DS-2003: Advanced Statistics

Spring 2024

In Class Activity

Question-1

If all possible samples of size $N = 16$ are drawn from a normal population with mean $\mu = 50$ and standard deviation $\sigma = 5$, what is the probability that a sample mean will fall in the interval from $\mu_{\bar{X}} - 1.9\sigma_{\bar{X}}$ to $\mu_{\bar{X}} - 0.4\sigma_{\bar{X}}$.

Question-2

If the standard deviation of the mean for the sampling distribution of random samples of size 36 from a large or infinite population is 2, how large must the sample size become if the standard deviation is to be reduced to 1.2?

Question-3

The average life of a bread-making machine is 7 years, with a standard deviation of 1 year. Assuming that the lives of these machines follow approximately a normal distribution, find

(a) the probability that the mean life of a random sample of 9 such machines falls between 6.4 and 7.2 years;

(b) the value of x to the right of which 15% of the means computed from random samples of size 9 would fall.

Question-4

A manufacturer of semiconductor devices takes a random sample of 100 chips and tests them, classifying each chip as defective or non-defective. Let $X_i = 0$ if the chip is non-defective and $X_i = 1$ if the chip is defective. The sample fraction defective is $\widehat{P} = \frac{\sum_{i=1}^{100} X_i}{100}$. What is the sampling distribution of the random variable $\widehat{P}$?

Question-5

Let three random samples of sizes $N_1 = 20$, $N_2 = 10$, and $N_3 = 8$ be taken from a population with mean μ and variance σ^2. Let S_1^2, S_2^2, and S_3^2 be the sample variances. Show that $S^2 = \frac{20S_1^2 + 10S_2^2 + 8S_3^2}{38}$ is an unbiased estimator of σ^2.