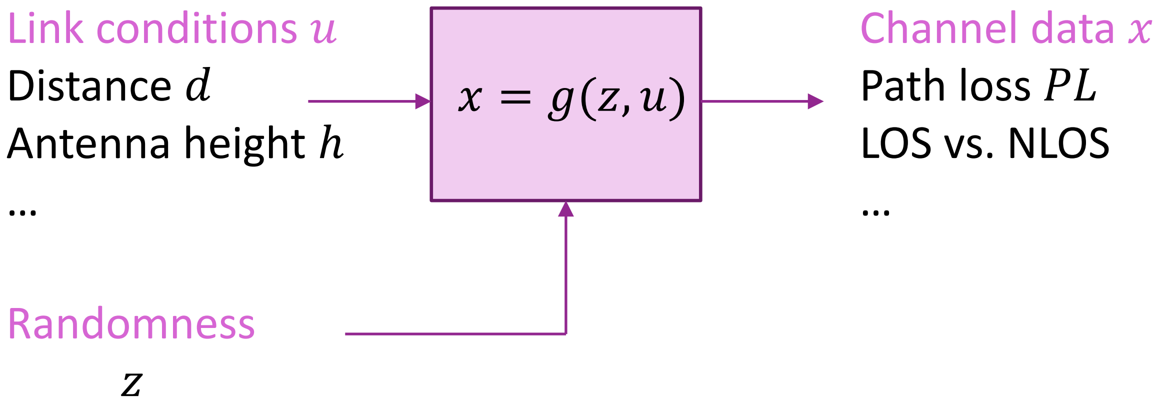




Outputs of commercial WinProp ray tracer

Statistical Channel Models

- Channel characteristics modeled as a random variable
  - Environmental effects are modeled as random
- Typically, a generative model:
  - $x = g(u, z)$
  - u: Link conditions
  - x: Channel state
  - z: Randomness
- Model fit for a type of environment
  - Eg. Urban, suburban, indoor, … Not site-specific
  - Based on data
  - Find function $g(u, z)$ to match conditional distribution $P(x|u)$

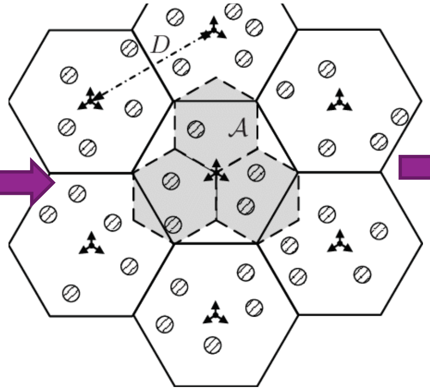


Simulation with a Channel Model

Randomness

Deployment Model (ex: urban micro)

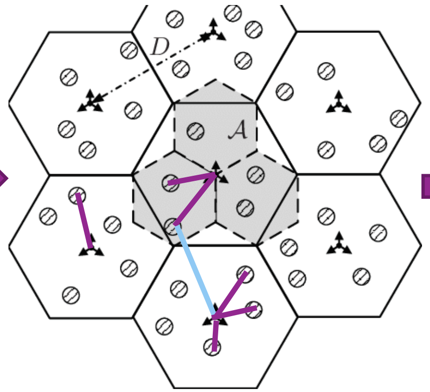
Parameters: ISD, density, …



Locations of UEs, gNBs, …

Randomness

Generative channel model



Channels between nodes

Performance evaluation

Linear Models

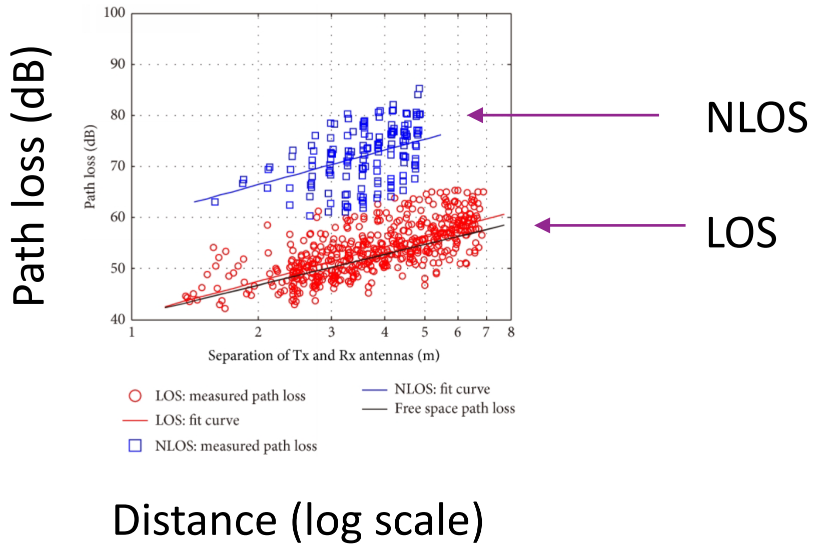
- Floating intercept model:

$$PL(d) = 10\alpha \log_{10} d + \beta + \xi, \qquad \xi \sim N(0, \sigma^2)$$

  - Parameters α, β, σ^2 fit from data

- Used widely in 3GPP, IEEE
  - Different models for different scenarios

- Caution in any fit model:
  - Do not use outside distances, frequencies it was derived



28 and 73 GHz measurements in:

Akdeniz, M. R., Liu, Y., Samimi, M. K., Sun, S., Rangan, S., Rappaport, T. S., & Erkip, E. (2014). Millimeter wave channel modeling and cellular capacity evaluation. *IEEE journal on selected areas in communications*, *32*(6), 1164-1179.

Related Models

- Close in (CI) model
  - Match free space at some fixed reference distance d_0

$$PL(d) = FSPL(d_0) + 10\alpha \log_{10} \frac{d}{d_0} + \xi$$

  - One less parameter to fit
  - Matches true path loss at d_0

- Hata model, ….
- Multi-slope models
- 3GPP NLOS / LOS hybrid models

Outage Probability

- Consider transmission with fixed MCS with rate R
  - No adaptation!
- Requires $SNR \geq SNR_{min}$
  - Outage: Event that $SNR < SNR_{min}$
  - Results in zero rate
- With variable path loss, SNR, is a random variable
- Outage probability:

$$\begin{aligned} P_{out} &= P(SNR < SNR_{min}) \\ &= P(P_{TX} - PL(d) - P_{noise} < SNR_{min}) \end{aligned}$$

In-Class Exercise

Problem 3: Simulating Path Loss Variations

Suppose the TX power is `Ptx` and the path loss can be modeled as a random variable as follows:

- The channel is either LOS or NLOS with probability `ProbLos` or `1-ProbLos`.
- If the channel is LOS, the path loss is lognormally distributed with mean `PLLos` and standard deviation `stdLos`.
- If the channel is NLOS, the path loss is lognormally distributed with mean `PLNlos` and standard deviation `stdNlos`.

Generate `n=10000` channel instances and plot the histogram of the received power. You may use the MATLAB `histogram` function.

```
Ptx = 15;         % Tx power in dBm
ProbLos = 0.4;    % LOS probability
PLNlos = 80;      % Path loss for LOS (dB)
PLLos = 100;      % Path loss for NLOS (dB)
stdLos = 8;       % Path loss std dev for LOS (dB)
stdNlos = 4;       % Path loss std dev for LOS (dB)
```

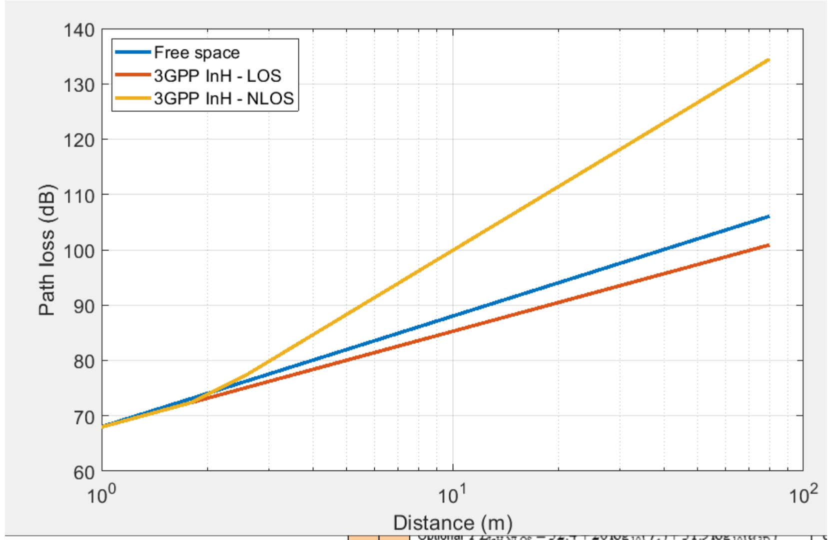
Outline

- ❑Noise and Interference
- ❑Communication Requirements and Link Budget Analysis
- ❑Non-LOS Propagation
- ❑Statistical Models for Path Loss
- ❑Demo: Estimating Rates with a 3GPP model

