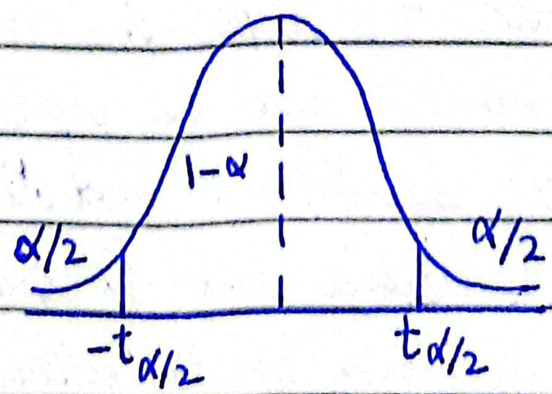
Case 2: Unknown Variance σ^2

If $\bar{X}$ and s are the mean and the standard deviation of a random sample of size n from a population with unknown standard deviation σ, a $100(1-\alpha)\%$ confidence interval for μ is given by:

$$P\left(-t_{\alpha/2} < \frac{\bar{X}-\mu}{s/\sqrt{n}} < t_{\alpha/2}\right)$$

$= 1-\alpha$



$$P\left(\bar{X}-t_{\alpha/2}\frac{s}{\sqrt{n}} < \mu < \bar{X}+t_{\alpha/2}\frac{s}{\sqrt{n}}\right)$$

$= 1-\alpha$

Where $t_{\alpha/2}$ is the t-value with $(n-1)$ degrees of freedom (v).

One-sided Confidence Interval:

$$\mu < \bar{x} + t_{\alpha} \frac{s}{\sqrt{n}}$$ upper bound

$$\bar{x} - t_{\alpha} \frac{s}{\sqrt{n}} < \mu$$ lower bound

Case: 3 Large Sample Confidence Interval

Assume that the sample size n is greater than 30 and the population is not too skewed. We may utilize both the z-values and the sample standard deviation s for estimating the population mean μ:

$$\bar{x} - Z_{\alpha/2} \frac{s}{\sqrt{n}} < \mu < \bar{x}\, Z_{\alpha/2} \frac{s}{\sqrt{n}}$$

Note:

This can be regarded as a normal approximation (t-value becomes z-value when n is sufficiently large); the quality of the approximation becomes better as the sample size gets larger.

Hypothesis Testing:

A statistical hypothesis is an assumption about a population parameter. For example, we may assume that the mean height of a male in the U.S. is 70 inches. The assumption about the height is the statistical hypothesis and the true mean height of a male in the U.S. is the population parameter.

So, a Hypothesis Test is a formal statistical test we use to reject or accept a statistical hypothesis.

The statements of claim we are testing are expressed as two hypotheses:

1) Null Hypothesis H_0:

The statement being

tested in a statistical test is called the null hypothesis, denoted by H_0. The test is designed to assess and determine the strength of evidence in the data against the null hypothesis H_0.

2) Alternative Hypothesis. H_1:

What you want to claim. This is what you want to accept as true when you reject the null hypothesis.

One should reach out at one of the following conclusions:

1) Reject H_0 in favour of H_1 because of sufficient evidence in the data.

2) Fail to reject H_0 because of insufficient evidence in the data.

For hypothesis tests involving a population mean, we let μ_0 denote the hypothesized value and we must choose one of following three forms for the hypothesis test:

$H_0 : \mu \geq \mu_0$; $\mu \leq \mu_0$; $\mu = \mu_0$
$H_1 : \mu < \mu_0$; $\mu > \mu_0$; $\mu \neq \mu_0$

Testing a Hypothesis:

Test Statistics:

A test statistic estimates the parameter that appears in the hypothesis

→ When H_0 is true, we expect the estimate to take a value near the parameter value specified by H_0.

→ Values of the estimate far from the parameter value specified by H_0 give evidence against H_0.

→ The alternative hypothesis determines which direction count against H_0.

There are two test statistics,

1) Z-test, $Z_0 = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$

when σ is known

2) t-test: $t_0 = \dfrac{\bar{x} - \mu_0}{S/\sqrt{n}}$

when σ is unknown

Level of Significance α :

As, hypothesis tests are based on sample information, we must allow for the possibility of errors. There are two types of errors that can be made in hypothesis testing:

1) Type I Error: Rejecting the null hypothesis when it is true. It is denoted as α.

