

Advanced Statistics (DS2003)
BDS-4A, 4B, 4C
Spring 2024

Linear Combination of Random Variables

Practice Problems

1. If X and Y are independent, normal random variables with $E(X)=5$, $\text{Var}(X)=4$, $E(Y)=10$, and $\text{Var}(Y)=9$. We let $W=2X+3Y$. Determine the following: $E(W)$, $\text{Var}(W)$ and $\Pr(W < 30)$
2. A plastic casing for a magnetic disk is composed of two halves. The thickness of each half is normally distributed with a mean of 2 millimeters and a standard deviation of 0.1 millimeter and the halves are independent.
 - a. Determine the mean and standard deviation of the total thickness of the two halves.
 - b. What is the probability that the total thickness exceeds 4.3 millimeters?
3. In the manufacturing of electroluminescent lamps, several different layers of ink are deposited onto a plastic substrate. The thickness of these layers is critical if specifications regarding the final color and intensity of light of the lamp are to be met. Let X and Y denote the thickness of two different layers of ink. It is known that X is normally distributed with a mean of 0.1 millimeter and a standard deviation of 0.00031 millimeter and Y is also normally distributed with a mean of 0.23 millimeter and a standard deviation of 0.00017 millimeter. Assume that these variables are independent.
 - a. If a particular lamp is made up of these two inks only, what is the probability that the total ink thickness is less than 0.2337 millimeter?
 - b. A lamp with a total ink thickness exceeding 0.2405 millimeters lacks the uniformity of color demanded by the customer. Find the probability that a randomly selected lamp fails to meet customer specifications.
4. The width of a casing for a door is normally distributed with a mean of 24 inches and a standard deviation of 0.125 inch. The width of a door is normally distributed with a mean of 23.875 inches and a standard deviation of 0.0625 inch. Assume independence.
 - a. Determine the mean and standard deviation of the difference between the width of the casing and the width of the door.
 - b. What is the probability that the width of the casing minus the width of the door exceeds 0.25 inch?
 - c. What is the probability that the door does not fit in the casing?