

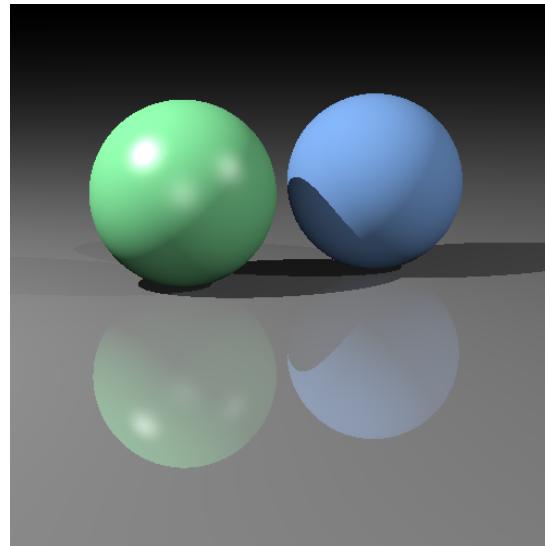
CENG 477

Introduction to Computer Graphics

Texture Mapping

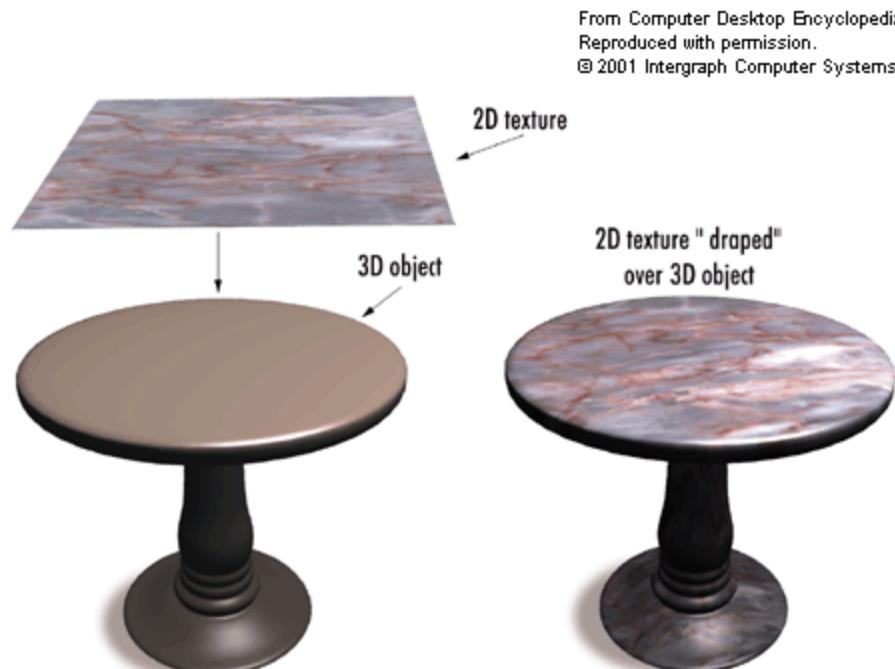
Until Now

- We assumed that objects have **fixed R, G, B reflectance** values
- Surface **orientation** and light **direction** determined the shading of objects
- Our best image so far is something like:



Texture Mapping

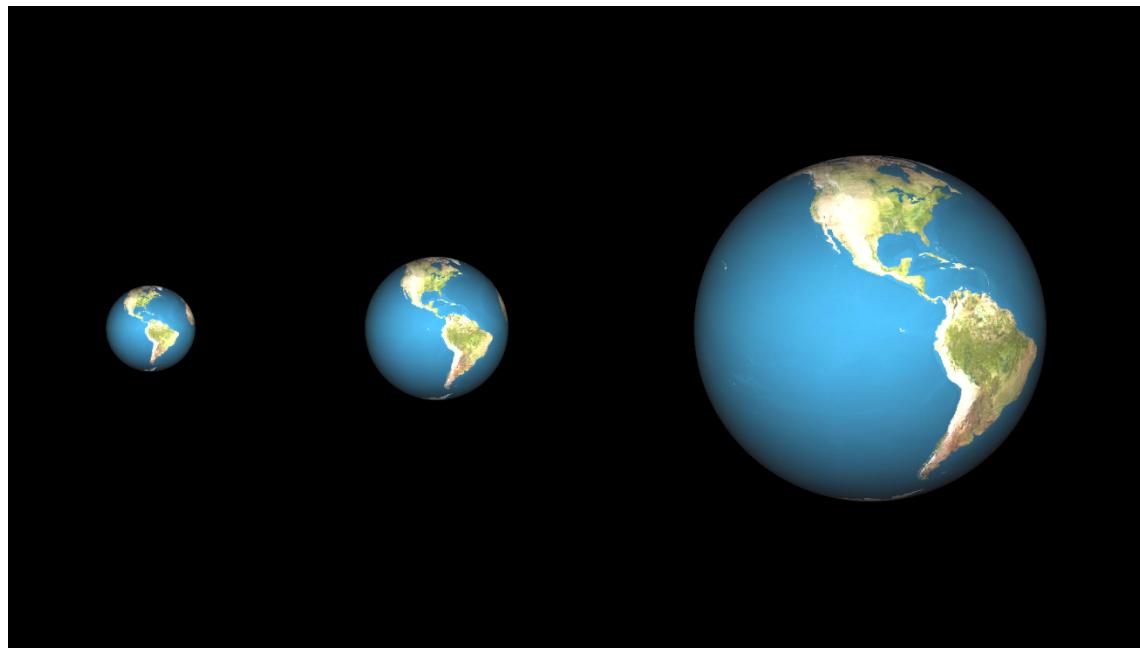
- **Goal:** Increase visual realism by using **images** to simulate reflectance characteristics of objects.
- A cheap and effective way to **spatially vary** surface reflectance.