2) Type II Error: Not rejecting the null hypothesis when it is false. It is denoted by β.

The level of significance gives or represents the probability of making a Type I error, which is rejecting the null hypothesis when it is true.

The significance level is manually

set to 0.05 or 5%.

Critical Region:

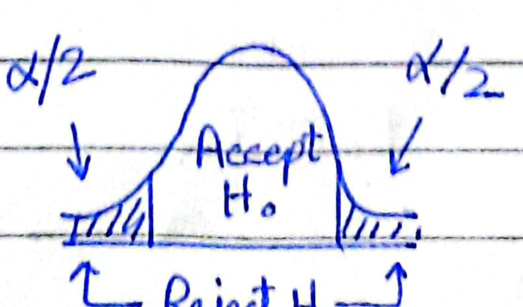
The significance level helps specify the size of the region where the null hypothesis should be rejected. We call that region, the critical region.

Suppose that we test.

(Two tailed)

→ $H_0 : \mu = \mu_0$ v.s. $H_1 : \mu \neq \mu_0$

The critical region is in the two extreme regions (both tails)



⇒ $Z_0 < -Z_{\alpha/2}$ or $Z_0 > Z_{\alpha/2}$

⇒ $t_0 < -t_{\alpha/2}$ or $t_0 > t_{\alpha/2}$

(One tailed)

→ $H_0 : \mu \geq \mu_0$ v.s. $H_1 : \mu < \mu_0$

The critical region is in the extreme left region (left tail).

$$Z_0 < -Z_\alpha$$

or

$$t_0 < -t_\alpha$$

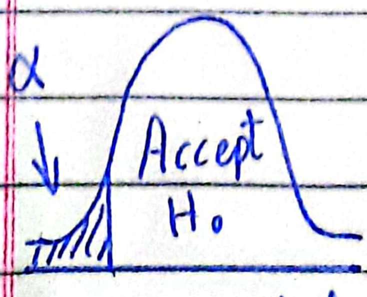
→ $H_0 : \mu \leq \mu_0$ v.s. $H_1 : \mu > \mu_0$

The critical region is in the extreme right region (right tail).

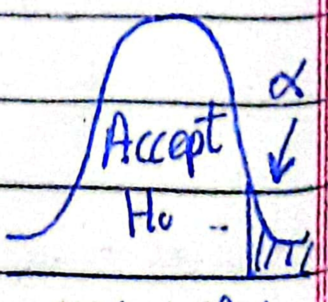
$$Z_0 > Z_\alpha$$

or

$$t_0 > t_\alpha$$



Left tailed



Right tailed

Decision criterion using Critical Regions:

→ If the test statistic falls within the critical region we reject H_0 at a level of significance α.

→ If the test statistic does not fall in the critical region, we fail to reject H_0 at an α level.

P-value Definition:

The P-value is known as the probability value. It is defined as the probability of getting a result that is either the same or more extreme than the actual observations. The P-value is known as the level of marginal significance within the hypothesis testing that represents that the probability of occurrence of the given event. The p-value is used as an alternative to the rejection point to provide the least significance at which the null hypothesis would be rejected. If the P-value is small, then there is stronger evidence in favour of the alternative hypothesis.

To test a population mean μ when σ is known:

→ $2P(Z \geq |z_0|)$ for $H_1: \mu \neq \mu_0$

→ $P(Z \geq z_0)$ for $H_1: \mu > \mu_0$

→ $P(Z \leq z_0)$ for $H_1: \mu < \mu_0$

To test a population mean μ when σ is unknown:

→ $2P(T \geq |t_0|)$ for $H_1: \mu \neq \mu_0$

→ $P(T \geq t_0)$ for $H_1: \mu > \mu_0$

→ $P(T \leq t_0)$ for $H_1: \mu < \mu_0$

⇒ If the P-value $\leq \alpha$, we reject H_0 at the level of significance.

⇒ If the P-value $> \alpha$, we fail to reject the H_0.

