Chapter 3 pg 81,-84 definitions.

* Discrete probability Distributions.

(1) $f(x) \geq 0$ prob lies between 0-1
(2) $\sum f(x) = 1$ sum of prob = 1
(3) $P(X=x) = f(x)$ denoted as $P(X), P(x), f(x)$

Ex 3.1

X is discrete.
Y is ~~discre~~ continuous
M is continuous.
N is discrete
P is discrete
Q is continuous

* Random variable must be denoted by capital X and its values always by small x

Ex 3.2

total = 5
Blemished = 2 = 2B
non-blemished = 3 = 3N
sample space = S = {NNN, BNN, NBN, NNB, BBN, BNB, NBB}

let X be random variable (R.V) for blemishes
then $x = 0, 1, 2$

Ex 3.4

$S = \{HHH, THHH, HTHHH, TTHHH, THTHHH, HTTHHH, TTTHHH, HHTHHH\}$

Yes this is a discrete sample space because head and tail are whole numbers

Ex 3.3

$n(S) = (2)^3 = 8$

$S = \{TTT, THH, HTH, HHT, THH, HTT, HHH, THT\}$

W = no. of heads − no. of tails

$TTT = 0-3 = -3$

$THT = 1-2 = -1$

$TTH = 1-2 = -1$

$HTT = 1-2 = -1$

$HHT = 2-1 = 1$

$HTH = 2-1 = 1$

$THH = 2-1 = 1$

$HHH = 3-0 = 3$

$W = -3, -1, 1, 3$

Ex 3.8

$P(H) = \frac{2}{3}$, $P(T) = \frac{1}{3}$

S = {TTT, THT, TTH, HTT, HHT, HTH, THH, HHH}

TTT $= (1/3)^3$
THT $= (2/3)(1/3)^2$
TTH $= (2/3)(1/3)^2$
HTT $= (2/3)(1/3)^2$
HHT $= (2/3)^2(1/3)$
HTH $= (2/3)^2(1/3)$
THH $= (2/3)^2(1/3)$
HHH $= (2/3)^3$

Q 3.3 w	Q 3.8 $f(w)$
-3	$1/27$
-1	$3(2/27)$
1	$3(4/27)$
3	$8/27$

* uniform distribution: if all elements of a samp set have equal probability to appear or be selected.

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Q3.11

$N = 7$, $n = 3$, $n(S) = {}^{N}C_n = {}^{7}C_3 = 35$

2 defective, $K = 2$ (interested in defective)

5 good

'0' D & '3' G = 3

1 D & 2 G = 3

2 D & 1 G = 3

r.v $X \Rightarrow x = 0, 1, 2$

Prob. dist of x

x	$f(x) = ({}^{2}C_x \times {}^{5}C_{3-x}) / {}^{7}C_3 = ({}^{K}C_x \times {}^{N-K}C_{n-x}) / {}^{N}C_n$
0	$({}^{2}C_0 \times {}^{5}C_3) / {}^{7}C_3 = 1 \times 10/35 = 10/35$
1	$({}^{2}C_1 \times {}^{5}C_2) / {}^{7}C_3 = 20/35$
2	$({}^{2}C_2 \times {}^{5}C_1) / {}^{7}C_3 = 5/35$.

$$\boxed{\Sigma f(x) = \frac{35}{35} = 1}$$

Q: Shipment of 8 computers contains 3 defectives. 4 comps are selected at random. If x is the number of defective sets, find prob dist of X.

$N = 8$, $n = 4$

3 defectives, $K = 3$

5 goods.

'0' D & '4' G = 4

1 D & 3 G = 4

2 D & 2 G = 4

3 D & 1 G = 4.

r.v $X \Rightarrow x = 0, 1, 2, 3$

Prob of dist of X

x	$f(x) = ({}^3C_x \times {}^5C_{4-x}) / {}^8C_4$
0	$({}^3C_0 \times {}^5C_4) / {}^8C_4 \Rightarrow 1/14$
1	$({}^3C_1 \times {}^5C_3) / {}^8C_4 \Rightarrow 3/7$
2	$({}^3C_2 \times {}^5C_2) / {}^8C_4 \Rightarrow 3/7$
3	$({}^3C_3 \times {}^5C_1) / {}^8C_4 \Rightarrow 1/14$

$\Sigma f(x) = 1$

Q 3.24

Jazz = 5 CDs $\quad$ K = 5 (interested)

Classical = 2 CDs

Rock = 3 CDs

n = 4 (selected CDs) $\quad$ N = 10 (all CDs)

4 J, 0 C, 0 R = 4

3 J, 1 C, 0 R = 4

3 J, 0 C, 1 R = 4

2 J, 1 C, 1 R = 4

2 J, 2 C, 0 R = 4

2 J, 0 C, 2 R = 4

1 J, 2 C, 1 R = 4

1 J, 1 C, 2 R = 4

1 J, 0 C, 3 R = 4

0 J, 2 C, 2 R = 4

0 J, 1 C, 3 R = 4

$x = 0, 1, 2, 3, 4$ $\qquad$ $f(x) = {}^{K}C_x \times {}^{N-K}C_{n-x} /$

Prob dist of x

x	$f(x) = ({}^{5}C_x \times {}^{5}C_{4-x}) / {}^{10}C_4$
0	$({}^{5}C_0 \times {}^{5}C_4) / {}^{10}C_5$
1	$({}^{5}C_1 \times {}^{5}C_3) / {}^{10}C_5$
2	$({}^{5}C_2 \times {}^{5}C_2) / {}^{10}C_5$
3	$({}^{5}C_3 \times {}^{5}C_1) / {}^{10}C_5$
4	$({}^{5}C_4 \times {}^{5}C_0) / {}^{10}C_5$

Q

A box contain 5 red balls, 3 white balls and 4 black balls. 4 balls are selected at random without replacement. Find prob dist for the number of black balls.

$N = 12$, $n = 4$

$K = 4$

$f(x) = ({}^{K}C_x \times {}^{N-K}C_{n-x}) / {}^{N}C_n$

$x = 0, 1, 2, 3, 4$

Prob dist of x

x	$f(x) = ({}^4C_x \times {}^8C_{4-x}) / {}^{12}C_4$
0	$({}^4C_0 \times {}^8C_4) / {}^{12}C_4 = 14/99$
1	$({}^4C_1 \times {}^8C_3) / {}^{12}C_4 = 224/495$
2	$({}^4C_2 \times {}^8C_2) / {}^{12}C_4 = 56/165$
3	$({}^4C_3 \times {}^8C_1) / {}^{12}C_4 = 32/495$
4	$({}^4C_4 \times {}^8C_0) / {}^{12}C_4 = 1/495$

$\Sigma f(x) = 1$

Q: A box contains 4 nickels & 3 dimes. 3 coins are selected at random without replacement. Find prob dist for total T of 3 coins.

nickels = 4

dimes = 3

$n = 3$ (selected)

$N = 7$ (total)

Lets assume $K = 3$ (interest in dimes)

$x = 0, 1, 2, 3$

Prob dist of x, $f(x) = ({}^{K}C_x \times {}^{N-K}C_{n-x}) / {}^{N}C_n$

x	$f(x) = ({}^{3}C_x \times {}^{4}C_{3-x}) / {}^{7}C_3$
0	$({}^{3}C_0 \times {}^{4}C_3) / {}^{7}C_3 = 4/35$
1	$({}^{3}C_1 \times {}^{4}C_2) / {}^{7}C_3 = 18/35$
2	$({}^{3}C_2 \times {}^{4}C_1) / {}^{7}C_3 = 12/35$
3	$({}^{3}C_3 \times {}^{4}C_0) / {}^{7}C_3 = 1/35$

$\Sigma f(x) = 35/35 = 1$

Q3·26: (Binomial distribution for ~~non~~ with replacement)

$$P(X=x) = f(x) = {}^nC_x \, p^x q^{n-x}$$

p : prob of success
q : prob of failure

$n = 3$, $p = P(\text{Green}) = \frac{2}{6} = \frac{1}{3}$

$q = P(\text{Black}) = \frac{4}{6} = \frac{2}{3}$

possible values $= x = 0, 1, 2, 3$

x	$f(x) = {}^3C_x (1/3)^x (2/3)^{3-x}$
0	${}^3C_0 \times (1/3)^0 \times (2/3)^3 = 8/27$
1	${}^3C_1 \times (1/3)^1 \times (2/3)^2 = 12/27$
2	${}^3C_2 \times (1/3)^2 \times (2/3)^1 = 6/27$
3	${}^3C_3 \times (1/3)^3 \times (2/3)^0 = 1/27$

$$\Sigma f(x) = \frac{27}{27} = 1.$$

parameters of binomial dist are n and p

Q:

5 red balls
3 white balls

$$\boxed{f(x) = {}^nC_x\, p^x\, q^{n-x}}$$

~~...~~
$n = 3$
$N = 8$
$p = P(\text{red balls}) = \frac{5}{8}$

$q = P(\text{white balls}) = \frac{3}{8}$

possible values of $x = 0, 1, 2, 3$

Prob. dist of x

x	$f(x) = 3C_x \times (5/8)^x \times (3/8)^{3-x}$
0	$3C_0 \times (5/8)^0 \times (3/8)^3 = 27/512$
1	$3C_1 \times (5/8)^1 \times (3/8)^2 = 135/512$
2	$3C_2 \times (5/8)^2 \times (3/8)^1 = 225/512$
3	$3C_3 \times (5/8)^3 \times (3/8)^0 = 125/512$

$\Sigma f(x) = 512/512 = 1$

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Poissen Distribution

Q 3.35:

$f(x) = \frac{e^{-\mu} \mu^{x}}{x!}$ where μ = mean of population

Given $f(x) = \frac{e^{-6} 6^{x}}{x!}$ for $x = 0, 1, 2, \ldots$

(a) $P(x > 8) = 1 - P(x \leq 8)$

$x = 9, 10, \ldots$

change let $P(x > x) = 1 - P(x \leq 3)$

$x = 0, 1, 2, 3$

$= 1 - [P(x=0) + P(x=1) + P(x=2) + P(x=3)]$

$= 1 - \left[\frac{e^{-6}(6)^0}{0!} + \frac{e^{-6}(6)}{1!} + \frac{e^{-6}(6)^2}{2!} + \frac{e^{-6}(6)^3}{3!}\right]$

$= 1 - e^{-6}\left[\frac{1}{1} + \frac{6}{1} + \frac{36}{2} + \frac{216}{6}\right]$

$= 1 - 0.0025[1 + 6 + 18 + 36]$

$= 1 - 0.0025(61)$

$= 0.8475$

(b) $P(X=2) = \frac{e^{-6}(6)^2}{2!} = \frac{0.0025(36)}{2!} = 0.045$

Cummulative Distribution Function (CDF)

$F(x) = P(X \leq x)$.

