National University of Computer and Emerging Sciences, Lahore Campus

Course Name:	Discrete Structures	Course Code:	CS-211
Program:	Computer Science	Semester:	Fall 2018
Duration:	3 Hrs	Total Marks:	90
Paper Date:	December 26, 2018.	Weight	
Section:	ALL	Page(s):	6
Exam Type:	Final		

Student : Name:_______________________________ Roll No._________________ Section:_______

Instruction/Notes:
1. Solve the exam on this question paper. You can get extra sheets for rough work but they will NOT be marked or graded.
2. Sharing calculators is strictly NOT allowed.
3. 1 A4 handwritten cheat sheet is allowed in the exam.

Question 1 **(Marks: 10)**

Prove using mathematical induction: $\sum_{k=0}^{n}\binom{n}{k} = 2^n$. Also, write down all identities you use.

Question 2 **(Marks: 5+5)**

i. Assume that there are 51 different types of animals having integral weights less than 100 kg (starting from one kg) and all weights are unique. Using the concept of Pigeon hole principle, prove that there is a pair whose sum of weights is 100 kg.

Q2, ii. For section A,C,D,E,F,G

A bag has 200 socks. There are 60 red, 60 blue, 60 orange, and 20 green ones. If socks are taken out one at a time, what is the minimum number of socks one must draw from the bag to ensure that at least 20 of them are of the same color. Show working/formula.

Q2, ii For section B (Dr. Khalid)

Three cards are selected from a bag containing 3 non-identical red, 4 non-identical white and 5 non-identical green cards. What are the possible number of ways of selecting cards if:

a. All three have the same color?

b. All three have a different color?

Question 3 **(Marks: 4+4)**

i. It is required that a number plate has three English capital letters, followed by four digits. In how many ways can a policeman trace a car whose **number starts from L and ends with digit 5 if:** **(for each part give formula/reasoning)**

a. Letters and digits both can be repeated.

b. Letters and digits both are distinct.

c. Letters can be repeated but digits have to be unique.

d. Letters are distinct but digits can be repeated.

ii. How many arrangements of the letters A,S,T,I,O,N,M can be made with no repetition if: **(for each part give formula/reasoning)**

a. A is to be first letter in each arrangement

b. A and T is fixed at first and last place respectively

c. MAS appears as a string at any place

d. A is fixed at second place and MT, NS appear as strings

Question 4 **(Marks: 10)**

Prove that $\sqrt{7}$ is an irrational number by giving a proof by contradiction.

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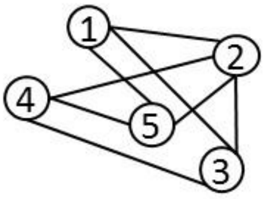
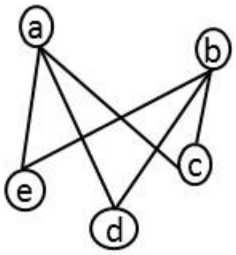
Question 5 **(Marks: 3+3+3+3+3)**

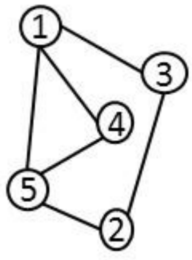
i. Is the following graph isomorphic to W_4 (wheel of order 4). If yes, transform the given graph to W_4 or show mapping. If not, then explain why?



ii. Are the following graphs G_1 and G_2 isomorphic? If yes, show their mapping. If not, then explain why?