Two Samples: Tests on Two Means:

Hypothesis testing for two samples involves comparing the means of two different groups to determine if there is a significant difference between them.

$$\mu_1 , \sigma_1^2 , N_1 , \bar{X}_1$$
$$\mu_2 , \sigma_2^2 , N_2 , \bar{X}_2$$

Formulating Hypothesis:

Null Hypothesis H_0 :

$$H_0 : \mu_1 - \mu_2 = d_0$$

Alternative Hypothesis H_1 :

① $H_1 : \mu_1 - \mu_2 \neq d_0$
② $H_1 : \mu_1 - \mu_2 > d_0$
③ $H_1 : \mu_1 - \mu_2 < d_0$

Choosing Appropriate Test:

When σ_1^2 and σ_2^2 are known:

$$Z_0 = \frac{(\bar{X}_1 - \bar{X}_2) - d_0}{\sqrt{\frac{\sigma_1^2}{N_1} + \frac{\sigma_2^2}{N_2}}}$$

Let $\sigma_1^2 = \sigma_2^2 = \sigma^2$ (Same)

$$Z_0 = \frac{(\bar{X}_1 - \bar{X}_2) - d_0}{\sigma\sqrt{\frac{1}{N_1} + \frac{1}{N_2}}}$$

Critical Region:

1. $|Z_0| > Z_{\alpha/2}$

   $Z_0 > Z_{\alpha/2}$ or $Z_0 < -Z_{\alpha/2}$

2. $Z_0 > Z_\alpha$
3. $Z_0 < -Z_\alpha$

P_value: $P_r(|z| > \text{test value})$
$P_r(z > \text{test\_value})$
$P_r(z < -\text{test\_value})$

σ_1^2 and σ_2^2 are unknown and equal:

$$t_0 = \frac{(\bar{X}_1 - \bar{X}_2) - d_0}{S_p\sqrt{\frac{1}{N_1} + \frac{1}{N_2}}}$$

where,

$$S_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

$$v = n_1 + n_2 - 2$$

Critical Region.

1. $|t_0| > t_{\alpha/2}$
   $t_0 > t_{\alpha/2}$ or $t_0 < -t_{\alpha/2}$

(2) $t > t_\alpha$

(3) $t < -t_\alpha$

P_value : $Pr(|t_0| > \text{test\_value})$

$Pr(t_0 > \text{test\_value})$

$Pr(t_0 < -\text{test\_value})$

σ_1^2 and σ_2^2 are unknown and un-equal.

$$t_0' = \frac{(\bar{X}_1 - \bar{X}_2) - d_0}{\sqrt{S_1^2/n_1 + S_2^2/n_2}}$$

where,

$$v = \frac{(S_1^2/n_1 + S_2^2/n_2)^2}{\frac{(S_1^2/n_1)^2}{n_1 - 1} + \frac{(S_2^2/n_2)^2}{n_2 - 1}}$$

Paired Observations:

A paired sample t-test is used to compare the means of two samples when each observation in one sample can be paired with an observation in the other sample.

It is commonly used when a measurement is taken on a subject before and after some treatement.

Let,
N sample from population 1
$X_{i,1} \rightarrow$ ith sample

N samples from population 2
$X_{i,2} \rightarrow$ ith sample

$$\bar{D} = \frac{1}{N} \sum_{i=1}^{N} D_i$$

$$S_D = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (D_i - \bar{D})^2}$$

μ_D is true mean of D_i

Hypothesis Formulation:

$H_0: \mu_D = \mu_1 - \mu_2 = d_0$

$H_1: \mu_D = \mu_1 - \mu_2 \neq d_0$ or

$\mu_D = \mu_1 - \mu_2 > d_0$ or

$\mu_D = \mu_1 - \mu_2 < d_0$

Statistical Testing:

$$t_0 = \frac{\bar{D} - d_0}{S_D / \sqrt{n}}$$

Critical Region:

1) $|T| > t_{\alpha/2}$
2) $T > t_\alpha$
3) $T < -t_\alpha$ with $v = n-1$

Chi-Square: Goodness of Fit test

A chi-square (X^2) goodness of fit is a goodness of fit test for a categorical variable. Goodness of fit is a measure of how well a statistical model fits a set of observations.

→ When goodness of fit is high, the values expected based on the model are close to the observed values.

→ When goodness of fit is low, the values expected based on the model are far from the observed values.

