CENG 477
Introduction to Computer Graphics

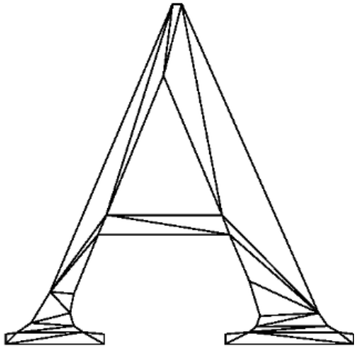
Data Structures for Graphics

Until Now

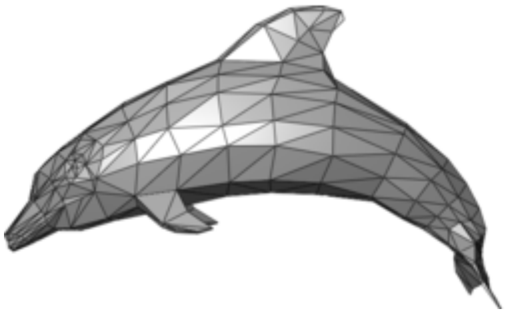
- We rendered virtual objects
  - Ray tracing
  - Ray are easy: $\mathbf{r}(t) = \mathbf{o} + d\mathbf{t}$
  - Mathematical objects also easy: $x^2 + y^2 + z^2 = R^2$
  - How about arbitrary objects embedded in 2D/3D scenes?
- Today we will learn about
  - explicitly representing those objects
    - triangle meshes
  - organizing them for efficiency
    - spatial structures

Triangle Meshes

- The most popular way of representing geometry embedded in 2D or 3D
- A triangle mesh is a piecewise linear surface representation



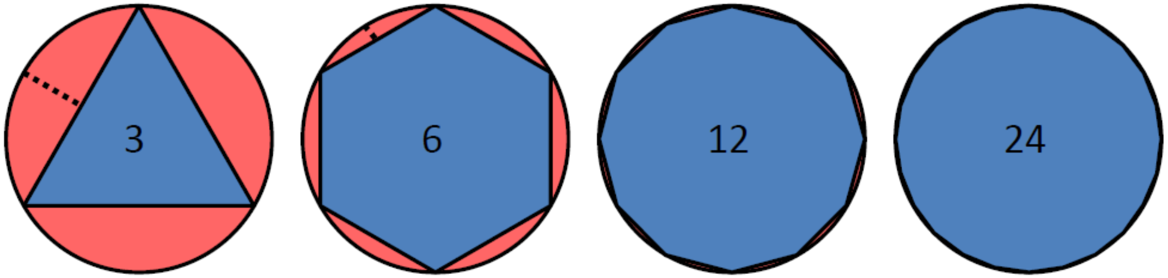
2D mesh

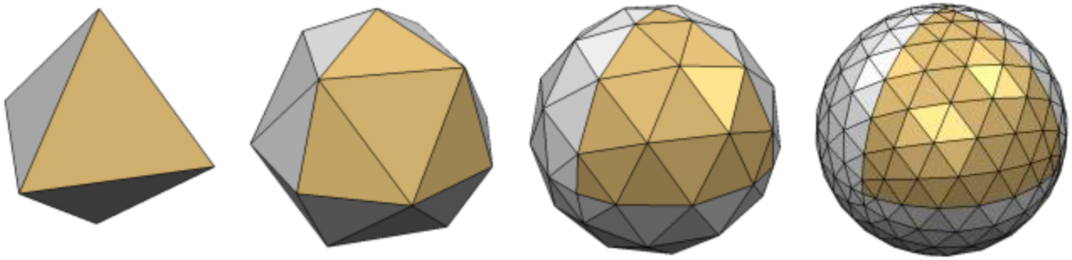


3D mesh

Triangle Meshes

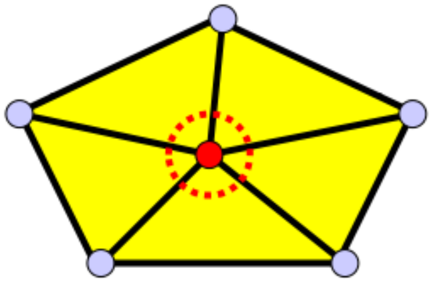
- Smoothness may be achieved by using a *larger* number of *smaller* pieces





Manifolds

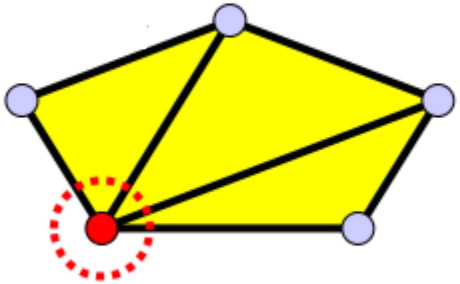
- Our meshes will be manifolds:
  – Each edge is shared by at most two faces



Closed fan

https://www.cs.mtu.edu/~shene

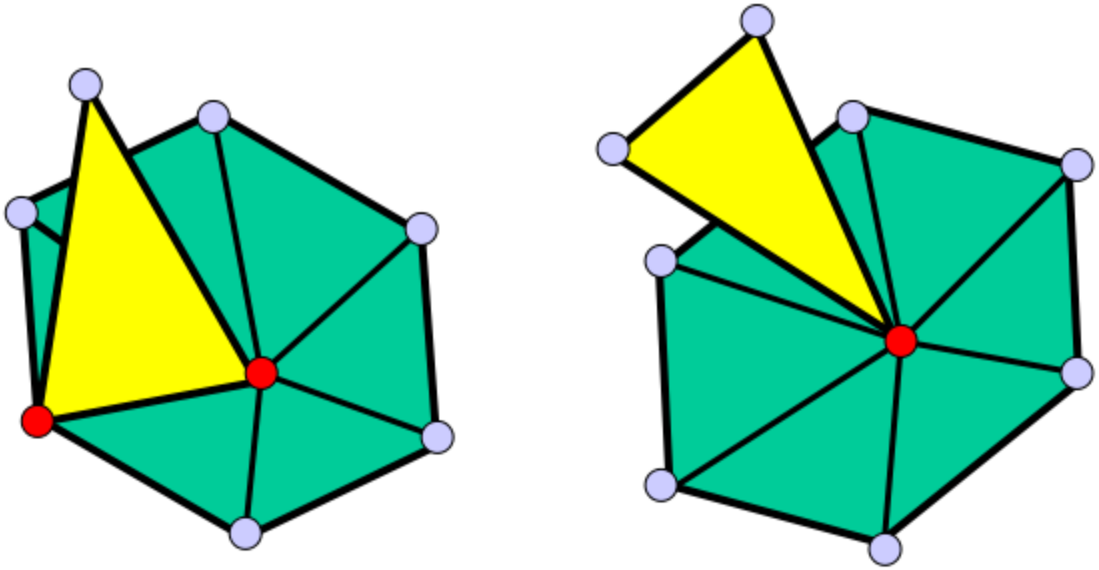
  – And faces containing a vertex form a closed or open fan:



Open fan

Non-manifolds

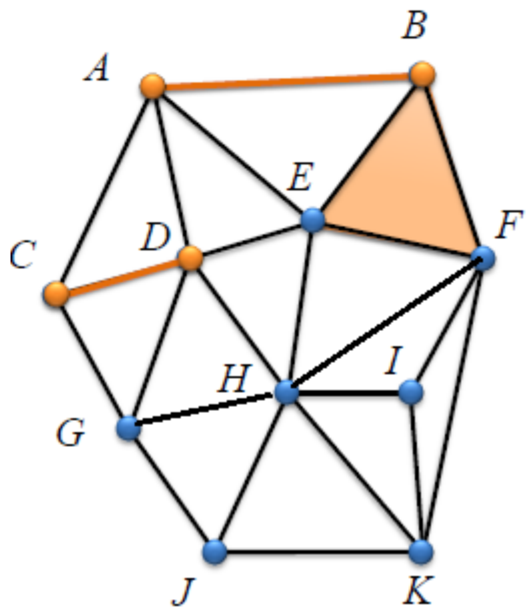
- The following meshes are non-manifolds:



https://www.cs.mtu.edu/~shene

Triangle Meshes

- A triangle mesh is an undirected graph with triangle faces
  - It has vertices, edges, and faces
  - The degree or valance of a vertex is the # of incident edges
  - A mesh is called k-regular if the degree of all vertices are k

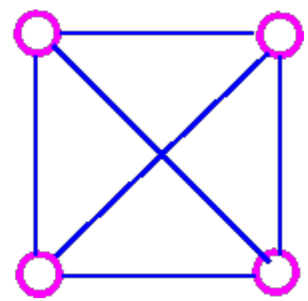


G = <V, E, F>
V = {A, B, C, …, K}
E = {(A, B), (A, E), …}
F = {(A, E, B), (B, E, F), …}

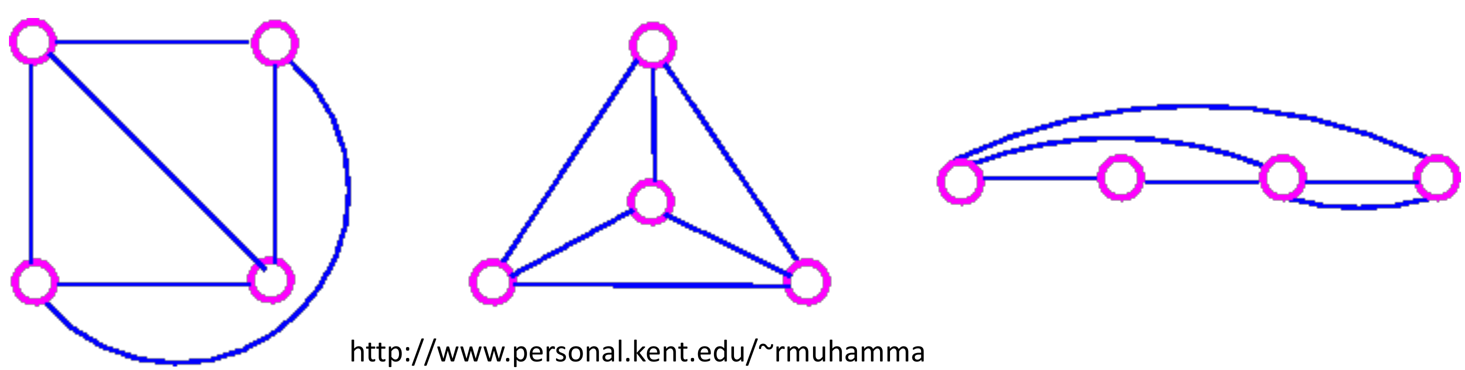
deg(A) = 4
deg(E) = 5

Graph Planarity

- A graph is planar if it can be drawn in the plane such that no two edges cross each other (except at the vertices):