From <http://img.tfd.com>

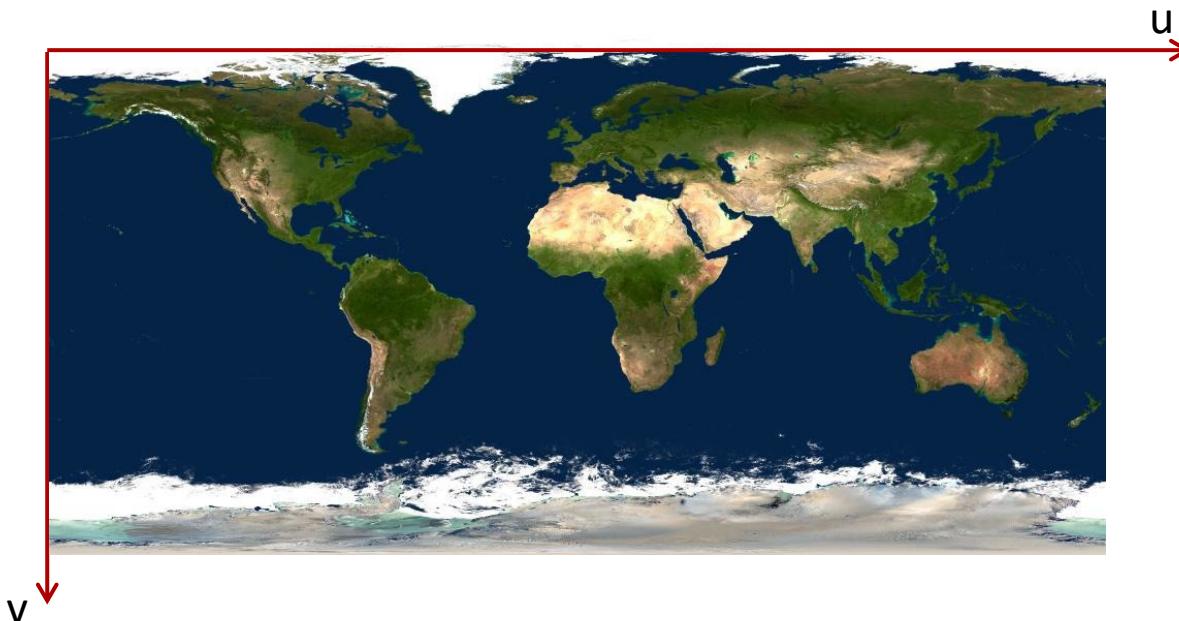
Texture Mapping

- Image (texture) sizes and object sizes may vary
- We need a uniform way so that any object can be mapped by any texture



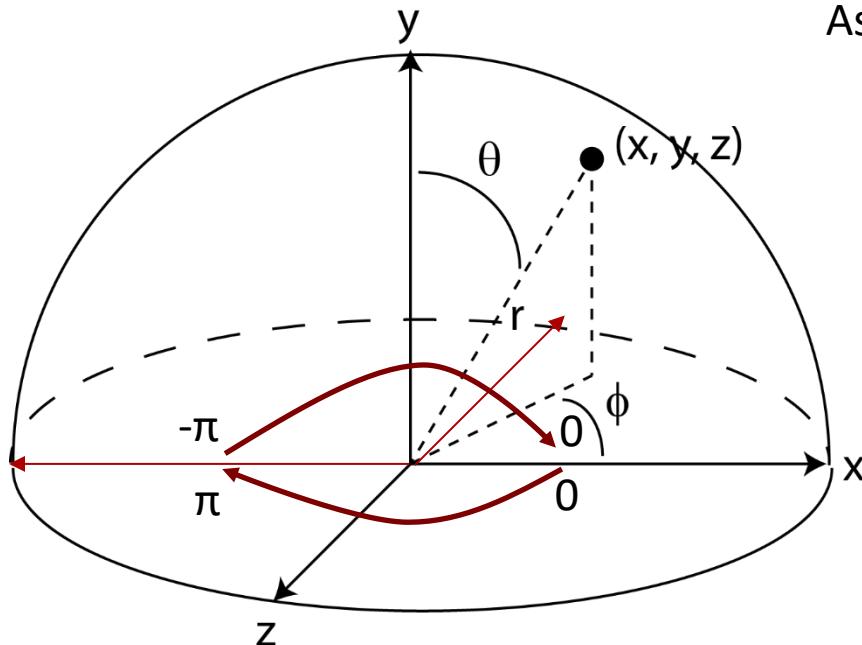
Texture Mapping

- **Step 1:** Associate a (u, v) coordinate system with the texture image where $(u, v) \in [0,1] \times [0,1]$



Texture Mapping

- **Step 2:** Parameterize the surface to be texture mapped using **two coordinates**. For instance, **a sphere**:



atan2(+0, x) returns $+\pi$ for $x < 0$.

atan2(-0, x) returns $+0$ for $x > 0$.

Assuming that the center is at $(0, 0, 0)$:

$$x = r\sin\Theta\cos\varphi$$

$$y = r\cos\Theta$$

$$z = r\sin\Theta\sin\varphi$$

$$\Theta = \arccos(y / r)$$

$$\varphi = \arctan(z / x)$$

In practice, $\varphi = \text{atan2}(z, x)$

$$(\Theta, \varphi) \in [0, \pi] \times [-\pi, \pi]$$

Texture Mapping

- Assume you want to wrap the image such that:

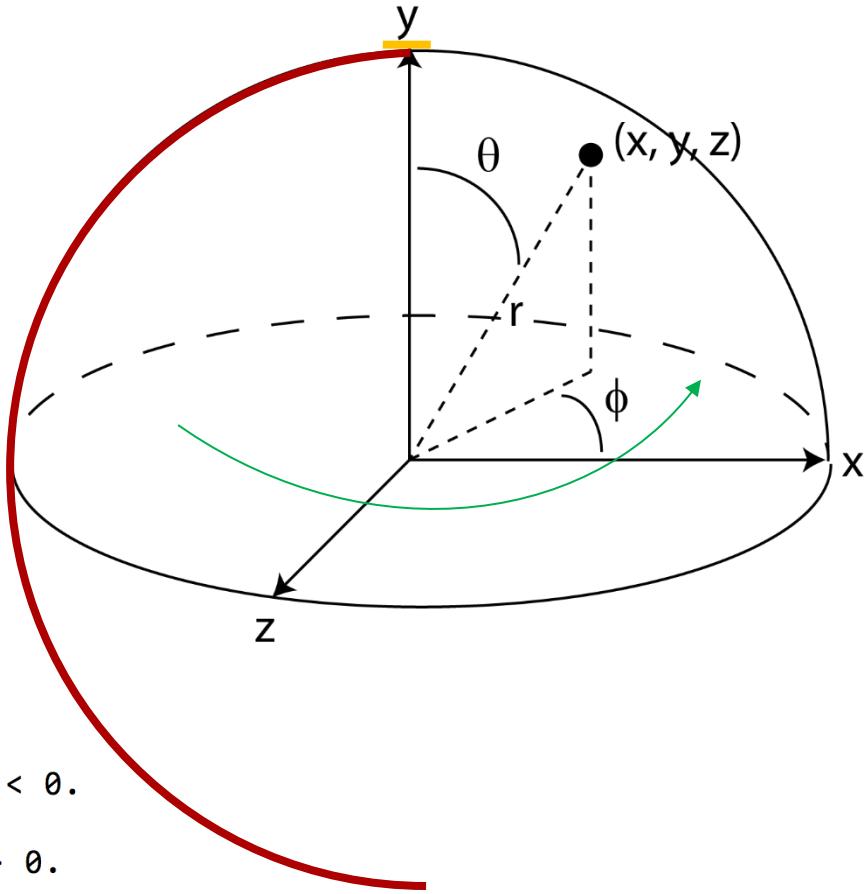


$$u = (-\phi + \pi) / (2\pi)$$

$$v = \Theta / \pi$$

atan2($\pm 0, x$) returns $\pm \pi$ for $x < 0$.

atan2($\pm 0, x$) returns ± 0 for $x > 0$.