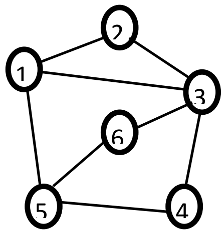


G_1

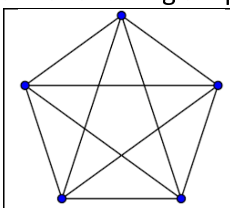
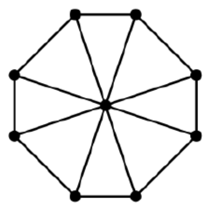
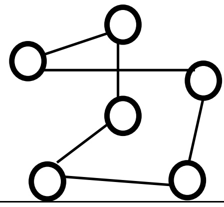


G_2

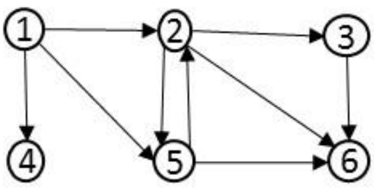
iii. Is the following graph bi-partite? If yes prove it by redrawing the graph, otherwise explain why it isn't?



iv. Name the following simple graphs:

		
a.	b.	c.

v. Write down all the simple paths from **1** to **6.**



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Question 6 FOR SECTIONS A & C (Miss Masooma) (Marks: 10)

Find the total number of bit strings of length 10 that satisfy:

i. Begin and end with a bit 1

ii. Begin with two 0s or end with three 1s

iii. Three consecutive 0s and two consecutive 1s

iv. Begin with two 1s or have four consecutive 0s

v. Begin with two 0s and end with three 1s and contain the string 101

Question 6 FOR SECTIONS B, D, E, F & G (Marks: 2+8)

i. Suppose the roots of the characteristic equation of the associated linear homogeneous recurrence relation are {1,2,2,2} for some constants c_1, c_2, c_3, c_4. The recurrence relation is given by:

$$a_n = c_1 a_{n-1} + c_2 a_{n-2} + c_3 a_{n-3} + c_4 a_{n-4} + n2^n$$

What is the form of the particular solution for the above recurrence? You are not required to solve the recurrence?

ii. Solve the following recurrence relation and give the final solution for initial conditions: $a_0=3, a_1=6$

$a_n = 6a_{n-1} - 9a_{n-2}$ NOTE: Clearly write down the final answer also.

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Question 7 (Marks: 4+2+2+4+3+2+5+5)

For all questions (where applicable) you can give the formula and do not have to compute the exact number.

i. Tick the correct options. C is the choose function:

a. C(5000,100) = C(4999,99)+C(4999,100)	☐ true	☐ false
b. C(5000,100) = C(5000,4900)	☐ true	☐ false
c. P(5000,100) = P(5000,4900)	☐ true	☐ false
d. C(5000,100) = C(100,90)*C(4900,10)	☐ true	☐ false

ii. GCD(100,190) =

iii. Give the smallest positive integer x that satisfies the following congruence:

$3x \equiv 2 \pmod 8$

iv. Tick the correct option?

a. {apples, oranges, bananas}	☐ countable	☐ uncountable
b. $\{x \mid 0 \le x \le 1$ and x is a real number with 100 digits after the decimal$\}$	☐ countable	☐ uncountable
c. 2.2222 <= x <= 0.2223 and x is a real number	☐ countable	☐ uncountable
d. $\{2^x \mid x \in Z\}$	☐ countable	☐ uncountable

v. Find the transitive closure of the following relation R defined on {a,b,c,d}:

R = {(a,b),(a,d),(c,b),(d,b),(d,c),(c,a)}

vi. Find the inverse of the following function $f : Z \to Z$

f(x) = x + 5

vii. Given the following knowledge base

cat(mano); cat(chotto); puppy(kalu); puppy(ragy); puppy(goldy);
color(mano,black); color(chotto,black);
color(kalu,black); color(ragy,brown); color(goldy,black)

Tick the correct option given the above facts.

a. $\forall x$ ($\neg$color(x,black) $\to$ $\neg$cat(x))	☐ true	☐ false
b. $\forall x$ (puppy(x) $\wedge$ (color(x,brown) $\vee$ color(x,black)))	☐ true	☐ false
c. $\forall x$ (color(x,black) $\to$ cat(x))	☐ true	☐ false
d. $\exists x$ (puppy(x) $\wedge$ color(x,brown))	☐ true	☐ false
e. $\forall y \exists x$ (puppy(y) $\to$ color (y,x))	☐ true	☐ false

viii. Use modular exponentiation algorithm to calculate the value of 4^{281} mod 11. No marks without proper working.