CS 301: Theory of Automata
Midterm Exam 2: Solutions
Fall 2019

PROBLEM 1 **(Marks: 10)**

Write a context free grammar for the following language and clearly indicate the start symbol.

L = $\{0^n 1^m \mid m \neq n, m \geq 0, n \geq 0\}$

SOLUTION

S is the start symbol

S → ZE | EA (zeros followed by $0^k 1^k$ or $0^k 1^k$ followed by ones)

Z → 0 | 0Z (all strings of zeros with at least one zero)

A → 1 | 1A (all strings of 1 with at least one 1)

E → ϵ | 0E1 ($0^k 1^k$ where $k \geq 0$)

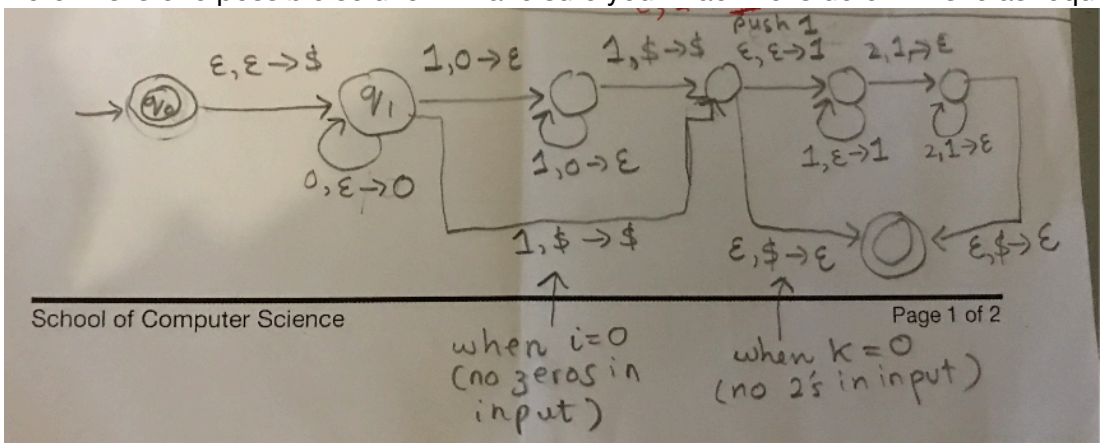
PROBLEM 2 **(Marks: 10)**

Construct a **deterministic PDA** to accept the following language. Show its state transition diagram.

L = $\{0^i 1^j 2^k \mid i \geq 0, j \geq 0, i \geq 0, j = i + k\}$

SOLUTION

Note this is one possible solution. Make sure your machine is deterministic as required.



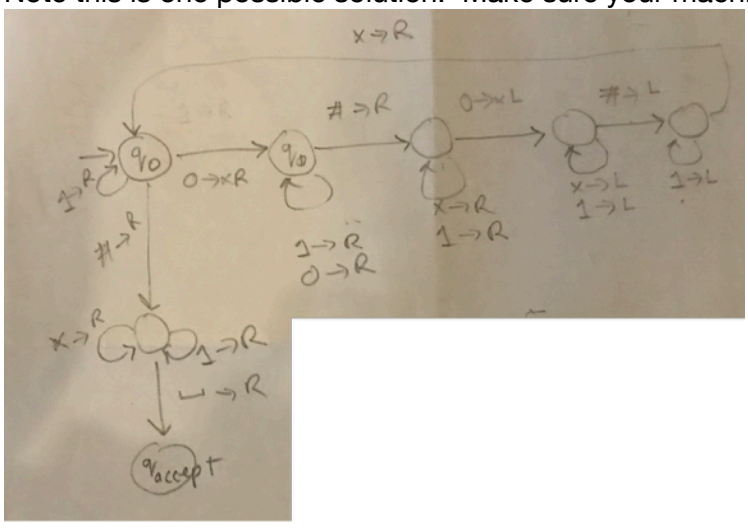
PROBLEM 3 **(Marks: 10)**

Show the transition diagram of a **single tape deterministic Turing machine** to decide the following:

L = {x#y | x and y have the same number of zeros and x ∈{0,1}* and y∈{0,1}* }

SOLUTION

Note this is one possible solution. Make sure your machine is deterministic as required.



PROBLEM 4 **(Marks: 5+5)**

1. Prove that the following is an ambiguous grammar using right most derivations:
   S -> S0S | 0SS | 0

SOLUTION

Take the string 000, two possible right most derivations are:

$S \Rightarrow S0S \Rightarrow S00 \Rightarrow 000$

$S \Rightarrow 0SS \Rightarrow 0S0 \Rightarrow 000$

As there are two derivations for the same string, hence the grammar is ambiguous.

2. Remove unit productions from the following grammar and write the final grammar. S is the start symbol.

   S -> AB
   A -> C
   B -> D
   C -> C0 | D
   D -> 0 | 1 | AB

SOLUTION

S -> AB
A -> C0 | 0 | 1 | AB
B -> 0 | 1 | AB
C -> C0 | 0 | 1 | AB
D -> 0 | 1 | AB