- Planarize:

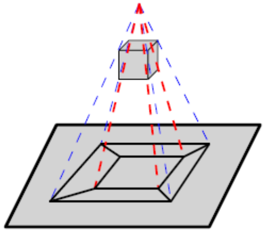


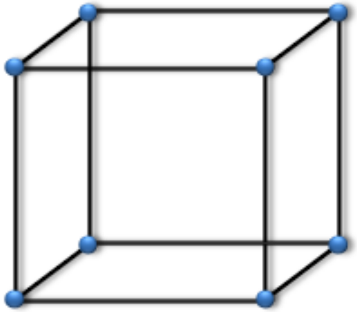
http://www.personal.kent.edu/~rmuhamma

Mesh Statistics

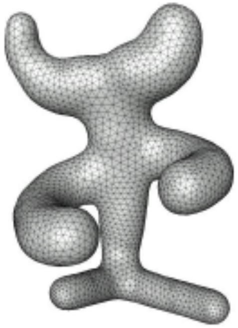
- Almost every mesh is a planar graph
- For such graphs Euler formula holds:

$$\#V - \#E + \#F = 2$$





$V = 8$
$E = 12$
$F = 6$
$\chi = 8 + 6 - 12 = \mathbf{2}$



$V = 3890$
$E = 11664$
$F = 7776$
$\chi = \mathbf{2}$

Mesh Statistics

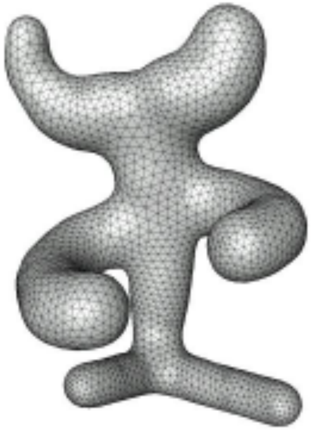
- Based on Euler's formula:

#F ~ 2V

#E ~ 3V

AVD ~ 6

Average
vertex degree



$V = 3890$

$E = 11664$

$F = 7776$

Mesh Structures

- How to store geometry & connectivity of a mesh

3D vertex coordinates   Vertex adjacency

- Attributes are also stored: normals, colors, texture coordinates, labels, …
- Efficient algorithms on meshes to get:
  - All vertices/edges of a face
  - All incident vertices/edges/faces of a vertex

Face-based Structures

- Face-Set Data Structure (.stl format)
  - Aka polygon soup as there is no connectivity information

Triangles		
x_{11} y_{11} z_{11}	x_{12} y_{12} z_{12}	x_{13} y_{13} z_{13}
x_{21} y_{21} z_{21}	x_{22} y_{22} z_{22}	x_{23} y_{23} z_{23}
...	...	...
x_{F1} y_{F1} z_{F1}	x_{F2} y_{F2} z_{F2}	x_{F3} y_{F3} z_{F3}

  - Vertices and associated data replicated ☹
  - Using 32-bit single precision numbers to represent vertex coords, we need 32/8 (bytes) * 3 (x-y-z coords) * 3 (# vertices) = 36 bytes per face
  - By Euler formula (F ~ 2V), each vertex consumes 72 bytes on average

Face-based Structures

- Indexed Face-Set Data Structure (.obj, .off, .ply, our XML format)
  - Aka shared-vertex data structure

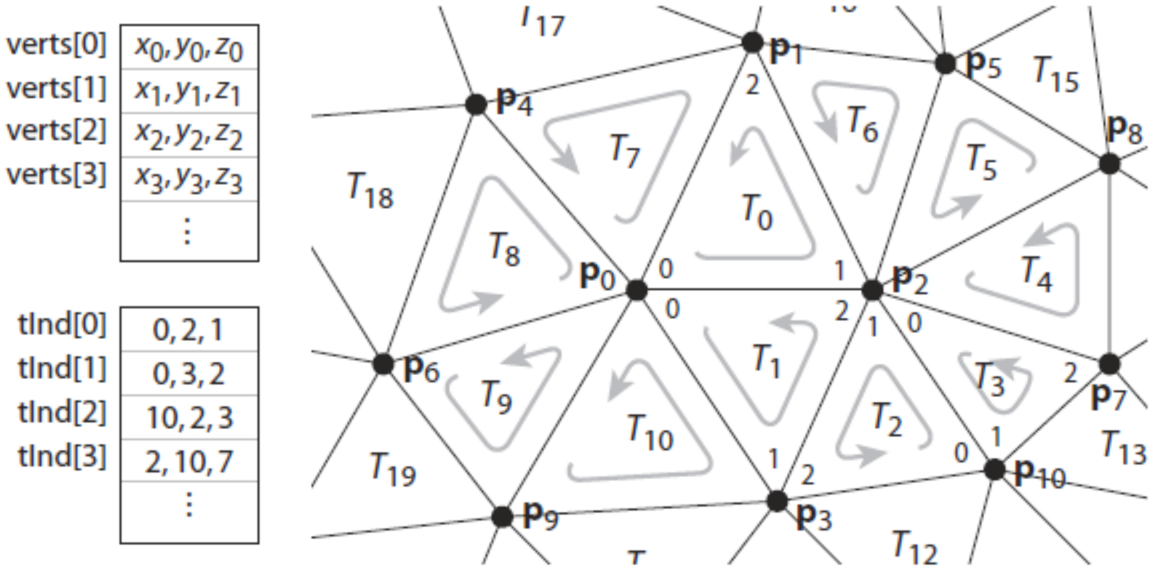
Vertices
x_1 y_1 z_1
...
x_V y_V z_V

Triangles
i_{11} i_{12} i_{13}
...
...
...
...
i_{F1} i_{F2} i_{F3}

  - No vertex replication ☺
  - We need 4 (bytes) * 3 (# indices) = 12 bytes per face (24 bytes per vertex)
  - We also need 4 (bytes) * 3 (x-y-z coords) = 12 bytes per vertex
  - Total = 36 bytes per vertex, half of Face-Set structure ☺

Face-based Structures

- Regardless of the structure, triangle vertices must be stored in a consistent order
  - Mostly counterclockwise (CCW)

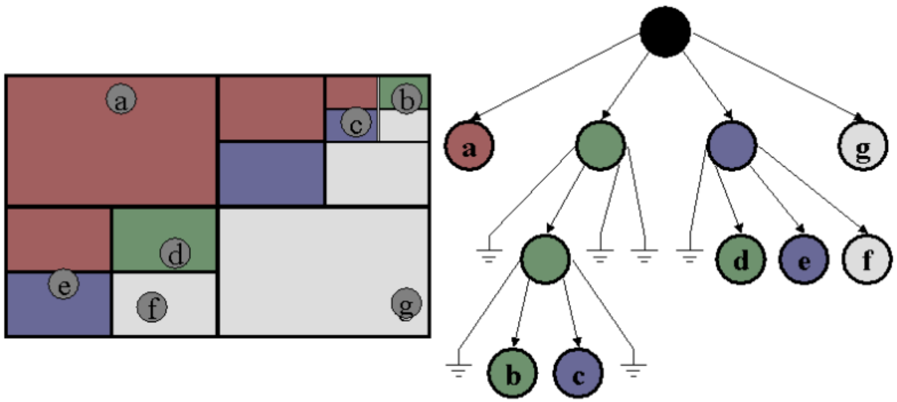


Data Structures for Graphics

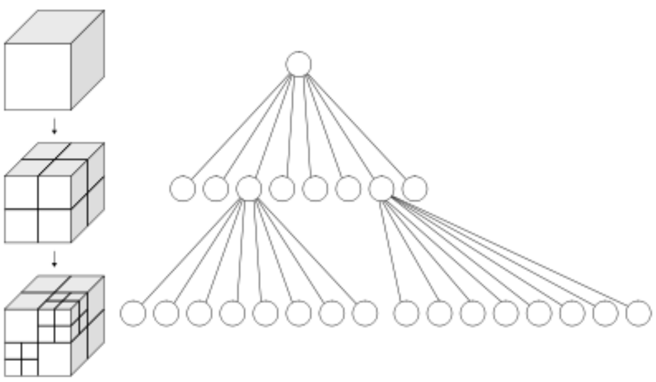
- With large number of triangles, rendering tasks tend to take too long if data is not properly organized
- Many data structures exist
  - Quadtree
  - Octree
  - BSP tree
  - k-D tree
  - BVH

