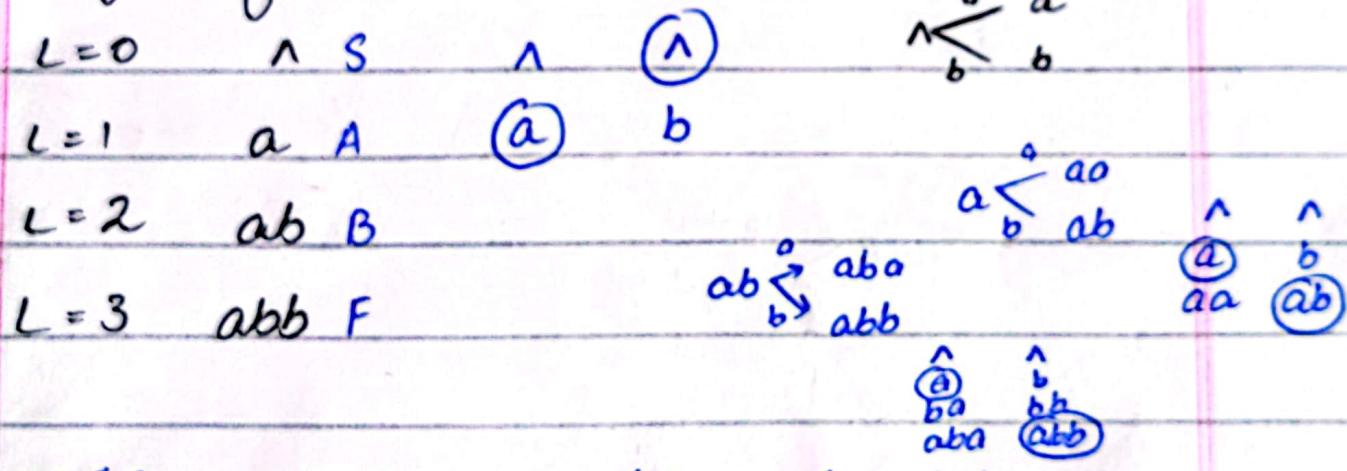


Theory of Automata Assig. 1

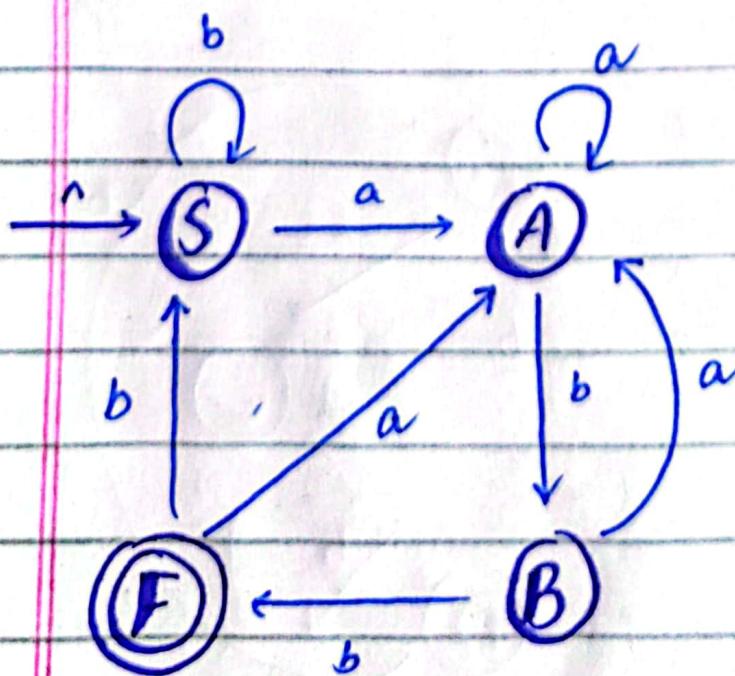
Q1 - $L = \{w \in \{a, b\}^* \mid w \text{ ends with } abb\}$
 $L = \{abb, aabb, babb, aaabb, bbabb, ababb, \dots\}$

Prefix of abb



Divide pattern in prefix and string
in suffix.

$abb \xrightarrow{a} abba$
 $abb \xrightarrow{b} abbb$



\wedge	(\wedge)
a	b
ba	bb
bba	bbb
$abba$	$abbb$

Q2

$L = \{ w \in \{a,b\}^* \mid w \text{ contain substring } abbaab \}$
 $L = \{ abbaab, aabbaab, abbaaba, babbaab, abbaabb \}$.

Prefix of abbaab :-

$L=0 \quad \wedge \quad S$

$L=1 \quad a \quad A \quad \begin{matrix} \wedge & \wedge \\ @ & b \end{matrix} \quad \begin{matrix} a \\ b \end{matrix} \xrightarrow{a} aa$

$L=2 \quad ab \quad B \quad aa \quad @b \quad ab$

$L=3 \quad abb \quad C$

$L=4 \quad abba \quad D \quad ab \quad \begin{matrix} \wedge & \wedge \\ b & b \end{matrix} \xrightarrow{a} aba$

