CENG 331 Take-Home Final

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Q 2.2)

For more accurate analysis, I extended the algorithm to loop through 100 times for each index.

```
for(int i=1; i<100; i++){
        for(int j=1; j<100; j++){
            c[i,j]=0;
            for(int k=1; k<100; k++){
                c[i,j]=c[i,j] + a[i,k]*b[k,j];
            }
        }
}
```

As I proved in the following pages, the misprediction rates are as follows:

1-bit ⇒ 0,019797

2-bit first ⇒ 0,010028

2-bit second ⇒ 0,010199

As we can see, the 2-bit predictors outperformed the 1-bit predictor, and the first 2-bit predictor made slightly less errors than the second 2-bit predictor for input size 100. These rates are of course subject to change for different input sizes.

For the 1-bit predictor, I assumed the initial state to be 0 (taken). Here is a summary of the desicions made:

T ⇒ Taken, NT ⇒ Not taken

	Guess	Actual	Result
i=1	T	T	✓
J=1	T	T	✓
k=1	T	T	✓
⋮	⋮	⋮	
k=99	T	T	✓
k=100	T	NT	X
J=2	NT	T	X
...			
J=99	NT	T	X
J=100	NT	NT	✓
i=2	NT	T	X
J=1	T	T	✓
J=2	NT	T	X
⋮	⋮		
J=99	NT	T	X
J=100	NT	NT	✓
⋮			
i=99	NT	T	X
J=1	T	T	✓
J=99	NT	T	X
J=100	NT	NT	✓
i=100	NT	NT	✓

$w_i \Rightarrow$ Count of wrong desicions on loop i

$t_i \Rightarrow$ total count of desicions on loop i

<u>i=1</u>

$w_i \Rightarrow 99 + 98$

$t_i \Rightarrow 100.99 + 100 + 1$

<u>i=2...99</u>

$w_i \Rightarrow 99 + 98 + 1$

$t_i \Rightarrow 100.99 + 100 + 1$

⋮

<u>i=100</u>

$w_i \Rightarrow 0$

$t_i \Rightarrow 1$

Then, the total misprediction rate for input size 100 is:

$$r = \sum_{i=1}^{100} \frac{w_i}{t_i} = \frac{99+98 + 98.(99+98+1) + 0}{100.99+100+1 + 98(100.99+100+1)+1} = 0{,}019797$$

For the first 2-bit predictor, I assumed the initial state to be 0 (strongly taken). Note that the input size is kept unchanged to observe the effect of predictors on misprediction rate. Here is a summary of the desicions made:

ST → strongly taken T → taken

SNT → strongly not taken NT → not taken

	Guess	Actual	New State
$i=1$	ST	taken	ST
$J=1$	ST	taken	ST
$k=1$	ST	taken	ST
⋮			
$k=99$	ST	taken	ST
$k=100$	ST	not taken	T
$J=2$	T	taken	ST
$k=1$	ST	taken	ST
⋮			
$k=99$	ST	taken	ST
$k=100$	ST	not taken	T
⋮			
$J=100$	T	not taken	NT
$i=100$	NT	not taken	SNT

$w_i \Rightarrow$ count of wrong desicions on loop i

$t_i \Rightarrow$ total count of desicions on loop i

$i=1$

$w_i \Rightarrow 99 \cdot 1 + 1$

$t_i \Rightarrow 99 \cdot 100 + 100 + 1$

$i = 2 \dots 99$

$w_i = 99 \cdot 1 + 1 + 1$

$t_i = 99 \cdot 100 + 100 + 1$

$i=100$

$w_i = 0$

$t_i = 1$

Then, the total misprediction rate for input size 100 is:

$$r = \sum_{i=1}^{100} \frac{w_i}{t_i} = \frac{99+1+98(99+1+1)+0}{99 \cdot 100 + 100 + 1 + 98(99 \cdot 100 + 101) + 1} = 0.010098$$

For the second 2-bit predictor, I assumed the inital state to be 00 (not taken). The input size is kept unchanged to observe the effect of predictors on misprediction rate. Here is the summary of the descisions made:

T → taken
NT → not taken

	Guess	Actual	New State
i=1	00(NT)	T	01(NT)
J=1	01(NT)	T	11(T)
k=1	11(T)	T	11(T)
k=2	11(T)	T	11(T)
⋮			
k=99	11(T)	T	11(T)
k=100	11(T)	NT	10(T)
J=2	10(T)	T	11(T)
k=1	11(T)	T	11(T)
⋮			
k=99	11(T)	T	11(T)
k=100	11(T)	NT	10(T)
J=3	10(T)	T	11(T)
⋮			
J=99	10(T)	T	11(T)
J=100	10(T)	NT	00(NT)
i=2	00(NT)	T	01(NT)
⋮			
i=99	00(NT)	T	01(NT)
i=100	00(NT)	NT	00(NT

$w_i \Rightarrow$ count of wrong desicions on loop i

$t_i \Rightarrow$ count of total desicions on loop i

$i = 1 \ldots 99$

$w_i = 99 + 1 + 1 + 1$

$t_i = 99.100 + 100 + 1$

$i = 100$

$w_i = 0$

$t_i = 1$

Then, the total misprediction rate for input size 100 is:

$$r = \sum_{i=1}^{100} \frac{w_i}{t_i} = \frac{(99+1+1+1).99 + 0.1}{(99.100+100+1)99 + 1} = 0{,}010199$$