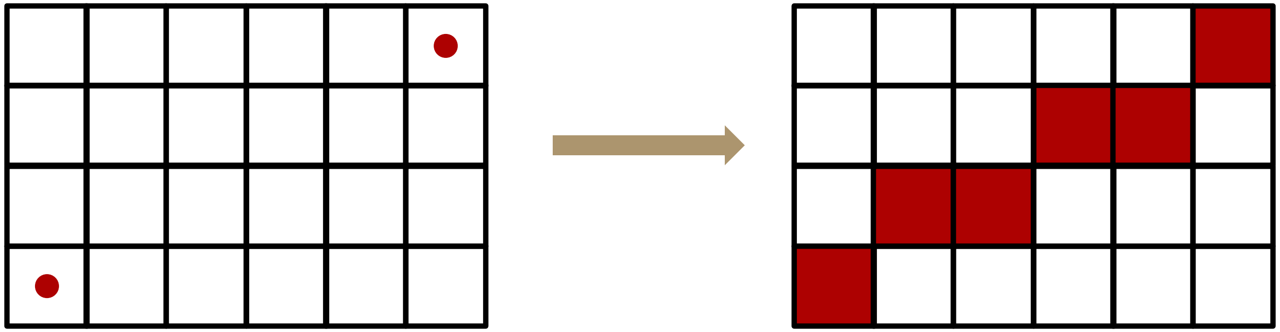
CENG – 477
Introduction to Computer Graphics

Rasterization

Rasterization

- Rasterization is concerned with creating fragments from vertices
- It works in screen coordinates – thus it is the next step after the viewport transform
- **Goal:** Given a set of vertices which fragments must be "turned on" to create the primitive:



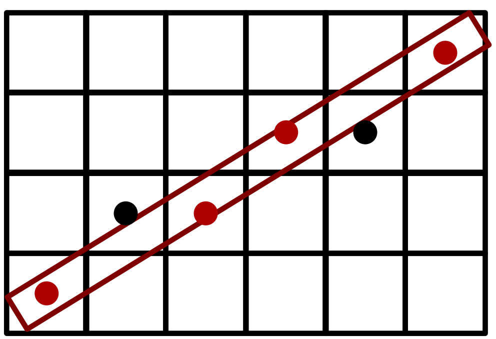
Rasterization

- **Subproblems:**
  - How to deal with different primitives
  - How to make it fast
  - How to interpolate color and other attributes
- We'll start with line rasterization

Line Rasterization

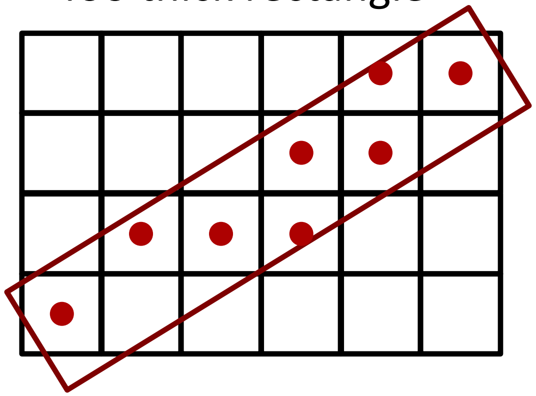
- Several methods exist
- **Option 1:** Treat the line as a thin rectangle and turn on pixels that are inside this rectangle

Too thin rectangle



- May cause gaps in the line

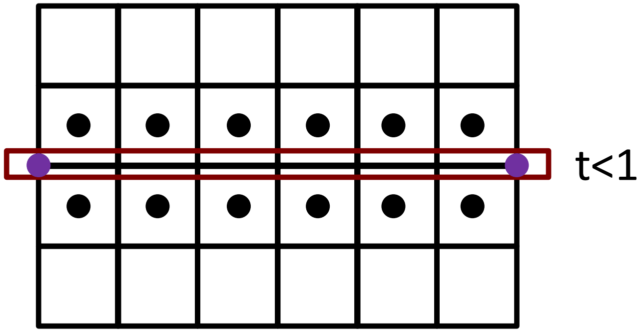
Too thick rectangle



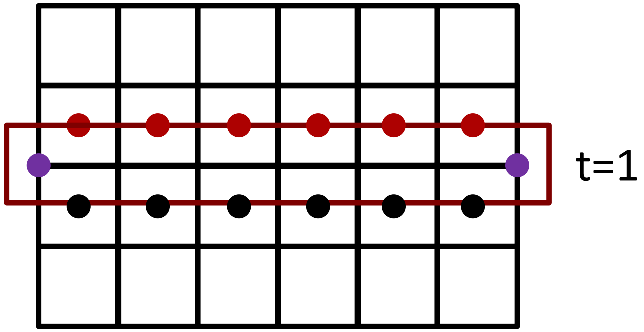
- Line thickness varies

Line Rasterization

- What must be the minimum thickness?