$L=5 \quad abbaa \quad E$

$L=6 \quad abbaab \quad F$

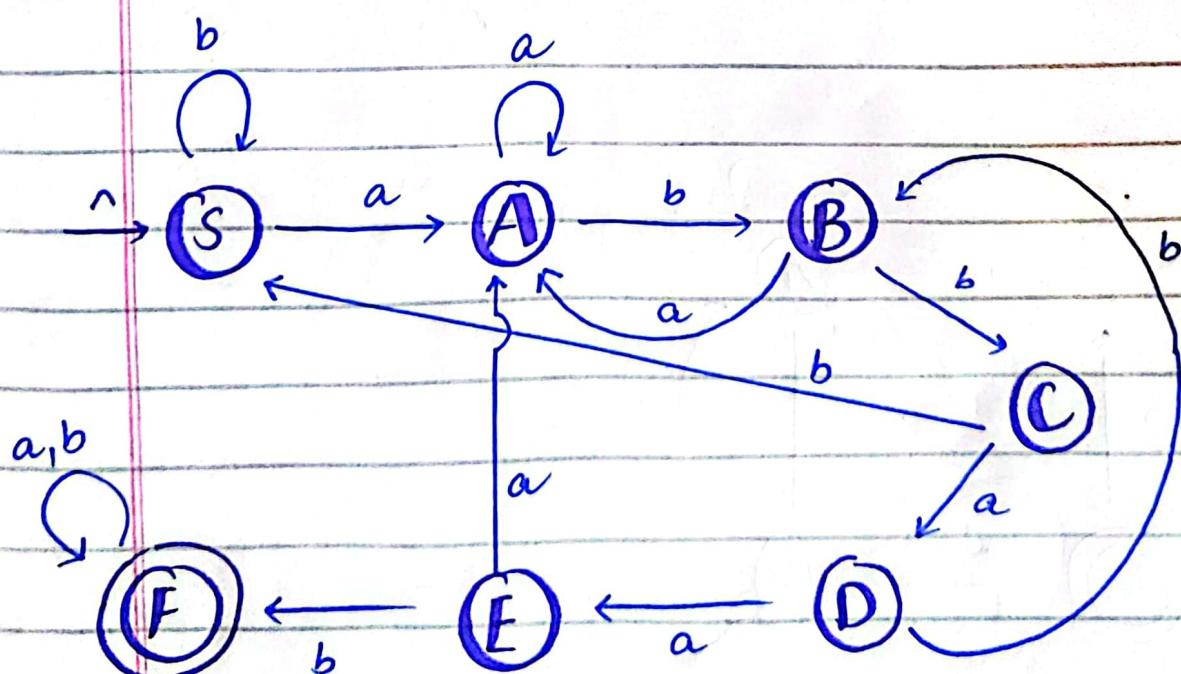
$\begin{matrix} abba \\ abba \end{matrix} \xrightarrow{a} abbaa$

$\begin{matrix} a \\ o \\ aa \\ baa \\ bbaa \\ abbaa \end{matrix} \quad \begin{matrix} \wedge & \wedge \\ b & b \end{matrix} \quad abba \quad bab \quad bbab$

$\begin{matrix} @ \\ ba \\ aba \end{matrix} \quad \begin{matrix} \wedge & \wedge \\ b & b \end{matrix} \quad bb \quad abb$

$\begin{matrix} ab \\ b \\ abb \end{matrix} \quad \begin{matrix} \wedge & \wedge \\ a & b \end{matrix} \quad ab \quad abb$

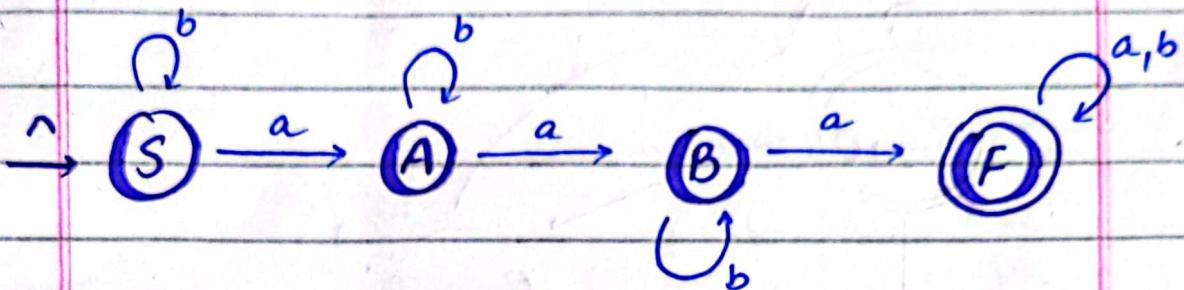
$\begin{matrix} a \\ ba \\ bb \\ abba \end{matrix} \quad \begin{matrix} \wedge & \wedge \\ a & b \end{matrix} \quad bb \quad bbb \quad abbb$