Q 3.13 :-

The P.D of X is

x	$f(x)$	$F(x) = P(X \leq x)$
0	0.41	0.41
1	0.37	$0.37 + 0.41 = 0.78$
2	0.16	$0.78 + 0.16 = 0.94$
3	0.05	$0.05 + 0.94 = 0.99$
4	0.01	$0.99 + 0.01 = 1$

$$
P(x) = \begin{cases} 0, & \text{for } x < 0 \\ 0.41, & \text{for } 0 \leq x < 1 \\ 0.78, & \text{for } 1 \leq x < 2 \\ 0.94, & \text{for } 2 \leq x < 3 \\ 0.99, & \text{for } 3 \leq x < 4 \\ 1, & \text{for } x \geq 4 \end{cases}
$$

Q 3.12:

~~x~~ t	F(x) (cummulative)
1	1/4
3	1/2
5	3/4
7	1

$\mu = \frac{16}{4} \Rightarrow 4$

(a) $P(T = 5) = F(5) - F(4)$

$= \frac{3}{4} - \frac{2}{4} \Rightarrow \frac{1}{4} \Rightarrow 0.25$

$\cancel{f(x) = \frac{e^{-\mu}\mu^x}{x!}}$

$\cancel{f(x) = \frac{e^{-4}(4)^5}{5!} = 0.1562}$

<u>Formula:</u>

$$P(a < x < b) = F(b) - F(a)$$

(b) $P(T > 3) = 1 - P(T \leq 3)$

$= 1 - F(3)$

$= 1 - \frac{2}{4}$

$= \frac{2}{4} \Rightarrow \frac{1}{2}$

solve ... practise

(c) $P(1{\cdot}4 < T < 6) = F(6) - F(1{\cdot}4)$

$$= \frac{3}{4} - \frac{1}{4} = \frac{3-1}{4}$$

$$= \frac{1}{2}.$$

(d) $P(T \leq 5 / T \geq 2)$

$\therefore P(A|B) = \frac{P(A \cap B)}{P(B)}$ (formula).

$$P(T \leq 5 | T \geq 2) = \frac{P(2 \leq T \leq 5)}{P(T \geq 2)}$$

$$= \frac{F(5) - F(2)}{1 - P(T \leq 1)}$$

$$= \frac{\frac{3}{4} - \frac{1}{4}}{1 - F(1)}$$

$$= \frac{2/4}{1 - 1/4} \Rightarrow \frac{2}{3}.$$

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Prob

Joint distribution (x & y)

Definition 3.8

① $f(x,y) \geq 0$ for all (x,y)

② $\sum_x \sum_y f(x,y) = 1$

③ $P(X=x, Y=y) = f(x,y)$

Q. 3.37

Joint P.D of x and y is $f(x,y)$

$$\boxed{f(x,y) = cxy}$$

~~x \ y~~	~~1~~	~~2~~	~~3~~
~~1~~	~~c(1·1)=c~~		
~~2~~			
~~3~~			

x \ y	1	2	3	f
1	$c(1\times1)=c$	$c(1)(2)=2c$	$3c$	$6c$
2	$2c$	$4c$	$6c$	$12c$
3	$3c$	$6c$	$9c$	$18c$
total $g(y)$	$6c$	$12c$	$18c$	

$36c$

$\sum\sum f(x,y) = 1$

$\therefore 36c = 1$

$c = 1/36$

© 3.37

(b) $f(x, y) = c|x - y|$, for $x = -2, 0, 2$
$y = -2, 3$

x \ y	-2	3	total x
-2	$-2c$ $(-2-(-2))$	$(3+2)c = 5c$	$5c$
0	$-2-0 = -2c$	$3-0 = 3c$	c
2	$-2-2 = 4c$	$3-2 = c$	$3c$
total y	$6c$	$9c$	$15c = \sum_x \sum_y f(x,y)$

property:

$$\sum_x \sum_y f(x,y) = 1$$

$$15c = 1$$

$$\boxed{c = \frac{1}{15}}$$

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Q 3.38:

$$f(x,y) = \frac{x+y}{30}$$

$x \backslash y$	0	1	2	total
0	0	1/30	2/30	3/30
1	1/30	2/30	3/30	6/30
2	2/30	3/30	4/30	9/30
3	3/30	4/30	5/30	12/30
total	6/30	10/30	14/30	30/30 = 1

$$\boxed{\sum_x \sum_y f(x,y) = 1}$$

find.

(a) $P(X \leq 2, Y = 1)$

$= f(0,1) + f(1,1) + f(2,1)$

$= \frac{1}{30} + \frac{2}{30} + \frac{3}{30}$

$= \frac{6}{30} = \frac{1}{5}.$

(b) $P(X > 2, Y \leq 1)$

$= f(3,0) + f(3,1)$

$= \frac{3}{30} + \frac{4}{30}$

$= \frac{7}{30}.$

(c) $P(X > y)$

$= f(1,0) + f(2,0) + f(3,0) + f(2,1) + f(3,1) + f(3,2)$

$= \frac{1}{30} + \frac{2}{30} + \frac{3}{30} + \frac{3}{30} + \frac{4}{30} + \frac{5}{30}$

$= \frac{18}{30}.$

(d) $P(X + Y) = 4$

$= f(2,2) + f(3,1)$

$= \frac{4}{30} + \frac{4}{30}$

$= \frac{8}{30}$

Q3.39:

3 oranges $\quad N = 8$
3 apples $\quad n = 4$
3 bananas $\quad n(S) = {}^8C_4 = 70$

x = no. of oranges
y = no. of apples

Joint Dist of x & y in $f(x,y)$

$$f(x,y) = \left({}^3C_x \times {}^2C_y \times {}^3C_{4-x-y}\right) / {}^8C_4$$

x \ y	0	1	2	total
0	NP	2/70	3/70	5/70
1	3/70	18/70	9/70	30/70
2	9/70	18/70	3/70	30/70
3	3/70	2/70	NP.	5/70
total	13/70	38/70	15/70	70/70 $= 1 = \sum_x \sum_y f(x,y)$.

cant select 3 out of 4.

$$0,0 = \frac{{}^3C_0 \cdot {}^2C_0 \cdot {}^3C_4}{{}^8C_4}$$ = Not possible.

$$0,1 = \frac{{}^3C_0 \cdot {}^2C_1 \cdot {}^3C_3}{{}^8C_4} = 2/70$$

$$0,2 = \frac{{}^3C_0 \times {}^2C_2 \times {}^3C_2}{{}^8C_4} = 3/70$$

$$1,0 = \frac{{}^3C_1 \times {}^2C_0 \times {}^3C_3}{{}^8C_4} = \frac{1}{70}$$

$$1,1 = \frac{{}^3C_1 \times {}^2C_1 \times {}^3C_2}{{}^8C_4} = \frac{18}{70}$$

$$1,2 = \frac{{}^3C_1 \times {}^2C_2 \times {}^3C_1}{{}^8C_4} = \frac{9}{70}$$

$$2,0 = \frac{{}^3C_2 \times {}^2C_0 \times {}^3C_2}{{}^8C_4} = \frac{9}{70}$$

$$2,1 = \frac{{}^3C_2 \times {}^2C_1 \times {}^3C_1}{{}^8C_4} = \frac{18}{70}$$

$$2,2 = \frac{{}^3C_2 \times {}^2C_2 \times {}^3C_0}{{}^8C_4} = \frac{1}{70}$$

$$3,0 = \frac{{}^3C_3 \times {}^2C_0 \times {}^3C_1}{70} = \frac{3}{70}$$

$$3,1 = \frac{{}^3C_3 \times {}^2C_1 \times {}^3C_0}{70} = \frac{2}{70}$$

$3,2$ = Not possible $\frac{{}^3C_3 \times {}^2C_1 \times {}^3C_{-1}}{{}^8C_4}$ ×

Marginal Dist of x is

$$g(x) = \sum_{y} f(x,y)$$

Marginal Dist of y is

$$h(y) = \sum_{x} f(x,y)$$

Conditional Prob. of random variable Y given that $X = x$ is

$$f(y|x) = \frac{f(x,y)}{g(x)}$$

Similarly conditional probability of R.V X given that $Y = y$ is

$$f(x|y) = \frac{f(x,y)}{h(y)}$$

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Q 3.49

Joint PD of X and Y is

$f(x,y)$		$x = 1$	2	3	$h(y)$
y	1	0.05	0.05	0.10	0.2
	3	0.05	0.10	0.35	0.50
	5	0.00	0.20	0.10	0.30
	$g(x)$	0.10	0.35	0.55	1 $\Sigma\Sigma f(x,y)$

(a) Marginal Dist of x is

x	$g(x)$
1	0.10
2	0.35
3	0.55

$\Sigma g(x) = 1$

(b) Marginal Dist of y is

y	$h(x)$
1	0.2
3	0.5
5	0.3

$1 = \Sigma h(x)$

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(c) $P(Y=3 \mid X=2) = ?$

$$f(y|x) = \frac{f(x,y)}{g(x)}$$

$$f(3|2) = \frac{f(2,3)}{g(2)}$$

$$f(3|2) = \frac{0.10}{0.35}$$

$$f(3|2) = 0.2857$$

(d) $P(x=2 \mid y=5)$

$$\cancel{f(5|2) = f(5,2)}$$

$$\cancel{P(y|x) = f(x,y)}$$

$$P(x|y) = f(x|y)$$

$$f(x|y) = \frac{f(x,y)}{h(y)} \Rightarrow \frac{f(2,5)}{h(5)}$$

$$= \frac{0.20}{0.3} = \frac{2}{3}$$

(e) $P(y=1 \mid x=3)$

$$f(y|x) = \frac{f(x,y)}{g(x)}$$

$$f(1|3) = \frac{0.05}{0.55}$$

$$= \frac{1}{11}$$

Q 3.50.

(a) Marginal dist of X

x	$g(x)$
2	0.4
4	0.6
	1

$1 = \sum_x f(x,y)$

(b) Marginal dist of Y

y	$h(y)$
1	0.25
3	0.5
5	0.25
	1

$1 = \sum_y f(x,y)$.

(c) $P(Y=5 \mid X=4)$

$$f(y|x) = \frac{f(x,y)}{g(x)} = \frac{f(4,5)}{g(4)} \Rightarrow \frac{0.15}{0.6} = \frac{1}{4}$$

(d) $$f(x|y) = \frac{f(x,y)}{h(y)} = \frac{f(2,1)}{h(1)} = \frac{0.10}{0.25}$$

$$= 0.4 = \frac{2}{5}$$

Chap 4 (Stats)
(Mathematical Expectations)

* Expected value of x or mean of x

$$E(x) = \mu = \Sigma x f(x)$$

* variance of the random variable X

$$Var(X) = \delta^2 = E(X-\mu)^2$$
$$= \Sigma (x-\mu)^2 f(x)$$

OR $$\delta^2 = E(X^2) - [E(x)]^2$$

where $E(X^2) = \Sigma x^2 f(x)$

* Standard deviation of R.V X

$$SD(X) = \delta = \sqrt{E(X^2) - [E(x)]^2}$$

Q 4.1

Prob Dist

x^2	x	$f(x)$	$x \times f(x)$	$x^2 f(x)$
0	0	0.41	0	0
1	1	0.37	0.37	0.37
4	2	0.16	0.32	0.64
9	3	0.05	0.15	0.45
16	4	0.01	0.04	0.16
			$0.88 = E(x)$	1.62

Average = ?

$$\boxed{E(x) = \Sigma x f(x)}$$

(b) Var $= \delta^2 = E(x^2) - [E(x)]^2$

$$\delta^2 = 1.62 - [0.88]^2$$

$$\boxed{\delta^2 = 0.8456}$$

$E(x^2) = \Sigma x^2 f(x) = 1.62$

(c) SD $= \delta = \sqrt{\delta^2} = \sqrt{0.8456}$

$$\boxed{\delta = 0.9195}$$

Chapter 4

Q 4.17 $g(x) = (2x+1)^2$

x	$f(x)$	$g(x)$	$g(x)f(x)$
-3	1/6	25	25/6
6	1/2	169	169/2
9	1/3	361	361/3

$\Sigma g(x)f(x) = 209$

$\mu_{g(x)} = ?$

$\mu_x = \Sigma x f(x)$

So, $\mu_{g(x)} = \Sigma g(x) f(x)$

$\boxed{\mu_{g(x)} = 209}$

$var[g(x)] = E\{[g(x)]^2\} - [E\{g(x)\}]^2$

$SD[g(x)] = \sqrt{var[g(x)]}$

Q 4.51

$y \backslash x$	0	1	2	3	$h(y)$	$y(h(y))$	$y^2 h(y)$
0	0	3/70	9/70	3/70	15/70	0	0
1	2/70	18/70	18/70	2/70	40/70	40/70	40/70
2	3/70	9/70	3/70	0	15/70	30/70	60/70
$g(x)$	5/70	30/70	30/70	5/70	$\sum_x\sum_y f(x,y) = 1$	$\Sigma y h(y) = 1$	$\Sigma y^2 h(y) =$
$xg(x)$	0	30/70	60/70	15/70	$\Sigma x g(x) = 105/70$		
$x^2 g(x)$	0	30/70	120/70	45/70	$\Sigma x^2 g(x) = 195/70$		

$E(x) = \Sigma x g(x) = \frac{105}{70} = 1.5$

$E(x^2) = \Sigma x^2 g(x) = \frac{195}{70} = 2.785$

$E(y) = \Sigma y h(y) = 1$

$E(y^2) = \Sigma y^2 h(y) = \frac{100}{70} = 1.428$

$$Var(x) = E(x^2) - (E(x))^2$$
$$= 2.785 - (1.5)^2$$
$$= 0.535$$

* range of co-relation coefficient
-1 to 1

two variables x & y are in-dependant if
$E(XY) = E(X)E(Y)$

$$Var(y) = E(y^2) - (E(y))^2$$
$$= 1.428 - (1)^2$$
$$= 0.428$$

$$SD(x) = \sqrt{0.535} \Rightarrow 0.731$$

$$SD(y) = \sqrt{0.428} \Rightarrow 0.654$$

$$E(XY) = \sum_x \sum_y xyf(x,y)$$

$$E(xy) = 18/70 + 36/70 + 6/70 + 18/70 + 12/70 + 0$$
$$= \frac{90}{70} \approx 1.2857$$

* co-variance

$$Cov(x,y) = E(xy) - E(x) \cdot E(y)$$
$$= \frac{9}{7} - (1.5)(1)$$

$$Cov(x,y) = -0.2142$$

* co-relation coefficient b/w x & y.

$$\rho_{xy} = \frac{Cov(x,y)}{\delta_x \delta_y}$$
$$= \frac{-0.2142}{(0.7314)(0.6542)}$$

$$\rho_{xy} = -0.4478$$

Q: what type of corelation is this?