Quadtree and Octree

- Quadtrees are used in 2D and octrees in 3D

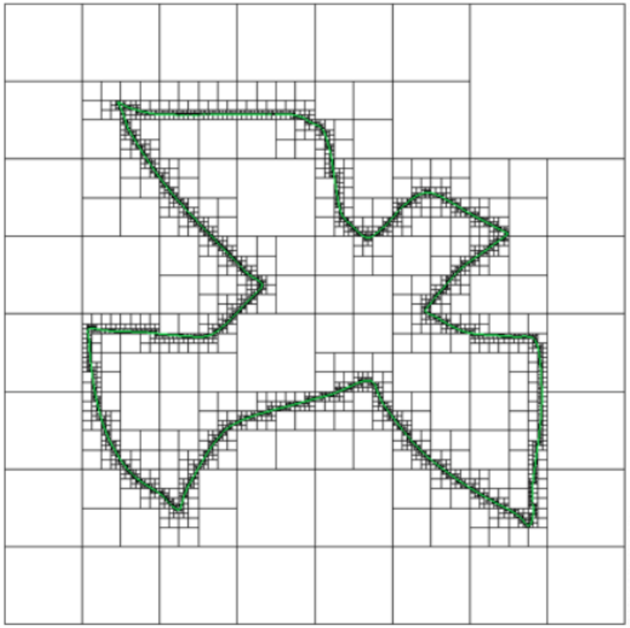


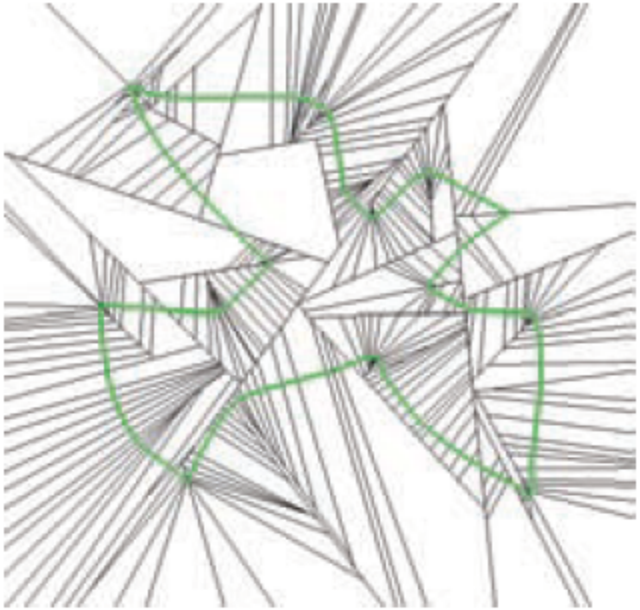
Quadtree


Octree

Binary Space Partitioning (BSP)

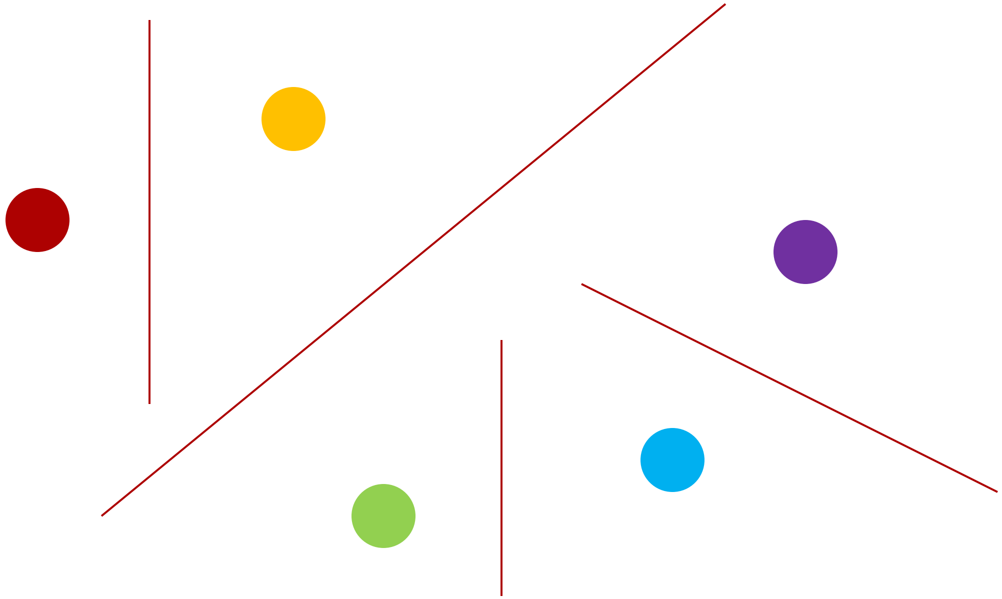
- Divide the space with freely oriented lines (2D) and planes (3D)

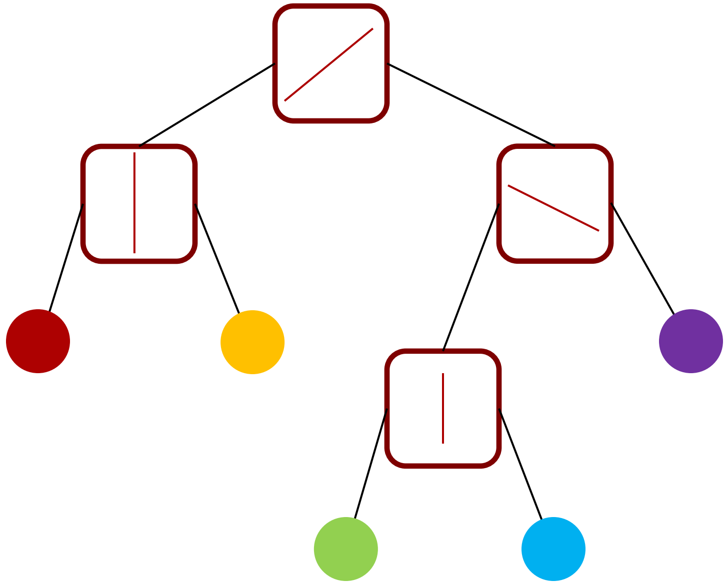




Binary Space Partitioning (BSP)

- BSP trees are view-independent (no need to reconstruct if the viewer moves)
- BSP trees can be used to draw a scene in back-to-front order with respect to the viewer

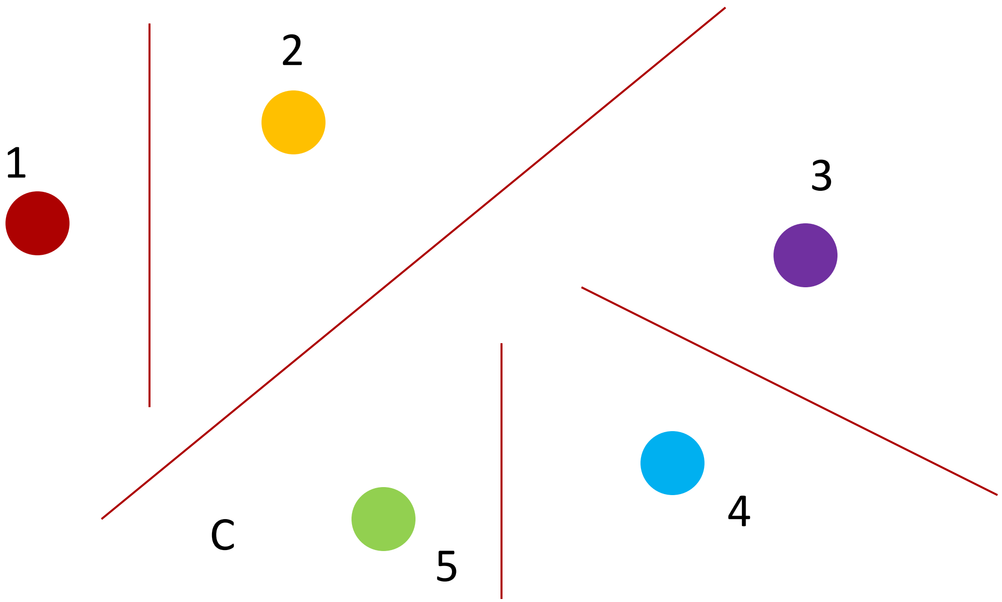


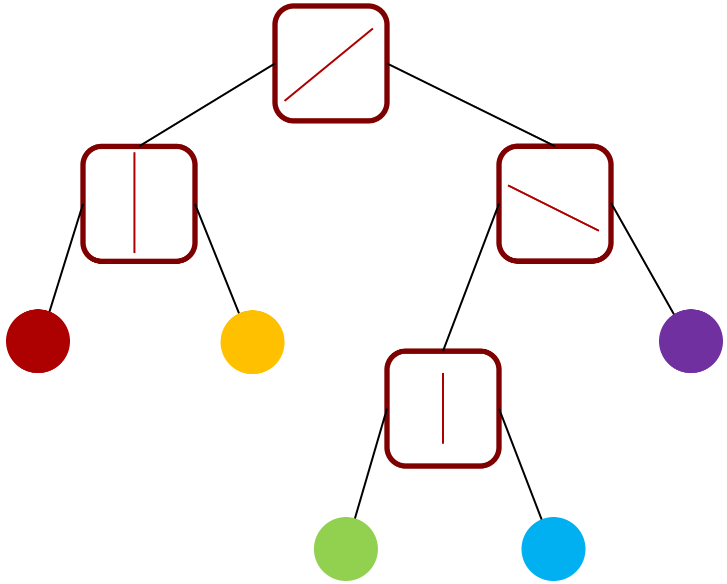


Corresponding BSP tree

Binary Space Partitioning (BSP)

- Assume camera is at point C
- Always traverse the half-space first that does not contain C
- This guarantees back-to-front traversal w.r.t. the camera



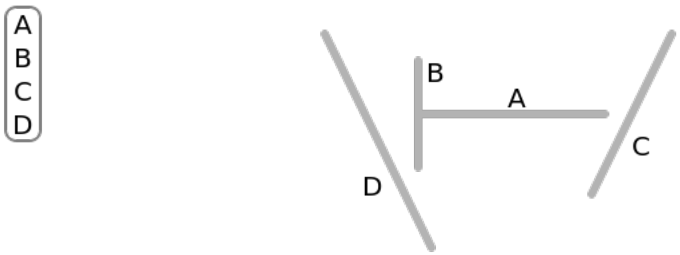


Corresponding BSP tree

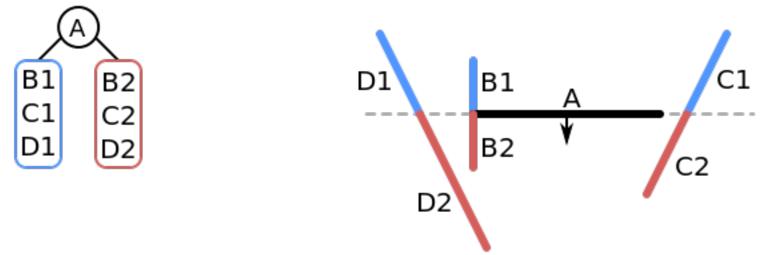
Binary Space Partitioning (BSP)