Texture Mapping

- **Step 3:** Compute a (u, v) value for every surface point. For instance, a sphere:

$$u = (-\varphi + \pi) / (2\pi)$$

$$v = \Theta / \pi$$

- **Step 4:** Find the texture image coordinate (i, j) at the given (u, v) coordinate:

$$i = u \cdot n_x$$

$$j = v \cdot n_y$$

Note that i, j can be fractional!

n_x = texture image width

n_y = texture image height

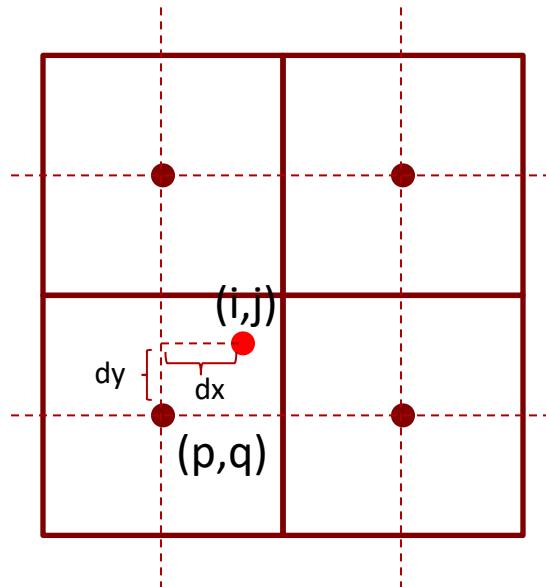
Texture Mapping

- **Step 5:** Choose the **texel** color using a suitable **interpolation strategy**

- **Nearest Neighbor:** fetch texel at nearest coordinate (faster)

$$\text{Color}(x, y, z) = \text{fetch}(\text{round}(i, j))$$

- **Bilinear Interpolation:** Average four closest neighbors (smoother):



$$p = \text{floor}(i)$$

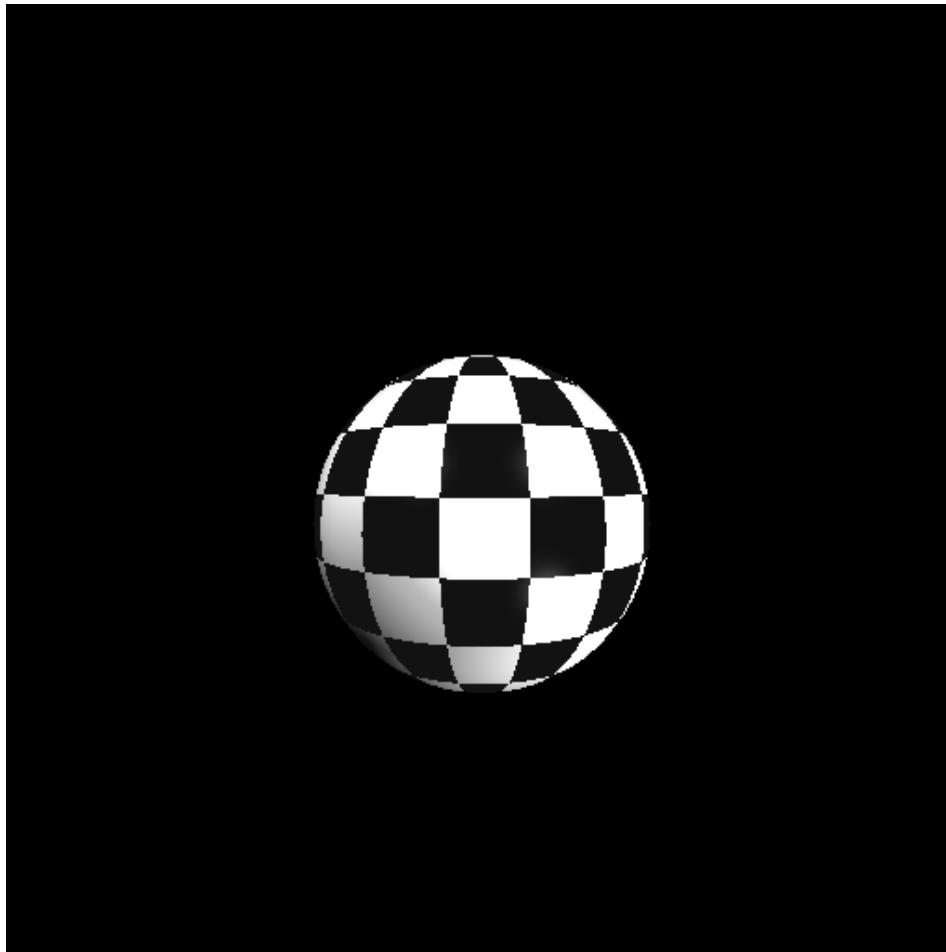
$$q = \text{floor}(j)$$

$$dx = i - p$$

$$dy = j - q$$

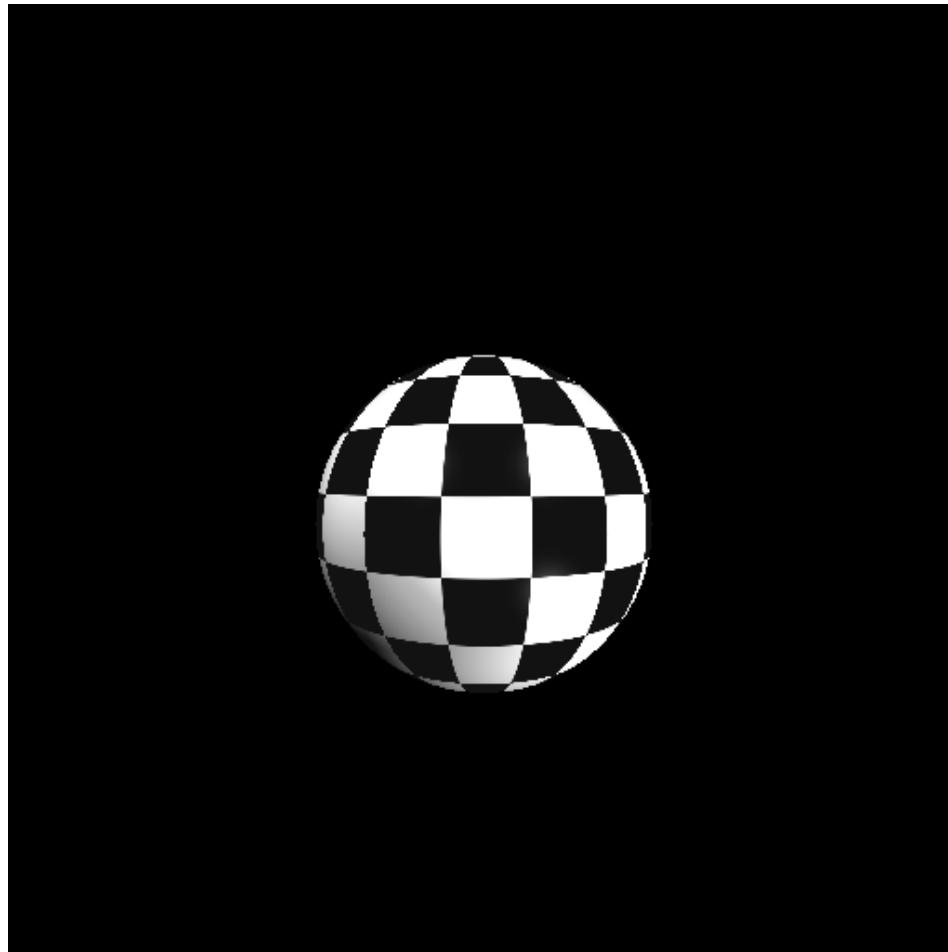
$$\begin{aligned}\text{Color}(x, y, z) = & \text{fetch}(p, q).(1 - dx).(1 - dy) + \\ & \text{fetch}(p+1, q).(dx).(1 - dy) + \\ & \text{fetch}(p, q+1).(1 - dx).(dy) + \\ & \text{fetch}(p+1, q+1).(dx).(dy)\end{aligned}$$

