Independent Component Analysis

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Cocktail Party Problem

Given two microphones placed at two locations not far apart and two people speaking at the same time, we record a signal on each microphone, that signal represents the combination of the two coming from these individuals and we would like to recover the original ones. Formally, we record $x_1 = a_{11}s_1 + a_{12}s_2$ and $x_2 = a_{21}s_1 + a_{22}s_2$ with s_1 and s_2 being the source signals we want to recover.

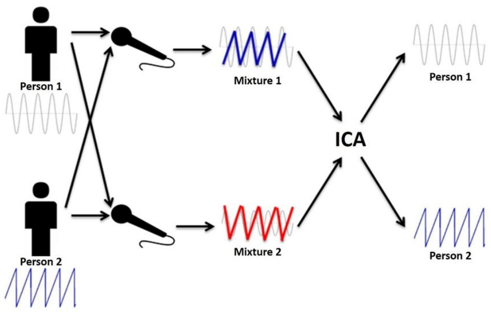

Figure 1: Cocktail Party Problem

ICA: Non-Gaussianity

Let's take the problem our ICA model is trying to solve: x=As. The Central Limit Theorem stipulates says the sum of random variables i.i.d tends towards a Gaussian variable. As a result, the linear combination of independent sources s_i generally has a distribution closer to the Gaussian than any of the original sources.
Let w be a vector, and $y = w^T x = w^T As = z^T s$ with $z^T = w^T A$, y is then a linear combination of the i.i.d sources s_i. Thus according to CLT, y is closer to a Gaussian than each s_i unless it's equal to one of them.
Ideally, we would like w to be one of the row of the inverse of A, so that way, y should be equal to a source s_i. Therefore, the ICA problem is equivalent to finding the vectors w that maximise the non-Gaussianity of y so that it is as close as possible to the sources s_i.
Kurtosis or Negentropy are potential measures of non-gaussianity.
E. O. A. Hyvarinen (2000)

Reduction of noise in images

To denoise images with ICA , we use the Sparse Code Shrinkage method defined in E. O. A. Hyvarinen P. Hoyer (1999).

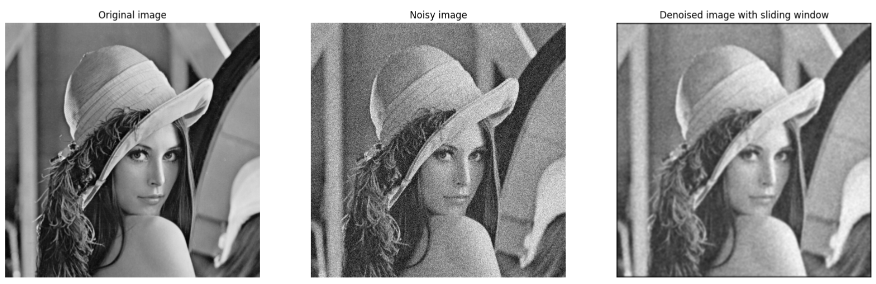

Figure 2: Original image, noisy image and denoised image using ICA

Heartbeat removal from MEG

Artefacts in MEG signals can arise from various sources, such as environmental noise, eye movements, muscle activity, heartbeats ... We retrieved 12 MEG signals showing clear signs of heartbeats and artifacts. It is difficult to identify the origin of the artifacts, however, the signal corresponding to the cardiac artifact is easily identifiable.

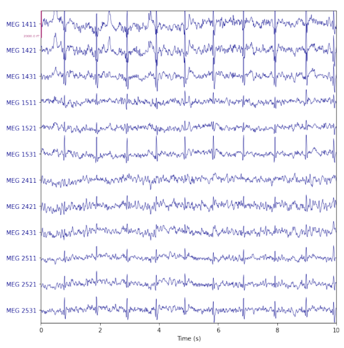

Figure 3: MEG signals with

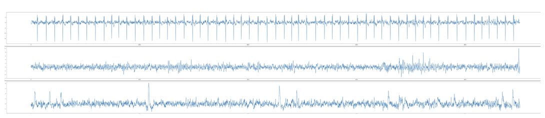

Figure 4: Independent components extracted from MEG signals

Blind source separation

ICA was introduced to solve blind source separation problems. We repeated the cocktail party experience by mixing 3 audio sources and applying ICA to extract them. When listening, the audios and the ICA source estimates correspond perfectly.

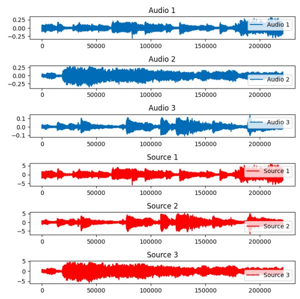

Figure 5: Original audios and extracted sources by FastICA

Face recognition

Another application of ICA is face recognition Deniz et al. (2001). The YaleB 32*32 face dataset contains 2414 facial images of 38 individuals. The features used for the classification are the mixing matrix coefficients $W = X * S^{-1}$ with S^{-1} the generalized inverse of the non-square matrix S. A SVM classifier was trained to identify the individual. We perform histogram equalisation beforehand, reducing the effects of brightness differences. We obtain 99,6% of accuracy on the training set and 95% on the testing set.

Figure 6: Examples from the dataset YaleB32*32

Limitations of ICA

The ICA method is based on two assumptions: the original sources are non-Gaussian (at most one can be) and independent. Furthermore, this method is somewhat unstable. When the number of sources is greater than the number of observations, it is almost impossible to find these sources. We need at least as many sources as observations for ICA to work.

Comparaison PCA and ICA: reduction of dimensionality

PCA algorithm seeks to preserve the greatest amount of information by projecting the data into a space with the greatest variability. This notion of preserved information is not specified in the ICA, but the assumption of independence and non-orthogonality of the sources gives greater freedom in the choice of directions returned.

Figure 7: On the first line, the images from the testing set, on the line below, the reconstruction after compression by PCA (left) and by ICA (right)

References

A. Hyvarinen, E. O. (2000). *Independent component analysis: algorithms and applications.*

A. Hyvarinen, E. O., P. Hoyer. (1999). *Image denoising by sparse code shrinkage.*

Deniz, O., Castrillón Santana, M., & Hernández, M. (2001, 08). Face