Ans: moderate -ve corelation b/w x & y

Q4.60:

$f(x,y)$		x: 2	4	$h(y)$	$yh(y)$
	1	0.10	0.15	0.25	0.25
y	2	0.20	0.30	0.50	1
	3	0.10	0.15	0.25	0.75
$g(x)$		0.40	0.60	~~$\Sigma x g(x)$~~	
$xg(x)$		0.80	2.4	$\Sigma x g(x) = 3.2$	

(a) $E(2X-3Y) = \sum_x \sum_y (2x-3y) f(x,y)$

$= (4-3)(0.10) + (8-3)(0.15) + (4-6)(0.20) + (8-6)(0.3)$
$+ (4-9)(0.10) + (8-9)(0.15)$
$= 0.10 + 0.75 + (-0.4) + (0.6) + (-0.5) + (-0.15)$
$= 0.4$

(b) $E(XY) = \sum_x \sum_y xy f(x,y)$

$= (2)(1)(0.10) + (2)(2)(0.20) + (2)(3)(0.10) + (4)(1)(0.15)$
$+ (4)(2)(0.30) + (4)(3)(0.15)$
$= 0.2 + 0.8 + 0.6 + 0.6 + 2.4 + 1.8$
$= 6.4$

(c) Show that $E(2X-3Y) = 2E(X) - 3E(Y)$

$0.4 = 2E(X) - 3E(Y)$

$E(X) = \Sigma x g(x) \Rightarrow 3.2$
$E(Y) = \Sigma x h(y) \Rightarrow 2$

$0.4 = 2(3.2) - 3(2)$
$0.4 = 6.4 - 6$
$\boxed{0.4 = 0.4}$ Hence proved.

(d) $E(XY) = E(X)E(Y)$

$E(XY) = \sum\sum xy\, f(x,y)$

$E(XY) = 6.4$

$6.4 = (3.2)(2)$

$6.4 = 6.4$

Hence X and Y are independant variables.

Q4.10

$f(x,y)$		$y=1$	$y=2$	$y=3$	$g(x)$	$x\,g(x)$
x	1	0.10	0.05	0.02	0.17	0.17
	2	0.10	0.35	0.05	0.50	1
	3	0.03	0.10	0.20	0.33	0.99
$h(y)$		0.23	0.50	0.27	~~0.81~~	2.16
$y\,h(y)$		0.23	1	0.81	$= \Sigma = 2.04$	

$\cancel{\mu_x = \frac{\Sigma X}{N} = \frac{6}{3} \Rightarrow 2}$ $\cancel{\mu_y = \frac{\Sigma Y}{N} = \frac{6}{3} \Rightarrow 2}$

$\mu_x = E(X) = \Sigma x\, g(x) \Rightarrow 2.16$

$\mu_y = E(Y) = \Sigma y\, h(y) \Rightarrow 2.04$

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Q4.23:

$f(x,y)$		$x=2$	$x=4$	$h(y)$	$yh(y)$
y	1	0.10	0.15	0.25	0.25
	3	0.20	0.30	0.50	1.5
	5	0.10	0.15	0.25	1.25
$g(x)$		0.40	0.60		3
$xg(x)$		0.80	2.4 = 3.2		

(a) $E(g(x,y)) = ?$ where $g(x,y) = XY^2$

$$E(g(X,Y)) = \sum_x \sum_y g(x,y) f(x,y)$$
$$= \sum_x \sum_y xy^2 f(x,y)$$

$$E(g(x,y)) = (2)(1)^2(0.10) + (2)(3)^2(0.20) + (2)(5)^2($$
$$+ (4)(1)^2(0.10) + (4)(3)^2(0.30) + (4)(5)^2(0.$$

$$E(g(x,y)) = 0.2 + 3.6 + 5 + 0.4 + 10.8 + 15$$
$$= 35$$

(b) $\cancel{\mu_x = \frac{\sum X}{N} \Rightarrow \frac{6}{2} \Rightarrow 3}$

$\cancel{\mu_y = \frac{\sum Y}{N} \Rightarrow \frac{9}{2} \Rightarrow 4.5}$

$\mu_x = E(X) = \sum x g(x) = 3.2$

$\mu_y = E(Y) = \sum y h(y) = 3$

Chapter 5

"Some Discrete Probability Distributions"

① Discrete uniform distribution:

The prob function is

$$f(x) = \frac{1}{K} \text{ for } x = 1, 2, \dots, K$$

Example: when a dice is rolled. write formula for prob dist, then find

(a) P(odd numbers)

(b) P(even numbers)

The formula is

$$f(x) = \frac{1}{6}, \text{for } x = 1, 2, 3, 4, 5, 6$$

i.e $f(1) = \frac{1}{6}$, $f(2) = \frac{1}{6}$, ..., $f(6) = \frac{1}{6}$

$$P(\text{odd nums}) = f(1) + f(3) + f(5)$$
$$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$$
$$= \frac{3}{6} \Rightarrow \frac{1}{2}$$

$$P(\text{even nums}) = f(2) + f(4) + f(6)$$
$$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} \Rightarrow \frac{3}{6} \Rightarrow \frac{1}{2}$$

Exercise pg (150)

Q 5.3:

$$f(x) = \frac{1}{K} \quad \text{for } x = 1, 2, \dots, K$$

$K = 10$

$$f(x) = \frac{1}{10}, \text{ for } x = 1, 2, \dots, 10$$

$$P(X < 4) = f(1) + f(2) + f(3)$$
$$= \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{3}{10}$$

$\longleftrightarrow$

The Bernoulli Process pg 144

① Repeated trials
② each trial has ~~two~~ outcomes either success or failure
③ p is constant
④ n is fixed (number of trials)

Binomial Distribution

$$b(x, n, p) = P(X = x) = {}^{n}C_{x}\, p^{x} q^{n-x} \text{ for } x = 0, 1, \dots, n$$

where n = no. of trials and
p = probability of success
$q = 1 - p$: probability of failure
x = values of random variable X

$\longleftrightarrow$

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Q5.2:

$n = 12$, $p = \frac{1}{2}$, $q = \frac{1}{2}$

Using Binomial distribution:

$P(X=x) = {}^nC_x \, p^x q^{n-x}$

$P(X=3) = {}^{12}C_3 \, (\frac{1}{2})^3 (\frac{1}{2})^{12-3}$

$= 220 \, (\frac{1}{8})(\frac{1}{512})$

$= \frac{220}{4096}$

$= \frac{55}{1024} = 0.0537$

$P(X=2) = {}^{12}C_2 \, (1/2)^2 (1/2)^{12-2}$

$= 66 \, (1/4)(1/1024)$

$= \frac{66}{4096} = \frac{33}{2048} = 0.0161$

Q5.4:

$p = 0.75$, $q = 1 - 0.75 \Rightarrow 0.25$

$n = 5$, $x = 0, 1, 2, 3, 4, 5$

$f(x) = {}^nC_x \, p^x q^{n-x}$

Binomial distribution of x

x	$f(x) = {}^5C_x (0.75)^x (0.25)^{5-x}$
0	${}^5C_0 (0.75)^0 (0.25)^5 = 0.000976$
1	${}^5C_1 (0.75)^1 (0.25)^4 = 0.01464$
2	${}^5C_2 (0.75)^2 (0.25)^3 = 0.0878$ (a)
3	${}^5C_3 (0.75)^3 (0.25)^2 = 0.2636$
4	
5	

$P(X \leq 3) = f(3) + f(2) + f(1) + f(0)$

$= 0.367016$

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Q5.6:

$p = \frac{1}{2}$, $q = \frac{1}{2}$, $n = 6$

(a) $P(2 \cancel{\leq} x \leq 5) = f(2) + f(3) + f(4) + f(5)$

(b) $P(x < 3) = f(2) + f(1) + f(0)$

$f(x) = {}^nC_x \, p^x \, q^{n-x}$

$f(0) = {}^6C_0 (0.5)^0 (0.5)^6 = 0.015625$

$f(1) = {}^6C_1 (0.5)^1 (0.5)^5 = 0.09375$

$f(2) = {}^6C_2 (0.5)^2 (0.5)^4 = 0.234375$

$f(3) = {}^6C_3 (0.5)^3 (0.5)^3 = 0.3125$

$f(4) = {}^6C_4 (0.5)^4 (0.5)^2 = 0.234375$

$f(5) = {}^6C_5 (0.5)^5 (0.5)^1 = 0.09375$

(a) $P(2 \leq x \leq 5) = 0.875$

(b) $P(x < 3) = 0.34375$

$\longleftrightarrow$

Q5.7: $p = 0.7$, $q = 0.3$, $n = 10$

(a) $P(x < 5)$

(b) $P(x < 10)$

Q5.8: $p = 0.6$, $q = 0.4$, $n = 8$

(a) $P(X=3) = {}^8C_3 (0.6)^3 (0.4)^5$
$= 0.1238$

(b) $P(X \geq 5) = f(5) + f(6) + f(7) + f(8)$
$= 0.594$

Multinomial Distribution

Having 3 or more possible outcomes

$$f(x_1, x_2, \ldots, x_k ; p_1, p_2, \ldots, p_k)$$
$$= {}^nC(x_1, x_2, \ldots, x_k)\, p_1^{x_1} \cdot p_2^{x_2} \cdot \ldots \cdot p_k^{x_k}$$

Q5.22:

Red		Black		White
8	:	4	:	4

$p_1 = P(\text{Red}) = 8/16 = 0.5$
$p_2 = P(\text{Black}) = 4/16 = 0.25$
$p_3 = P(\text{White}) = 4/16 = 0.25$
$n = 8$, $x_1 = 5$, $x_2 = 2$, $x_3 = 1$

Using multinomial Dist

$$f(x_1, x_2, x_3 ; p_1, p_2, p_3) = {}^nC(x_1, x_2, x_3)\, p_1^{x_1} \cdot p_2^{x_2} \cdot p_3^{x_3}$$

$= \binom{8}{5,2,1} (0.5)^5 (0.25)^2 (0.25)$

$= \frac{8!}{5!\,2!\,1!} (0.00049) \Rightarrow 168 (0.00049)$

$= 0.0820$

Q 5.23: $p_1 = 0.4$, $p_2 = 0.2$, $p_3 = 0.3$, $p_4 = 0.1$

By air, $x_1 = 3$ $\quad n = 9$
By bus, $x_2 = 3$
By AM, $x_3 = 1$
By train, $x_4 = 2$

$$f(3,3,1,2; 0.4, 0.2, 0.3, 0.1) = \binom{9}{3,3,1,2}(0.4)^3(0.2)^3(0.3)^1(0.1)^2$$

$$= \frac{9!}{3!\,3!\,1!\,2!}(0.000001536)$$

$$= 5040(0.000001536) \Rightarrow \boxed{0.00774144}$$

~~0.270144~~

Hypergeometric Probability distribution:

1. Items are selected without replacement
2. Two outcomes of success & failure
3. trials are independant

The prob function is:

$$h(x, N, n, K) = P(X = x) = \frac{{}^{K}C_{x} \cdot {}^{N-K}C_{n-x}}{{}^{N}C_{n}}$$

where N = Total items (population size)
n = items selected (sample size)
K = No. of successes in population
x = values of the random variable 'X'

'N, n, k are parameters of hypergeometric prob. dist.'