NN vs Bilinear Interpolation



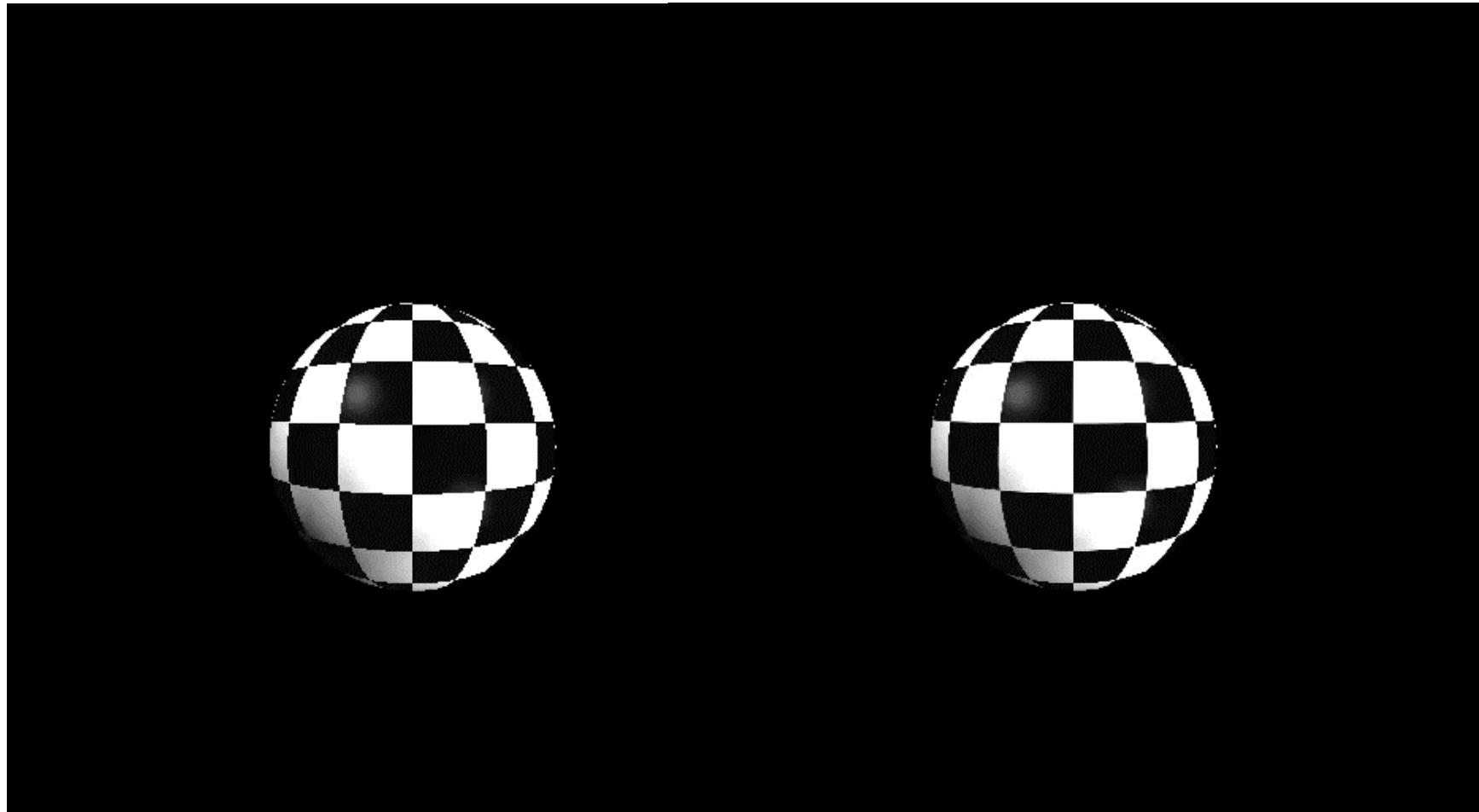
Nearest-neighbor

NN vs Bilinear Interpolation



Bilinear

NN vs Bilinear Interpolation

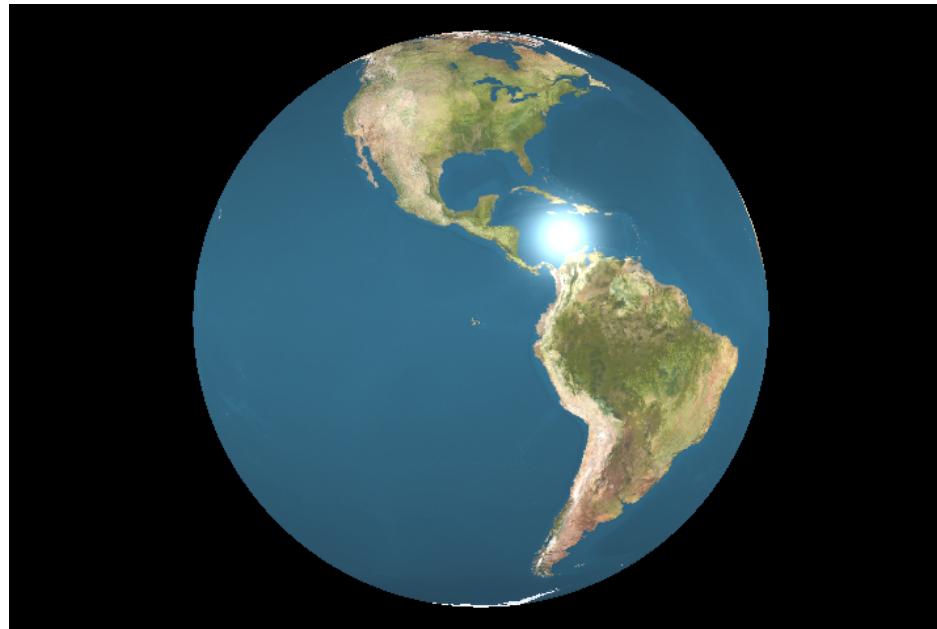


Nearest-neighbor

Bilinear

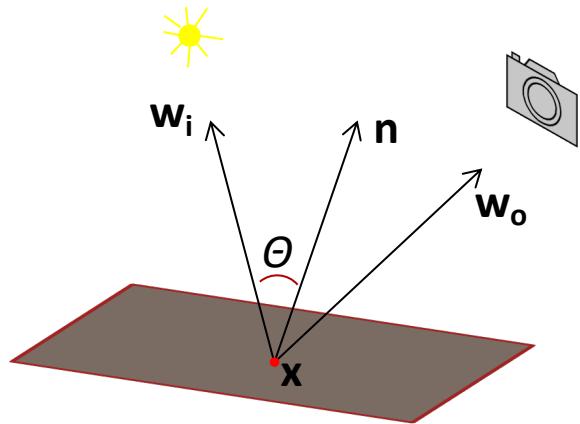
Texture Color

- Once we have the texture color, what to do with it?
- Several options possible: replace shading color, blend with shading color, replace k_d component, etc.



Texture Color

- For instance, to replace the k_d component:



$$L_o^d(x, wo) = k_d \cos\theta' E_i(x, w_i)$$

Outgoing
radiance

Received
irradiance

$$\cos\theta' = \max(0, w_i \cdot n)$$

where

$$k_d = \text{textureColor} / 255$$

Texture Color

