Assignment 2: Search Problems in AI - Probabilistic Reasoning

Fatima AlSaadeh
fatima.alsaadeh@rutgers.edu
fya7

Ashley Dunn
ashley.dunn@rutgers.edu
apd109

March 29, 2020

1 Problem 1

city	f(city)	g(city)	h(city)
Lugoj	244	0	244
Mehadia	311	70	241
Drobeta	387	145	242
Craiova	425	265	160
Timisoara	440	111	329
Pitesti	503	403	100
Bucharest	504	504	0

2 Problem 2

2.a

Breadth First Search: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11
Depth-limited Search (Limit 3): 1, 2, 4, 8, 9, 5, 10, 11
Iterative Deepening Search: 1, 1, 2, 3, 1, 2, 4, 5, 3, 6, 7, 1, 2, 4, 8, 9, 5, 10, 11

2.b

Forward: 1, 2, 3
Backward: 11, 5, 2

The branching factor is 2. The depth is 3. This means that the time complexity would both be $O(2^{\frac{3}{2}})$. This is better than BFS, depth-limited, and iterative deepening, all of which are $O(2^3)$ in this case.

3 Problem 3

3.a Breadth-first search is a special case of uniform-cost search

Correct

3.b Depth-first search is a special case of best-first tree search

Correct

3.c Uniform-cost search is a special case of A* search

Correct

3.d Depth-first graph search is guaranteed to return an optimal solution

Wrong

3.e Breadth-first graph search is guaranteed to return an optimal solution

Wrong (unless all edge costs are equal)

3.f Uniform-cost graph search is guaranteed to return an optimal solution

Correct

3.g A graph search is guaranteed to return an optimal solution if the heuristic is consistent

Correct

3.h A* graph search is guaranteed to expand no more nodes than depth-first graph search if the heuristic is consistent

Wrong

3.i A* graph search is guaranteed to expand no more nodes than uniform-cost graph search if the heuristic is consistent

Correct

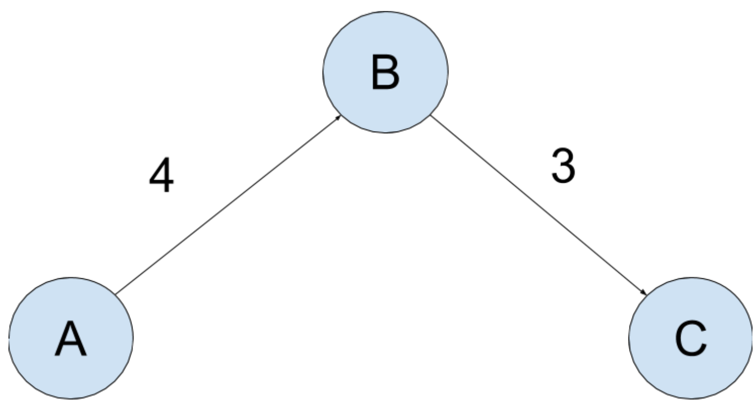
4 Problem 4

One advantage is that the space complexity of iterative deepening is smaller, only $O(bd)$ versus $O(b^d)$ for BFS. One disadvantage is that iterative deepening has to revisit nodes each iteration, while BFS does not revisit any nodes.

5 Problem 5

Let k(n) be cost of the optimal path from n to the goal.
Proof by induction:

Base Case: When n is the goal node, there are 0 steps to the goal and $h(n) = 0$, therefore $h(n) \leq k(n)$
Induction Step: When n is i steps from the goal, there is a successor n' reached by action a along the optimal path from n to the goal, which is i-1 steps from the goal. Using consistency, $h(n) \leq c(n, a, n') + h(n')$. By the induction hypothesis, $h(n') \leq k(n')$ which means that we can substitute in $k(n')$: $h(n) \leq c(n, a, n') + k(n')$. We know that a is along the optimal path, so that means $c(n, a, n') + k(n') = k(n)$, therefore $h(n) \leq k(n)$. QED



