

Q 2.2)

For more accurate analysis, I extended the algorithm to loop through 100 times for each index.

```
for(int i=1; i<100; i++) {  
    for(int j=1; j<100; j++) {  
        c[i,j]=0;  
        for(int k=1; k<100; k++) {  
            c[i,j]=c[i,j] + a[i,k]*b[k,j];  
        }  
    }  
}
```

As I proved in the following pages, the misprediction rates are as follows:

1-bit  $\Rightarrow 0,019727$

2-bit first  $\Rightarrow 0,010028$

2-bit second  $\Rightarrow 0,012228$

As we can see, the first 2-bit predictor performed the best, with around 1% misprediction rate. Other two predictors yielded similar results for input size 100.

I suppose in bigger input sizes, the 1-bit predictor would outperform the second 2-bit predictor.

But the superiority of the first 1-bit predictor would continue.

For the 1-bit predictor, I assumed the initial state to be 0 (taken). Here is a summary of the decisions made:

	Guess	Actual	Result	
$i=1$	T	T	✓	
$\quad j=1$	T	T	✓	
$\quad \quad k=1$	T	T	✓	
$\quad \quad \quad \vdots$	$\vdots$	$\vdots$	$\vdots$	
$\quad \quad k=99$	T	T	✓	
$\quad \quad k=100$	T	NT	X	
$\quad j=2$	NT	T	X	
$\quad \quad \vdots$	$\vdots$	$\vdots$	$\vdots$	
$\quad j=99$	NT	T	X	
$\quad j=100$	NT	NT	✓	
$i=2$	NT	T	X	
$\quad j=1$	T	T	✓	
$\quad j=2$	NT	T	X	
$\quad \quad \vdots$	$\vdots$	$\vdots$	$\vdots$	
$\quad j=99$	NT	T	X	
$\quad j=100$	NT	NT	✓	
$\quad \quad \vdots$	$\vdots$	$\vdots$	$\vdots$	
$i=99$	NT	T	X	
$\quad j=1$	T	T	✓	
$\quad j=99$	NT	T	X	
$\quad j=100$	NT	NT	✓	
$i=100$	NT	NT	✓	

$w_i \Rightarrow$  count of wrong decisions on loop  $i$

$t_i \Rightarrow$  total count of decisions on loop  $i$

$i=1$

$w_1 \Rightarrow 99 + 98$

$t_1 \Rightarrow 100 \cdot 99 + 100 + 1$

$i=2 \dots 99$

$w_i \Rightarrow 99 + 98 + 1$

$t_i \Rightarrow 100 \cdot 99 + 100 + 1$

$i=100$

$w_i \Rightarrow 0$

$t_i \Rightarrow 1$

Then, the total misprediction rate for input size 100 is:

$$r = \sum_{i=1}^{100} \frac{w_i}{t_i} = \frac{99+98 + 98 \cdot (99+98+1) + 0}{100 \cdot 99 + 100 + 1 + 98(100 \cdot 99 + 100 + 1) + 1} = 0,019797$$

For the first 2-bit predictor, I assumed the initial state to be 0 (strongly taken.). Note that the input size is kept unchanged to observe the effect of predictors on misprediction rate. Here is a summary of the decisions made:

	Guess	Actual	New State	ST $\rightarrow$ strongly taken T $\rightarrow$ taken SNT $\rightarrow$ strongly not taken NT $\rightarrow$ not taken
$i=1$	ST	taken	ST	
$J=1$	ST	taken	ST	
$k=1$	ST	taken	ST	$w_i \Rightarrow$ count of wrong decisions on loop $i$
$\vdots$				
$k=99$	ST	taken	ST	
$k=100$	ST	not taken	T	$t_i \Rightarrow$ total count of decisions on loop $i$
$J=2$	T	taken	ST	
$k=1$	ST	taken	ST	
$\vdots$				
$k=99$	ST	taken	ST	$i=1$
$k=100$	ST	not taken	T	$w_i \Rightarrow 99 \cdot 1 + 1$
$\vdots$				$t_i \Rightarrow 99 \cdot 100 + 100 + 1$
$J=100$	T	not taken	NT	$i=2 \dots 99$
$\vdots$				$w_i \Rightarrow 99 \cdot 1 + 1 + 1$
$i=100$	NT	not taken	SNT	$t_i \Rightarrow 99 \cdot 100 + 100 + 1$
				$i=100$
				$w_i = 0$
				$t_i = 1$

Then, the total misprediction rate for input size 100 is:

$$r = \sum_{i=1}^{100} \frac{w_i}{t_i} = \frac{99 + 1 + 98(99 + 1 + 1) + 0}{99 \cdot 100 + 100 + 1 + 98(99 \cdot 100 + 100 + 1) + 1} = 0.010098$$

For the second 2-bit predictor, I assumed the initial state to be 00 (not taken). The input size is kept unchanged to observe the effect of predictors on misprediction rate. Here is the summary of the decisions made:

	<u>Guess</u>	<u>Actual</u>	<u>New State</u>	T $\rightarrow$ taken NT $\rightarrow$ not taken
$i=1$	00(NT)	T	01(NT)	
$J=1$	01(NT)	T	11(T)	
$k=1$	11(T)	T	11(T)	$w_i \Rightarrow$ count of wrong decisions on loop $i$
$k=2$	11(T)	T	11(T)	
$\vdots$				$t_i \Rightarrow$ count of total decisions on loop $i$
$k=99$	11(T)	T	11(T)	
$k=100$	11(T)	NT	10(T)	
$J=2$	10(T)	T	11(T)	$i = 1 \dots 99$
$k=1$	11(T)	T	11(T)	$w_i = 99 \cdot 1 + 1 + 1 + 99$
$\vdots$				$t_i = 99 \cdot 100 + 100 + 1$
$k=99$	11(T)	T	11(T)	
$k=100$	11(T)	NT	10(T)	
$J=3$	10(T)	T	11(T)	$i = 100$
$\vdots$				$w_i = 0$
$J=99$	10(T)	T	11(T)	$t_i = 1$
$J=100$	10(T)	NT	00(NT)	
$i=2$	00(NT)	T	01(NT)	
$\vdots$				
$i=99$	00(NT)	T	01(NT)	
$i=100$	00(NT)	NT	00(NT)	

Then, the total misprediction rate for input size 100 is:

$$r = \sum_{i=1}^{100} \frac{w_i}{t_i} = \frac{(99+1+1+99) \cdot 22 + 0 \cdot 1}{(99 \cdot 100 + 100 + 1)99 + 1} = 0,019998$$