- Replacing k_d typically improves realism over replacing the entire diffuse color:



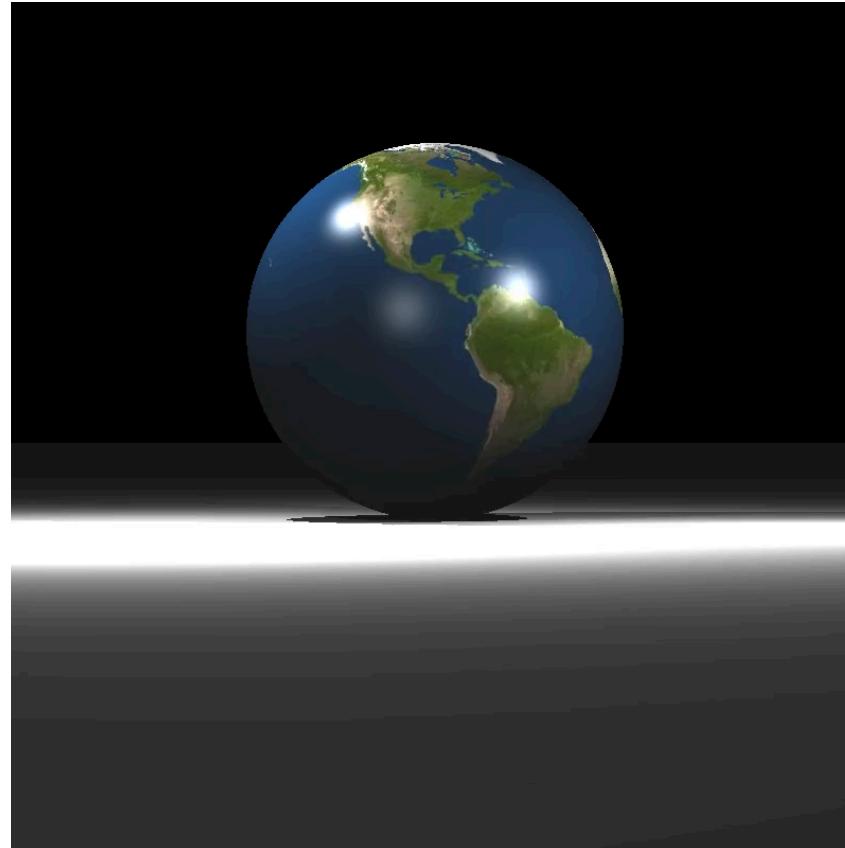
Diffuse color replaced



k_d component replaced

Animation

- Animation, shading, and texture mapping enhances realism:



Texture Mapping Triangles

- First, we must associate (u, v) coordinates for each vertex
- The (u, v) coordinates inside the triangle can be found using the **barycentric coordinates**
- Recall that the position of point p at **barycentric coordinates** (β, γ) is equal to:

$$p(\beta, \gamma) = a + \beta(b - a) + \gamma(c - a)$$

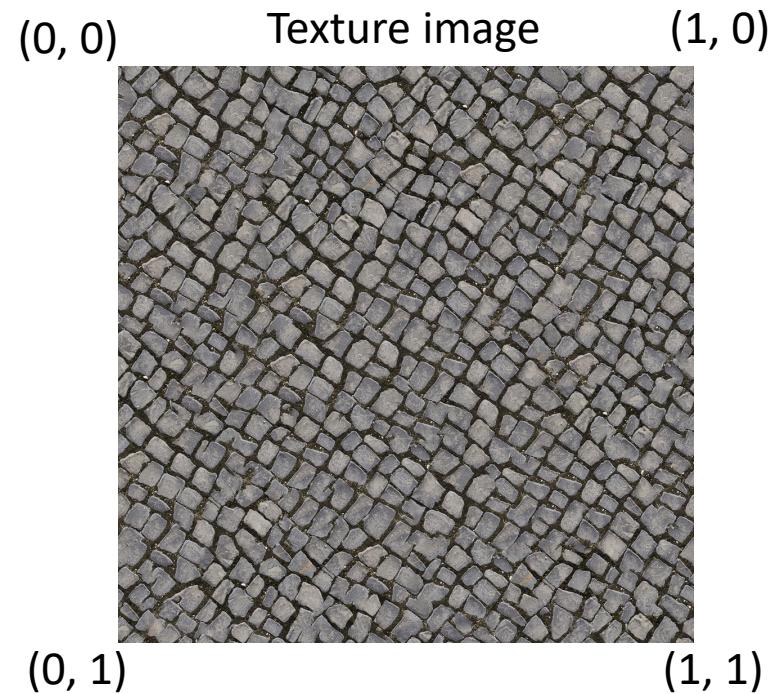
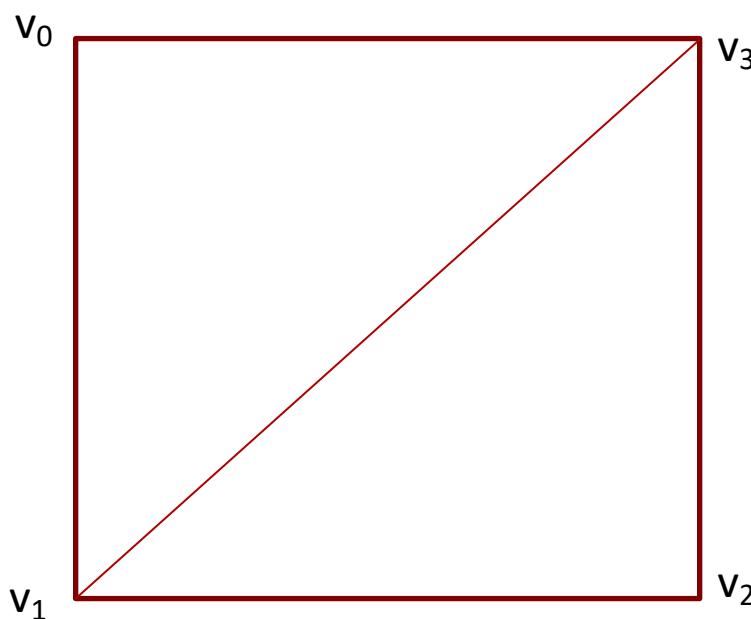
- Texture coordinates can be interpolated similarly:

$$u(\beta, \gamma) = u_a + \beta(u_b - u_a) + \gamma(u_c - u_a)$$

$$v(\beta, \gamma) = v_a + \beta(v_b - v_a) + \gamma(v_c - v_a)$$

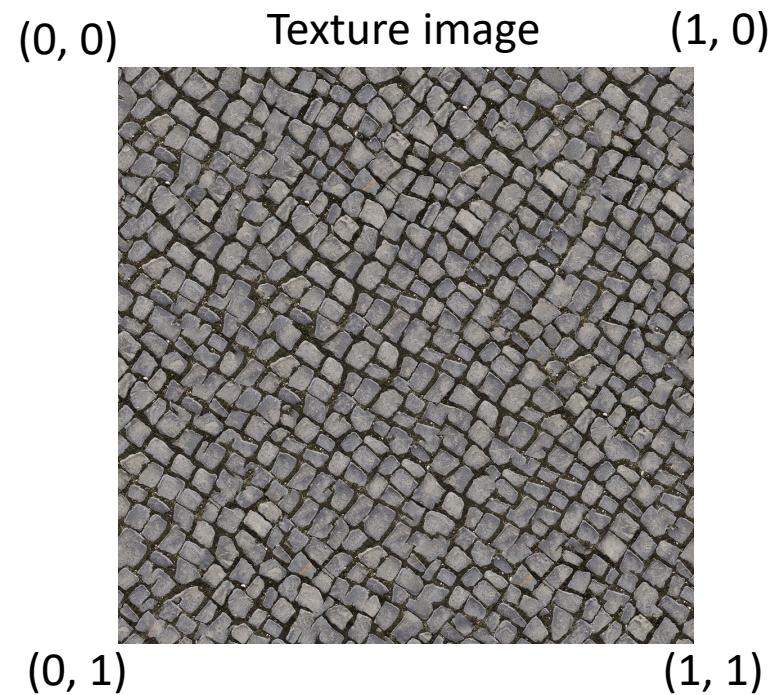
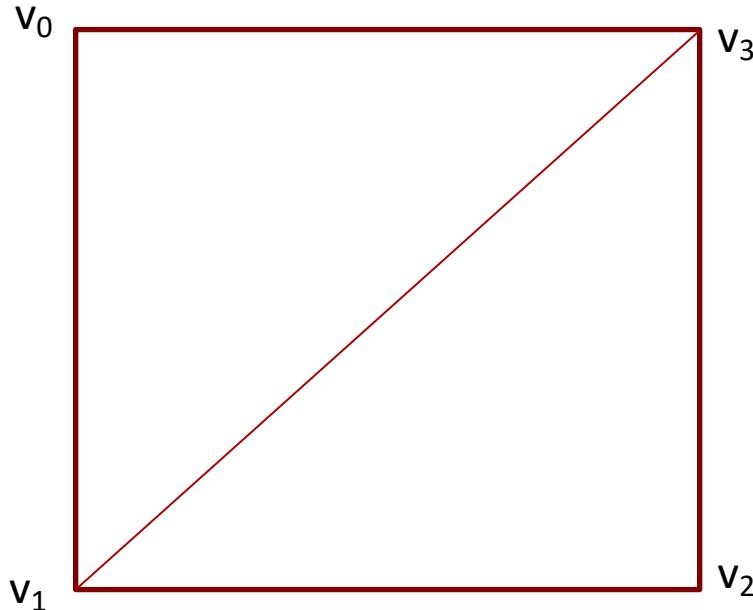
Texture Mapping Triangles

- Assume we want the texture map a rectangle made of two triangles:
 - v_3, v_1, v_2
 - v_1, v_3, v_0



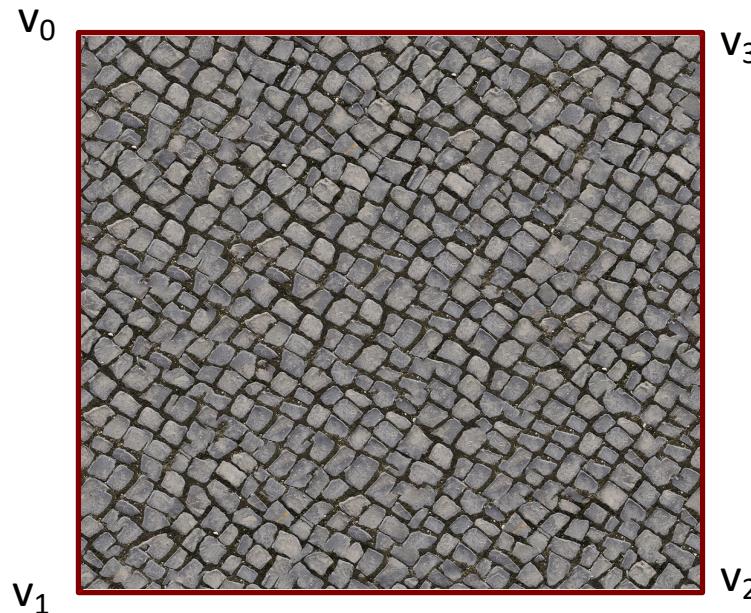
Texture Mapping Triangles

- We associate a texture coordinate with each vertex of each triangle:
 - v_3, v_1, v_2 : $(1, 0), (0, 1), (1, 1)$
 - v_1, v_3, v_0 : $(0, 1), (1, 0), (0, 0)$



Texture Mapping Triangles

- This way, the texture image gets stretched to match the triangles' positions
 - v_3, v_1, v_2 : $(1, 0), (0, 1), (1, 1)$
 - v_1, v_3, v_0 : $(0, 1), (1, 0), (0, 0)$