Q3-

$L = \{ w \in \{a,b\}^* \mid w \text{ contains at least three } a's \}$

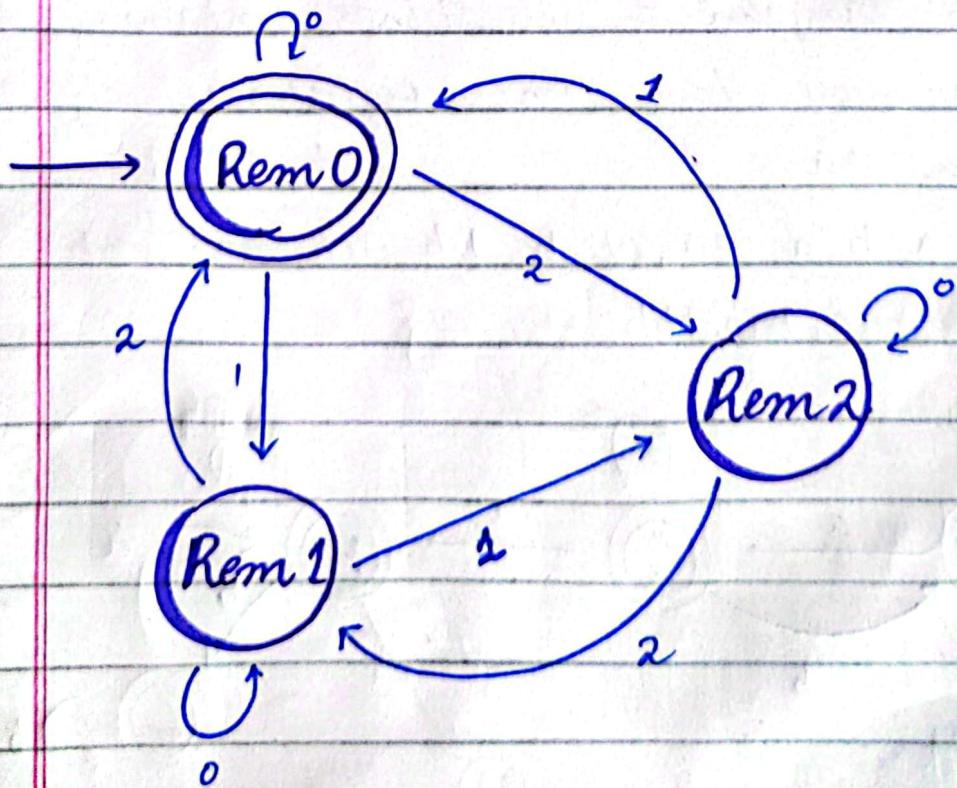
$L = \{ aaa, aaab, aaaa, baaa, abaa, aaba \dots \}$



Q4-

$L = \{ w \in \{0,1,2\}^* \mid \text{sum of digits in } w \text{ are divisible by 3} \}$

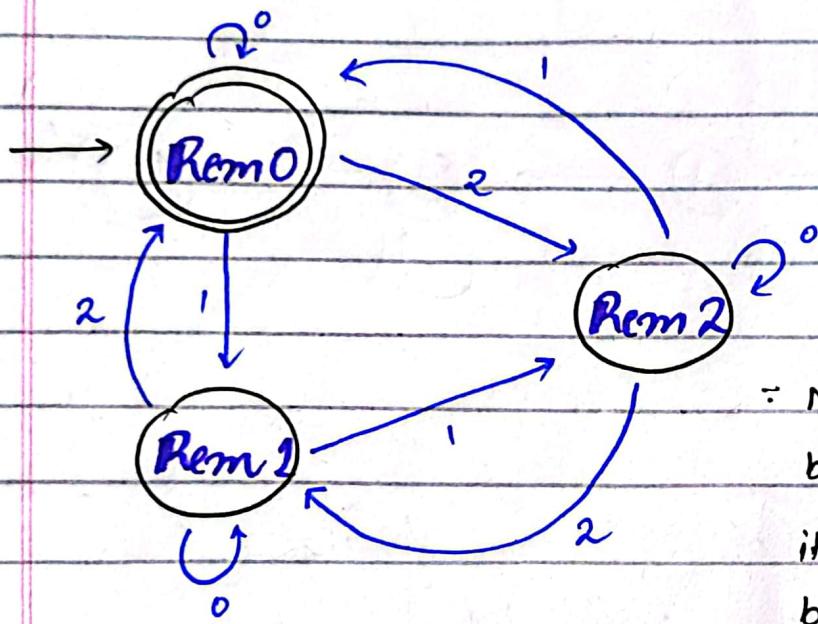
$L = \{ 0, 00, 12, 21, 000, 111, 012, 021, 201, 102, 120 \dots \}$



Q5-

$L = \{ w \in \{0, 1, 2\}^* \mid \text{number is divisible by } 3 \}$

$L = \{ 0, 00, 12, 21, 000, 012, 021, 102, 111, \dots \}$

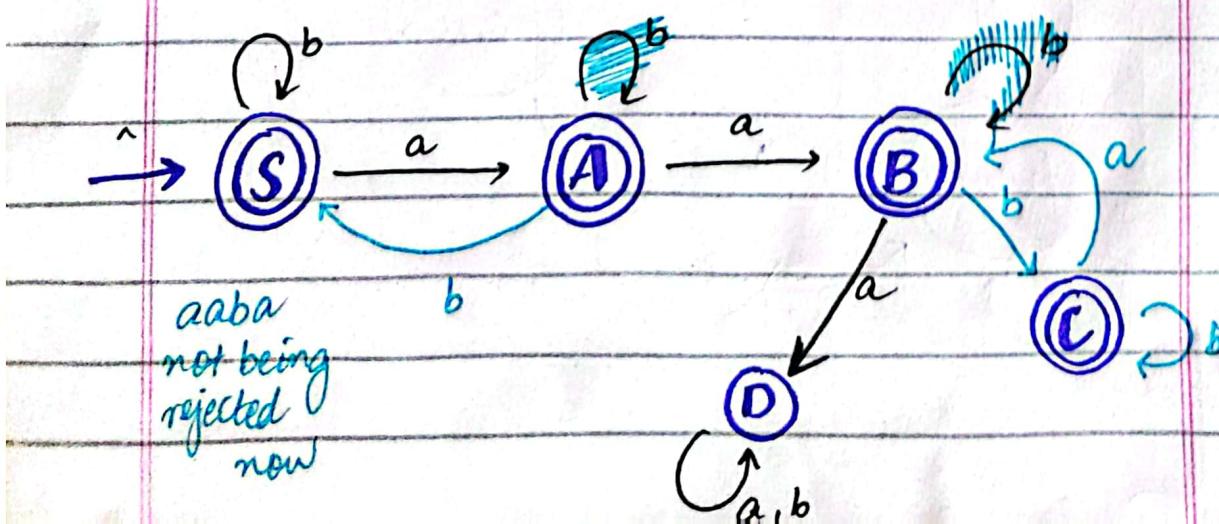


= Number is divisible by 3 if sum of its digits is divisible by 3.