Example 1: (Discrete)

A school principal would like to know which days of the week students are most likely to be absent. The principal expects that students will be absent equally during the 5-day school week. The principle selects a random sample of 100 teachers asking them which day of week they had the highest number of student absences. The observed and expected results are shown in the table below. Based on the results, do the days for the highest number of absences occur with equal frequencies? (Use a 5% significance level)

	Mon	Tue	Wed	Thurs	Friday
Observed	23	16	14	19	28
Expected	20	20	20	20	20

H_0 : Equal Frequencies
H_1 : Unequal Frequencies

So, $\chi^2_c = \dfrac{\Sigma (O-E)^2}{E}$

=>

$$\chi_c^2 = \frac{3^2}{20} + \frac{(-4)^2}{20} + \frac{(-6)^2}{20} + \frac{(-1)^2}{20} + \frac{8^2}{20}$$

$$= \frac{126}{20} = 6.3$$

Now using Chi-square table:

$$\chi_\alpha^2 = \chi_{0.05}^2 = 9.49$$

where $v = k - 1 = 5 - 1 = 4$

Here k is the number of cells or columns.

As $\chi_c^2 < \chi_\alpha$

$6.3 < 9.49$

So we will accept Null hypothesis.

Example 2: (Continuous)

Class Boundaries	O_i		e_i	
1.45 - 1.95	2		0.5	
1.95 - 2.45	1	7	2.1	8.5
2.45 - 2.95	4		5.9	
2.95 - 3.45	15		10.3	
3.45 - 3.95	10		10.7	
3.95 - 4.45	5	8	7.0	10.5
4.45 - 4.95	3		3.5	

Here given: $\mu = 3.5$ and $\sigma = 0.7$

The expected frequencies for the 7 classes/cells are obtained using the following formula.

(Finding for 4th class)

First,

$$Z_1 = \frac{2.95 - 3.5}{0.7} = -0.79$$

$$Z_2 = \frac{3.45 - 3.5}{0.7} = -0.07$$

Second,

Now find area between z_1 and z_2 using z_table:

$$\text{Area} = P(-0.79 < Z < -0.07)$$
$$= 0.4721 - 0.2148$$
$$= 0.2573$$

Total observed frequency

Hence, $e_4 = (0.2573)(40)$
$= 10.3$

It is compulsory to round these to one decimal.

Now,

$$\chi_c^2 = \frac{(7-8.5)^2}{8.5} + \frac{(15-10.3)^2}{10.3} + \frac{(10-10.7)^2}{10.7} + \frac{(8-10.5)^2}{10.5}$$

$= 3.05$

Note: We have combined adjacent classes where we see the expected frequencies are less than 5.

So, total classes reduced from

7 to 4 means $v = 3$.

Hence, $\chi^2_{0.05} = 7.815$

which is greater than χ^2_c

Hence, H_0 is accepted

Chi-Square: Test for Independence

The chi-square test of independence is a statistical method used to determine if there's a relationship between two categorical variables.

Example:

The table given shows the average number of hours students spend studying for classes each day in a high school. Is the average number of hours dependent on the type of student? (Use a 5% significance level)

Observed Results

Student	0-2 Hrs	2-4 Hrs	4-6 Hrs	Total
Freshman	76	143	91	310
Seniors	147	109	64	320
Total	223	252	155	630

Null Hypothesis: H_0 : Independent

H_1 : Dependent

Here $V = (r-1)(c-1)$

$V = (2-1)(3-1)$

$V = 2$

Now lets find the expected values using the following formula:

$$E = \frac{(\text{column total}) \times (\text{row total})}{\text{grand total}}$$

Expected Results

Student	0-2 Hrs	2-4 Hrs	4-6 Hrs	Total
Freshman	110	124	76	310
Seniors	113	128	79	320
Total	223	252	155	630

$E_{1,1} = \frac{(310)(223)}{630}$ $E_{1,2} = \frac{(310)(252)}{630}$

$E_{1,3} = \frac{(310)(155)}{630}$ $E_{2,1} = \frac{(320)(223)}{630}$

$E_{2,2} = \frac{(320)(252)}{630}$ $E_{2,3} = \frac{(320)(155)}{630}$

Now,

$$\chi^2_c = \frac{\Sigma(O-E)^2}{E}$$

$$= \frac{(76-110)^2}{110} + \frac{(143-124)^2}{124}$$

$$+ \frac{(91-76)^2}{76} + \frac{(147-113)^2}{113}$$

$$+ \frac{(109-128)^2}{128} + \frac{(64-79)^2}{79}$$

$\chi_c^2 = 32.28$

Now,

$$\chi_\alpha^2 = \chi_{0.05}^2 = 5.99$$

As, $\chi_c^2 > \chi_\alpha^2$

$32.28 > 5.99$

Hence we will reject H_0

Note: When $v = 1$

We use this formula:

$$\chi_c^2 = \frac{\Sigma(|O_i - E_i| - 0.5)^2}{E_i}$$

→ Linear Regression:

Many problems involve exploring the relationships between two or more variables. For example, in a chemical process, suppose that the yield of the product is related to the process-operating temperature. Regression analysis can be used to build a model to predict yield at a given temperature level.