Ex: 3GPP Indoor Home Office Model

- 3GPP has models for many statistical models
  - 38.900: Path loss models for above 6 GHz
- Example: Indoor home office
  - Separate models for LOS and NLOS
  - Plotted is the median path loss vs. distance

```
% Parameters
fc = 60e9;   % Frequency
vp = physconst('lightspeed');  % speed of light
lambda = vp/fc;    % wavelength

% Compute LOS and NLOS path loss
dist = linspace(1,80,100)';
pllos = 32.4 + 17.3*log10(dist) + 20*log10(fc/1e9);
plnlos = 17.3 + 38.3*log10(dist) + 24.9*log10(fc/1e9);
plnlos = max(pllos, plnlos);
plfs = fspl(dist,lambda);
```

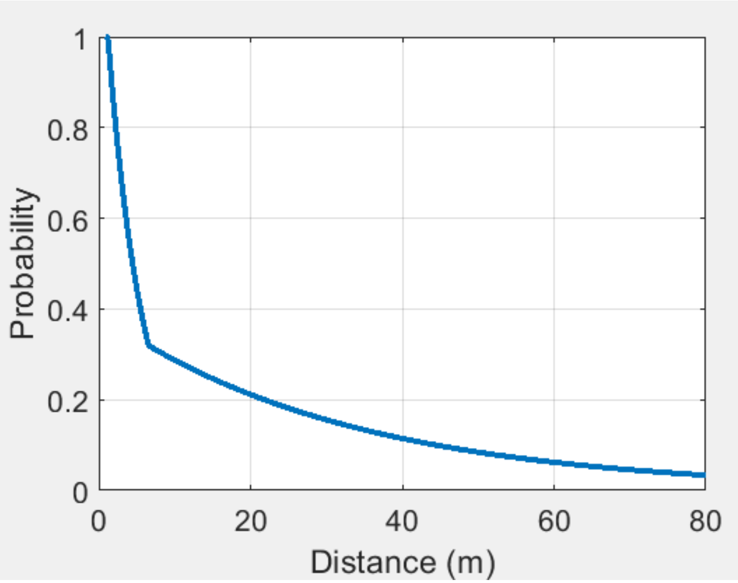


InH - Office	LOS	$PL_{\text{InH-LOS}} = 32.4 + 17.3\log_{10}(d_{3D}) + 20\log_{10}(f_c)$	$\sigma_{SF} = 3$	$1\text{m} \le d_{3D} \le 100\text{m}$
	NLOS	$PL_{\text{InH-NLOS}} = \max(PL_{\text{InH-LOS}}, PL'_{\text{InH-NLOS}})$ $PL'_{\text{InH-NLOS}} = 38.3\log_{10}(d_{3D}) + 17.30 + 24.9\log_{10}(f_c)$	$\sigma_{SF} = 8.03$	$1\text{m} \le d_{3D} \le 86\text{m}$
		Optional $PL'_{\text{InH-NLOS}} = 32.4 + 20\log_{10}(f_c) + 31.9\log_{10}(d_{3D})$	$\sigma_{SF} = 8.29$	$1\text{m} \le d_{3D} \le 86\text{m}$

Ex: Probability of LOS

- Model has a probability of LOS
- Function of distance
  - As distance is larger, probability of LOS is smaller

```
%% Plot the probability of LOS
plosl = min( exp(-(dist-1.2)/4.7), 1);
plos = min( 0.32*exp(-(dist-6.5)/32.6), plosl);
plot(dist, plos, 'Linewidth', 3);
```



Indoor - Mixed office	$P_{LOS} = \begin{cases} 1 & , d_{2D} \le 1.2\text{m} \\ \exp\left(-\dfrac{d_{2D}-1.2}{4.7}\right) & , 1.2\text{m} < d_{2D} < 6.5\text{m} \\ \exp\left(-\dfrac{d_{2D}-6.5}{32.6}\right)\cdot 0.32 & , 6.5\text{m} \le d_{2D} \end{cases}$