Q6 language of all strings containing no more than one occurrence of string aa.

$$\Sigma = \{a, b\}$$

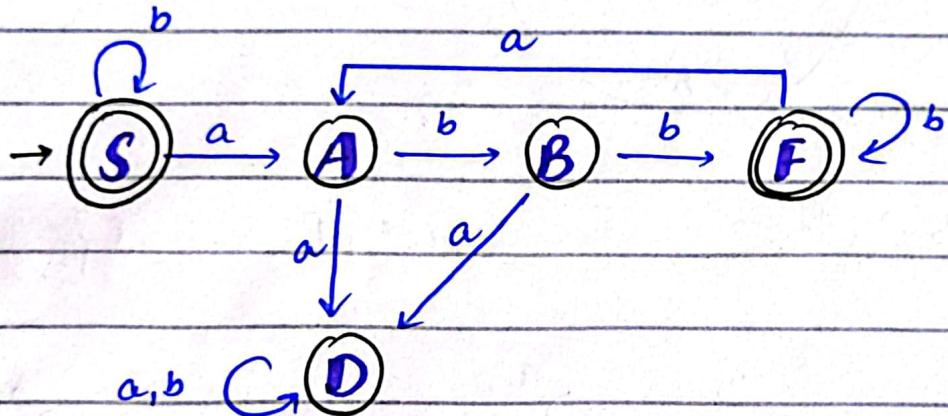
$L = \{ \lambda, b, a, aa, ab, ba, bb, abb, aba, aab, baa, bab, bbb, bba, \dots \}$



Q7-

language of all strings in which every a (if any) is followed immediately by bb.

$$L = \{ \lambda, b, bb, abb, bbb, abbb, babb, bbbb \dots \}$$



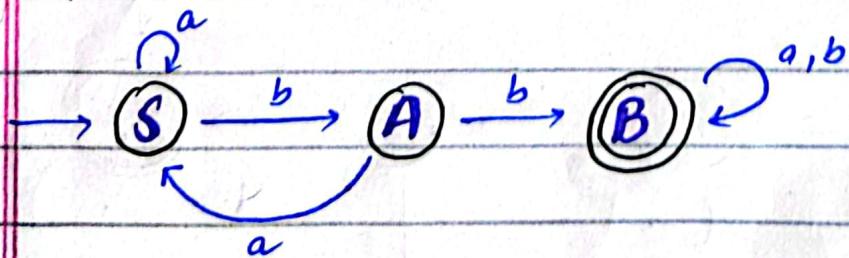
Q8-

language of all strings containing both bb and aba as substring.