$\longleftrightarrow$

Exercise (pg 157)

Q5.29: $n = 6$, $N = 5 + 4 = 9$, $K = 4$

$x = 0, 1, 2, 3, 4$

Using H.P.D:

$$P(X = x) = \frac{{}^{K}C_{x} \cdot {}^{N-K}C_{n-x}}{{}^{N}C_{n}}$$

(a)

$$P(X = 2) = ({}^{4}C_{2} \cdot {}^{5}C_{4}) / {}^{9}C_{6}$$

$$P(X = 2) = \frac{30}{84} = 0.3571$$

Q5.30

$N = 15, \quad n = 3, \quad K = 6, \quad x = 1, 2, 3$

HPD of x

x	$P(X=x) = ({}^{K}C_x \times {}^{N-K}C_{n-x}) / {}^{N}C_n$
1	$({}^{6}C_1 \times {}^{9}C_2)/455 = 216/455$
2	$({}^{6}C_2 \times {}^{9}C_1)/455 = 135/455$
3	$({}^{6}C_3 \times {}^{9}C_0)/455 = 20/455$

$\Sigma = 371/455 = 0.815$

Q5.31. $n = 3, \quad N = 6, \quad K = 4$

HPD of x

formula:

$$P(X = x) = ({}^{4}C_x \times {}^{2}C_{3-x}) / {}^{6}C_3$$

$$P(2 \le X \le 3) = P(3) + P(2)$$

$$P(2) = ({}^{4}C_2 \times {}^{2}C_1) / {}^{6}C_3$$

$$= 12/20 \Rightarrow 0.6$$

$$P(3) = ({}^{4}C_3 \times {}^{2}C_0)/20$$

$$= 4/20 = 0.2$$

$$P(2 \le X \le 3) = 0.2 + 0.6$$

$$= 0.8$$

QS.33: $N = 52$, $n = 7$,

(a) $P(\text{exactly 2 face cards}) = \dfrac{{}^{12}C_2 \times {}^{40}C_5}{{}^{52}C_7}$

$x = 2$

$= 0.3246$

(b) $P(\text{atleast 1 queen}) = 1 - P(\text{no queen})$

$= 1 - \dfrac{{}^{4}C_0 \times {}^{40}C_7}{{}^{52}C_7}$

$x = 1, 2, 3, 4$

$= 1 - \dfrac{18643560}{133784560}$

$\boxed{= 0.86064}$

QS.34: $N = 9$, $n = 5$, $K = 4$

$P(X = 2) = \dfrac{{}^{4}C_2 \times {}^{5}C_3}{{}^{9}C_5}$

$= \dfrac{6 \times 10}{126} = \dfrac{60}{126} = \dfrac{10}{21}$

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Geometric Distribution

geometric dist is used when first success occurs.

$g(x;p) = pq^{x-1}$ for $x = 1, 2, 3, \ldots$

Q5.59:

$p = 0.8$, $q = 1 - p = 0.2$

(b) Hence , $x = 3$

$g(x;p) = pq^{x-1} = (0.8)(0.2)^{3-1} = 0.032$

Q5.54

$p = \frac{2}{3}$, $q = 1 - p \Rightarrow \frac{1}{3}$

(a) $x = 5$

$g(x;p) = pq^{x-1} \Rightarrow (2/3)(1/3)^4 \Rightarrow \frac{2}{243}$

Poisson Distribution (pg 162)

$$p(x;\lambda t) = \frac{e^{-\lambda t}(\lambda t)^x}{x!} , x = 0, 1, 2, \ldots$$

$$\text{or } p(x;\mu) = \frac{e^{-\mu}(\mu)^x}{x!} , x = 0, 1, 2, \ldots$$

where μ = average number of occurences
and λ = average number of outcomes
and t = time or space

* μ is the parameter of poisson distribution
* This dist is also known as "rare event dist"

Q5.56:

$\mu = 3$

(a) $P(x=5) = \frac{e^{-\mu}\mu^{x}}{x!} \Rightarrow \frac{e^{-3}(3)^{5}}{5!} \Rightarrow \boxed{0.10081}$

(b) $P(x<3) = \frac{e^{-3}(3)^{0}}{0!} + \frac{e^{-3}(3)^{1}}{1!} + \frac{e^{-3}(3)^{2}}{2!}$

$= \cancel{(0.04978)+}$

$\boxed{P(x<3) = 0.4231}$

Q5.57: $\mu = 2$

(a) $P(x \geq 4) = 1 - P(x<4)$

$= 1 - (P(x=0) + P(x=1) + P(x=2) + P(x=3))$

$= 1 - e^{-2}\left[\frac{(2)^{0}}{0!} + \frac{(2)^{1}}{1!} + \frac{(2)^{2}}{2!} + \frac{(2)^{3}}{3!}\right]$

$= 1 - e^{-2}[1 + 2 + 2 + 1.334]$

$= 1 - e^{-2}(6.3334)$

$\boxed{= 0.1428}$

(b) $P(x=0) = \frac{e^{-2}(2)^{0}}{0!} \Rightarrow \boxed{0.13533}$

Q 5.58: $\mu = 6$

(a) $P(x < 4) = P(0) + P(1) + P(2) + P(3)$

$= e^{-6}\left[\frac{6^0}{0!} + \frac{6^1}{1!} + \frac{6^2}{2!} + \frac{6^3}{3!}\right]$

$= e^{-6}[1 + 6 + 18 + 36]$

$= 0.1512$

(b) $P(6 \le x \le 8) = P(8) + P(7) + P(6)$

$= e^{-6}\left[\frac{6^8}{8!} + \frac{6^7}{7!} + \frac{6^6}{6!}\right]$

$= e^{-6}(162)$

$= 0.40155$

Q5.66: $\lambda = 6$, $t = 1$, $\boxed{\mu = 6 = \lambda t}$

(a) $P(x=4) = e^{-6}\frac{6^4}{4!} \Rightarrow 0.13385$

(b) $P(x \ge 4) = 1 - P(x < 4)$

$= 1 - e^{-6}\left[\frac{6^0}{0!} + \frac{6^1}{1!} + \frac{6^2}{2!} + \frac{6^3}{3!}\right]$

$= 1 - e^{-6}(61)$

$= 1 - 0.1512 \Rightarrow 0.8487$

(c) $\mu = 6(12) \Rightarrow 72$

$P(x \ge 75) = 1 - P(X < 75)$

Q5.67: $\lambda = 7$, $t = 2$, $\mu = 7\times 2 = \lambda t = 14$

(a) $P(X > 10) = 1 - P(x \leq 10)$

(b) mean number of arrivals $= \mu = \lambda t$

$$= 7(2)$$

$$= 14$$

Q5.84: imp

$\lambda = 100 / t = 1$

(a) $t = \frac{3}{60}$, $\lambda t = 100\left(\frac{3}{60}\right) = 5$

$$P(X=0) = \frac{e^{-5}(5)^0}{0!} = 0.0067$$

(b) $t = 3/60$, $\lambda t = 5$

$$P(X > 5) = 1 - P(X \leq 5)$$

$$= 1 - e^{-5}\left[\frac{5^0}{0!} + \frac{5^1}{1!} + \frac{5^2}{2!} + \frac{5^3}{3!} + \frac{5^4}{4!} + \frac{5^5}{5!}\right]$$

$$= 1 - e^{-5}\cancel{[1 + 5 + 12.5 +}$$

$$= 1 - e^{-5}(65.375)$$

$$= 0.5595$$

Q: An interchange has a record that 150 cars cross it during 1 hour period, find prob that in 2 min period
(a) no car passes by
(b) 4 or more cars pass by

(a) $P(X=0)$, $t = \frac{2}{60}$, $\lambda t = \mu = 150\left(\frac{2}{60}\right)$
$\mu = 5$

$P(X=0) = e^{-5} \frac{5^0}{0!} \Rightarrow \boxed{0.0067}$

(b) $P(x \geq 4) = 1 - P(x<4)$
$= 1 - e^{-5}\left[\frac{5^0}{0!} + \frac{5^1}{1!} + \frac{5^2}{2!} + \frac{5^3}{3!}\right]$
$= 1 - e^{-5}[1 + 5 + 12.5 + 20.8334]$
$\boxed{= 0.7349}$

$\boxed{\text{Chapter 6}}$: Normal Distribution

The prob distribution function (pdf) is

$$n(x, \mu, \sigma) = f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$$

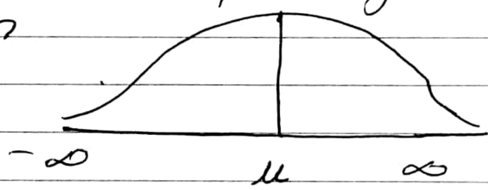
for $-\infty < x < \infty$

where μ = mean of normal dist
σ = SD " " "
$\pi = 22/7$
$e = 2.71828$

μ and σ are parameters of the normal distribution.

Properties of Normal Dist:

① It is a bell shaped symmetrical distribution

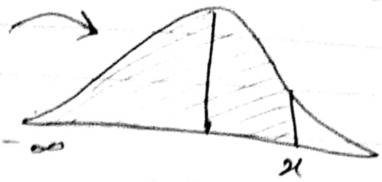


② Its range is from $-\infty$ to ∞
③ Hence, mean = median = mode = μ
④ Total area under the curve is 1
⑤ Standard normal variable:

we transform x variable into standard normal variable Z where

$$Z = \frac{X - \mu}{\sigma}$$

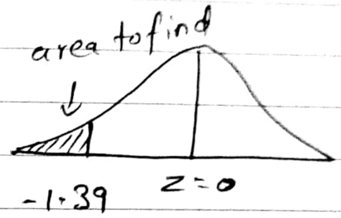
* left area in this graph will always be available or could be calculated.



Exercise Questions:

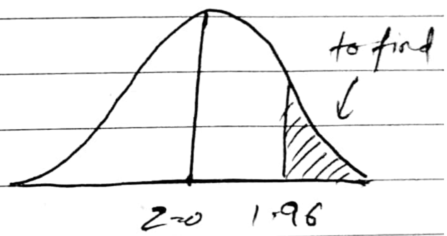
Q6.5:

(a) to the left of $z = -1.39$



$P(z < -1.39) = 0.0823$
(from table)

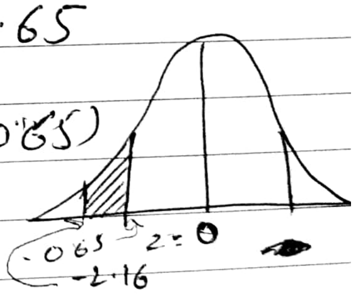
(b) to the right of $z = 1.96$



$P(z > 1.96) = 1 - 0.9750$
$= 0.025$

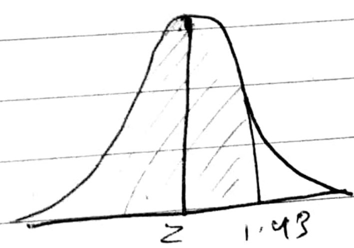
(c) between $z = -2.16$, $z = -0.65$

$P(-2.16 < z < -0.65)$
value from tab (-2.16) − v of table (-0.65)



$= \cancel{0.0154 - 0.2578}$
$= 0.2578 - 0.0154$
$\boxed{= 0.2424}$

(d) To the left of $z = 1.43$



$P(z < 1.43) = 0.9236$.

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(e) to the right of $z = -0.89$.

$P(z < -0.89) = 1 - 0.1867$
$= 0.8133$

~~(f) between $-z$ & z with $z > 0$, is 0.9500~~

(f) $z = -0.48$, $z = 1.74$ between

$P(1.74 > z > -0.48)$

$= 0.9591 - 0.3156$
$= 0.6435$

<u>Inverse use of area table</u>

<u>Q 6.6:</u>

(a) to the right of z is 0.3622

Area to left $= 1 - 0.3622$
$= 0.6378$

value of $Z = 0.35$ nearest was 0.6368

(b) to the left of z is 0.1131

$z = -1.21$

(c) b/w 0 & z, $z > 0$ is 0.4838

$\boxed{0.5162} = 1 - 0.4838$

~~$z = 0.04$~~ ~~nearest 0.4840~~

$\boxed{z = 0.04}$

nearest 0.5160

~~(d) b/w -z & z with z > 0, is 0.9500~~

~~z = 1.65 $\boxed{\text{nearest } 0.9505}$~~

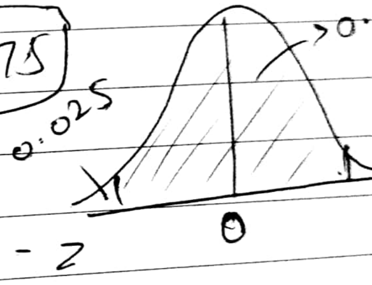
(c) $0.5 + 0.4838 = 0.9838$

$z = 2.14$ (exact)

(d) Area to left $= 0.5 - 0.4750 = 0.0250 \rightarrow z = -1.96$

to right $= 0.5 + 0.4750 = 0.9750 \rightarrow z = 1.96$

$\dfrac{0.9500}{2} = 0.475$



~~Q3/4 combined~~ Chap 3, 4 combined 1 Q
~~Q5~~ Chap 5 one long Q.

Normal approximation to the binomial

When 'n' becomes large (more than 20) in binomial distribution we use normal approximation

$\mu = np$

μ: mean of normal distribution. np: mean of binomial distribution

$\delta = \sqrt{npq}$

We use continuity correction $\boxed{\text{RULES}}$

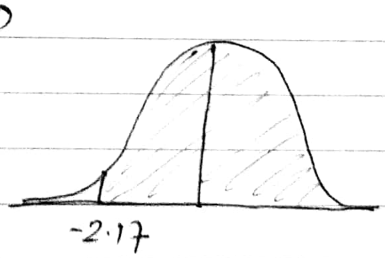
1. $P(X > 10) \Rightarrow$ By c.c $x = 10 + 0.5 = 10.5$
2. $P(X < 10) \Rightarrow$ " " $x = 10 - 0.5 = 9.5$
3. $P(8 < X < 12) \Rightarrow$ " " $x_1 = 7.5$, $x_2 = 12.5$
4. $P(X \geq 10) \Rightarrow$ " " $X = 10 - 0.5 = 9.5$
5. $P(X \leq 10) \Rightarrow$ " " $x = 10 + 0.5 = 10.5$
6. $P(X = 10) \Rightarrow$ " " $X_1 = 9.5$, $X_2 = 10.5$

Q6.8. $\mu = 30$, $\delta = 6$

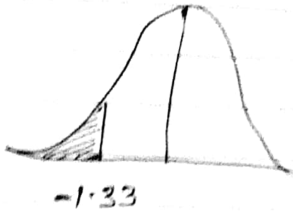
(a) $P(X > 17) = ?$

$$Z = \frac{x - \mu}{\delta} = \frac{17 - 30}{6} = -2.17$$

$$P(Z > -2.17) = 1 - P(Z \leq -2.17)$$
$$= 1 - 0.0150$$
$$= 0.9850$$



(b) $P(X < 22) = ?$



$$z = \frac{x-\mu}{\sigma} = \frac{22-30}{6} = -1.33$$

$P(Z < -1.33) = 0.0918$

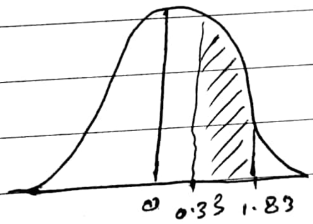
(c) $P(32 < x < 41) = ?$

$X_1 = 32, \quad X_2 = 41$

$$Z_1 = \frac{x_1 - \mu}{\sigma} \qquad Z_2 = \frac{x_2 - \mu}{\sigma}$$

$$= \frac{32-30}{6} \qquad = \frac{41-30}{6}$$

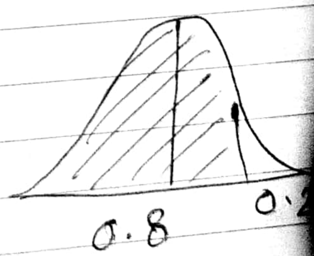
$$= 0.33 \qquad = 1.83$$



$P(0.33 < Z < 1.83) = 0.9664 - 0.6293$
$= 0.3371$

(d) Check 0.8000 in table
nearest are two values on
0.83 & 0.84 which are
0.7995 & 0.8023.
↑ closest so

$z = 0.83$.



$$z = \frac{x - \mu}{\delta}$$

$$0.84 = \frac{x - 30}{6}$$

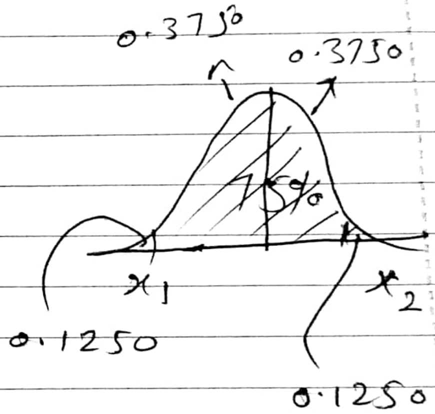
$$5.04 = x - 30$$

$$30 + 5.04 = x$$

$$\boxed{x = 35.04}$$

(e) $75\% = 0.7500$

$$\text{half} = \frac{0.7500}{2} = 0.3750$$



For area $= 0.1250$

$$z = -1.15$$

Area to the left of $x_2 = 1 - 0.1250$
$= 0.8750$

For area $= 0.8750$
$z = 1.15$

$$z_1 = \frac{x_1 - \mu}{\delta}$$

$$-1.15 = \frac{x_1 - 30}{6}$$

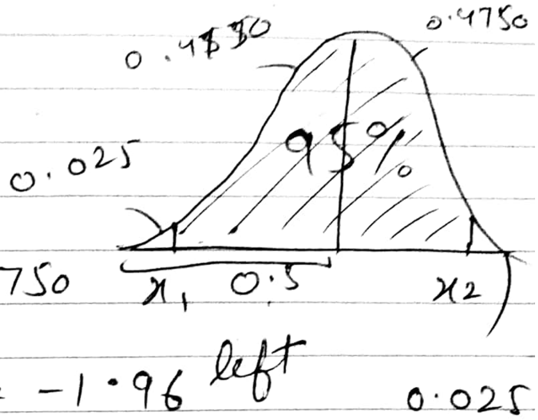
$$x_1 = 23.10$$

$$z_2 = \frac{x_2 - \mu}{\delta}$$

$$1.15 = \frac{x_2 - 30}{6}$$

$$x_2 = 36.9.$$

Q: The average life of an electric component is 1 year with SD of 3 months. Find 2 values of x that contain the middle 95% of the area.



$\mu = 12$, $\delta = 3$

$95\% = 0.95$

half of $0.95 = \frac{0.95}{2} = 0.4750$

For area $= 0.025$, $Z_1 = -1.96$ left

For area $= 0.975$, $Z_2 = 1.96$ right

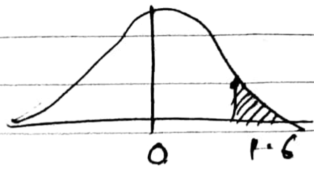
$Z_1 = \frac{X_1 - \mu}{\delta}$, $Z_2 = \frac{X_2 - \mu}{\delta}$

$-1.96 = \frac{X_1 - 12}{3}$, $1.96 = \frac{X_2 - 12}{3}$

$\boxed{x_1 = 6.12}$

$\boxed{x_2 = 17.88}$

Q 6.11 : $\mu = 200$, $\delta = 15$

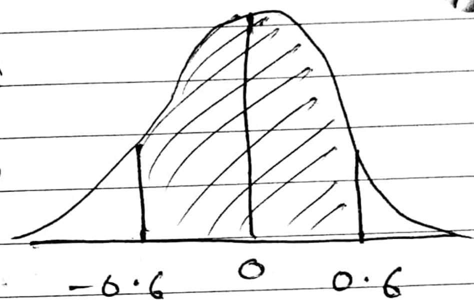


(a) $P(X > 224) = ?$

$$Z = \frac{X - \mu}{\delta} = \frac{224 - 200}{15} = 1.6$$

$$P(Z > 1.6) = 1 - P(Z \le 1.6) = 1 - 0.9452$$
$$= 0.0548$$

(b) $P(191 < Z < 209)$, $x_1 = 191$, $x_2 = 209$



$$Z_1 = \frac{x_1 - \mu}{\delta} = \frac{191 - 200}{15} = -0.6$$

$$Z_2 = \frac{x_2 - \mu}{\delta} = \frac{209 - 200}{15} = 0.6$$

$$P(-0.6 < Z < 0.6) = 0.7257 - 0.2743$$
$$= 0.4513$$

(c) $N = 1000$

$P(X > 230) = ?$

$$Z = \frac{X - \mu}{\delta} = \frac{230 - 200}{15} = 2$$

$$P(Z > 2) = 1 - P(Z \le 2)$$
$$= 1 - 0.9772 = 0.0228$$

No. of cups = N x prob.
$= 1000 \times 0.0228 = 22.8$
≈ 23

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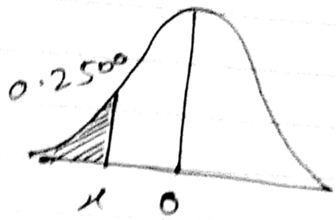
(d)

Area = 0.2500 , $z = -0.67$



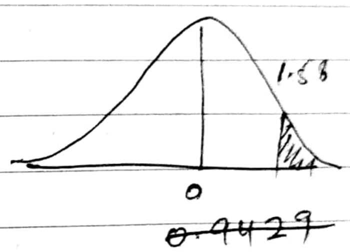
$$z = \frac{x - \mu}{\delta}$$

$$-0.67 = \frac{x - 200}{15}$$

$$-10.05 = x - 200$$

$$\boxed{x = 189.75}$$

Q 6.15 : $\mu = 24$, $\delta = 3.8$



(a) $P(X > 30) = ?$

$$z = \frac{x - \mu}{\delta} = \frac{30 - 2}{3.8} = 1.58$$

For $z = 1.58$, area at $1.58 = 1 - 0.9429$

$= \boxed{0.0571}$

(b) $P(X > 15) = ?$

$$z = \frac{15 - 24}{3.8} = -2.37.$$

for $z = -2.37$

area $= 0.0089$

$P(z > -2.37) = 1 - P(z \leq -2.37)$

$= 1 - 0.0089$

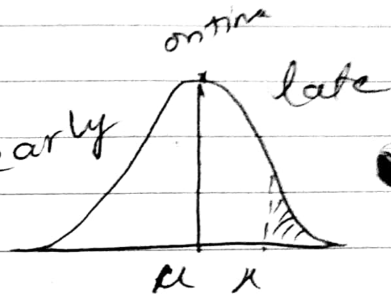
$= 0.9911$.

He will be late 99.11% of times.

(c) $P(X > 25) = ?$

$$z = \frac{x - \mu}{\sigma} = \frac{25 - 24}{3.8} = 0.26$$

$$P(z > 0.26) = 1 - P(z \le 0.26)$$
$$= 1 - 0.6026$$
$$= \boxed{0.3974}$$



(d) Area to left of x

$= 1 - 0.15 = 0.85$

area $= 0.8500$, $z = 1.04$

$$z = \frac{x - \mu}{\sigma}$$

$$1.04 = \frac{x - 24}{3.8}$$

$$\boxed{x = 27.95}$$

$0.8531 \overline{|1.05}$ 0.6736
0.7088

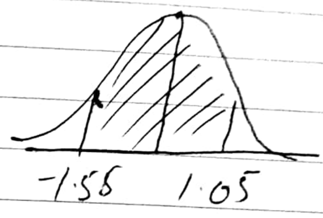
Normal approximation to the binomial

* if $n > 20$ then we use normal approximation to the binomial instead of binomial distribution.

$\therefore$ Equate $\mu = np$

$$\delta = \sqrt{npq}$$

then use continuity correction



Q 6.24:

$n = 400$, p = prob of head $= \frac{1}{2}$, $q = \frac{1}{2} = 1-p$

(a) P(b/w 185 & 210 heads inclusive) = ?

$\mu = np = 400(1/2) = 200$
$\delta = \sqrt{npq} = \sqrt{400(1/2)(1/2)} = \sqrt{100} = 10$

By continuity correction:

$X_1 = 185 - 0.5 \Rightarrow 184.5$
$X_2 = 210 + 0.5 \Rightarrow 210.5$

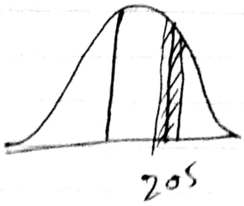
$$Z_1 = \frac{X_1 - \mu_1}{\delta} \Rightarrow \frac{184.5 - 200}{10} \Rightarrow -1.55$$

$$Z_2 = \frac{X_2 - \mu}{\delta} \Rightarrow \frac{210.5 - 200}{10} \Rightarrow 1.05$$

$P(-1.55 < z < 1.05) = P(Z < 1.05) - P(Z > 1.55)$
$= 0.8531 - 0.0606$
$= \boxed{0.7925}$

0.0071
-2.45 ~~0.9970~~
-2.75 0.0030

(b) $P(X = 205) = ?$



By C.C.

$x_1 = 204.5$
$x_2 = 205.5$

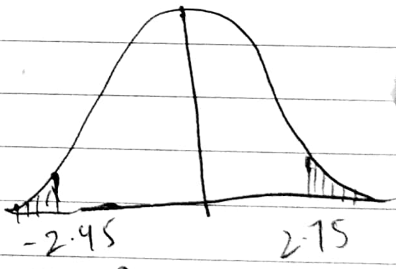
$z_1 = \dfrac{x_1 - \mu}{\delta} = 0.45$

$z_2 = \dfrac{x_2 - \mu}{\delta} = 0.55$

$P(0.45 < z < 0.55) = P(z < 0.55) - P(z > 0.45)$
$= 0.7088 - 0.6736$
$= \boxed{0.0352}$

(c) $P(x < 176 \text{ or } x > 227) = ?$

$x_1 = 176 - 0.5$, $x_2 = 227 + 0.5$
$x_1 = 175.5$, $x_2 = 227.5$



$z_1 = -2.45$
$z_2 = 2.75$

$P(z < -2.45) = 0.0071$
$P(z > 2.75) = 1 - (z \leq 2.75)$

$P(z < 2.75) = 0.9970$

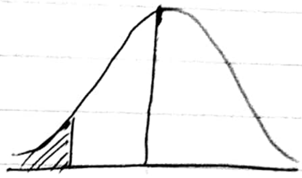
$= 0.0071 + (1 - 0.9970)$
$\boxed{= 0.0101}$

.0978

Q 6.33: $p = \frac{1}{10}$, $n = 400$, $q = 1 - p = \frac{9}{10}$

$\mu = np = \frac{400}{10} = 40$

$\delta = \sqrt{npq} = \sqrt{\frac{400}{10} \times \frac{9}{10}} = 6$



(a) $P(X < 32) = ?$

By C.C, $X = 32 - 0.5 = 31.5$

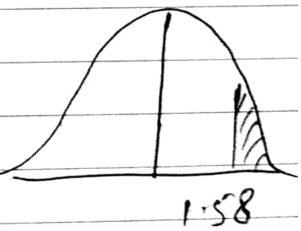
$Z = \frac{X - \mu}{\delta} = \frac{31.5 - 40}{6} = -1.42$

$P(Z < -1.42) = 0.0778$

(b) $P(X > 49) = ?$

By C.C $X = 49 + 0.5 = 49.5$

$Z = \frac{X - \mu}{\delta} = \frac{49.5 - 40}{6} = 1.58$



$P(Z > 1.58) = 1 - P(Z \leq 1.58)$

$= 1 - 0.9429$

$= 0.0571$

(c) $P(X \geq 35$ but $x < 47) = ?$

$X_1 = 35 - 0.5 = 34.5$

$X_2 = 47 - 0.5 = 46.5$

$z_1 = 0.92$, $z_2 = 1.08$

$0.8599 - 0.1788 = 0.6911$

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Q 6.34: $n = 180$

Sample points $= \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$

$p = \frac{6}{36} = \frac{1}{6}$, $q = 1 - p = \frac{5}{6}$

$\mu = np = (180)(1/6) \Rightarrow 30$

$\delta = \sqrt{npq} = \sqrt{(30)(5/6)} = 5$

(a) $P(X \geq 25) = ?$

$P(X \geq 25) = 1 - P(X < 25)$

$\cancel{P(X \geq 25) = 1 - P(X < 25)}$.

By C.C:

$x_1 = 25 - 0.5$

$x_1 = 24.5$

$Z = \frac{x - \mu}{\delta} \Rightarrow \frac{24.5 - 30}{5} \Rightarrow -1.1$

$\cancel{P(Z \geq -1.1) = 1 - P(Z \leq -1.1)}$

$\cancel{= 1 - 0.1357}$

$=$

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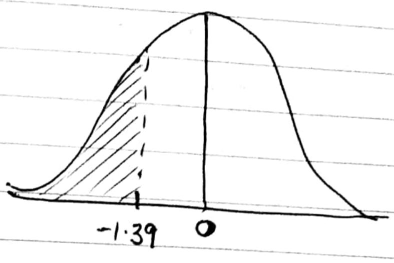
Practise Mid 2.

Chp 6:

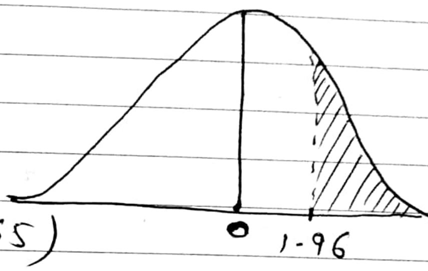
Q6.5 (a) to the left of $z = -1.39$

Area on left of $z = 0.0823$



(b) to the right of $z = 1.96$

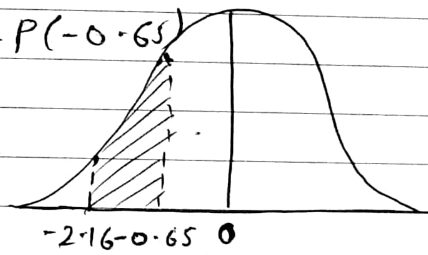
Area of right of $z = 1 - 0.9750$
$= 0.0250$



(c) b/w $z = -2.16$ & $z = -0.65$

Area b/w $= P(-2.16 < z < -0.65)$

$P(-2.16 < z < -0.65) = P(-2.16) - P(-0.65)$



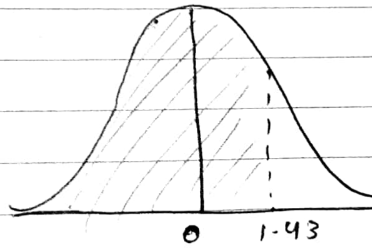
$P(z > -2.16) = 0.0154$

$P(z < -0.65) = 0.2578$

~~= 0.9846 - 0.2578~~
~~= 0.7268~~
$= 0.2578 - 0.0154$
$= 0.2424$

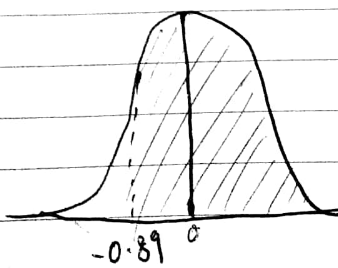
(d) to left of $z = 1.43$

$P(z < 1.43) = 0.9236$



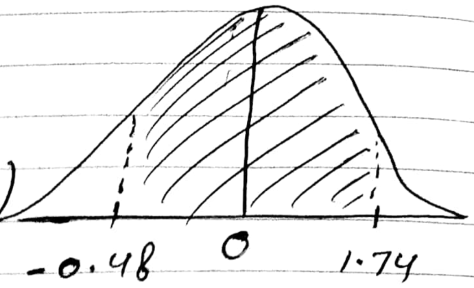
(e) to right of $z = -0.89$

$P(z > -0.89) = 1 - 0.1867$
$= 0.8133$



f) b/w $z = -0.48$ & $z = 1.74$

$P(1.74 > z > -0.48)$



$= P(Z < 1.74) - P(Z > 0.48)$
$= 0.9591 - 0.3156$
$= 0.6435$

Q6.6:

(a) to the right of z is 0.3622

$z = 0.35$

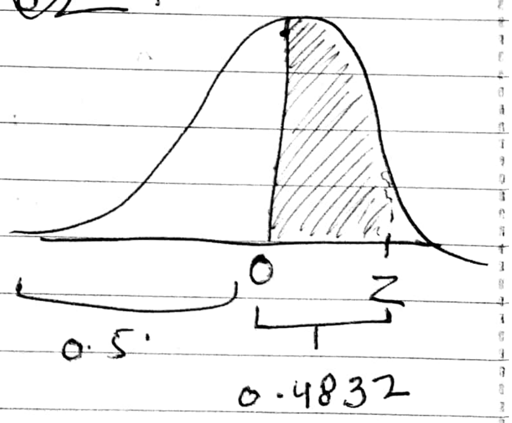
(b) to the left of z is 0.1131

$z = -1.21$

(c) b/w 0 & Z, $Z > 0$, is 0.4838

~~Z = -0.04~~ ~~0.5 - 0.4838 = 0.0162~~

~~z = 0.04~~

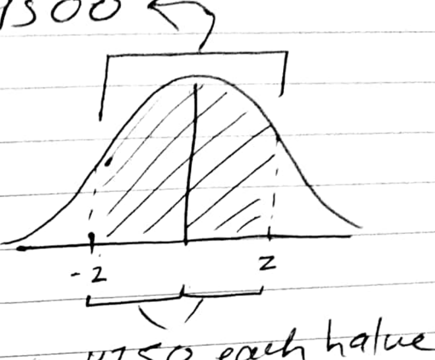


total area at left side

$= 0.5 + 0.4838$
$= 0.9838$

$z = 2.14$.

(d) b/w $-z$ & z, $z>0$, is 9500



for $-z$, $\frac{9500}{2} \Rightarrow 0.4750$

for $-z$ = check in table
$0.5 - 0.4750 \Rightarrow 0.0250$
$-z =$ ~~0.06~~ -1.96

for $z = 0.4750 + 0.5$
$= 0.9750$

check 0.9750 in table
$z =$ ~~0.08~~ 1.96

Q6.7 (a) $k = 0.54$

(b) $k = -1.72$

(c) for -0.93, $z = 0.1762$

$0.7235 + 0.1762 = 0.8997$.

check 0.8997 in table.

$k =$ ~~0.12~~ $k = 1.28$

Q6.8 $\mu = 30$, $\delta = -6$

$z = \frac{x - \mu}{\delta}$ ~~$z = \frac{17 - 30}{-6} = 2.17$~~

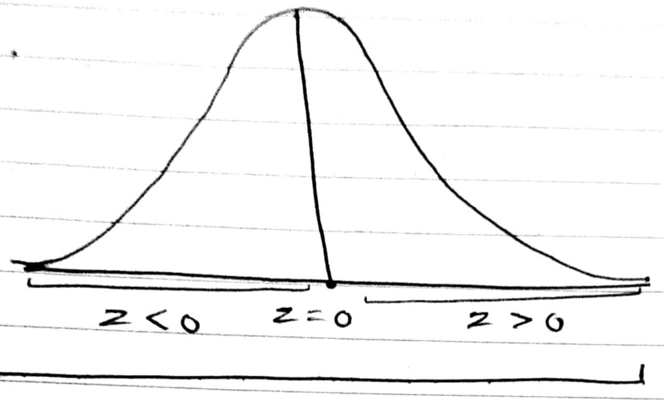
(a) ~~Area at $z = 2.17 = 0.9850$~~

~~Area at right $= 1 - 0.9850$~~
~~$= 0.0150$~~

~~Because~~

Basic concepts of chap 6

→ A normal distribution
also called "bell shaped distribution"

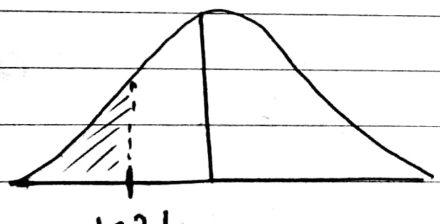


total area under curve = 1

(ar)ea of left side = 0·5   0·5 = area of right side

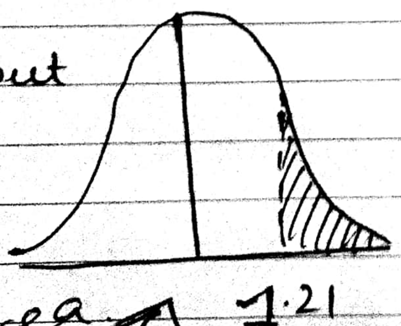
all the values of the area under the curve are given in table.

e.g



if someone asks you to calculate the area under curve to the left of a point -1·21, you just have to look it up the ~~graph~~. table.

but

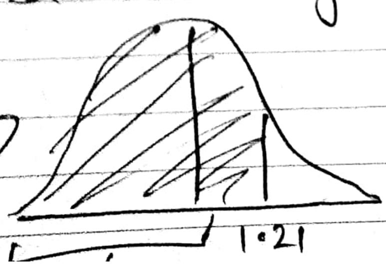


If someone asks you to find area under curve on "right" side of a point ~~m~~ 1·21 you will check the value up the table which would actually be

(t)his area

but we needed

this area



so what we will do is

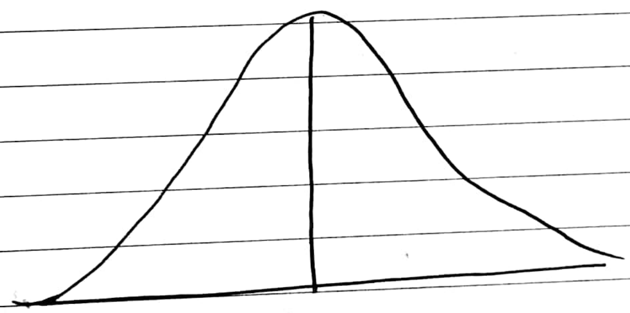
we will subtract the found value from table with 1 e.g

value on 1·21 in table is 0·8869

so the area on right of 1·21 will be

$$1 - 0.8869 = 0.1131$$

and that's it.



~~The mormal~~ distribution has its mean at centre.

formula when mean & SD are involved : $z = \dfrac{x - \mu}{\delta}$

Chp 10

0.0029

Note: Equality in H_0: [$H_0: \mu = 10$, $H_0: \mu \geq 10$, $H_0: \mu \leq 10$] There is a sign of in-equality H_A [$H_A: \mu \neq 10$, $H_A: \mu > 10$, $H_A: \mu < 10$]

For large n (i.e $n \geq 30$, $S \cong \sigma$).

Q 10.19 Pg 356 $\mu < 40$, $n = 64$, $\bar{x} = 38$, $S \cong \sigma = 5.8$

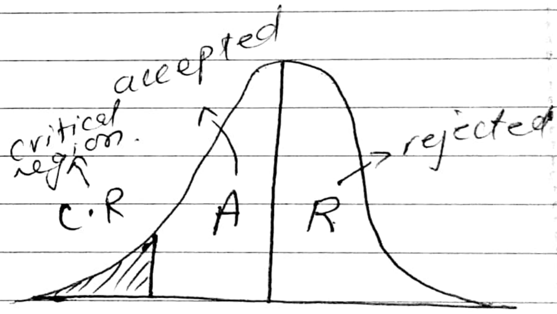
Step I: $H_0: \mu \geq 40$

$H_A: \mu < 40$ (one-tailed test)

Step II:

Let $\alpha = 0.05$

Step III: Test statistic



$$Z = \frac{(\bar{X} - \mu)\sqrt{n}}{\sigma}$$

Step IV

C.R $Z < -Z_{\alpha}$ → alpha.

$Z < -Z_{0.0500}$

$Z < -1.645$

Step V: Calculations

$$z = \frac{(38-40)\sqrt{64}}{5.8} = -2.76$$

Step VI: Conclusion:

Since, $Z_{cal} = -2.76$ which lies in C.R hence, reject H_0.

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α	Z_α (one tail)	$Z_{\alpha/2}$ (two tail)
0.05	± 1.65	± 1.96
0.01	± 2.33	± 2.58

Q10.20 $n = 64,\ \bar{X} = 5.23,\ S \cong \delta = 0.24$

Step I:

$H_0 : \mu = 5.5$

$H_A : \mu < 5.5$

Step II

Let $\alpha = 0.05$

Step III Test statistics

$$z = \frac{(\bar{X} - \mu)\sqrt{n}}{\delta} \Rightarrow \frac{(5.23 - 5.5)\sqrt{64}}{0.24} \Rightarrow -9$$

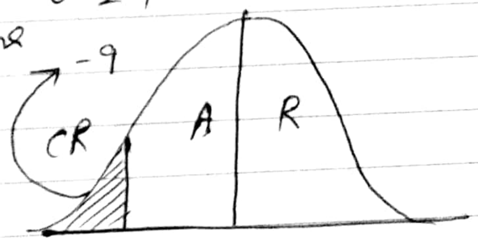
Step IV CR

alpha

$Z < -Z_\alpha$

$Z < -Z_{0.05}$

$Z < -1.645$



Step V

Rejected as it lies in CR

Q10.22 $n = 225,\ \bar{X} = 8.5,\ S \cong \delta = 2.25,\ \mu > 8$

$\alpha = 0.01$ $H_0 : \mu \le 8$

$H_A : \mu > 8$

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Q 10.25 $\mu > 20{,}000$, $n = 100$, $\bar{x} = 23{,}500$, $S = 3900$

Let $\alpha = 0.01$

Step I:

$H_0 : \mu \leq 20{,}000$

$H_A : \mu > 20{,}000$

Step II:

Let $\alpha = 0.01$

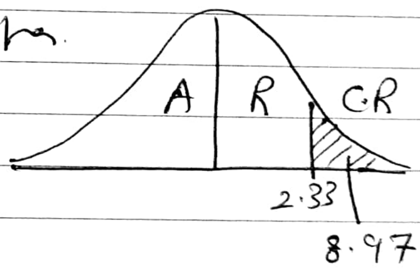
Step III: Test statistics

$$Z = \frac{(\bar{x} - \mu)\sqrt{n}}{S}$$

Step IV: C.R

$Z > Z_{\alpha}$ – alpha

$Z > 2.33$



Step V: Calculations

$$Z = \frac{(23{,}500 - 20{,}000)(\sqrt{100})}{3900} = 8.97$$

Step VI: Since, $Z_{cal} = 8.97$ which lies in C.R, hence reject H_0.

Mew μ
New ν

* Z-test is used when
  1. - when δ is known ($n \geq 30$ or $n < 30$)
  2. - when $n \geq 30$, then $\delta \cong S$

* t-test is used when
  1. - when $n < 30$
  2. - δ is unknown, we use s.

Q 10·26 $n = 20$, $\bar{x} = 244$, $s = 24.5$, $\alpha = 0.05$, $\mu > 220$

Step I: $H_0 : \mu = 220$ or $H_0 : \mu \leq 220$
$H_A : \mu > 220$

Step II: Given $\alpha = 0.05$

Step III: Test statistic

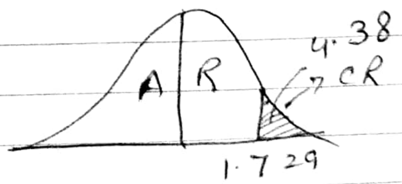
$$t = \frac{(\bar{x} - \mu)\sqrt{n}}{s}$$

Step IV: Critical region

degree of freedom

$t > t_{\alpha(\nu)}$ where $\nu = (n-1)$ d.f
$= (20-1)$ df $= 19$ df.

$t > t_{0.05(19)}$
$t > 1.729$



Step V: Calculations

$$t = \frac{(244 - 220)\sqrt{20}}{24.5} = 4.38$$

Step VI: Conclusion: Since $t_{cal} = 4.38$ which lies in C.R, hence reject H_0.

Q: It is claimed that average height of females in a college is 164 cm. A sample of 25 females is selected and average height is 163 cm with a standard deviation of 8 cms. Is there a reason to believe that average height is more than 164 cm. use $\alpha = 0.01$ (Pg 738)

Q The average life of an electronic component is atleast 10 years. A sample of 16 components is selected & mean is found to be 8.9 years with a standard deviation of 1.6 years. Use alpha = 0.05

$\alpha = 0.05$.

Sol $\mu \geq 10$, $n = 16$, $\bar{x} = 8.9$, $s = 1.6$, $\alpha = 0.05$

Step I:

$H_0 : \mu \geq 10$

$H_A : \mu < 10$

Step II:

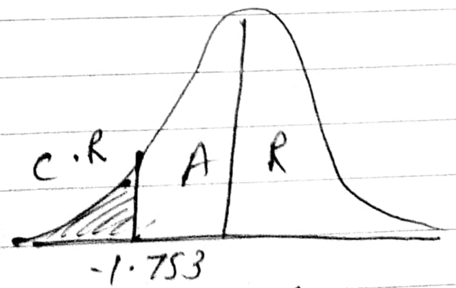
Given $\alpha = 0.05$

Step III: Test statistic

$$t = \frac{(\bar{x} - \mu)\sqrt{n}}{s}$$

Step IV:

Critical Region



$t < -t_{\alpha(\nu)}$ where $\nu = (n-1)$ df

$= 16 - 1 = 15$ df

$t < -t_{0.05(15)}$

$t < -1.753$

Rounding rules $2.7\underline{2}5 \rightarrow 2.72$ even

$2.7\underline{3}5 \rightarrow 2.74$ odd

Step V : Calculations:

$$t = \frac{(8.9 - 10)\sqrt{16}}{1.6} = \frac{-1.1(4)}{1.6} = -2.72$$

Step VI: Conclusion

Since, $t_{cal} = -2.72$, which lies in CR hence reject H_0.

$\longleftrightarrow$

Q: The avg marks obtained by students are less ~~more~~ than 12, a sample of 20 students is selected & avg is found to be 11.2 with S.D of 1.8. Use $\alpha = 0.01$.

$\mu < 12$, $n = 20$, $\bar{x} = 11.2$, $s = 1.8$
$\alpha = 0.01$.

Step I :

$H_A : \mu < 12$
$H_0 : \mu \geq 12$

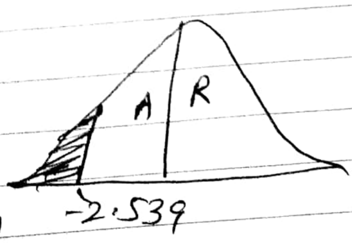
Step II :

Given $\alpha = 0.01$

Step III : Test statistic

$$t = \frac{(\bar{x} - \mu)\sqrt{n}}{s}$$

Step IV : Critical region



$t < -t_{\alpha(\gamma)}$ where $\gamma = (n-1)$ df
$\gamma = 19$ df

$t < -t_{(0.01)(19)}$

$t < -2.539$

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t- table pg 738

Step V Calculations.

$$t = \frac{(11.2 - 12)\sqrt{20}}{1.8}$$

$$t = -1.99$$

Step VI Conclusion

Since, $t_{cal} = -1.99$, so Ho is accepted.

Q10.23 $\mu = 10$, $n = 10$,

X	$(X-\bar{X})^2 = (X-10.06)^2$
10.2	
9.7	
10.1	
10.3	
10.1	
9.8	
9.9	
10.4	
10.3	
9.8	

$\Sigma X = 100.6$ $\quad \Sigma(X-\bar{X})^2 = 0.544$

$$s = \sqrt{\frac{\Sigma(X-\bar{X})^2}{n-1}}$$

$$= \sqrt{\frac{0.544}{10-1}}$$

$$= 0.246$$

Step I:

$H_0: \mu = 10$

$H_A: \mu \neq 10$

$$\bar{X} = \frac{\Sigma X}{n} = \frac{100.6}{10} = 10.06$$

Step II

Given $\alpha = 0.01$

$\frac{\alpha}{2} = \frac{0.01}{2} = 0.005$

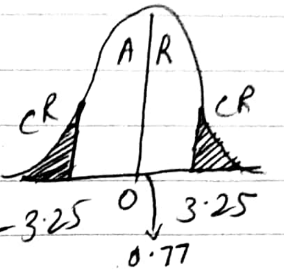
Step III: T.S $\quad t = \frac{(\bar{x}-\mu)\sqrt{n}}{s}$

Step IV: CR: $t < -t_{\alpha/2}(\nu)$ or $t < t_{\alpha/2}(\nu)$

where $\nu = (n-1)$ df $= (10-1)$ df $= 9$ df.

$t < t_{0.005(9)}$ or $t > t_{0.005(9)}$

$t < -3.250$ or $t > 3.250$



Step II: Calculations

$$t = \frac{(10.06 - 10)\sqrt{10}}{0.246} = 0.77$$

where lies in A.R, hence accepted.

Q The avg life of a heater is 8 years. A sample of 22 heaters in selected & mean is found to be 8.5 years with SD of 2.1 years. Use $\alpha = 0.05$.

Step I: $H_0 : \mu = 8$
$H_A : \mu \neq 8$

Independence of attributes (Qualitative)

Q10.87: Gender & TV watched

O = observed frequencies

	Gender Male	Female	Total
over 25 hours	15	29	44
under 25 hours	27	19	46
total	42	48	90 = n.

Step I: H_0: Attributes are independant
H_A: Attributes are associated (dependant)

Step II: Given $\alpha = 0.01$

Step III : T.S used in Chi-Square

$$\chi^2 = \sum\sum\left[\frac{(O-e)^2}{e}\right]$$

Step IV : Critical region

$$\chi^2 > \chi^2_{\alpha(\nu)} \quad \text{where } \nu = (\underset{\text{number of rows}}{r}-1)(\underset{\text{columns}}{c}-1)$$

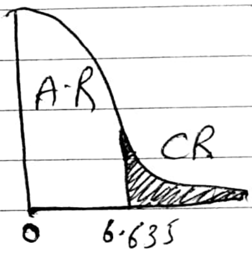
$$= (2-1)(2-1)$$

$$= 1$$

$$\chi^2 > \chi^2_{(0.01)(1)}$$

check in table A5 (pg 740)

$$\chi^2 > 6.635$$



Step V : Calculations

e = expected frequencies

$$= \frac{RT \times CT}{n} \rightarrow \frac{\text{row total} \times \text{cols total}}{\text{total}}$$

Gender

	Male	Female	total
over 25 hours	$\frac{44\times42}{90}$	$\frac{44\times48}{90}$	44
under 25 hours	$\frac{46\times42}{90}$	$\frac{46\times48}{90}$	46
total	42	48	$n=90$

O	e	$(O-e)^2/e$	$O-e$
15	20.53		
27	21.47		
29	23.47		
19	24.53		

$\chi^2 = 5.46$ $\quad \Sigma = 0$ always

Topics for finals : Chp 6 , Chp 10, 11, 13 , 3 ⟹ 1 question each
(1 ques) (3 questions)

Q10.86 O = observed frequencies

	Non smoker	moderate smoker	heavy smoker	Total
hypertension	21	36	30	87
No hypertension	48	26	19	93
Total	69	62	49	n=180

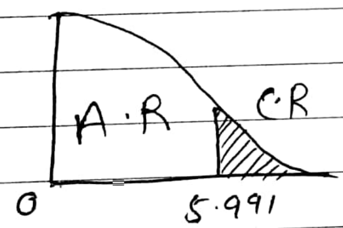
Step I : H_0 : Attributes are independant
H_A : Attributes are associated

Step II : Given $\alpha = 0.05$

Step III : T.S : $\chi^2 = \Sigma\Sigma\left[\frac{(o-e)^2}{e}\right]$

Step IV : C.R : $\chi^2 > \chi^2_{\alpha(\nu)}$ where $\nu = (r-1)(c-1)$
$> \chi^2_{0.05(2)}$ $= (2-1)(3-1) = 2$

> 5.991 from table



Step V : Calculations:

e = expected frequencies = $\frac{RT \times CT}{n}$

	non smoker	mod smoker	Heavy smoker	Total
hyper	$\frac{87\times69}{180}=33.5$	$\frac{87\times62}{180}=29.9$	$\frac{87\times49}{180}=23.68$	87
non hyper	35.65	32.03	25.32	93
total	69	62	49	180 = n

o	e	o - e	$(o-e)^2/e$
21	33.35	-12.35	4.57
48	35.65	12.35	4.27
36	29.97	6.03	1.21
26	32.03	-6.03	1.14
30	23.68	6.32	1.69
19	25.32	-6.32	1.58
		0	$\chi^2 = 14.48$

$$\chi^2 = \Sigma\Sigma \left[\frac{(o-e)^2}{e} \right] = 14.48$$

Finals sallybus

Chap 6 (1 Question)
Chap 10 (z test, t test) (chi-squared test)
total 3 questions
Chap 11 (regression & corelation) (1 question)
Chap 13 (analysis of variance) (1 question)
Chap 3 (1 question)

Chapter 11 : Regression & corelation

Regression: The dependence of one variable (Y) on the other variable (X) is called regression.

Least squares regression line is

$$y = a + bx$$

where Y = dependant variable
X = Independant variable
'a' and 'b' are unknown constants to be determined.

where $b = \dfrac{n\sum XY - (\sum X)(\sum Y)}{n\sum x^2 - (\sum x)^2}$ [reg. coeff of Y on X]

and $a = \bar{Y} - b\bar{X}$

Reg. coeff of X on Y is

$$b_{xy} = \frac{n\sum XY - (\sum X)(\sum Y)}{n\sum Y^2 - (\sum Y)^2}$$

corelation:

The interdependance between 2 variables X and Y is called corelation.

Corellation-coefficient:

The measure of inter-dependance b/w 2 variables X & Y is called corelation coefficient.

It is denoted by 'r' where

$$r = \frac{n\sum XY - (\sum X)(\sum Y)}{\sqrt{\left[(n\sum x^2 - (\sum x)^2)(n\sum y^2 - (\sum Y)^2)\right]}}$$

Properties:

1 - r lies b/w -1 and 1

2 - $r_{xy} = \sqrt{b_{yx} \cdot b_{xy}}$

Q: Find (a) least squares regression line

(b) estimate Y for $X = 12$

(c) correlation coefficient

(d) Show that $r_{xy} = \sqrt{b_{yx} \cdot b_{xy}}$

	X	Y	XY	X^2	Y^2
	1	10	10	1	100
	2	11	22	4	121
	3	12	36	9	144
	4	13	52	16	169
	5	14	70	25	196
Σ =	15	60	190	55	730

The least square regression line is

$y = a + bx$

where $b = \frac{n\sum XY - (\sum x)(\sum Y)}{n\sum x^2 - (\sum x)^2}$

$$b = \frac{950 - 900}{275 - 225} = \frac{50}{50} = 1$$

Quiz 4 χ^2 = Chp 10

Chp 11

and $a = \bar{y} - b\bar{x}$

where $\bar{x} = \frac{\Sigma x}{n} = \frac{15}{5} = 3$

$\bar{y} = \frac{\Sigma y}{n} = \frac{60}{5} = 12$

$a = 12 - 1(3) = 9$

Hence least squares regression line is

$\hat{y} = 9 + x$

(c) corelation coefficient

$$r_{xy} = \frac{n\Sigma xy - (\Sigma x)(\Sigma y)}{\sqrt{(n\Sigma x^2 - (\Sigma x)^2)(n\Sigma y^2 - (\Sigma y)^2)}}$$

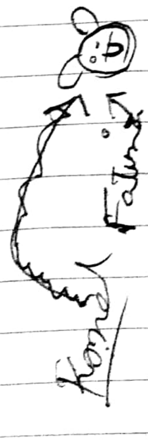
$$= \frac{5(190) - 15(60)}{\sqrt{(5(55) - 15^2)(5(730) - (60)^2)}}$$

$$= \frac{50}{\sqrt{50 \times 50}} = \frac{50}{50} = 1$$ ≫ perfect +ve corelation

0.83 : High +ve corelation
-0.21 : small -ve corelation
-0.61 : moderate -ve corelation
$r = 0$, no correlation.

$\longleftrightarrow$

↳ agay bhi hy.



Reg coeff of X on Y is

$$b_{xy} = \frac{n\sum XY - (\sum X)(\sum Y)}{n\sum y^2 - (\sum Y)^2}$$

$$= \frac{50}{50} = 1$$

Result :

$$r_{xy} = \sqrt{b_{yx} \cdot b_{xy}}$$

$$1 = \sqrt{1 \times 1}$$

$$1 = 1$$ hence proved.

Q2: Find a least squares regression line of
(a) Y on X for data below.
(b) Find trend values and show that $\Sigma Y = \Sigma \hat{Y}$
(c) Show that $\Sigma(Y-\hat{Y}) = 0$ & $\Sigma(Y-\hat{Y})^2$ = Least

	X	Y	XY	X^2	T-Values	$Y-\hat{Y}$	$(Y-\hat{Y})^2$
	25	22	550	625	18.6223	3.3777	11.4089
	28	18	504	784	19.0045	-1.0045	1.0090
	32	17	544	1024	19.5141	-2.5141	6.3207
	35	16	560	1225	19.8963	-3.8963	15.1812
	37	20	740	1369	20.1511	-0.1511	0.0228
	38	21	798	1444	20.2785	0.7215	0.5206
	40	24	960	1600	20.5333	3.4667	12.0180
$\Sigma =$	235	138	4656	8071	138.0001	0	46.4812

$\bar{X} = \frac{\Sigma X}{n} = \frac{235}{7} = \boxed{33.5714}$

$\bar{Y} = \frac{\Sigma Y}{n} = \frac{138}{7} = \boxed{19.7143}$

(a) Regression line of Y on X is $y = a + bX$
where

$$b = \frac{n\,\Sigma XY - (\Sigma X)(\Sigma Y)}{n\,\Sigma X^2 - (\Sigma X)^2}$$

$$b = \frac{(7)(4656) - (235)(138)}{(7)(8071) - (235)^2}$$

$$b = \frac{162}{1272} \Rightarrow \boxed{0.1274}$$

and $a = \bar{Y} - b\bar{X}$

$$a = 19.7143 - (0.1274)(33.5714)$$

$$a = 19.7143 - 4.2770$$

$$\boxed{a = 15.4373}$$

Hence, estimated line is,

$$\boxed{\hat{Y} = 15.4373 + 0.1274X}$$

(b) $\Sigma Y = \Sigma \hat{Y}$

$138 = 138$ Hence proved!

(c) $\Sigma(Y-\hat{Y}) = 0$, $\Sigma(Y-\hat{Y})^2 = 46.4812$

Q

X	Y	XY	X^2	Y^2	T values
10	1	10	100	1	1
11	2	22	121	4	2
12	3	36	124	9	3
13	4	52	169	16	4
15	6	90	225	36	6
61	16	210	759	66	16

$\bar{X} = \frac{61}{5} = \frac{\Sigma X}{n} = \boxed{12.2}$, $\bar{Y} = \frac{\Sigma Y}{n} = \frac{16}{5} = \boxed{3.2}$

(a) Regression line of Y on X is $\hat{Y} = a + bX$
where $X = 14$ so $\hat{Y} = a + 14b$

$$b = \frac{n\Sigma XY - (\Sigma X)(\Sigma Y)}{n(\Sigma X^2) - (\Sigma X)^2}$$

$$b = \frac{(5)(210) - (61)(16)}{(5)(759) - (61)^2} = \frac{74}{74}$$

$\boxed{b = 1}$ and

$a = \bar{Y} - b\bar{X}$

$a = 3.2 - (1)(12.2)$

$\boxed{a = -9}$

$$\boxed{\hat{Y} = -9 + X}$$

$x = 14$

So, $\hat{y} = 14 - 9 \Rightarrow 5$

(b) Find trend values:

$\Sigma(\text{Trend values}) = 16$

(c) Corelation coefficient b/w X & Y

$$r_{xy} = \frac{n\Sigma XY - (\Sigma X)(\Sigma Y)}{\sqrt{[n\Sigma x^2 - (\Sigma x)^2][n\Sigma y^2 - (\Sigma y)^2]}}$$

$$= \frac{(5)(210) - (61)(16)}{\sqrt{[(5)(759) - (61)^2][(5)(66) - (16)^2]}}$$

$$= \frac{74}{\sqrt{(74)(74)}} = \boxed{1}$$

ANOVA: analysis of variance

Chapter 13

pg 507

Analysis of variance (ANOVA)

* one-way classification

imp Q Following are three consecutive week's earnings of 3 departmental stores in thousands. Calculate F and, test that $\alpha = 5\%$ whether differences b/w stores are significant.

Stores

n = total values

	A	B	C	Total ($T_{i.}$)	$\sum x_{ij}^2$
	15 (225)	18 (324)	16 (256)	49	805
	17 (289)	17 (289)	13 (169)	47	747
	18 (324)	20 (400)	12 (144)	50	868
Total ($T_{.j}$)	50	55	41	$T_{..} = 146$	
$T_{.j}^2$	2500	3025	1681	$\sum T_{.j}^2 = 7206$	
$\sum x_{ij}^2$	838	1013	569	$\boxed{2420 = \sum\sum x_{ij}^2}$	

same

Step 1: $H_0: \mu_1 = \mu_2 = \mu_3$

H_A: all means are not equal

Step 2: Given

$\alpha = 5\%$

Step 3: Test Statistic: $\boxed{F = \frac{S_b^2}{S_w^2} = \frac{MSB}{MSW}}$

Step 4: Calculations

Correction factor: $\boxed{C \cdot F = \frac{(T_{..})^2}{n}} = \frac{(146)^2}{9} = 2368.4444$

MSB: mean square for between

MSW: mean square for within.

Total sum of squares: ~~TSS = ΣΣ(T.j)² − C.F~~

$$TSS = \Sigma\Sigma x_{ij}^2 - C.F$$
$$= 2420 - 2368.4444$$
$$\boxed{= 51.5556}$$

Between Sum of squares:

$$BSS = \frac{\Sigma T.j^2}{r} - C.F$$ where r is no. of rows.

$$= \frac{7206}{3} - 2368.44 \Rightarrow \boxed{33.56}$$

within SS or error SS:

$$WSS = TSS - BSS$$
$$WSS = 51.5556 - 33.5556$$
$$\boxed{WSS = 18}$$

df: degree of freedom

SOV = source of variations

ANOVA table

SOV	df	SS	MS (mean)	F. Ratio (→ MSB)
BSS	3-1=2	33.56	$\frac{33.56}{2}$ = 16.78	$F = \frac{MSB}{MSW} \Rightarrow \frac{16.78}{3} = \boxed{5.59}$
WSS	8-2=6	18	$\frac{18}{6}$ = 3 (MSW)	
Total				

Step 5: Critical region:

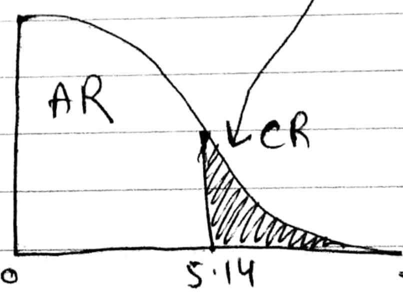
$$F \geq F_\alpha(\nu_1, \nu_2)$$
$$F \geq F_{(0.05)}(2, 6)$$
$$F \geq 5.14$$

table on page 741

(always ≥)

Step 6: Conclusion:

Ho is rejected because F = 5.59 which lies in CR.



Q Marks obtained by 3 students in 3 monthly test are given below.

A	B	C
12	16	11
15	14	15
18	11	17

calculate F and test at $\alpha = 5\%$ that there is no significant difference b/w means of 3 students.