- BSP trees do not need to be recreated if the camera moves
  - Their traversal depends on the camera position
- How to create a BSP tree?
- **Step 1:** Select a polygon and create a plane aligned with it
  - Put that polygon to your root
- **Step 2:** Separate the other polygons into two sets
  - One above the plane and other below the plane
  - If the plane intersects some polygons, split them and place them to their corresponding sets
- **Step 3:** Recursively apply Step 1 and 2 until you reach a desired stopping condition

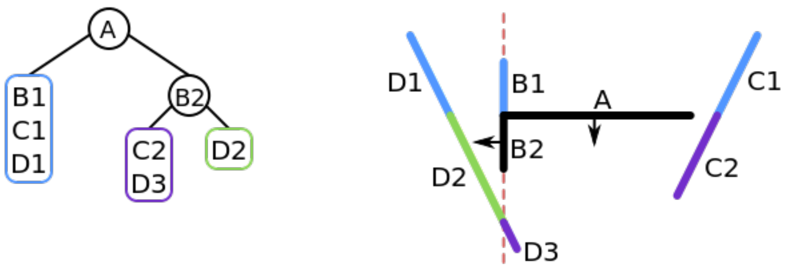
Binary Space Partitioning (BSP)



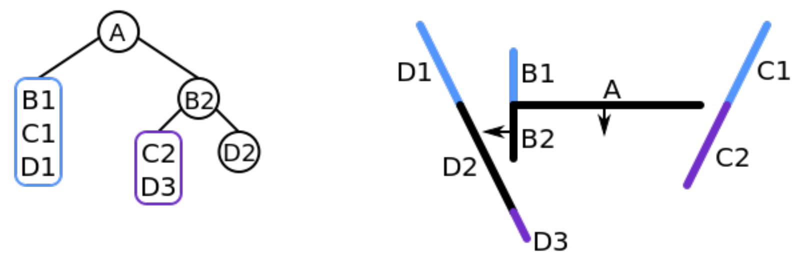
Sample Input



Step 1



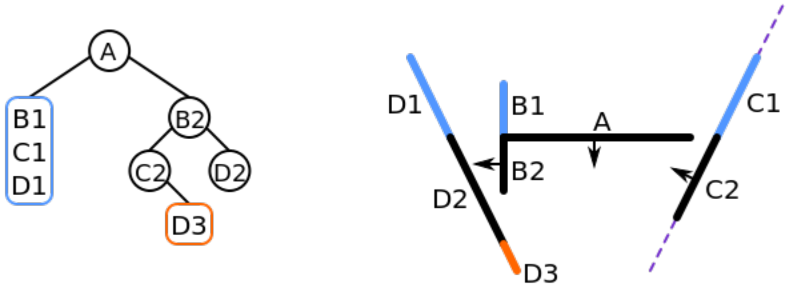
Step 2



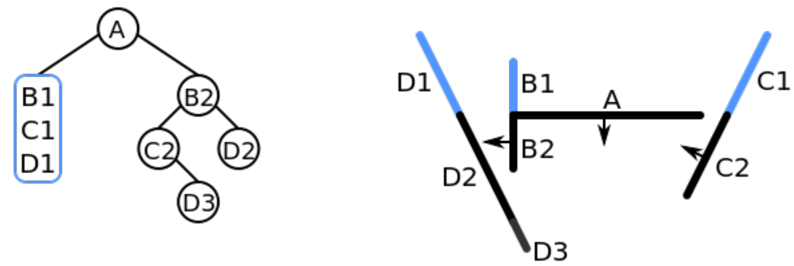
Step 3

See: https://en.wikipedia.org/wiki/Binary_space_partitioning

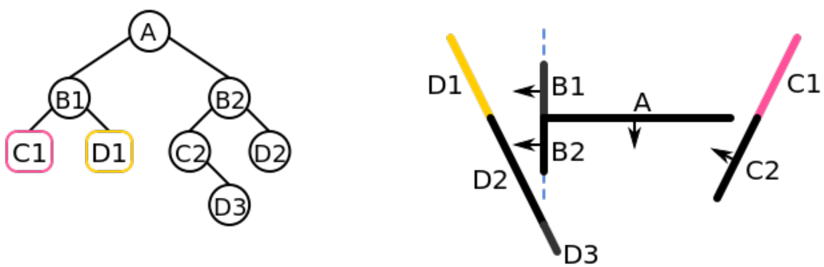
Binary Space Partitioning (BSP)



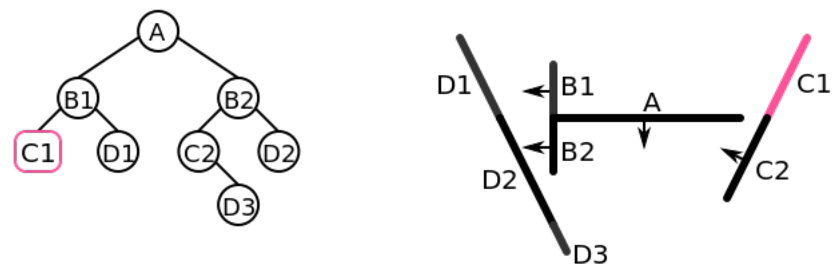
Step 4



Step 5



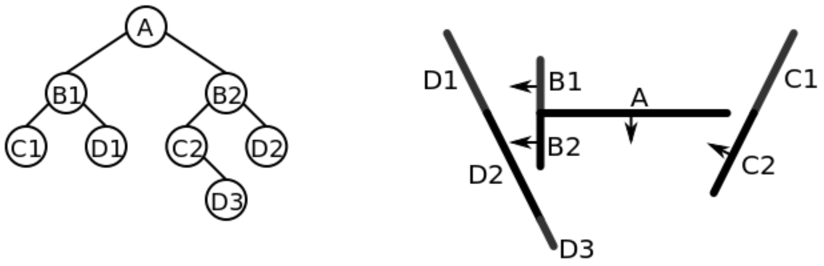
Step 6



Step 7

See: https://en.wikipedia.org/wiki/Binary_space_partitioning

Binary Space Partitioning (BSP)

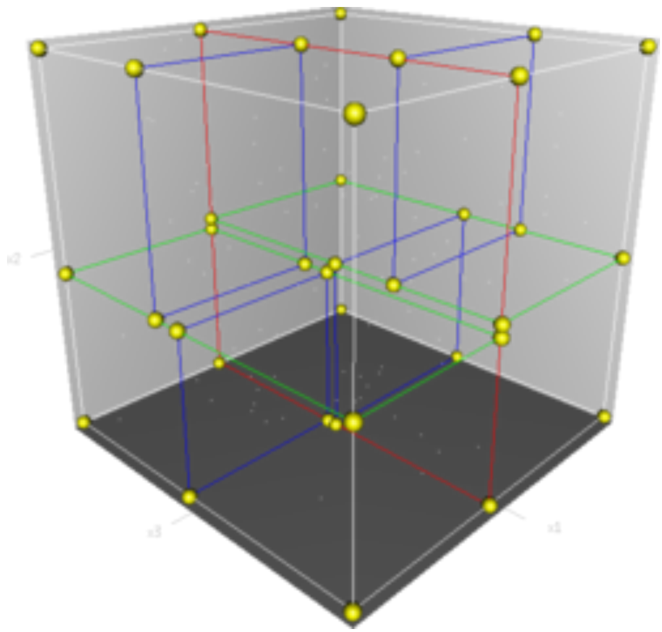


Step 9

See: https://en.wikipedia.org/wiki/Binary_space_partitioning

k-D Tree

- In a k-D tree, a scene is recursively divided into 2 convex sets by axis-aligned hyperplanes: a special case of BSP tree



wikipedia.com

k-D Tree

- k-D tree is a binary tree
- Each node is a k-dimensional point
- Splitting planes are alternatingly chosen between dimensions:
  - First x, then y, then z, back to x and so on (axis = (axis + 1) % 3)
- It will be a balanced tree if the median element is chosen at each split
  - $\text{Log}_2(n)$ depth in that case

k-D Tree

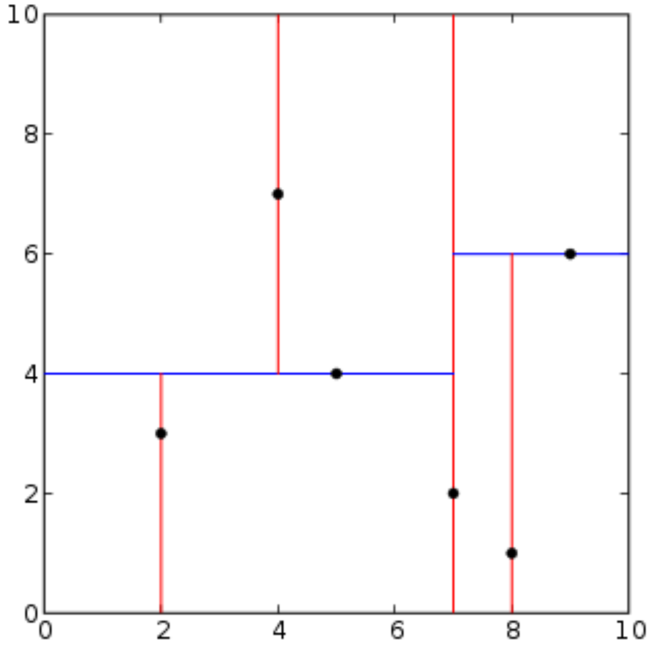
- Basic algorithm:

```
function kdtree (list of points pointList, int depth)
{
    // Select axis based on depth so that axis cycles through all valid values
    var int axis := depth mod k;

    // Sort point list and choose median as pivot element
    select median by axis from pointList;

    // Create node and construct subtrees
    var tree_node node;
    node.location := median;
    node.leftChild := kdtree(points in pointList before median, depth+1);
    node.rightChild := kdtree(points in pointList after median, depth+1);
    return node;
}
```

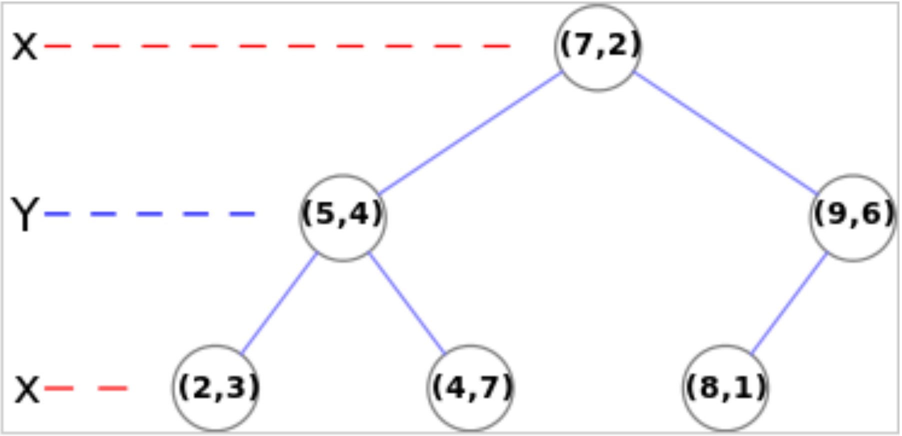
wikipedia.com

k-D Tree

- A 2-D example (k = 2):



k-d tree decomposition for the point set
(2,3), (5,4), (9,6), (4,7), (8,1), (7,2).