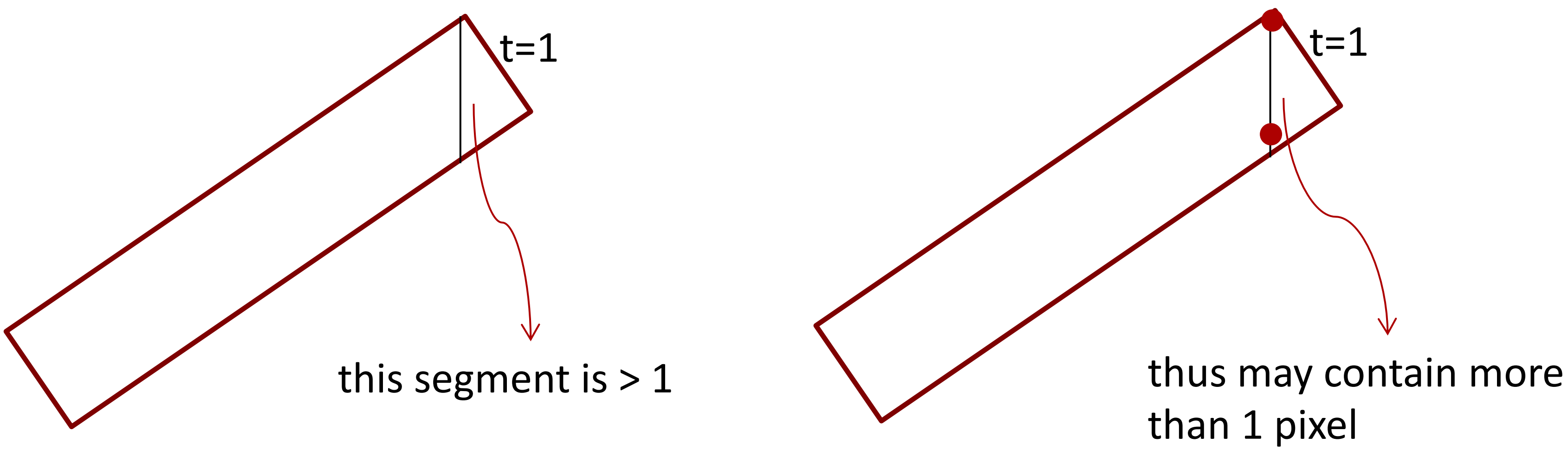
- May not draw anything if it is too thin



- Must be at least one to draw a horizontal line in some cases

Line Rasterization

- But a thickness of 1 may result in too thick lines

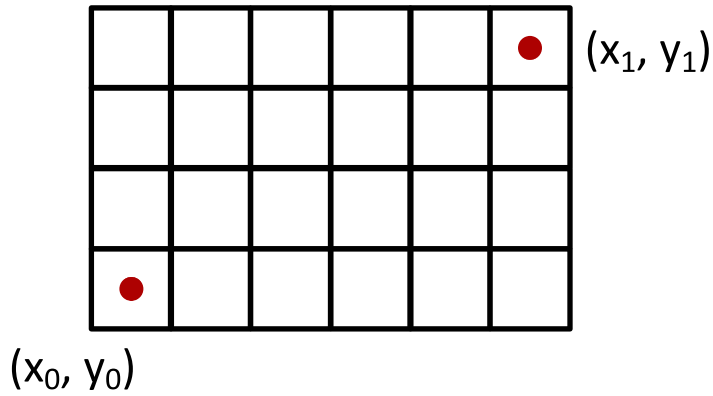


Line Rasterization

- Therefore we need another solution
- We can use line equations to decide which pixels belong the line
- Assume that we want to draw a line between two screen coordinates: (x_0, y_0) to (x_1, y_1)
- Assume that the slope of the line, *m*, is in range (0, 1]

$$m = \frac{y_1 - y_0}{x_1 - x_0}$$

Which pixels should we draw?

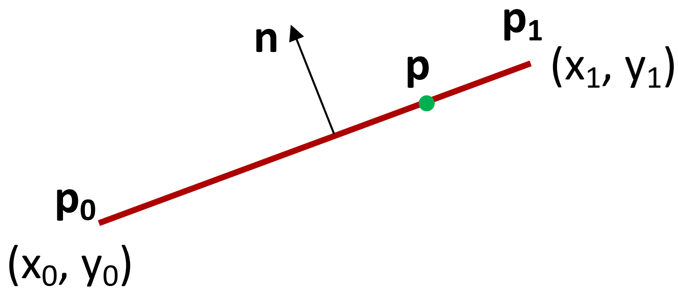


Line Equation

- Let's first remember the implicit line equation:

$$f(x, y) = x(y_0 - y_1) + y(x_1 - x_0) + x_0 y_1 - y_0 x_1$$

- We can derive it from geometry:



$$\boldsymbol{n}.[x_1 - x_0, y_1 - y_0]^T = 0$$

$$\boldsymbol{n} = [y_0 - y_1, x_1 - x_0]$$

$$f(\boldsymbol{p}) = (\boldsymbol{p} - \boldsymbol{p_0}).\boldsymbol{n} = 0$$

What is the meaning of f(**p**) = 0, f(**p**) > 0 and f(**p**) < 0

The Basic Algorithm

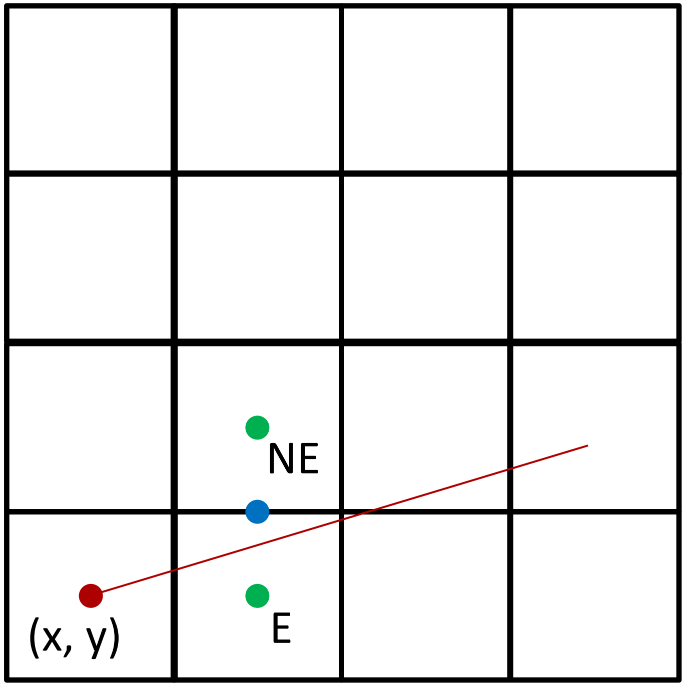
- The basic algorithm is as follows:

> $y = y_0$
> **for** $x = x_0$ to x_1 **do**:
>   draw(x, y)
>   **if** (some condition) **then**:
> $y = y + 1$

- Because the slope is in (0, 1], we always go right and sometimes go up (assuming that $x_0 < x_1$ and $y_0 < y_1$)

The Midpoint Algorithm

- Assume we have just drawn (x, y). Which pixel to draw next?



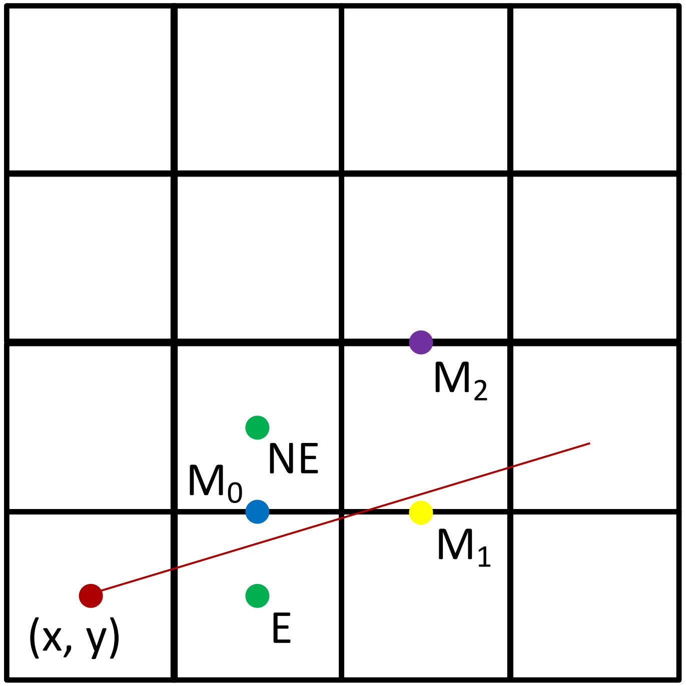
- Compute the midpoint of the next pixel
- If the midpoint is on or above the line choose the right pixel (E: east)
- If the midpoint is below the line, choose the top right pixel (NE: north-east)

$y = y_0$
for $x = x_0$ to x_1 **do**:
    draw(x, y)
    if $f(x+1, y+0.5) < 0$ **then**:
        $y = y + 1$

We call these pixels E and NE

The Midpoint Algorithm

- This algorithm works well but requires evaluating the line equation at every iteration
- Can be optimized with an incremental algorithm:



- If we selected E, in the next iteration we need the value of f(**M_1**)
- If we selected NE, in the next iteration we need f(**M_2**)
- Both can be computed from f(**M_0**)

$$f(M_1) - f(M_0) = y_0 - y_1$$

$$f(M_2) - f(M_0) = (y_0 - y_1) + (x_1 - x_0)$$

The Midpoint Algorithm

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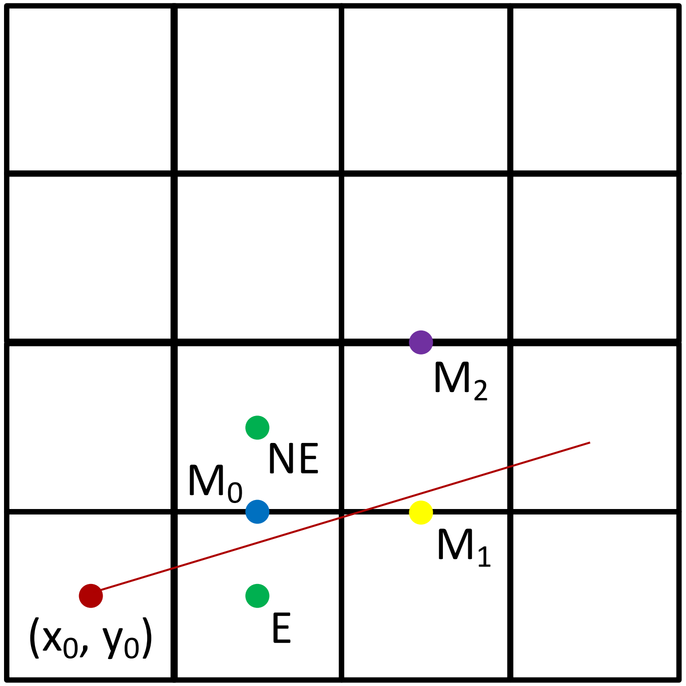

- In other words, if we know f(**M**), we can compute the next f(**M**) by simple integer arithmetic
- What is the first f(**M**)?
- Note that $f(x_0, y_0) = 0$ as it is the starting point of the line

$$f(M_0) = f(x_0 + 1, y_0 + 0.5)$$

$$= (y_0 - y_1) + 0.5(x_1 - x_0)$$

The Midpoint Algorithm

- So the overall algorithm is:

$y = y_0$
$d = (y_0 - y_1) + 0.5(x_1 - x_0)$
for $x = x_0$ to x_1 **do:**
    draw(x, y)
    if $d < 0$ **then:** // choose NE
        $y = y + 1$
        $d \mathrel{+}= (y_0 - y_1) + (x_1 - x_0)$
    else: // choose E
        $d \mathrel{+}= (y_0 - y_1)$

The Midpoint Algorithm

- For max efficiency, $f(x, y) = 0$ is written as $2f(x, y) = 0$. This entirely eliminates floating point operations:

```
y = y_0
d = 2(y_0 – y_1) + (x_1 – x_0)
for x = x_0 to x_1 do:
    draw(x, y)
    if d < 0 then: // choose NE
        y = y + 1
        d += 2[(y_0 – y_1) + (x_1 – x_0)]
    else: // choose E
        d += 2(y_0 – y_1)
```

The Midpoint Algorithm

- The presented algorithm works when the slope of the line, *m*, is in range (0, 1]
- For other slope ranges, minor modifications to the algorithm is required
- For instance if $m \in (1, \infty)$, we need to swap the roles of x and y
- Other cases must be adapted similarly

Float vs Integer

- The midpoint algorithm was originally developed by Pitteway in 1967
- Floating point arithmetic was very expensive at the time
- Does it still matter?
- We implemented both algorithms and drew one million lines each between 1000 and 1400 pixels long:
  - Test run on Intel Core i7 CPU at 3.2 GHz
  - Compiled with g++ and –O2 option
  - Basic algorithm: 7.2 seconds
  - Optimized algorithm: 3 seconds
  - So it still makes a difference!

Interpolating Attributes

- What if the two end points of the line have a different color?
- The color across the line must smoothly change:

Interpolating Attributes

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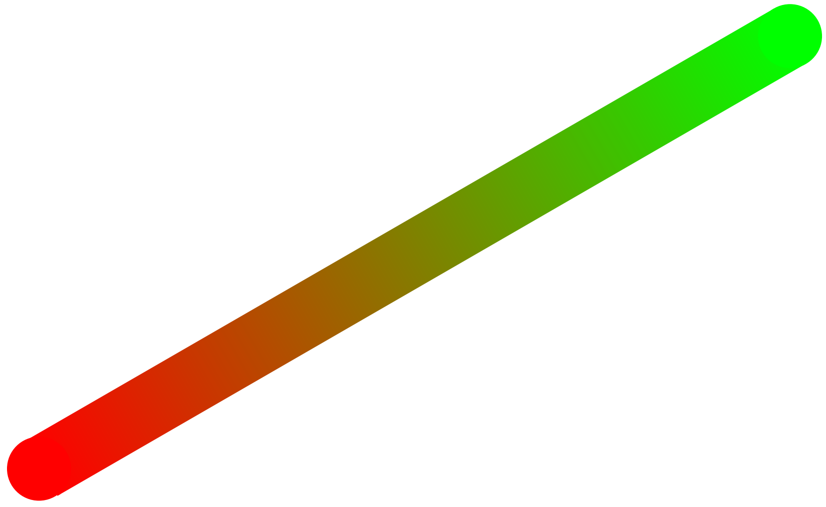
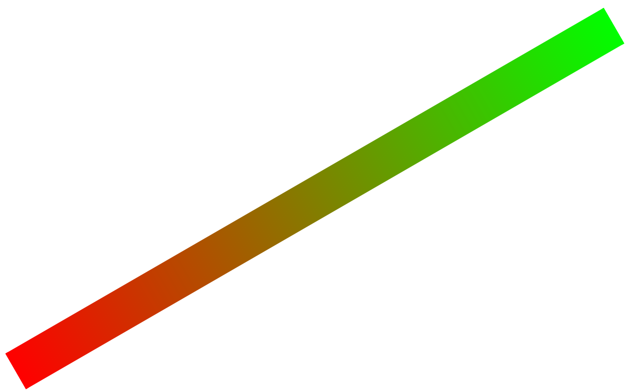
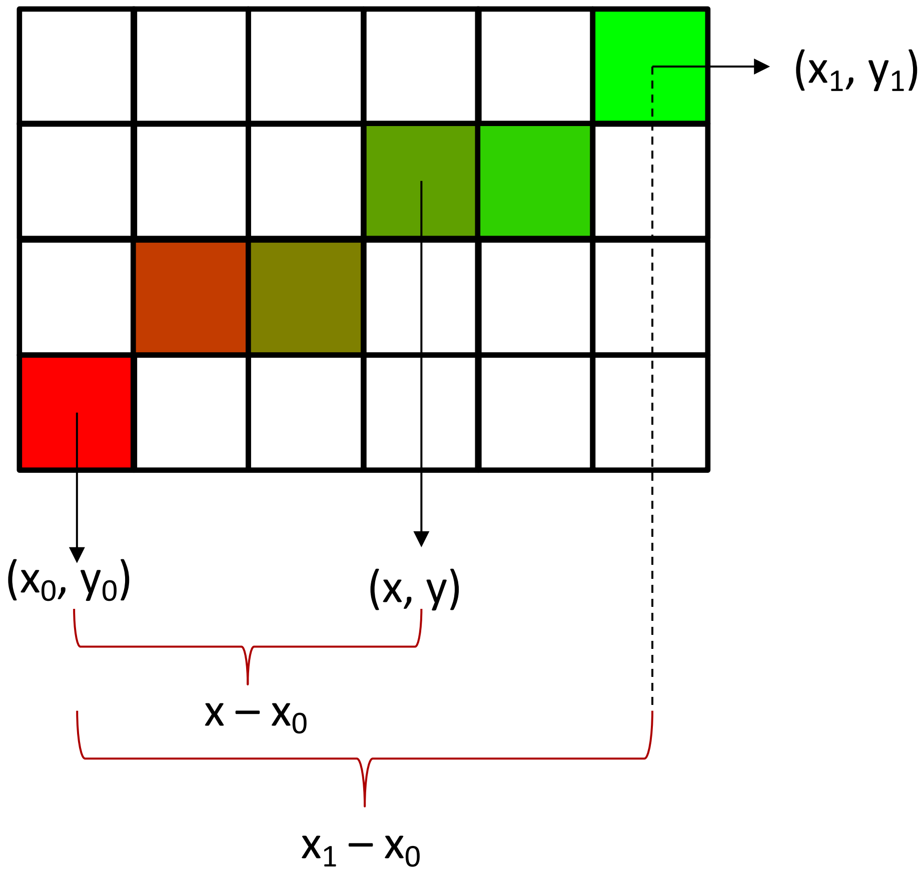


Interpolating Attributes





Interpolating Attributes

- Assume that the color of the endpoints are c_0 and c_1
- The color of an intermediate point should be:

$$c = (1 - \alpha)c_0 + \alpha c_1$$

where α is the interpolation variable

- At any pixel (x, y), we can compute it based on the horizontal or vertical distance of the pixel to the first endpoint:

$$\alpha = \frac{x - x_0}{x_1 - x_0}$$

For $m \in [0,1]$ it is better to interpolate in the x direction as as it will produce a smoother variation

Interpolating Attributes

- Assume that the color of the endpoints are c_0 and c_1
- The color of an intermediate point should be:

$$c = (1 - \alpha)c_0 + \alpha c_1$$

where α is the interpolation variable

- It can also be computed incrementally

$$\alpha(x_0 + 1) = \alpha_1 = \frac{1}{x_1 - x_0}$$

$$\alpha(x_0 + 2) = \alpha_2 = \alpha_1 + \frac{1}{x_1 - x_0}$$

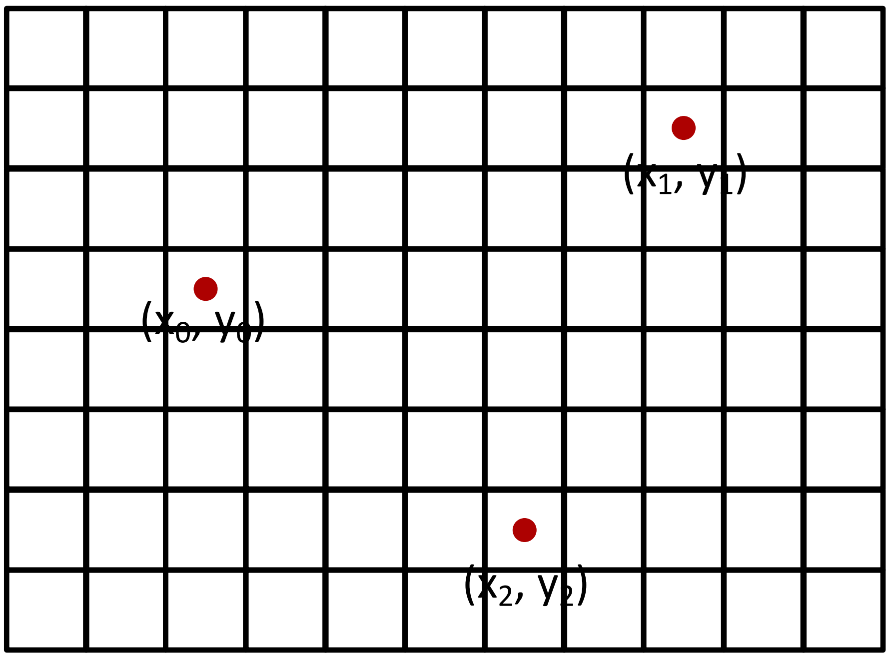
Algorithm with Interpolation

- Algorithm with color interpolation:

$y = y_0$
$d = (y_0 - y_1) + 0.5(x_1 - x_0)$
$c = c_0$
$dc = (c_1 - c_0) / (x_1 - x_0)$ // skip α; directly compute color increment
for $x = x_0$ to x_1 **do:**
    draw(x, y, round(c))
    if $d < 0$ **then:** // choose NE
        $y = y + 1$
        $d \mathrel{+}= (y_0 - y_1) + (x_1 - x_0)$
    else: // choose E
        $d \mathrel{+}= (y_0 - y_1)$
    $c \mathrel{+}= dc$

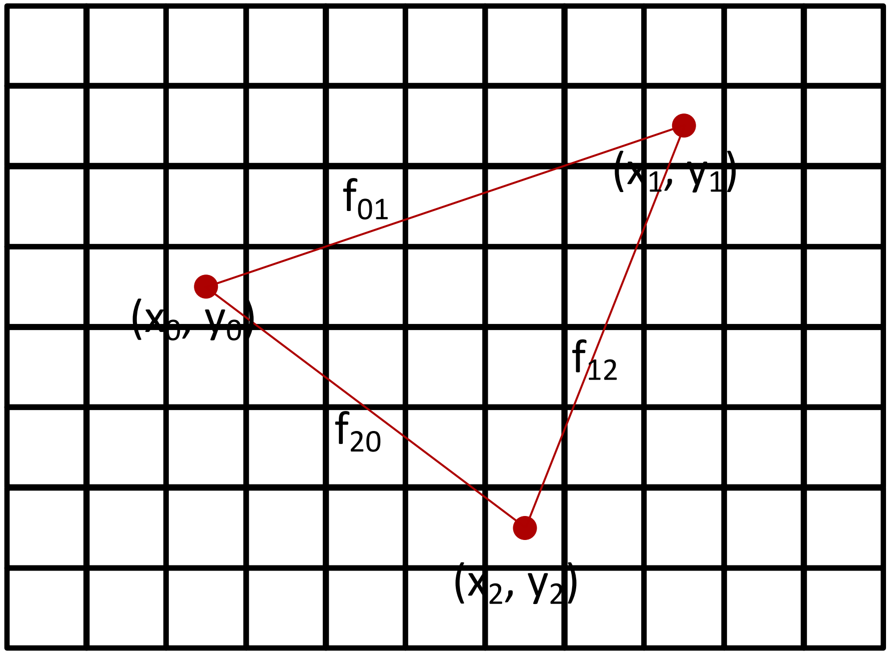
Triangle Rasterization

- Initially all we have is the screen coordinates of three vertices



Triangle Rasterization

- From them we compute line equations for the edges:



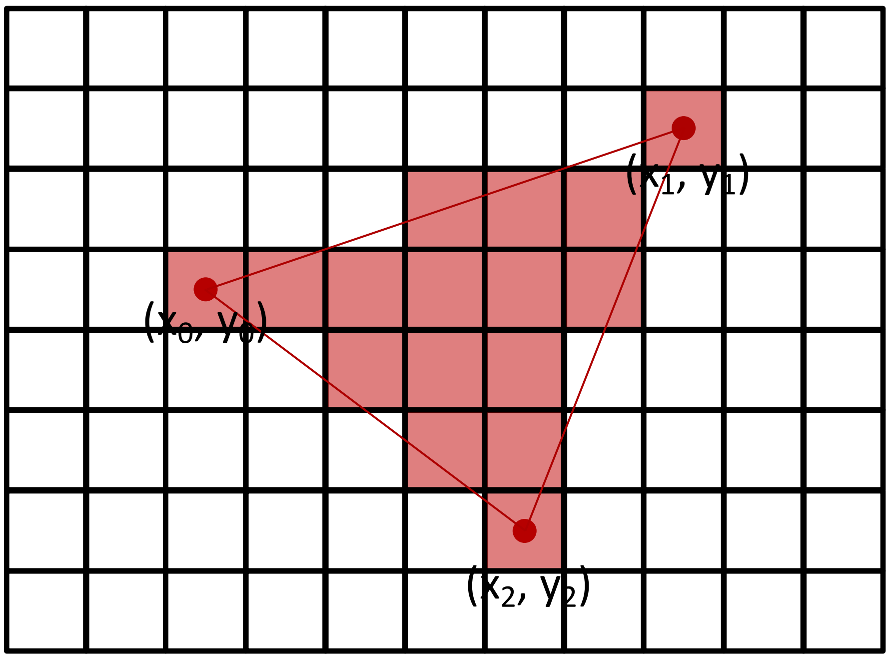
$$f_{01}(x,y) = x(y_0 - y_1) + y(x_1 - x_0) + x_0y_1 - y_0x_1$$

$$f_{12}(x,y) = x(y_1 - y_2) + y(x_2 - x_1) + x_1y_2 - y_1x_2$$

$$f_{20}(x,y) = x(y_2 - y_0) + y(x_0 - x_2) + x_2y_0 - y_2x_0$$

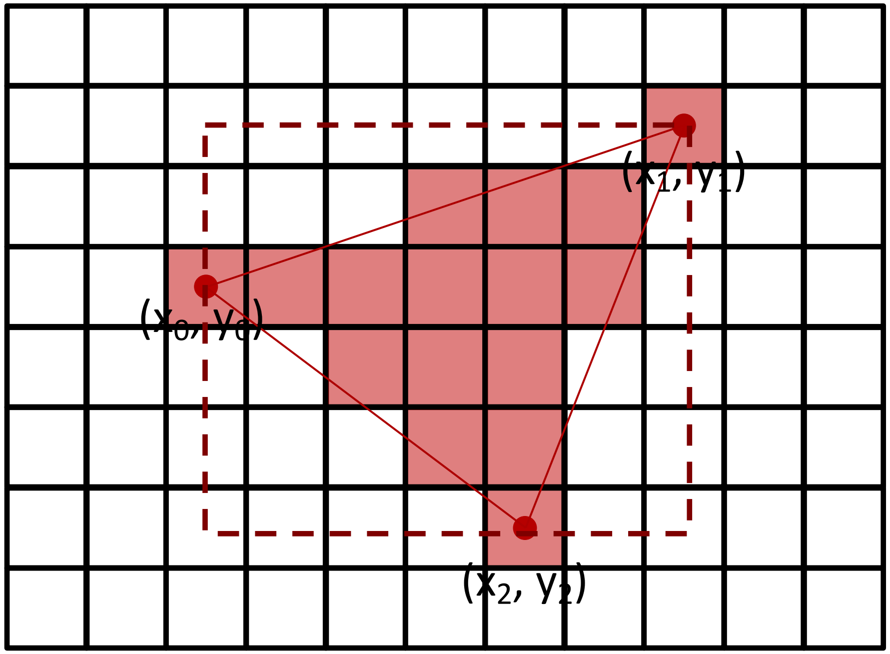
Triangle Rasterization

- We then walk across the viewport to determine inside pixels:



Triangle Rasterization

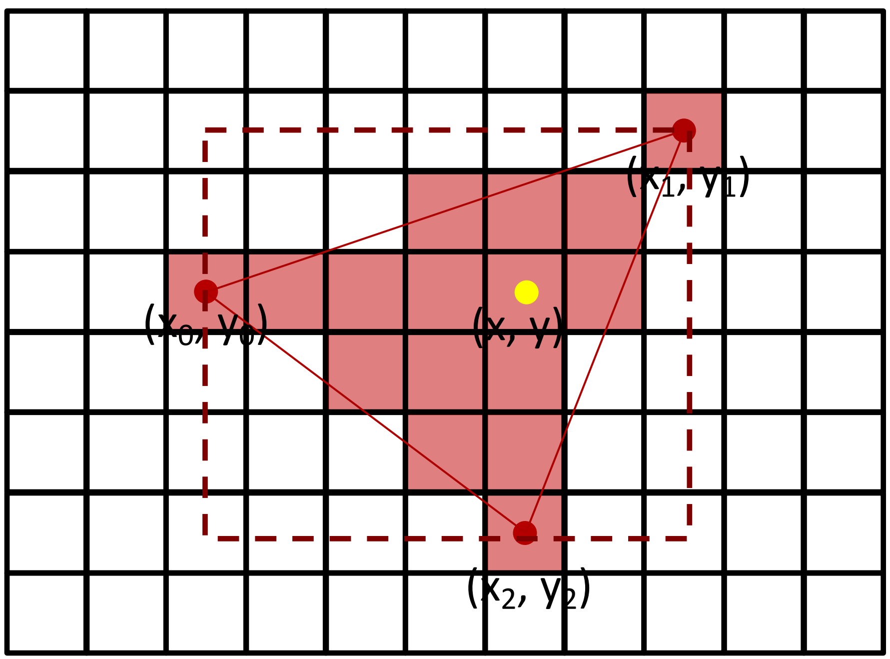
- For efficiency we may only walk within the bounding box of the triangle:



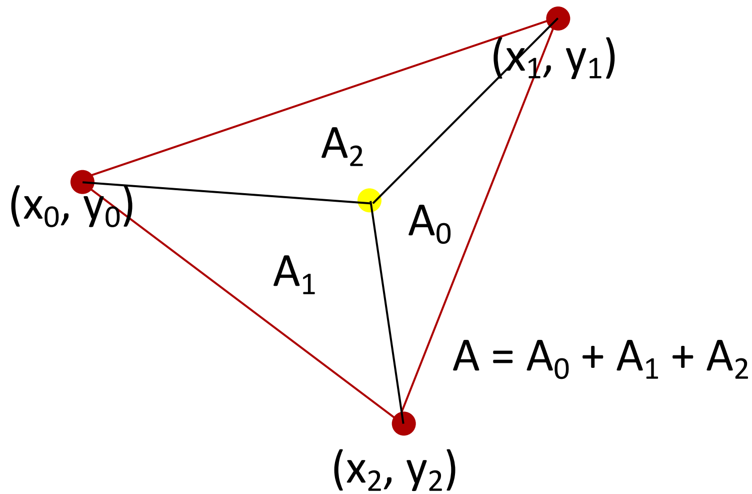
- For each pixel we visit, we must make an inside test with respect to all three edges
- We can simply plug-in the (x, y) value of the visited pixel to each line equation
- If all are negative, the pixel is inside the triangle

Triangle Rasterization

- However, using barycentric coordinates will help us with interpolation of attributes



What are the barycentric coordinates of (x, y)?



$$\alpha = \frac{A_0}{A}, \beta = \frac{A_1}{A}, \gamma = \frac{A_2}{A}$$

Triangle Rasterization

- We don't actually need to compute the areas as the bases are shared and will cancel out

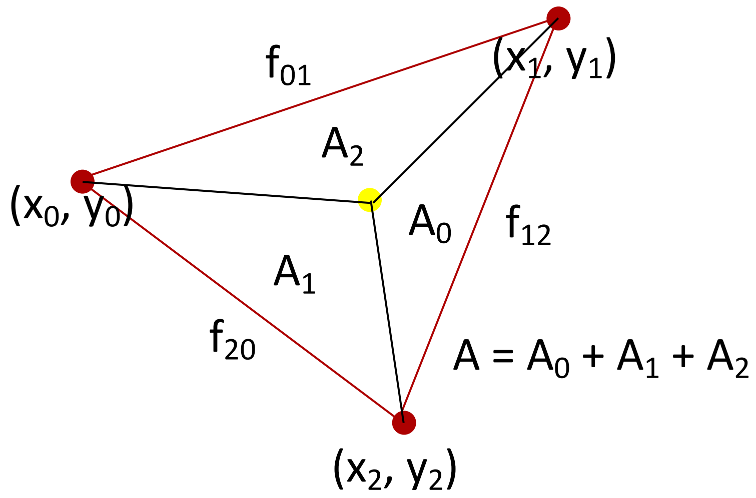
$$\alpha = \frac{A_0}{A}, \beta = \frac{A_1}{A}, \gamma = \frac{A_2}{A}$$

$$\alpha = \frac{f_{12}(x,y)}{f_{12}(x_0,y_0)}$$

$$\beta = \frac{f_{20}(x,y)}{f_{20}(x_1,y_1)}$$

$$\gamma = \frac{f_{01}(x,y)}{f_{01}(x_2,y_2)}$$

What are the barycentric coordinates of (x, y)?



Overall Algorithm

for y = y_{min} **to** y_{max} **do:**
  for x = x_{min} **to** x_{max} **do:**
    $\alpha = f_{12}(x, y)/f_{12}(x_0, y_0)$
    $\beta = f_{20}(x, y)/f_{20}(x_1, y_1)$
    $\gamma = f_{01}(x, y)/f_{01}(x_2, y_2)$
    if $\alpha \geq 0$ **and** $\beta \geq 0$ **and** $\gamma \geq 0$ **then:**
      c = $\alpha c_0 + \beta c_1 + \gamma c_2$
      draw(x, y, round(c))

- Note that the computation of the barycentric coordinates can also be made incremental for greater efficiency

Summary

- The result of rasterization is a set of fragments (pixel-to-be) for each primitive
- Each fragment has interpolated values of attributes:
  - Color values
  - Texture coordinates
  - Depth value
  - Normals
  - Or any user-defined attribute for vertices
- The next stage in the life of a fragment is to go through the fragment pipeline