Lo : bb substring

$$Lo = \{ bb, abb, bba, abba, bbb a, aabb, bb aa \dots \}$$

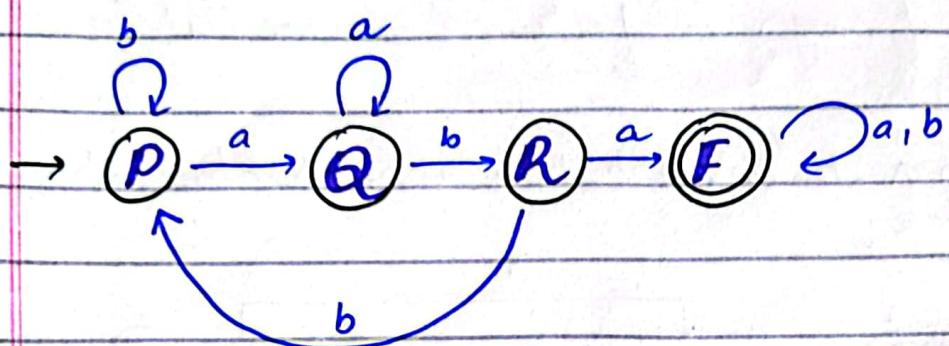
Mo :-



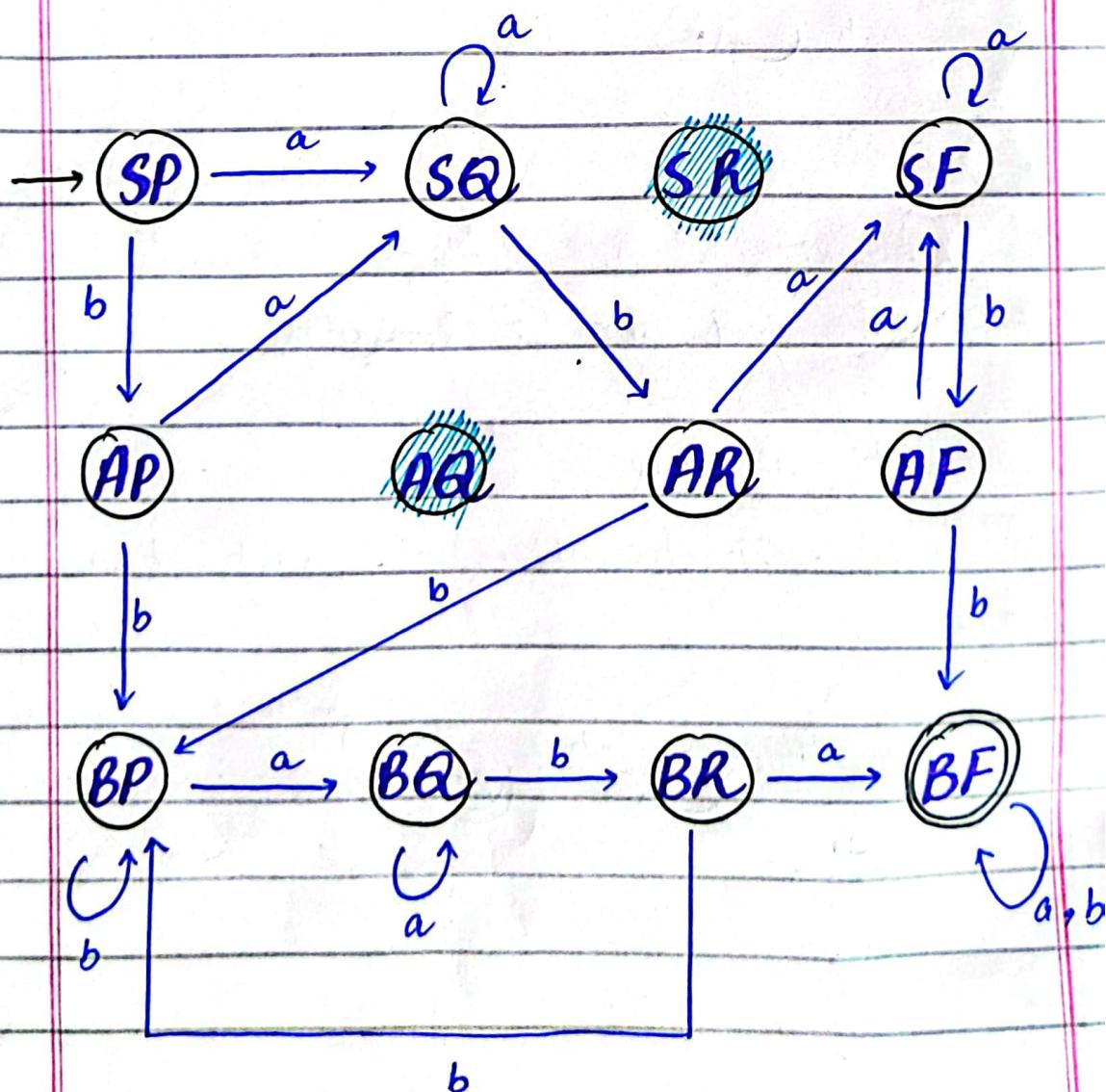
L_1 :- aba as substring

$$L_1 = \{ aba, aaba, baba, abaa, abab \dots \}$$

M_1 :-



$$M_1 \cap M_2 := \{ S, A, B \} \times \{ P, Q, R, F \}$$

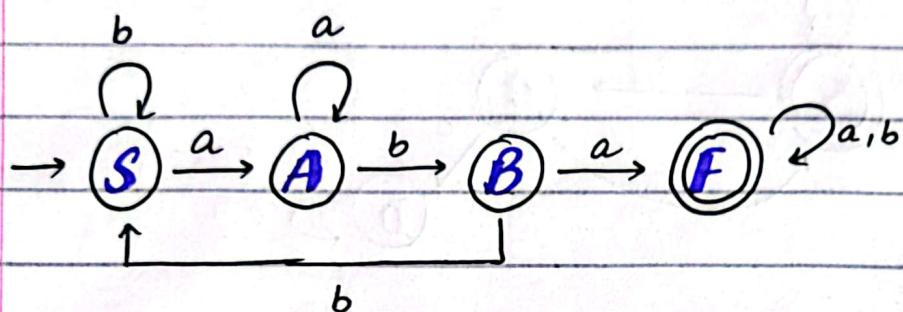


Q9

language of all strings containing both aba and bab as substring.