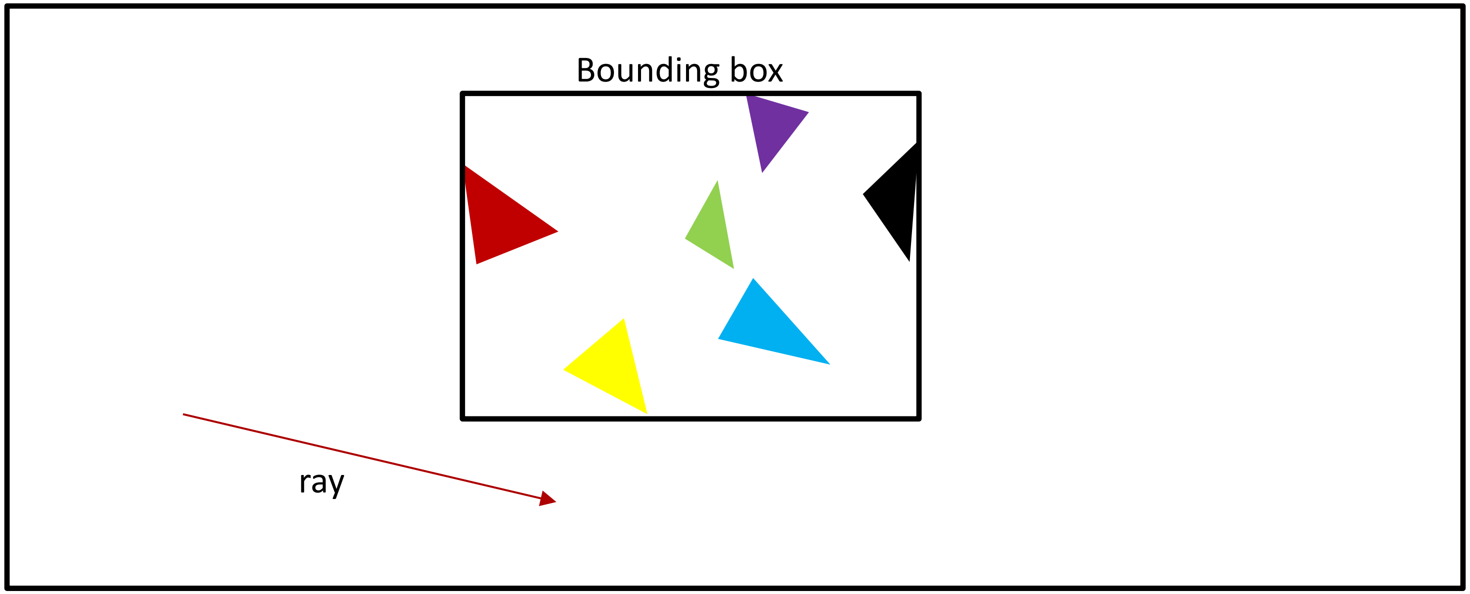


Bounding Volume Hierarchy (BVH)

- While k-D trees partition space into disjoint regions, a BVH partitions objects into disjoint polygons
  - k-D tree: **space** subdivision
  - BVH: **object** subdivision
- Objects are contained within bounding boxes (aka bounding volumes)

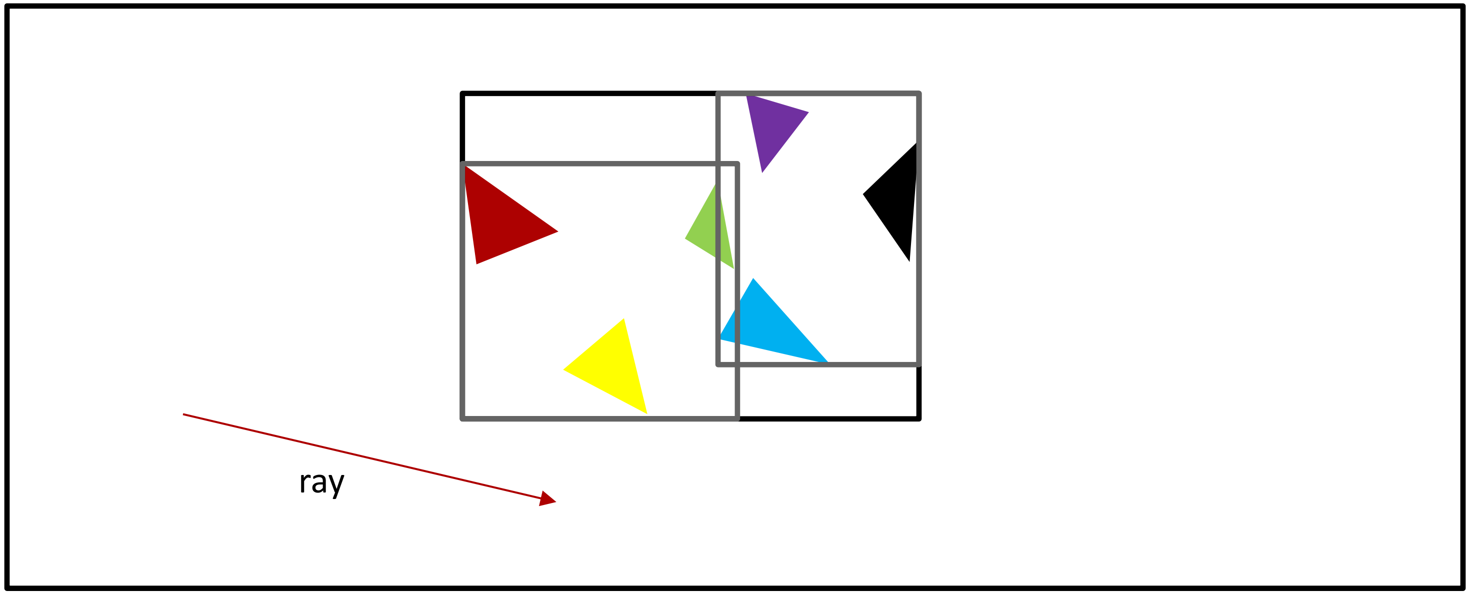
Bounding Volume Hierarchy (BVH)

- Rays missing the bounding boxes are not intersected with the actual objects



Bounding Volume Hierarchy (BVH)

- Bounding boxes can be made hierarchical
- Overlap between bounding boxes are possible (whereas in a k-D tree overlap between regions is not possible)