Ex: Generating Random Samples

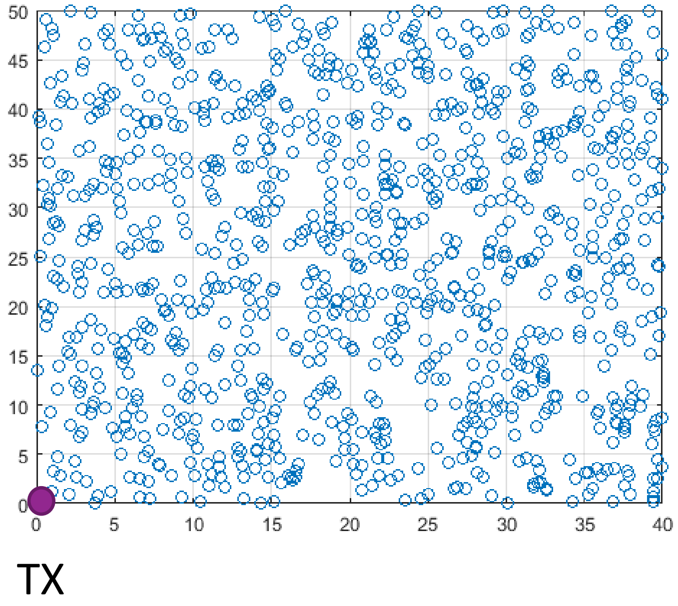
- The full model generate random path loss
  - Path loss is a function of distance
  - Samples from the conditional distribution $P(L|d)$
- Steps:
  - Compute median LOS and NLOS path loss
  - Add shadowing
  - Randomly select between LOS and NLOS
  - Use PLOS probability

```
function pl = pathLoss3GPPInH(dist,fc)
    % pathLoss3GPPInH:  Generates random path loss
    %
    % Samples the path loss using the 3GPP-InH model

    % Compute the median path losses for LOS and NLOS
    pllos = 32.4 + 17.3*log10(dist) + 20*log10(fc/1e9);
    plnlos = 17.3 + 38.3*log10(dist) + 24.9*log10(fc/1e9);

    % Add shadowing
    w = randn(size(dist));
    pllos = pllos + 3*w;
    plnlos = plnlos + 8.03*w;

    % Compute probability of being LOS or NLOS
    plos = min( exp(-(dist-1.2)/4.7), 1);
    plos = min( 0.32*exp(-(dist-6.5)/32.6), plos);

    % Select randomly between LOS and NLOS path loss
    u = (rand(size(dist)) < plos);
    pl = u.*pllos + (1-u).*plnlos;

end
```

Example Simple Simulation

- Simulations: Often used to estimate distribution of rates
  - Assume some statistical distribution on locations and propagation
- Illustrate with a simple simulation
  - RX is randomly located in a square region.
  - TX is located at origin
- 3GPP has much more realistic deployment models

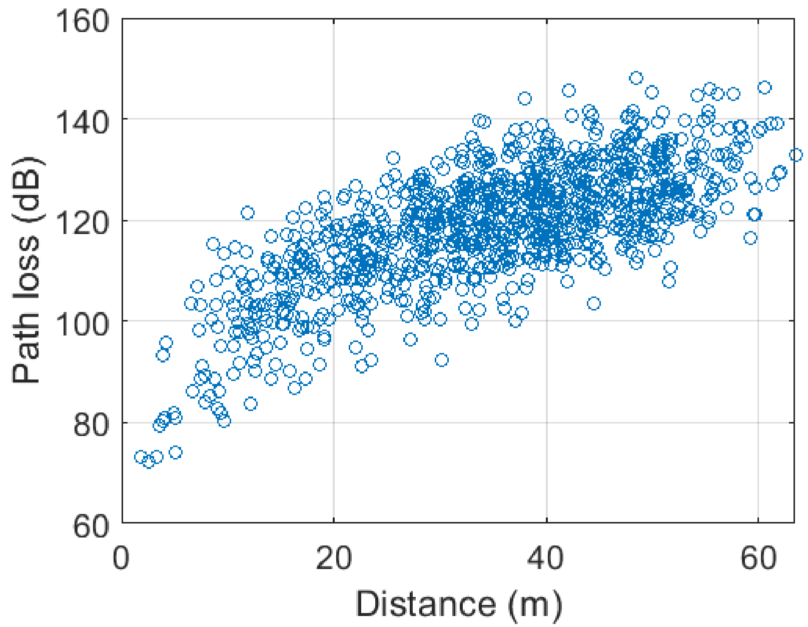
```
% Parameters
len = 40;  % length of region in m
wid = 50;  % width in m
nx = 1000;  % number of random points

% Generate random points in a square
x = rand(nx,2).*[len wid];

% Plot the random points
plot(x(:,1), x(:,2), 'o');
grid on;
```



