Statistical and Mathematical Methods for Data Science
Midterm 2 Solutions
Fall 2018

QUESTION 1

Given $\mathbf{x} \in \Re^3$ and $f(\mathbf{x}) = 2x_1^2 + 5x_2^3 + 10x_3x_1$. Write the gradient and Hessian of f. (Marks 3+4)

$\nabla_x f(\mathbf{x}) =$	$\nabla^2_x f(\mathbf{x}) =$
$\begin{pmatrix} 4x_1+10x_3 \\ 15x_2^2 \\ 10x_1 \end{pmatrix}$	$\begin{pmatrix} 4 & 0 & 10 \\ 0 & 30x_2 & 0 \\ 10 & 0 & 0 \end{pmatrix}$

QUESTION 2 (Marks: 6)

Use Gram Schmidt procedure to give an **orthonormal** set for the given set of vectors (the order of vectors should not be changed when applying the method).
$\mathbf{v}_1 = [1\ 0\ 0]^T$, $\mathbf{v}_2 = [2\ 1\ 1]^T$, $\mathbf{v}_3 = [1\ 2\ 3]^T$

SOLUTION

$\mathbf{a}_1 = \mathbf{v}_1$

$$\mathbf{a}_2 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} - (2/1) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$\mathbf{a}_3 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} - (1/1) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} - (5/2) \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1/2 \\ -1/2 \end{pmatrix}$$

Normalizing all basis vectors

$$\mathbf{u}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{u}_2 = 1/(\text{sqrt}(2)) * \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$\mathbf{u}_3 = 1/(\text{sqrt}(2)) * \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

QUESTION 3 (Marks: 3)

Given $[4\ -2\ 6]^T$

L_1 norm is: **12** L_2 norm is: **sqrt(56)** L_∞ norm is: **6**

QUESTION 4 **(Marks: 4)**

Write the complete expression for updating a when $f(a) = (1-a)^2$ is to be minimized using gradient descent. The expression should only contain the variable a. You can assume momentum $\alpha = 0$ and learning rate $\eta = 1/4$.

SOLUTION

$a^{(t)} \leftarrow a^{(t-1)} + 1/2 * (1 - a^{(t-1)})$

QUESTION 5 **(Marks: 5)**

Given the following values of the standard Normal variable:

$z_{0.10} = 1.282$, $z_{0.05} = 1.645$, $z_{0.025} = 1.960$, $z_{0.01} = 2.326$, $z_{0.005} = 2.576$.

Find the 90% confidence interval for the population mean when the sample measurements are: {0,0,0,1,2,2,3,5,5}. It is known that the population standard deviation is 2

SOLUTION

[18/9 - 1.645*2/3 18/9+1.645*2/3] = [0.903 3.097]

QUESTION 6 **(Marks: 4)**

When given the following predictions and corresponding labels, fill in the confusion matrix. +1 is the positive class

prediction	1	-1	1	-1	1	-1	1
label	+1	+1	+1	-1	-1	-1	-1

TP = 2	FP = 2
FN = 1	TN = 2

QUESTION 7 **(Marks: 6)**

Find the regression coefficients via the method of least squares for the following data:

x ->	2	0	-2
target	0	-1	1

SOLUTION (do the working yourself)

The coefficients are: $\beta_0 = 0$, $\beta_1 = -1/4$