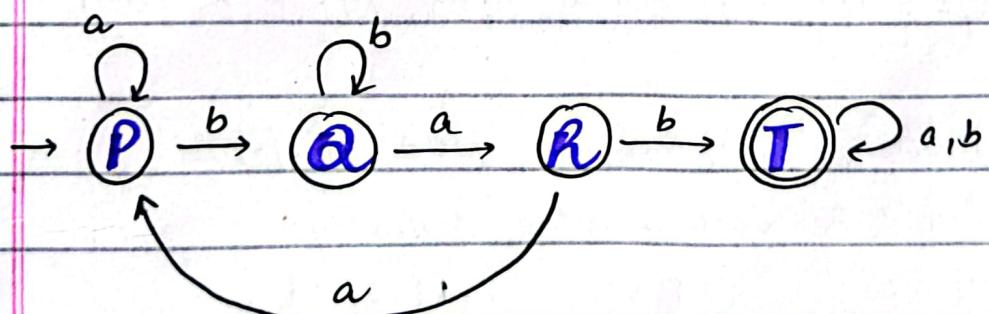
$$\Sigma = \{a, b\}$$

M₀ for aba :-

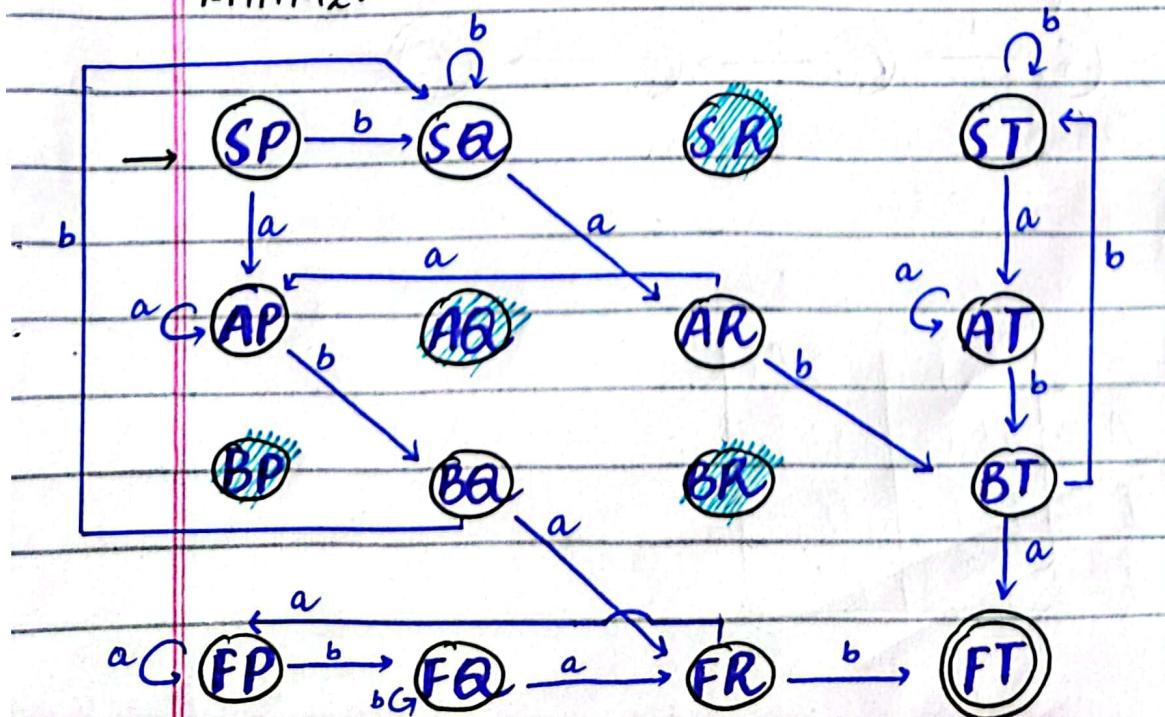


L₁ for bab :-

$$L_1 = \{bab, abab, baba, babb, \dots\}$$



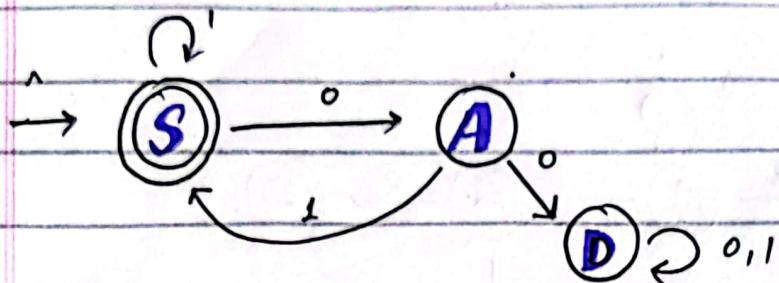
M₁ ∩ M₂ :-



Q₁₀

$L = \{w \in \{0,1\}^* \mid \text{every } 0 \text{ in } w \text{ is followed by at least one } 1\}$

$$L = \{\lambda, 1, 01, 11, 011, 101, 111, 0111, \dots\}$$



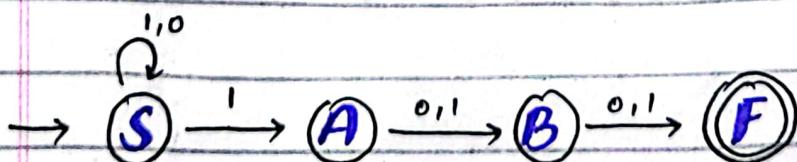
Q₁₁₋

$L = \{x \text{ over } \{0,1\}^* \mid \text{3rd last element must be } 0 \text{ or } 1\}$

$$L = \{100, 0100, 1100, 0110, 1111, 0101, 1101, \dots\}$$

First we will make NFA as DFA would be very complex initially.