Generate Random Path Loss

- Generate random path losses based on distances

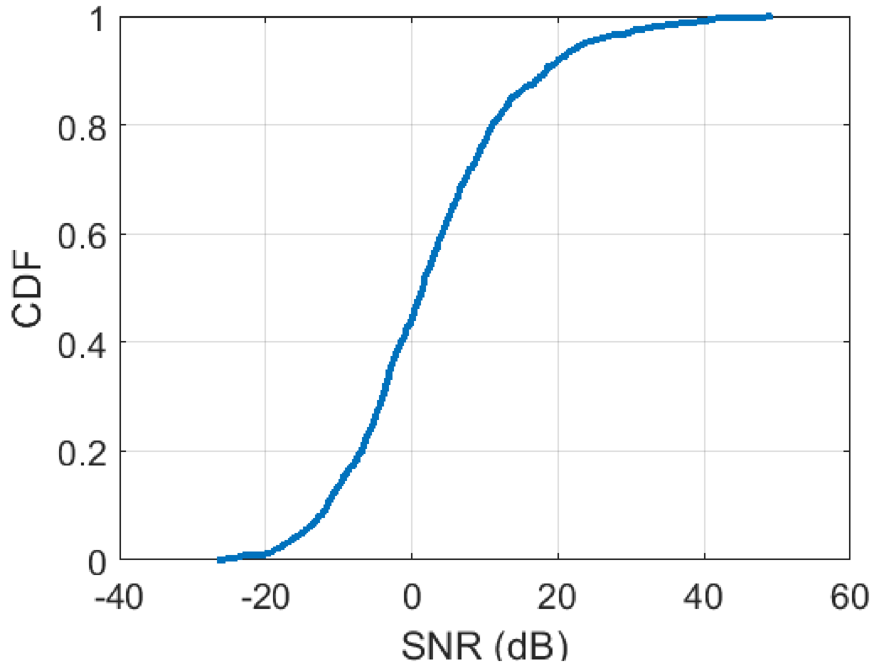
```
% We will make some simple assumptions for a wifi-like system
fc = 60e9;

% Compute the distances
dh = 1;  % Distances in height
dist = sqrt(sum(x.^2,2) + dh^2);

% We next generate random path losses to each
pl = pathLoss3GPPInH(dist, fc);

% Plot a scatter plot of the PL vs. distance
plot(dist, pl, 'o');
grid on;
xlabel('Distance (m)');
ylabel('Path loss (dB)');
set(gca, 'Fontsize', 16);
```



Compute SNR Distribution

- Make assumptions similar to an 802.11ad-like system
  - We assume very directive antennas
  - Later we show how to do this with beamforming

```
% Parameters
ptx = 20;      % transmit power
bw = 1.76e9;   % sample rate
nf = 6;        % noise figure
kt = -174;     % thermal noise
gaintx = 16;   % antenna gain
gainrx = 10;

% Compute SNR
snr = ptx - pl - nf - 10*log10(bw) - kt + gainrx + gaintx;

% Plot the SNR CDF
p = (1:nx)/nx;
plot(sort(snr), p,'Linewidth', 3);
grid on;
xlabel('SNR (dB)');
ylabel('CDF');
set(gca, 'Fontsize', 16);
```



Compute Rate Distribution

- Compute rate distribution assuming simple backoff from Shannon capacity
  - More realistic models are possible
- Note the large range of rates in this region

```
% Finally we compute the rate based on some simple simulatins
snrLoss = 6;
bwLoss = 0.2;
maxSE = 4.8;
rate = bw*(1-bwLoss)*min(log2(1 + 10.^(0.1*(snr-snrLoss))), maxSE);
rate = rate/1e6;

p = (1:nx)/nx;
semilogx(sort(rate), p,'Linewidth', 3);
grid on;
xlabel('Rate (Mbps)');
ylabel('CDF');
set(gca, 'Fontsize', 16);
```