Heuristic:
h(A) = 8
h(B) = 2
h(C) = 0

6 Problem 6

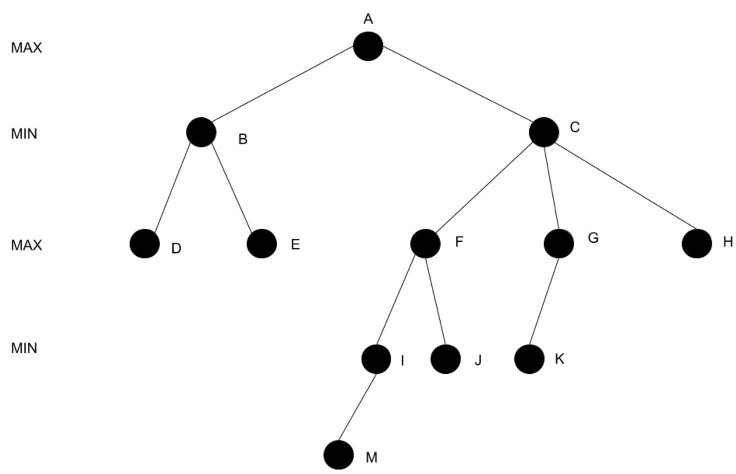
Choosing the most constrained variable will allow you to fail quickly and move on from a possible scenario, if the configuration will not work. In addition, using the least constraining value will allow you to work with scenarios that are the most likely to succeed.

7 Problem 7

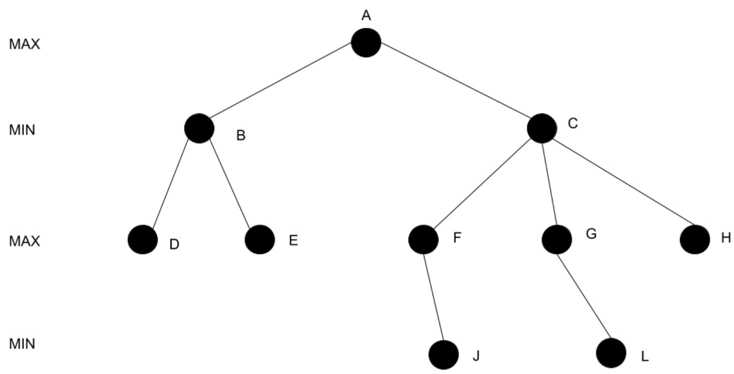
7.a

The best move is C

7.b



7.c



The right-to-left ordering of actions produces different pruning because it is based on the best value that the min and max player can currently guarantee, which is in turn based on the values that they have seen so far. Going from right to left gives us the optimum value immediately, thus allowing for much more pruning.

8 Problem 8

8.a

h(n) = min{h1(n), h2(n)} If h1 and h2 are consistent then for state i on the path
$h1(n) <= c(n, a, i) + h(i)$ and $h2(n) <= c(n, a, i) + h(i)$ then:
min{h1(n), h2(n)}$<= c(n, a, i) + h(i)$
is a consistent heuristic and because it's consistent it will be admissible min{h1(n), h2(n)}$< h^*(a)$.

8.b

if $0 <= w <= 1$ then $wh1 < h1$ and $(1 - w)h2 < h2$ since h1 and h2 already consistent wh1 and (1-w)h2 will also be consistent and admissible, so we will have
$w * h1(n) \leq w * c(n, a, n') + w * h1(n')$
and
$(1 - w) * h2(n) \leq (1 - w) * c(n, a, n') + (1 - w) * h2(n')$
adding these two equations

$w * h1(n) + (1 - w) * h2(n) \leq w * c(n, a, n') + w * h1(n') + (1 - w) * c(n, a, n') +$
$(1 - w) * h2(n')$
$w * h1(n) + (1 - w) * h2(n') \leq w * c(n, a, n') - w * c(n, a, n') + c(n, a, n') + w *$
$h1(n') + (1 - w) * h2(n')$
as we have $h(n) = wh1(n) + (1 - w)h2(n)$
$h(n) \leq c(n, a, n') + h(n)$
as a result b is consistent and admissible.

8.c

h(n) = max{h1(n), h2(n)}
$h1(n) <= c(n, a, i) + h(i)$ and $h2(n) <= c(n, a, i) + h(i)$ then:
max{h1(n), h2(n)}$<= c(n, a, i) + h(i)$
is a consistent heuristic and because it's consistent it will be admissible max{h1(n), h2(n)}$< h^*(a)$. we choose (c) If h1 and h2 are consistent then for state i on the path we need a strong heuristic that is admissible and consistent which isn't less than h1 and h2 and that what the max function give us.

9 Problem 9

9.a

It depends on the shape of the state-space landscape. If the solution in the neighborhood and the shape is without plateaus it's better to choose hill climbing.

9.b

when is the solution is on plateau or flat area.

9.c

the shape of the state-space landscape to use simulated annealing when the cost function has many local maximum with one global maximum, along with plateaus.

9.d

discovering the neighbours of the states we move to will help us enhance the performance of the discovery.

9.e

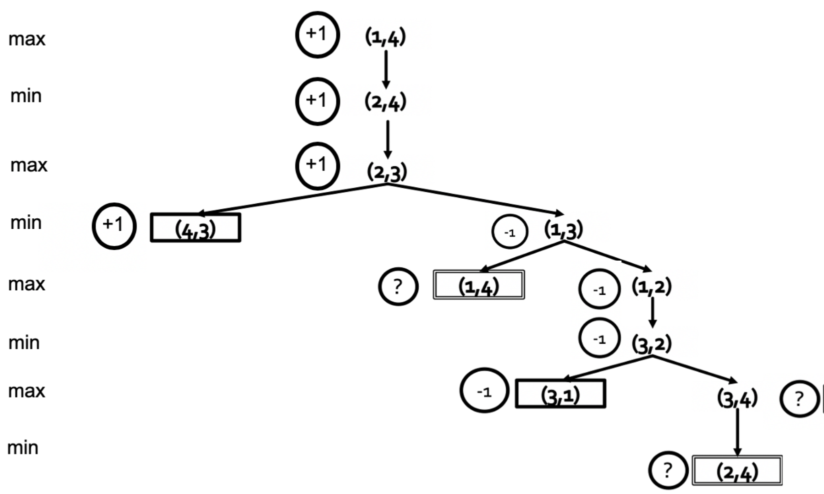
Use multiple searches running simultaneously, marking all the visited states of the cost function so they won't be visited by other simultaneous searches. When all of the searches have completed, we select the best out of all of the returned states.

9.f

The hill climbing version of simulated annealing moves in the direction of the best neighbor state to improve the cost function, and with a certain probability will move to a neighbor state that does not improve the cost function. A similar model can be used for gradient ascent, in which the algorithm will move in the direction of the steepest ascent, except with a certain probability it will move in a direction against the steepest ascent.

10 Problem 10

If player A start, she will represent the max and player B will represent the min in our minimax game, If it max and we need to choose between +1 and ? we will choose +1 of course so the Player A win If it min and we need to choose between -1 and ? we will choose -1 so Player A loose, as loops here are represented as loss, if loop is the only option the player will enter the loop.



11 Problem 11

11.a

$P(A, B, C, D, E) =$
$P(A) * P(D \mid A, B) * P(B) * P(E \mid B, C) * P(C) =$
$.2 * .1 * .5 * .3 * .8 = 0.0024$

11.b

$P(\bar{A}, \bar{B}, \bar{C}, \bar{D}, \bar{E}) =$
$P(\bar{A}) * P(\bar{D} \mid \bar{A}, \bar{B}) * P(\bar{B}) * P(\bar{E} \mid \bar{B}, \bar{C}) * P(\bar{C}) =$
$.8 * .1 * .5 * .8 * .2 = 0.0064$

11.c

$P(\bar{A} \mid B, C, D, E) =$
$\frac{P(\bar{A},B,C,D,E)}{P(\bar{A},B,C,D,E)+P(A,B,C,D,E)} =$
$\frac{P(\bar{A})*P(D|\bar{A},B)*P(B)*P(E|B,C)*P(C)}{P(\bar{A})*P(D|\bar{A},B)*P(B)*P(E|B,C)*P(C)+P(A)*P(D|A,B)*P(B)*P(E|B,C)*P(C)} =$
$\frac{0.0576}{0.0576+0.0024} = 0.96$

12 Problem 12

12.a

$$P\left(B|\, J, M\right) = \alpha \sum_E \sum_A P\left(B, J, M, E, A\right) = \frac{P\left(B, J, M\right)}{P\left(B, J, M\right) + P\left(\bar{B}, J, M\right)}$$

$$= \alpha P\left(B\right) \sum_E P\left(E\right) \sum_A P\left(A|\, B, E\right) P\left(J|\, A\right) P\left(M|\, A\right)$$

$$\begin{aligned} = \alpha P\left(B\right) \sum_E P\left(E\right) \Bigg(& P\left(J|\, A\right) P\left(M|\, A\right) * \begin{bmatrix} P\left(A|\, B, E\right) & P\left(A|\, \bar{B}, E\right) \\ P\left(A|\, B, \bar{E}\right) & P\left(A|\, \bar{B}, \bar{E}\right) \end{bmatrix} \\ & + P\left(J|\, \bar{A}\right) P\left(M|\, \bar{A}\right) * \begin{bmatrix} P\left(\bar{A}\middle|\, B, E\right) & P\left(\bar{A}\middle|\, \bar{B}, E\right) \\ P\left(\bar{A}\middle|\, B, \bar{E}\right) & P\left(\bar{A}\middle|\, \bar{B}, \bar{E}\right) \end{bmatrix} \end{aligned} \quad (1)$$

$$= \alpha P\left(B\right) \sum_E P\left(E\right) \left(.9 * .7 * \begin{bmatrix} .95 & .29 \\ .94 & .001 \end{bmatrix} + .05 * .01 * \begin{bmatrix} .05 & .71 \\ .06 & .999 \end{bmatrix} \right)$$

$$= \alpha P\left(B\right) \sum_E P\left(E\right) \left(\begin{bmatrix} .5985 & .1877 \\ .5922 & .0006 \end{bmatrix} + \begin{bmatrix} .000025 & .00035 \\ .00003 & .00049 \end{bmatrix} \right)$$

$$= \alpha P\left(B\right) \sum_E P\left(E\right) \left(\begin{bmatrix} .598525 & .187735 \\ .59223 & .00109 \end{bmatrix} \right)$$

$$= \alpha P\left(B\right) * \begin{bmatrix} P(E) & P(\bar{E}) \end{bmatrix} \begin{bmatrix} .598525 & .187735 \\ .59223 & .00109 \end{bmatrix} = \alpha P\left(B\right) * \begin{bmatrix} .002 & .998 \end{bmatrix} \begin{bmatrix} .598525 & .187735 \\ .59223 & .00109 \end{bmatrix}$$

$$= \alpha P\left(B\right) \quad * \quad \begin{bmatrix} .5922 \\ .00149 \end{bmatrix}$$

$$\alpha P(B) * .5922 = \alpha * .001 * .5922 = .000592\alpha$$

$$\alpha P(\overline{B}) * .00149 = \alpha * .999 * .00149 = .00148\alpha$$

$$P(B|J,M) = \frac{.000592\alpha}{.000592\alpha + .00148\alpha} = .2857$$

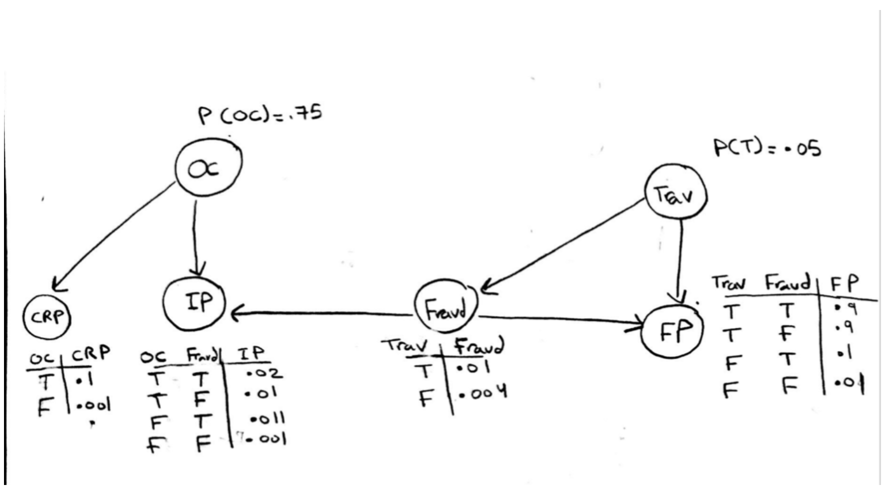
$$P(\overline{B}|J,M) = 1 - .2857 = .7142$$

12.b

using enumeration O(n*2^{n-2})
using elimination O(n)

13 Problem 13

13.a



13.b

$$P\left(Fraud\right) = P\left(Fraud, Trav\right) + P\left(Fraud, \left(\overline{Trav}\right)\right)$$
$$= P\left(Fraud|Trav\right)P(Trav) + P\left(Fraud|\overline{Trav}\right)P(\overline{Trav})$$
$$= .01*\ .05 + .004*\ .95$$
$$= .0043$$

$$\begin{aligned}P(Fraud|FP,\overline{IP},CRP) = (&P(Fraud,FP,\overline{IP},CRP,OC,Trav)\\ &+P(Fraud,FP,\overline{IP},CRP,\overline{OC},Trav)\\ &+P(Fraud,FP,\overline{IP},CRP,OC,\overline{Trav})\\ &+P(Fraud,FP,\overline{IP},CRP,\overline{OC},\overline{Trav}))\\ &*\frac{1}{P(Fraud,FP,\overline{IP},CRP)+P(\overline{Fraud},FP,\overline{IP},CRP)}\end{aligned} \tag{2}$$

$$P(Fraud,FP,\overline{IP},CRP,OC,Trav) =$$
$$P(Fraud|Trav)*P(FP|Trav,Fraud)*P(\overline{IP}|OC,Fraud)*P(CRP|OC)*P(OC)*P(T)$$
$$= .01*.9*.98*.1*.75*.05 = .33*10^{-4}$$

$$P(Fraud,FP,\overline{IP},CRP,\overline{OC},Trav) =$$
$$P(Fraud|Trav)*P(FP|Trav,Fraud)*P(\overline{IP}|\overline{OC},Fraud)*P(CRP|\overline{OC})*P(\overline{OC})*P(T)$$
$$= .01*.9*.989*.001*.25*.05 = .111*10^{-6}$$

$$P(Fraud,FP,\overline{IP},CRP,OC,\overline{Trav}) =$$
$$P(Fraud|\overline{Trav})*P(FP|\overline{Trav},Fraud)*P(\overline{IP}|OC,Fraud)*P(CRP|OC)*P(OC)*P(T)$$
$$= .004*.1*.98*.1*.75*.95 = .279*10^{-4}$$

$$P(Fraud,FP,\overline{IP},CRP,\overline{OC},\overline{Trav}) =$$
$$P(Fraud|\overline{Trav})*P(FP|\overline{Trav},Fraud)*P(\overline{IP}|\overline{OC},Fraud)*P(CRP|\overline{OC})*P(\overline{OC})*P(T)$$
$$= .004*.1*.989*.001*.25*.95 = .939*10^{-7}$$

$$P(Fraud,FP,\overline{IP},CRP,OC,Trav) = \alpha(.33*10^{-4}*.111*10^{-6}*.279*10^{-4}*.939*10^{-7})$$
$$= \alpha*6.12*10^{-5}$$

$$P(\overline{Fraud}, FP, \overline{IP}, CRP) = P(\overline{Fraud}|Trav) * P(FP|Trav, \overline{Fraud}) * P(\overline{IP}|OC, \overline{Fraud})$$

$$\begin{aligned} &= P(\overline{Fraud}|Trav) * P(FP|Trav, \overline{Fraud}) * P(\overline{IP}|OC, \overline{Fraud}) * P(CRP|OC) * O(OC) * P(T) \\ &+ P(\overline{Fraud}|Trav) * P(FP|Trav, \overline{Fraud}) * P(\overline{IP}|\overline{OC}, \overline{Fraud}) * P(CRP|\overline{OC}) * O(\overline{OC}) * P(T) \\ &+ P(\overline{Fraud}|Trav) * P(FP|Trav, \overline{Fraud}) * P(\overline{IP}|OC, \overline{Fraud}) * P(CRP|OC) * O(OC) * P(\overline{T}) \\ &+ P(\overline{Fraud}|Trav) * P(FP|Trav, \overline{Fraud}) * P(\overline{IP}|\overline{OC}, \overline{Fraud}) * P(CRP|\overline{OC}) * O(\overline{OC}) * P(\overline{T}) \end{aligned} \tag{3}$$

$$\begin{aligned} P(\overline{Fraud}, FP, \overline{IP}, CRP) &= .99 * .9 * .99 * .1 * .75 * .05 \\ &+ .99 * .9 * .999 * .001 * .25 * .05 \\ &+ .996 * .01 * .99 * .1 * .75 * .95 \\ &+ .996 * .01 * .999 * .001 * .25 * .95 \\ &= .00402 \end{aligned} \tag{4}$$

$$P(Fraud|FP, \overline{IP}, CRP) = \frac{\alpha * 6.12 * 10^{-5}}{\alpha * 6.12 * 10^{-5} + \alpha * .00402} = .01498$$