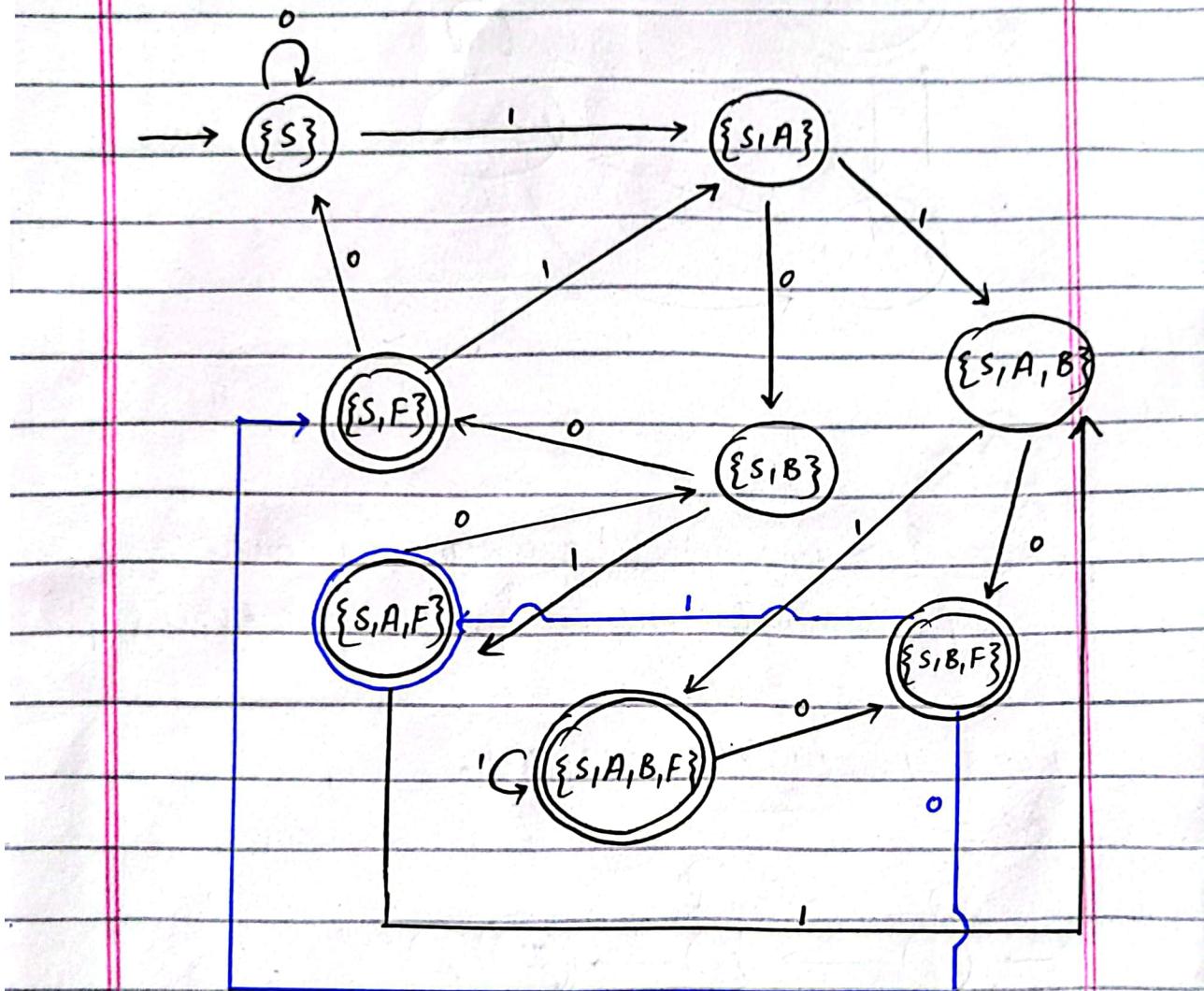
NFA:- $\{\{\lambda\}, \{0\}, \{1\}, \{0,1\}\}$



Subset construction

	0	1
-S	$\{S\}$	$\{S, A\}$
A	$\{B\}$	$\{B\}$
B	$\{F\}$	$\{F\}$
+F	$\{\}$	$\{\}$

DFA :-

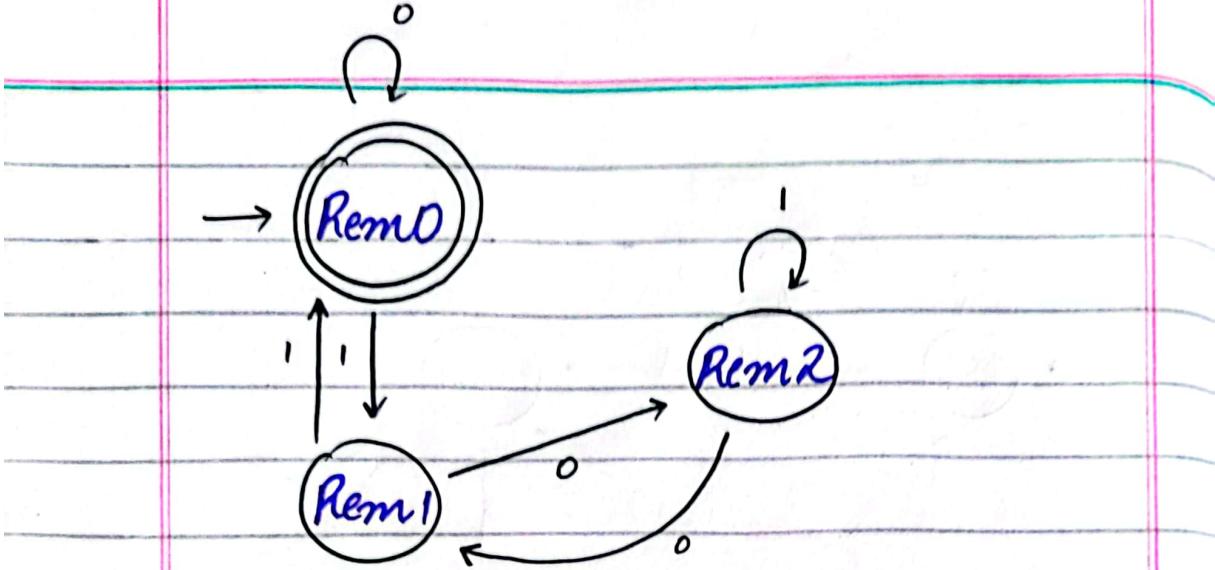


00110 accepted

Q₁₂

$L = \{x \text{ over } \{0, 1\} \mid x \text{ as binary number divisible by 3}\}$

