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LINEAR ALGEBRA
ASSIGNMENT # 2

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QUESTION # 1

What is a matrix determinant?

A matrix determinant is a scalar value specially defined for square matrices. It can be defined using the following expression

$$A = |a_{ij}| \text{ of order } m \times m$$

PROPERTIES OF MATRIX DETERMINANT

1- Reflection Property

The matrix remains unaltered if the rows of m×m matrix is changed to columns and vice versa.

Ex.

$$\begin{vmatrix} 1 & 3 & 9 \\ 4 & 6 & 1 \\ 2 & 5 & 0 \end{vmatrix} \Longrightarrow \begin{vmatrix} 1 & 4 & 2 \\ 3 & 6 & 5 \\ 9 & 1 & 0 \end{vmatrix}$$

Matrix A   matrix B

For matrix A, the determinant is

$$1(6.0 - 1.5) - 3(4.0 - 1.2) - 9(4.5 - 6.2)$$

$$1(0 - 5) - 3(0 - 2) - 9(20 - 12)$$

$$-5 + 6 - 72 = +73$$

For matrix B, the determinant is

$$1(6.0 - 5.1) - 4(3.0 - 5.4) - 2(3.1 - 6.9)$$

$$1(0 - 5) - 4(3 - 45) - 2(3 - 54) = +73$$

2- **Repetition Property**

If any two rows or colums of a matrix has the same value in every entry, then the determinant is zero.

Ex
$$\begin{bmatrix} 1 & 3 & 9 \\ 5 & 5 & 5 \\ 2 & 2 & 2 \end{bmatrix} = \begin{vmatrix} 1 & 3 & 9 \\ 5 & 5 & 5 \\ 2 & 2 & 2 \end{vmatrix} = 0$$

3- **All- zero Property**

If every entry of a row or column is zero, then the determinant is also zero

$$\begin{bmatrix} 1 & 2 & 5 & 9 \\ 0 & 0 & 0 & 0 \\ 3 & 6 & 2 & 9 \\ 7 & 4 & 3 & 1 \end{bmatrix} = \begin{vmatrix} 1 & 2 & 5 & 9 \\ 0 & 0 & 0 & 0 \\ 3 & 6 & 2 & 9 \\ 7 & 4 & 3 & 1 \end{vmatrix} = 0$$

4- **Switching Property**

Switching a row or column with another row/column will change the sign of the determinant

$$\begin{bmatrix} 1 & 4 & 1 \\ 3 & 6 & 9 \\ 9 & 8 & 7 \end{bmatrix} = \begin{vmatrix} 1 & 4 & 1 \\ 3 & 6 & 9 \\ 9 & 8 & 7 \end{vmatrix} = 150$$

Interchanging column 1 with 3

$$\begin{bmatrix} 9 & 8 & 7 \\ 3 & 6 & 9 \\ 1 & 4 & 1 \end{bmatrix} = \begin{vmatrix} 9 & 8 & 7 \\ 3 & 6 & 9 \\ 1 & 4 & 1 \end{vmatrix} = -150$$

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5- Sum Property

If the entries of a matrix are expressed as sum of two or more elements, then the determinant can be expressed as the sum of two or more determinants

Ex

$$\begin{vmatrix} x_1 & x_2 & x_3 \\ x_1+y_1 & x_2+y_2 & x_3+y_3 \\ z_1 & z_2 & z_3 \end{vmatrix} = \begin{vmatrix} x_1 & x_2 & x_3 \\ x_1 & x_2 & x_3 \\ z_1 & z_2 & z_3 \end{vmatrix} + \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix}$$

6- Scalar Multiple Property

If any row or colum is multiplied by any non-zero constant, then the determinant also gets multiplied by the same constant

$$\begin{vmatrix} 1 & 4 & 1 \\ 3 & 6 & 9 \\ 9 & 8 & 7 \end{vmatrix} = 150$$

$$\begin{vmatrix} 2\times1 & 2\times4 & 2\times1 \\ 3 & 6 & 9 \\ 9 & 8 & 7 \end{vmatrix} = 2(150) = 300$$

7- Invariance

If any scalar multiples of corresponding elements of other rows/column are added to every element of any row/column, the value of determinant will remain same.

$$\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix} = \begin{vmatrix} x_1+(kz_1) & x_2+(kz_2) & x_3+(kz_3) \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix}$$

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8. Triangle Property

If the entries above or below the diagonal are zeros (triangular matrix), then the determinant is the product of diagonal entries

Ex $\begin{vmatrix} 1 & 0 & 0 \\ 3 & 4 & 0 \\ 9 & 2 & 6 \end{vmatrix} = \begin{vmatrix} 1 & 3 & 9 \\ 0 & 4 & 2 \\ 0 & 0 & 6 \end{vmatrix} = (1)\cdot(4)(6) = 24$

9. Factor Property

If the Δ of a determinant becomes zero when we insert $x = \alpha$ then $x - \alpha$ is a factor of Δ

Ex $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \Rightarrow \Delta_1 = \begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix}$

C_{ij} denotes the cofactor of the element a_{ij} in Δ

→ $(x - \alpha)$ is a cofactor of Δ

10. Determinant of co-factor Matrix

Ex $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$\Delta_1 = \begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix}$