Combined Result



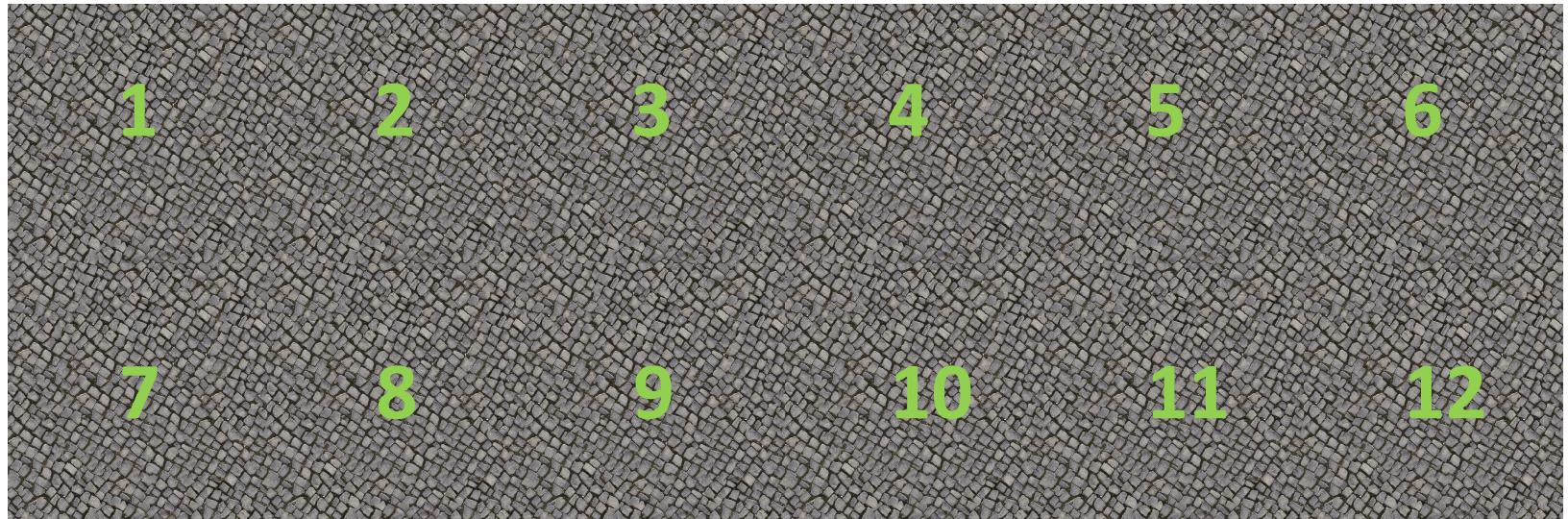
Tiling

- For large surfaces, stretching may excessively distort the texture image making it unrealistic



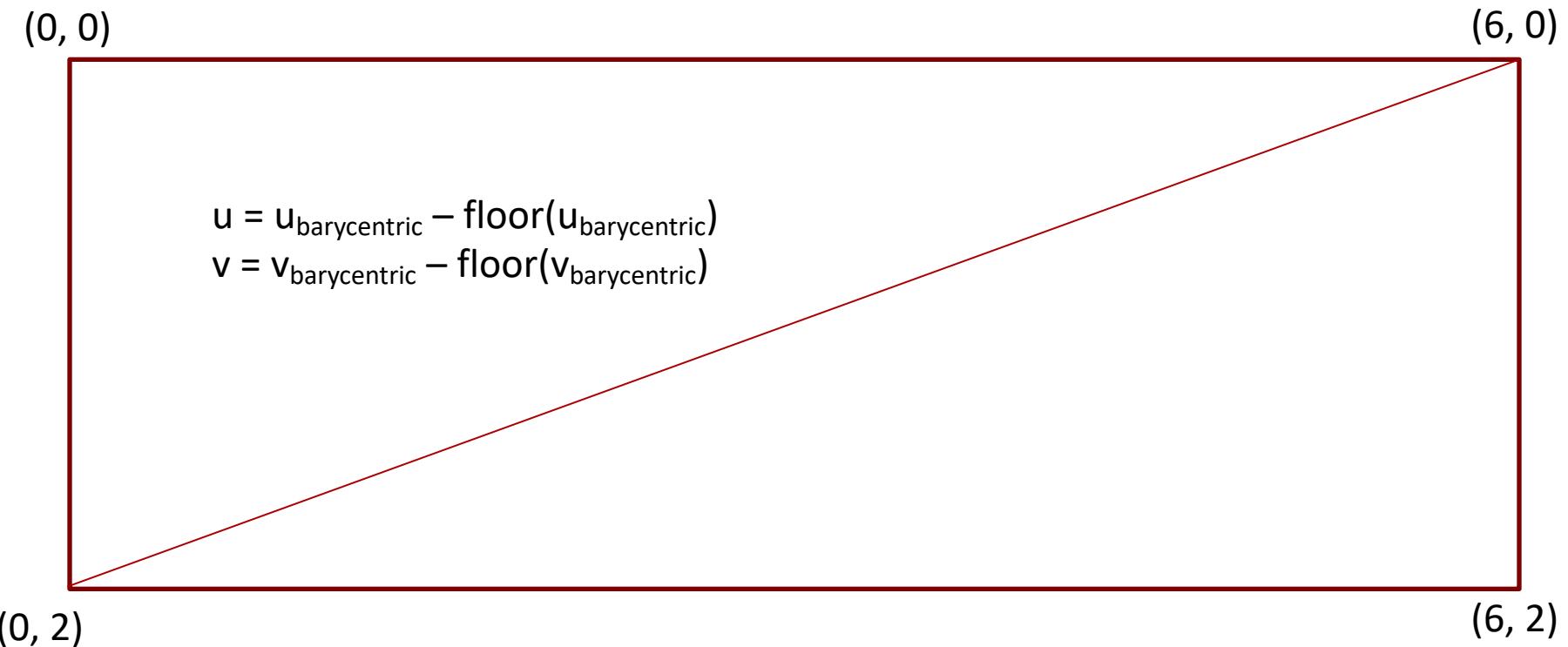
Tiling

- Tiling is the process of repeating a texture instead of stretching
- Multiple copies of the texture fill the surface
- Tiling may also look unrealistic if repetition is clear



Tiling

- To support tiling, tex. coords. are not limited to [0, 1] range



Textures Atlases

- Most objects in CG will have a texture applied to it
- This requires using multiple textures, sometimes thousands or more, for a moderately complex environment
- Many times text is also retrieved from texture images
- Instead of using multiple texture, a texture atlas combines textures of many objects into a single image