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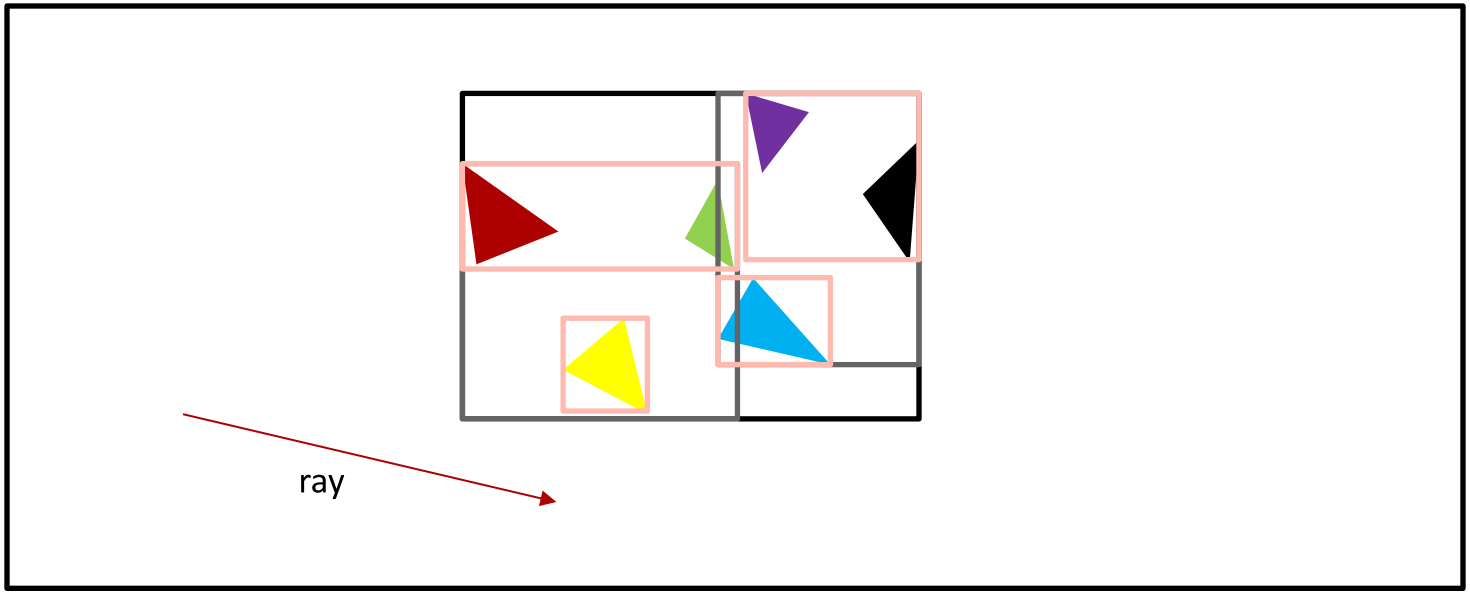
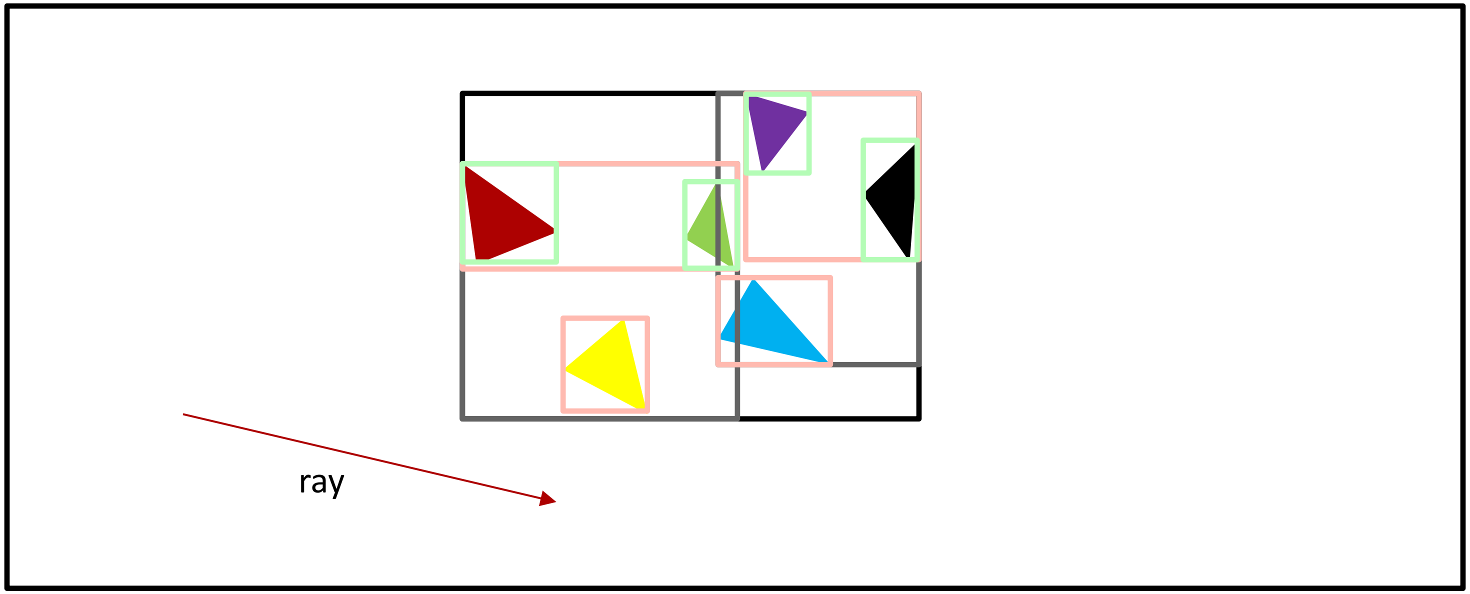


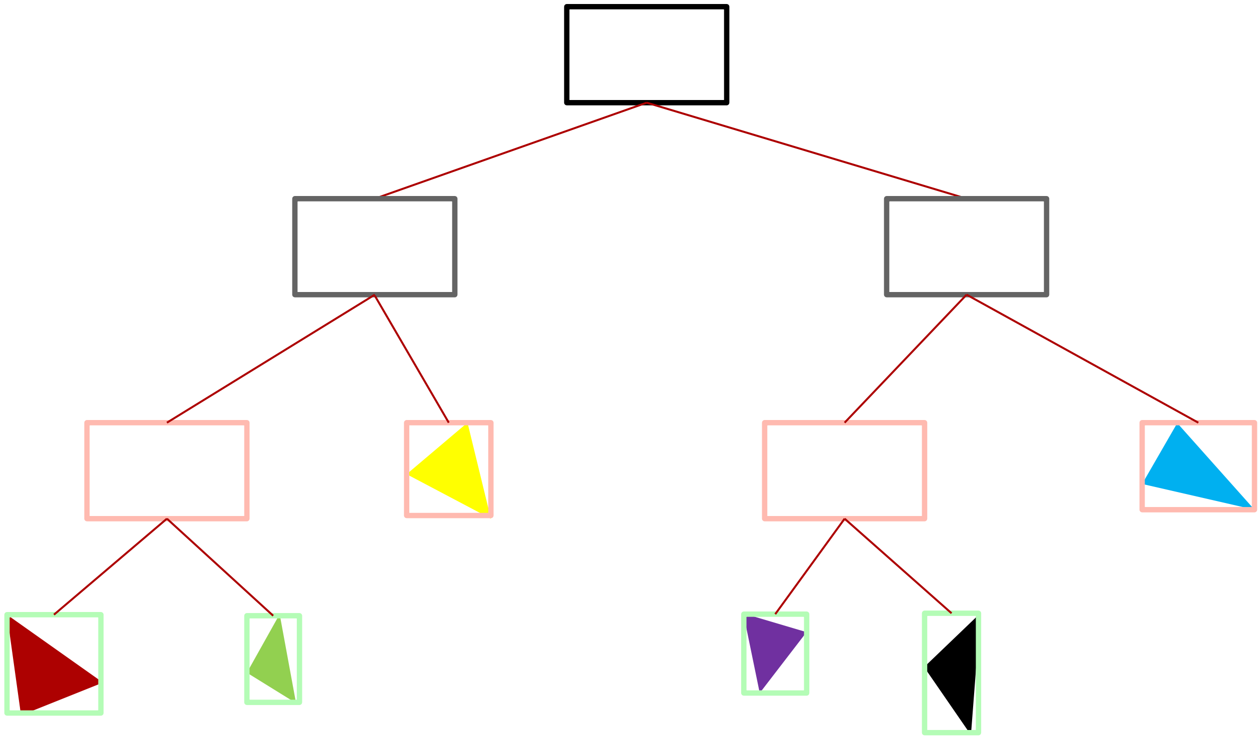
Bounding Volume Hierarchy (BVH)

- BBs may go all the way down to individual primitives (triangles)



Bounding Volume Hierarchy (BVH)

- This is represented as a binary tree where only the leaf BBs contain the actual objects



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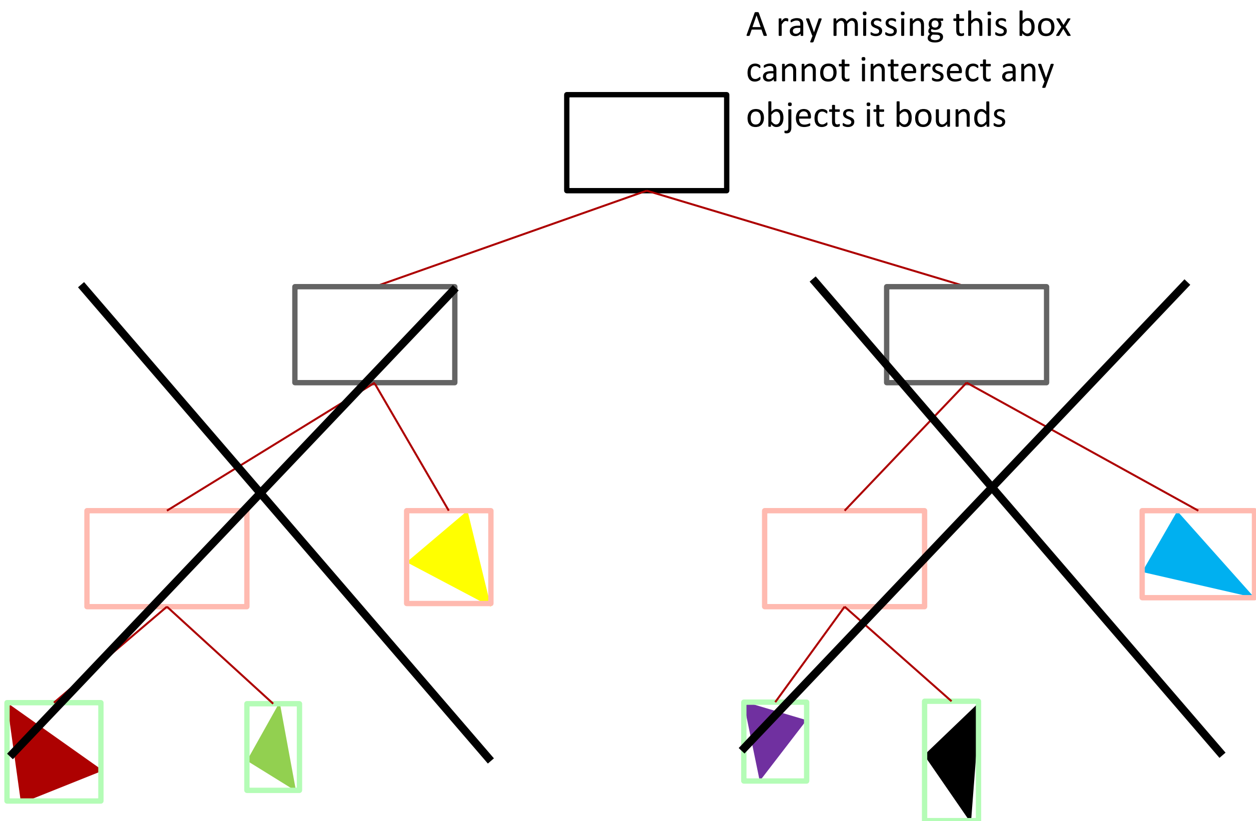