→ Single Linear Regression:

Single linear regression is a simple and commonly used model that explores the relationship between two continuous quantitative variables. It is used to explain the behaviour of a dependent variable (Y) based on a single independent variable (X).

There is are straight-line relation-ship between X and Y:

$$\hat{y} = \beta_0 + \beta_1 x$$

Here, $\hat{y}$ is the predicted dependent variable

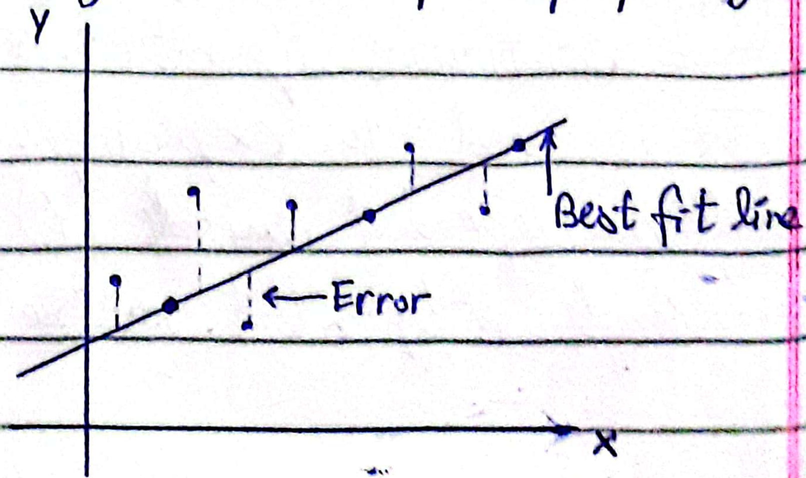
x is independent variable

β_0 is the intercept

β_1 is the slop

Calculating the Linear Best Fit:

We want to find a straight line that best fits our data points. But, it's impossible to have one line pass through all the points perfectly.



Method of least Squares:

We aim to minimize the prediction errors. We look at each data point and see how much our line's prediction is off from the actual value. Our goal is to find the line with the smallest errors - that's called the "linear best fit".
To measure the error or residual we use this formula:

$$e_i = y_i - \hat{y}_i$$

The method of least squares is used to reduce the sum of squares (SSE) errors as much as possible.

$$SSE = \sum_{i=1}^{i=N} (e_i)^2$$

$$SSE = \sum_{i=1}^{i=N} (y_i - \hat{y}_i)^2$$

Here are the derived formulas to calculate the required parameter values (intercept and slope) for the best-fit line:

$$\hat{\beta}_0 = \bar{Y} - \beta_1 \bar{X}$$

$$\hat{\beta}_1 = \frac{n \sum XY - \sum X \sum Y}{n \sum X^2 - (\sum X)^2}$$

Note: Each pair of observations satisfies the relationship:

$$y_i = \hat{\beta}_0 + \hat{\beta}_1 x_i + e_i$$

where $i = 1, 2, \ldots, n$

Coefficient of Determination:

It is denoted by r^2. It provides insight into how we a regression model fits the data and quantifies the proportion of the total variability in the dependent (Y) that can be explained by the independent (X).

$$r^2 = \frac{\text{Explained Variation}}{\text{Total Variation}} = \frac{SSR}{SST}$$

$$1 - \frac{SSE}{SST} = 1 - \frac{\text{Unexplained Variation}}{\text{Total Variation}}$$