Textures Atlases



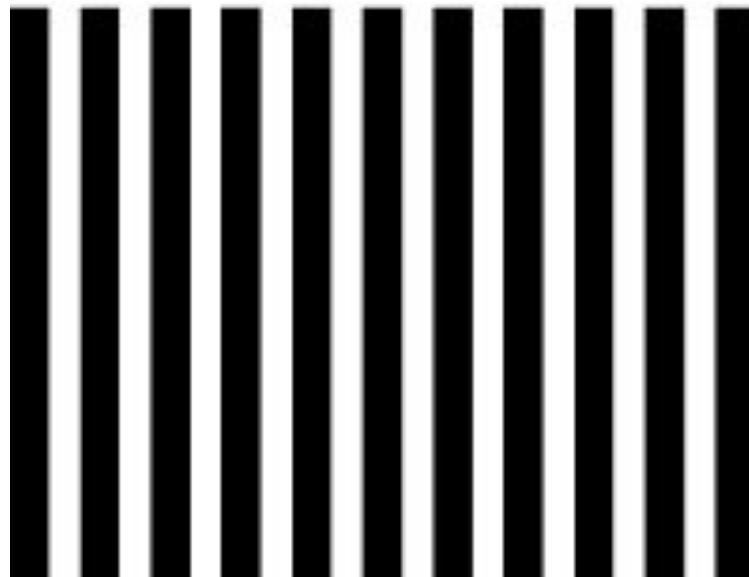
<http://jeapostrophe.github.io/2013-05-06-texture--post.html>

Procedural Textures

- The previous approach requires us to use an image as the texture to be mapped
- We can also achieve spatial variation without using any texture image at all
- **Solution:**
 - Generate textures procedurally for each surface point
- **Problem:**
 - How to make them look natural?

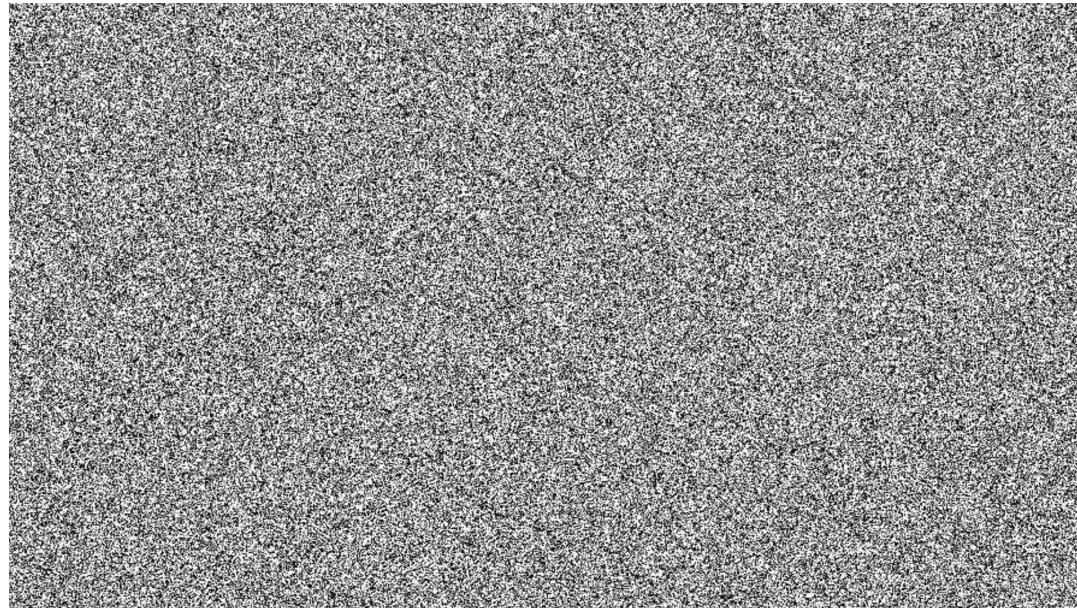
Procedural Textures

- Periodic patterns are easy to generate but they do not look natural (e.g. evaluate a sine function at every point):



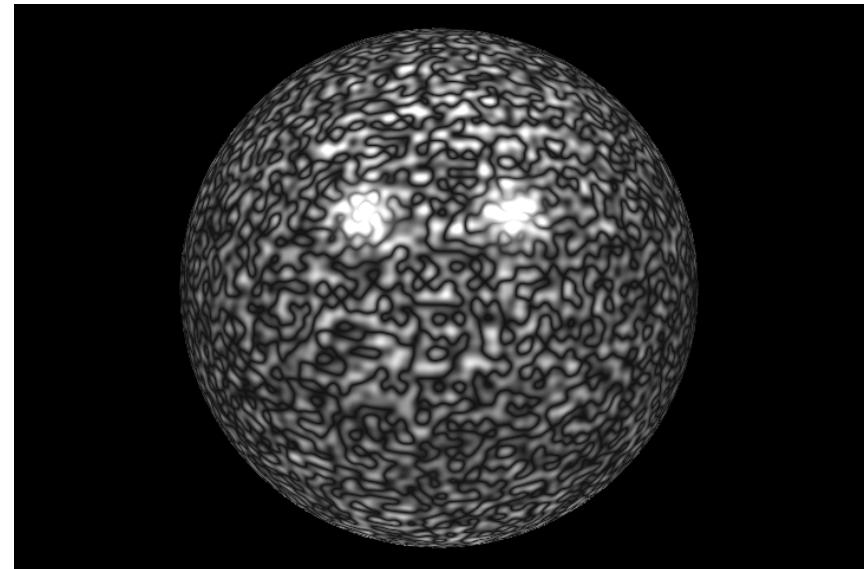
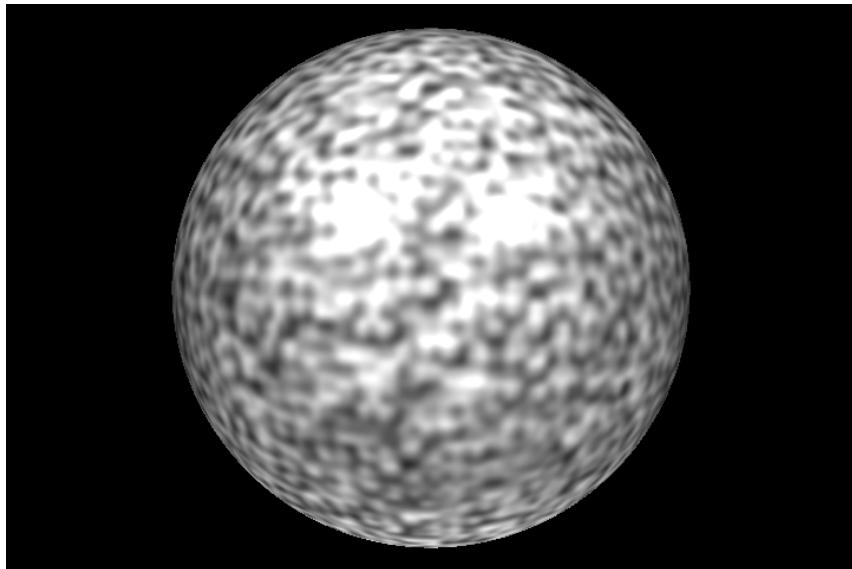
Procedural Textures

- We need a function that returns a **random value** for each (x, y, z) point in space
- However, **too random** is not good as it will appear as **noise**:



Procedural Textures

- There are various techniques such as **Perlin noise** that allow on-the-fly computation of controlled random patterns



Procedural Textures

- Image and procedural textures can be used together:

