Advanced Statistics (DS2003)
BDS-4A, 4B, 4C
Spring 2024

Summary of Some Important Random Variables

1. Discrete Uniform Random Variable

$$S_X = \{a_1, a_2, \ldots a_N\}$$
$$p_X(x) = \frac{1}{N}$$
$$E(X) = \frac{1}{N}\sum_{i=1}^{N} a_i; \quad Var(X) = \frac{1}{N}\sum_{i=1}^{N}[a_i - E(X)]^2$$

2. Bernoulli Random Variable

$$S_X = \{0, 1\}$$
$$p_X(x) = \begin{cases} p & when\ x = 1 \\ 1-p & when\ x = 0 \end{cases}$$
$$E(X) = p; \quad Var(X) = p(1-p)$$

3. Binomial Random Variable

$$S_X = \{0, 1, 2, 3, \ldots, n\}$$
$$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k}$$
$$E(X) = np; \quad Var(X) = np(1-p)$$

4. Geometric Random Variable

$$S_X = \{1, 2, 3, \ldots\}$$
$$p_X(n) = p(1-p)^{n-1}$$
$$E(X) = \frac{1}{p}; \quad Var(X) = \frac{1-p}{p^2}$$

5. Negative Binomial Random Variable

$$S_X = \{r, r+1, r+2, r+3, \ldots\}$$
$$p_X(n) = \binom{n-1}{r-1} p^r (1-p)^{n-r}$$
$$E(X) = \frac{r}{p}; \quad Var(X) = r\left(\frac{1-p}{p^2}\right)$$

6. Poisson Random Variable

$$S_X = \{0, 1, 2, 3, \ldots\}$$
$$p_X(k) = \frac{(\lambda t)^k}{k!} e^{-\lambda t}$$
$$E(X) = \lambda t; \quad Var(X) = \lambda t$$

7. Continuous Uniform Random Variable

$$f_X(x) = \frac{1}{b-a}; \quad a \leq x \leq b$$
$$E(X) = \frac{a+b}{2}; \quad Var(X) = \frac{(b-a)^2}{12}$$

8. Exponential Random Variable

$$f_X(x) = \lambda e^{-\lambda x}; \quad x \geq 0$$
$$E(X) = \frac{1}{\lambda}; \quad Var(X) = \frac{1}{\lambda^2}$$

9. Gaussian Random Variable

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}; \quad -\infty \leq x \leq \infty$$
$$E(X) = \mu; \quad Var(X) = \sigma^2$$