$$SSE = \Sigma(Y - \hat{Y})^2$$
$$= \Sigma Y^2 - \beta_0 Y - \beta_1 \Sigma XY$$

$$SST = \Sigma(Y - \bar{Y})^2$$
$$= \Sigma Y^2 - \frac{(\Sigma Y)^2}{n}$$

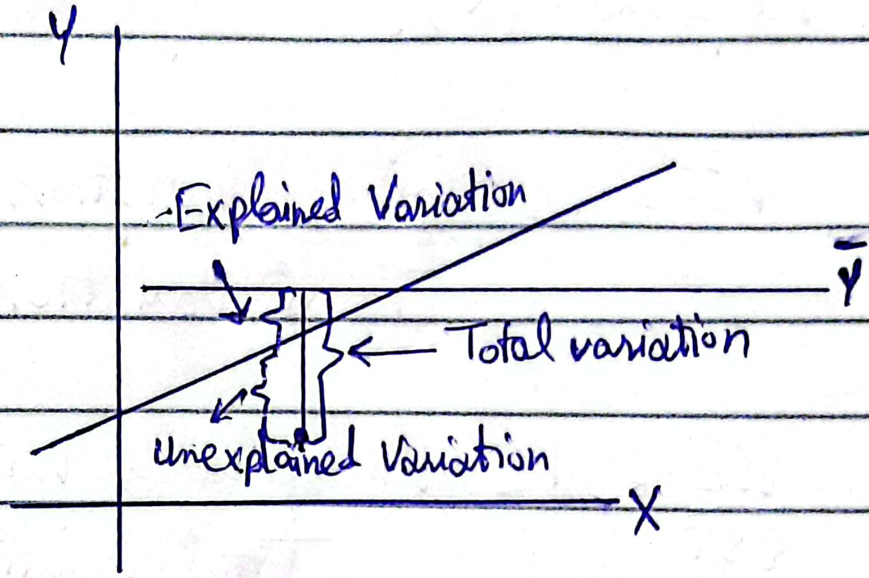
$$\text{Explained Variation} = \boxed{\text{Total Variation}} - \boxed{\text{Unexplained}}$$

→ $0 \leq r^2 \leq 1$

→ $0 \leq r^2 < 0.5$ weak

$0.5 \leq r^2 < 0.8$ medium

$0.8 \leq r^2 \leq 1$ strong



Inferences Concerning the Regression Coefficients:

→ **Confidence Interval for β_1, the slope parameter:**

A $(1-\alpha)\,100\%$ confidence interval for the parameter β in the regression line $\hat{y} = \beta_0 + \beta_1 x$ is

$$\Rightarrow \hat{\beta}_1 - t_{\alpha/2} \frac{s}{\sqrt{S_{xx}}} < \beta_1 < \hat{\beta}_1 + t_{\alpha/2} \frac{s}{\sqrt{S_{xx}}}$$

where $t_{\alpha/2}$ is a value of the t-distribution with $n-2$ degrees of freedom

Here, $s^2 = \dfrac{SSE}{n-2} = \dfrac{S_{yy} - \hat{\beta}_1 S_{xy}}{n-2}$

$\Rightarrow S_{xx} = \sum (x-\bar{x})^2$

$\Rightarrow S_{yy} = \sum (y-\bar{y})^2$

$\Rightarrow S_{xy} = \sum (x-\bar{x})(y-\bar{y})$

→ Hypothesis testing on the slop parameter, β_1 :

For hypothesis testing, use the following test statistic :

$$t = \frac{\hat{\beta}_1 - \beta_{1,0}}{S/\sqrt{S_{xx}}}$$

→ $H_0 : \beta_1 = \beta_{1,0}$

→ 1) $H_1 : \beta_1 \neq \beta_{1,0}$

2) $H_1 : \beta_1 > \beta_{1,0}$

3) $H_1 : \beta_1 < \beta_{1,0}$

→ Confidence Interval for β_0 :

A $100(1-\alpha)\%$ confidence interval for the parameter β_0 in the regression line :

$$\hat{\beta}_0 - t_{\alpha/2} \frac{S}{\sqrt{n S_{xx}}} \sqrt{\Sigma x^2} < \beta_0 < \hat{\beta}_0 + t_{\alpha/2} \frac{S}{\sqrt{n S_{xx}}} \sqrt{\Sigma x^2}$$

where, $t_{\alpha/2}$ is a value of the t-distribution with $n-2$ degrees of freedom.

→ **Hypothesis testing on the intercept parameter, β_0:**

For hypothesis testing, use the following test statistic:

$$t = \cancel{\frac{\hat{\beta}_0 - \beta_{0,0}}{S/\sqrt{S_{xx}}}} = \frac{\hat{\beta}_0 - \beta_{0,0}}{S\sqrt{\frac{\sum x^2}{(n S_{xx})}}}$$

→ $H_0 : \beta_0 = \beta_{0,0}$

→ 1) $H_0 : \beta_0 \neq \beta_{0,0}$

2) $H_0 : \beta_0 > \beta_{0,0}$

3) $H_0 : \beta_0 < \beta_{0,0}$

→ **Special Case for β_1 :**

if, $H_0 : \beta_1 = 0$

$\downarrow$

It shows ~~data~~ data is not linear

$\beta_1 = 0$, means there is no randomness in the variables.

Example: Consider the table:

X	Y
3	5
7	11
11	21
15	16
18	16
27	28
29	27
30	25
30	35

X	Y	X	Y
31	30	46	46
31	40	47	49
32	32	50	51
33	34		
33	32		
34	34		
36	37		
36	38		
36	34		
37	36		
38	38		
39	37		
39	36		
39	45		
40	39		
41	41		
42	40		
42	44		
43	37		
44	44		
45	46		

a) Estimate the regression line:

$\sum X = 1104$, $\sum Y = 1124$

$\sum XY = 41355$, $\sum X^2 = 41086$

Now,

$$\hat{\beta}_1 = \frac{n \sum XY - \sum X \sum Y}{n \sum X^2 - (\sum X)^2}$$

$$= \frac{(33)(41355) - (1104)(1124)}{(33)(41086) - (1104)^2}$$

$$= 0.903643$$

$$\hat{\beta}_0 = \overline{Y} - \beta_1 \overline{X}$$

$$= 34.0606 - (0.903643)(33.4545)$$

$$= 3.829633$$

Hence,

$$\hat{y} = 3.8296 + 0.9036\, x$$

b) Find a 95% confidence interval for β_1 in the regression line.

$S_{xx} = 4152.18$
$S_{xy} = 3752.09$
$S_{yy} = 3713.88$
$\hat{\beta}_1 = 0.903643$

$$s^2 = \frac{S_{yy} - \hat{\beta}_1 S_{xy}}{n-2} = \frac{3713.88 - (0.9036)(3752.09)}{31}$$

$$= 10.4299$$

$s = 3.2295$

And now,

$$\Rightarrow \hat{\beta}_0 - t_{\alpha/2} \frac{s}{\sqrt{S_{xx}}} < \beta_1 < \hat{\beta}_1 + t_{\alpha/2} \frac{s}{\sqrt{S_{xx}}}$$

$$\Rightarrow 0.903643 - \frac{(2.045)(3.2295)}{\sqrt{4152.18}} < \beta <$$

$$0.903643 + \frac{(2.045)(3.2295)}{\sqrt{4152.18}}$$

$\Rightarrow 0.8012 < \beta_1 < 1.0061$

c) Using the estimated value $\hat{\beta}_1 = 0.903643$. Test the hypothesis that $\beta_1 = 1.0$ against the alternative that $\beta_1 < 1.0$:

$$t_0 = \frac{0.903643 - 1.0}{3.2295/\sqrt{4152.18}} = -1.92$$

Now,

$$t_0 < -t_\alpha$$
$$-1.92 < -t_{0.05,31}$$
$$-1.92 < -1.697$$

Hence we will reject the null hypothesis.

d) Find a 95% confidence interval for β_0 :

$S_{xx} = 4152.18$, $s = 3.2295$

$\sum x^2 = 41086$, $\hat{\beta}_0 = 3.829633$

$$\Rightarrow 3.829633 - \frac{(2.045)(3.2295)\sqrt{41086}}{\sqrt{(33)(4152.18)}}$$

$$< \beta_0 <$$

$$3.829633 + \frac{(2.045)(3.2295)\sqrt{41086}}{\sqrt{(33)(4152.18)}}$$

$$\Rightarrow \quad 0.2132 < \beta_0 < 7.4461$$

e) Using the estimated value $\hat{\beta}_0 = 3.829633$, test the hypothesis that $\beta_0 = 0$ at 0.05 level of significance against $\beta_0 \neq 0$:

$H_0 : \beta_0 = 0$

$H_1 : \beta_0 \neq 0$

$$t_0 = \frac{3.829633 - 0}{3.2295\sqrt{\frac{41086}{(33)(4152.18)}}}$$

$= 2.17$

Now,

$t_0 > t_{0.025}$ or $t_0 < -t_{0.025}$

$2.17 > 2.355$ or $2.17 < -2.355$

Hence, we will reject null hypothesis.