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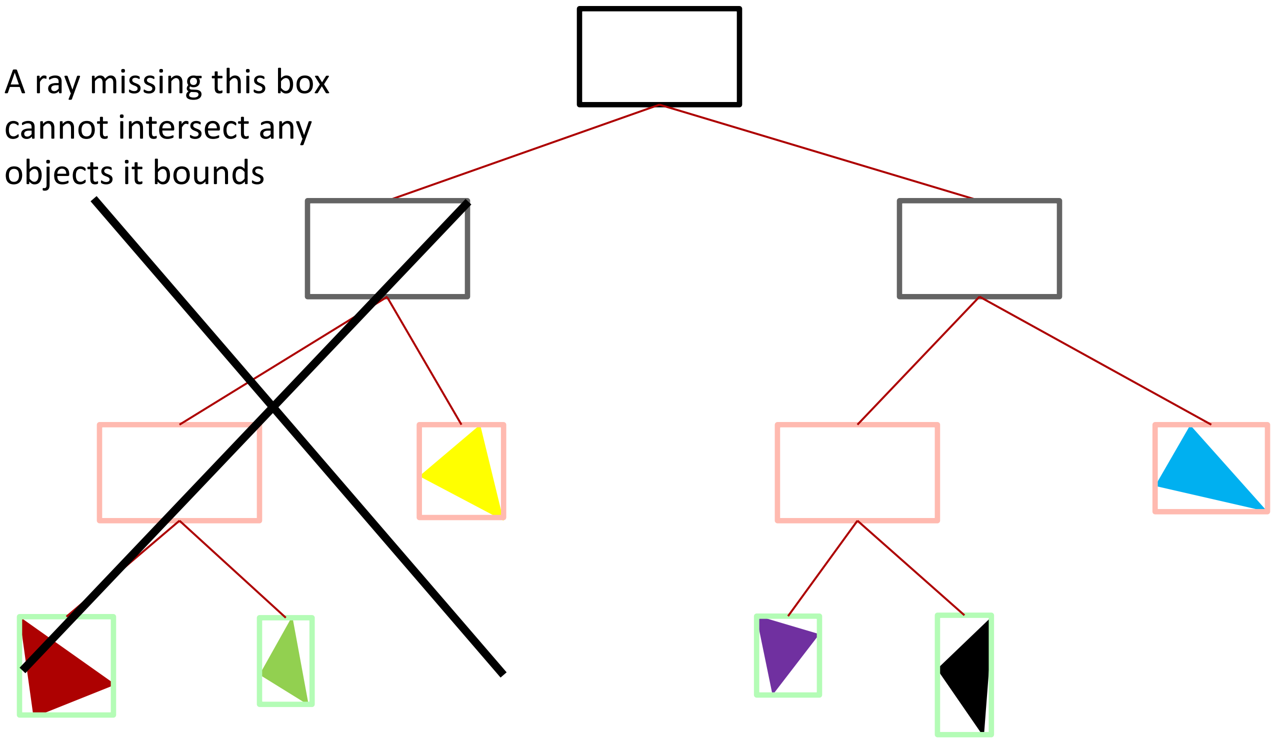
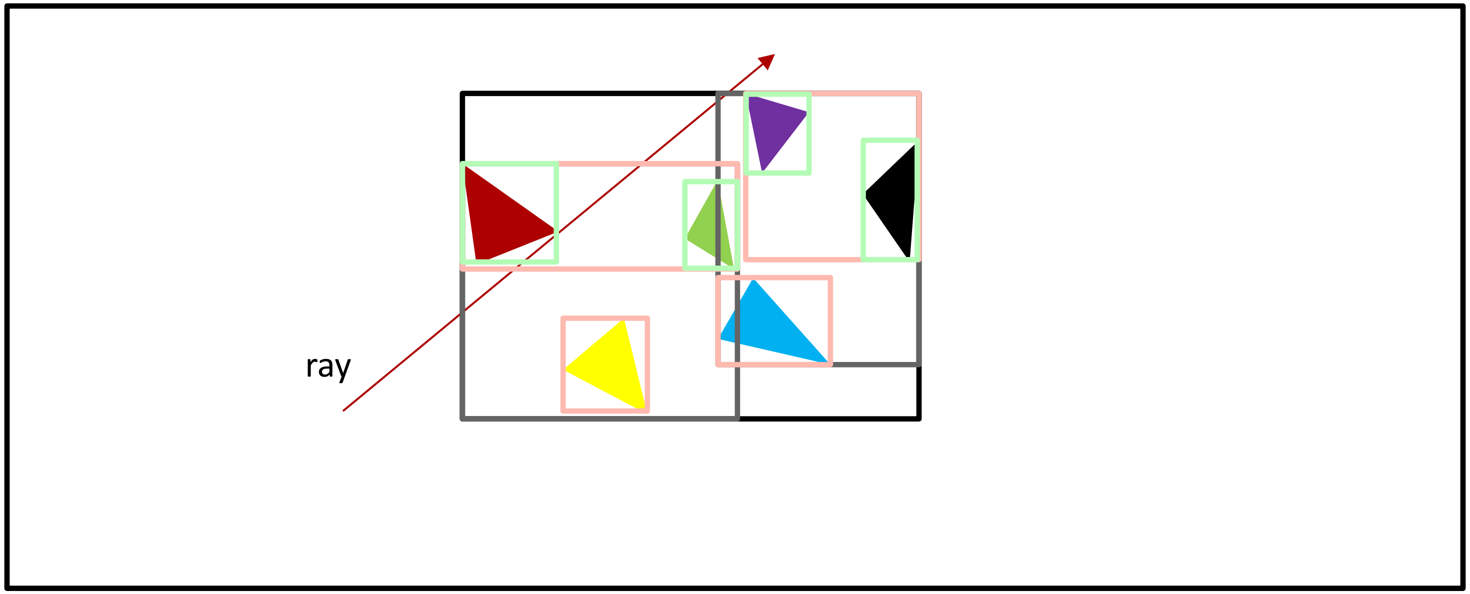


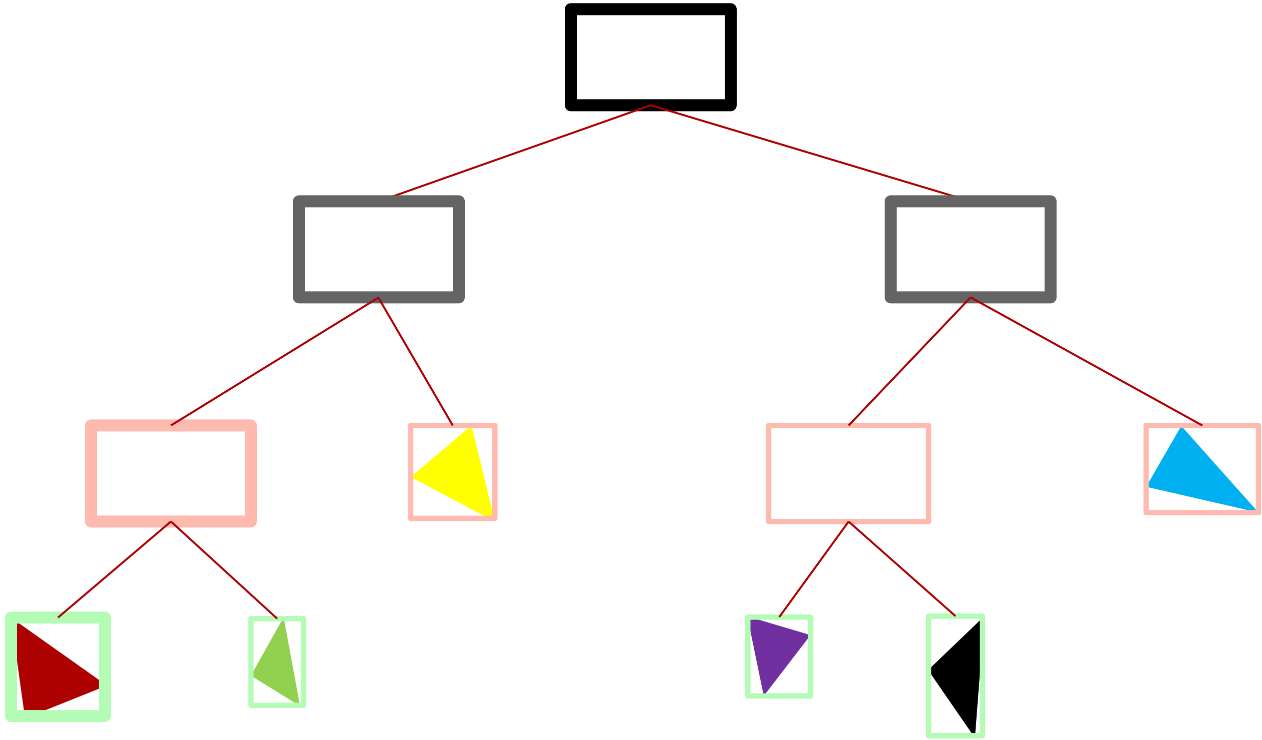
Bounding Volume Hierarchy (BVH)

- Note that a ray may intersect with multiple bounding boxes



Bounding Volume Hierarchy (BVH)

- Intersecting boxes are shown in bold
- Note that this ray does not intersect with any real object



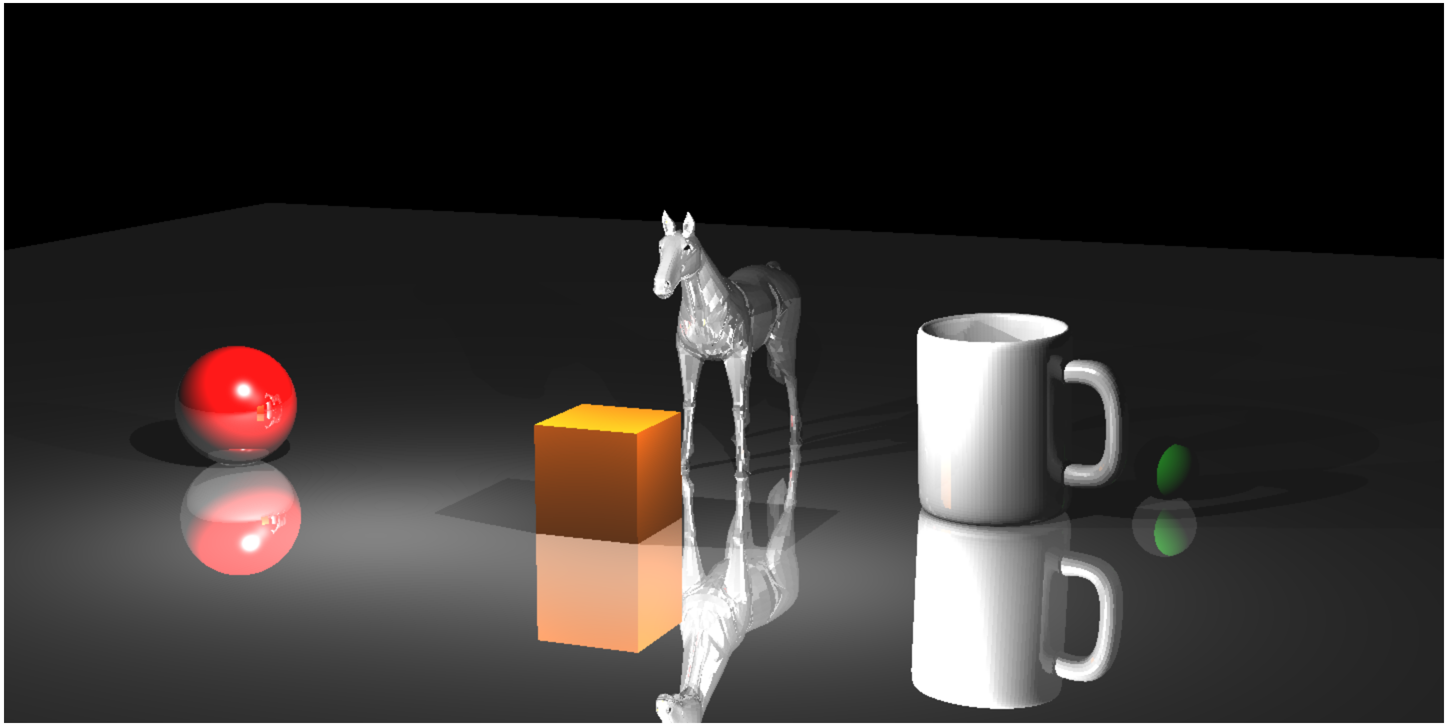
Bounding Volume Hierarchy (BVH)

- Basic traversal algorithm:

```
bool BVH::intersect(const Ray& r, HitRecord& rec) const
{
    if (bbox.rayIntersect(r) == false)
    {
        // This ray entirely misses this bounding box
        return false;
    }

    HitRecord rec1, rec2;

    rec.t = INFINITY;

    bool hitLeft = left->intersect(r, rec1);
    bool hitRight = right->intersect(r, rec2);

    if (hitLeft)
    {
        rec = rec1;
    }

    if (hitRight)
    {
        rec = rec2.t < rec.t ? rec2 : rec;
    }

    return (hitLeft || hitRight);
}
```

Bounding Volume Hierarchy (BVH)

- BVH affords significant speed-up in ray tracing:
  - This scene contains 31584 objects (all triangles except 2 spheres)



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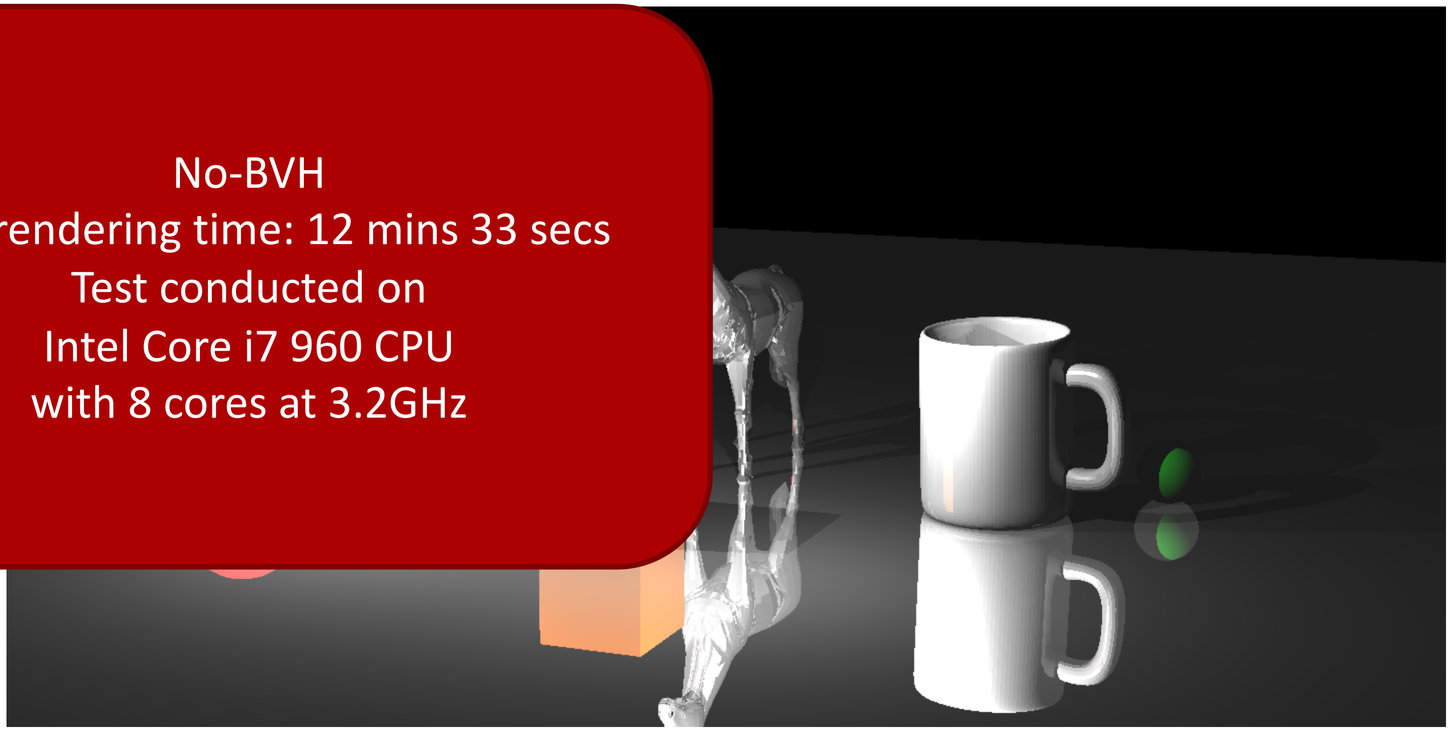

No-BVH
Total rendering time: 12 mins 33 secs
Test conducted on
Intel Core i7 960 CPU
with 8 cores at 3.2GHz

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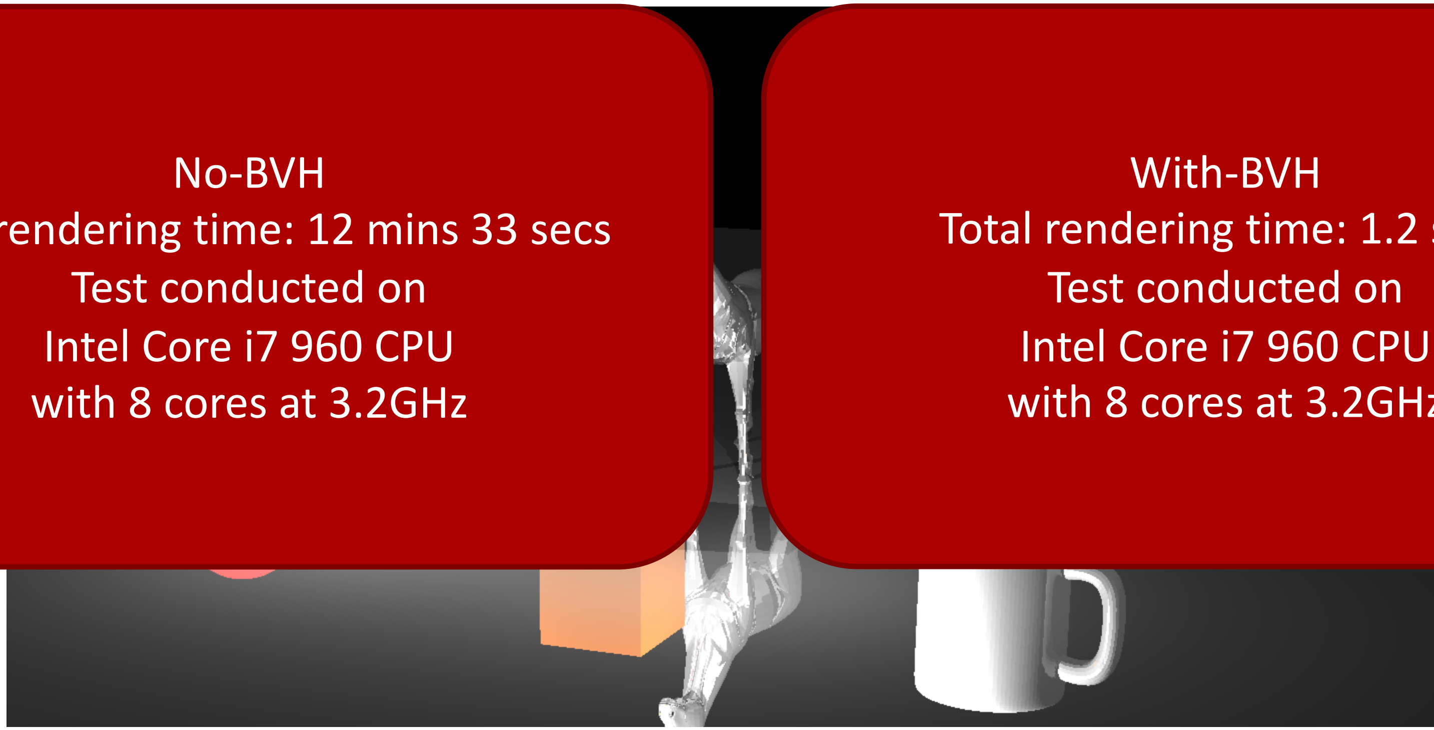

No-BVH
Total rendering time: 12 mins 33 secs
Test conducted on
Intel Core i7 960 CPU
with 8 cores at 3.2GHz

With-BVH
Total rendering time: 1.2 secs
Test conducted on
Intel Core i7 960 CPU
with 8 cores at 3.2GHz