$L = \{0, 00, 11, 000, 011, 110, \dots\}$



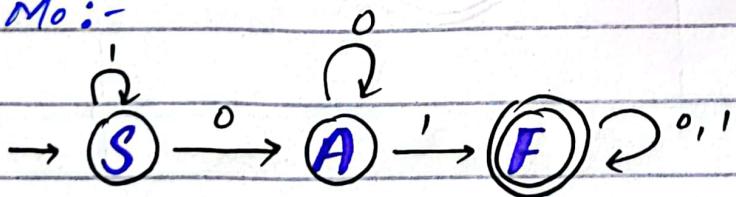
Q₁₃-

$L = \{ x \text{ over } \{0,1\} \mid x \text{ contain } 01 \text{ but does not contain } 010 \text{ as substring} \}$

$L_0 := 01 \text{ as substring}$

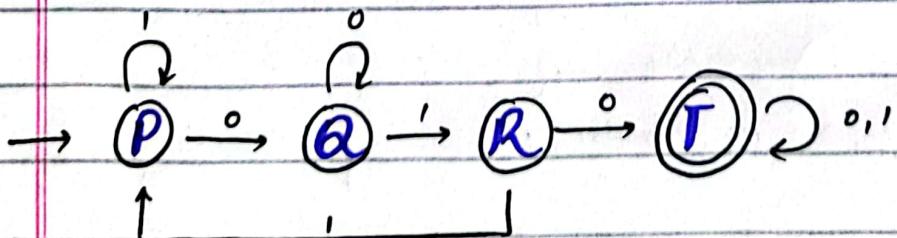
$L_0 = \{ 01, 001, 010, 011, 101, \dots \}$

$M_0 :=$



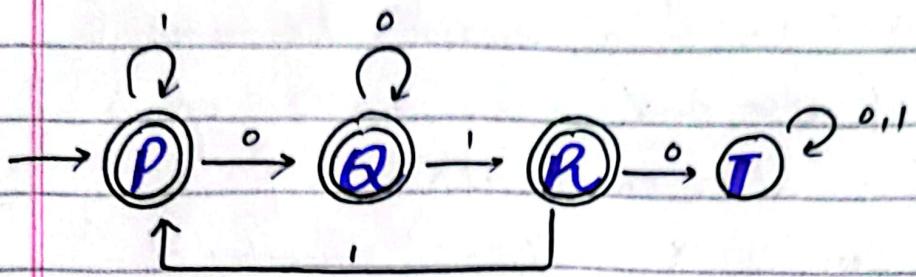
$L_1 := 010 \text{ as substring}$

$L_1 = \{ 010, 0010, 0100, 1010, 0101, \dots \}$

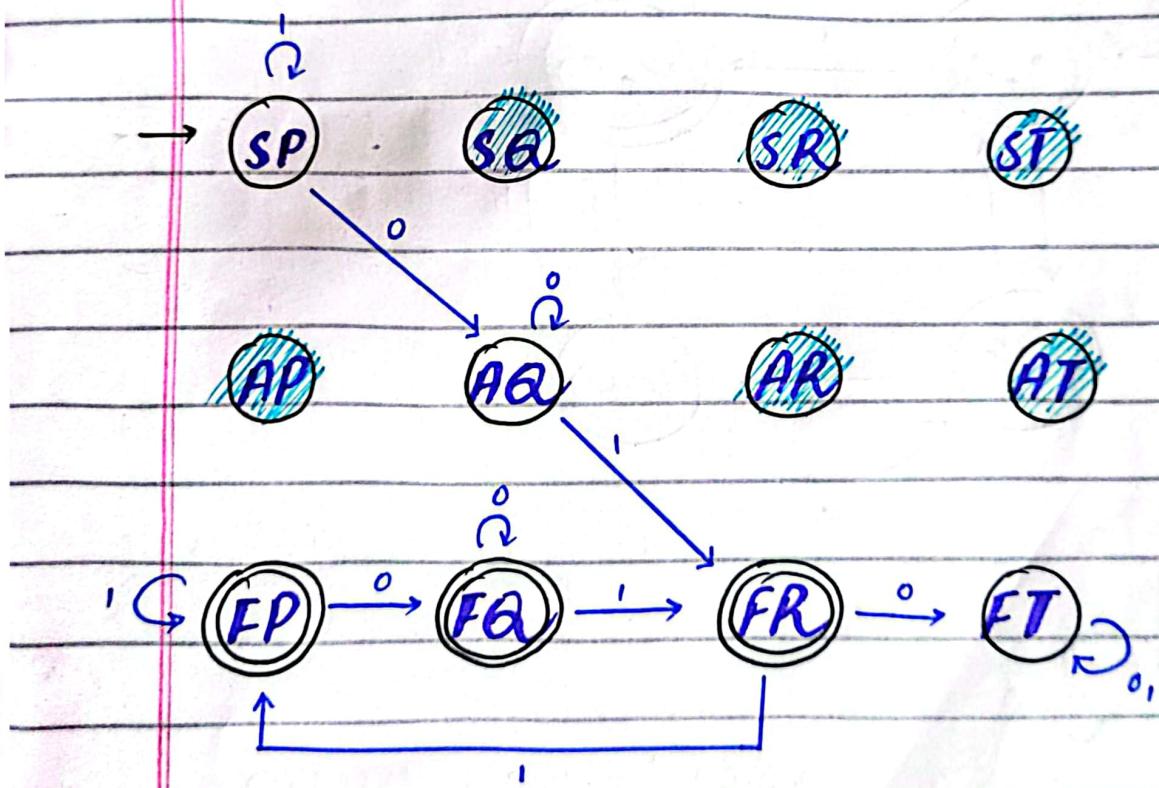


$\lambda_2 = \lambda_1$

$M_2 :=$



$M_0 \cap M_2 :=$

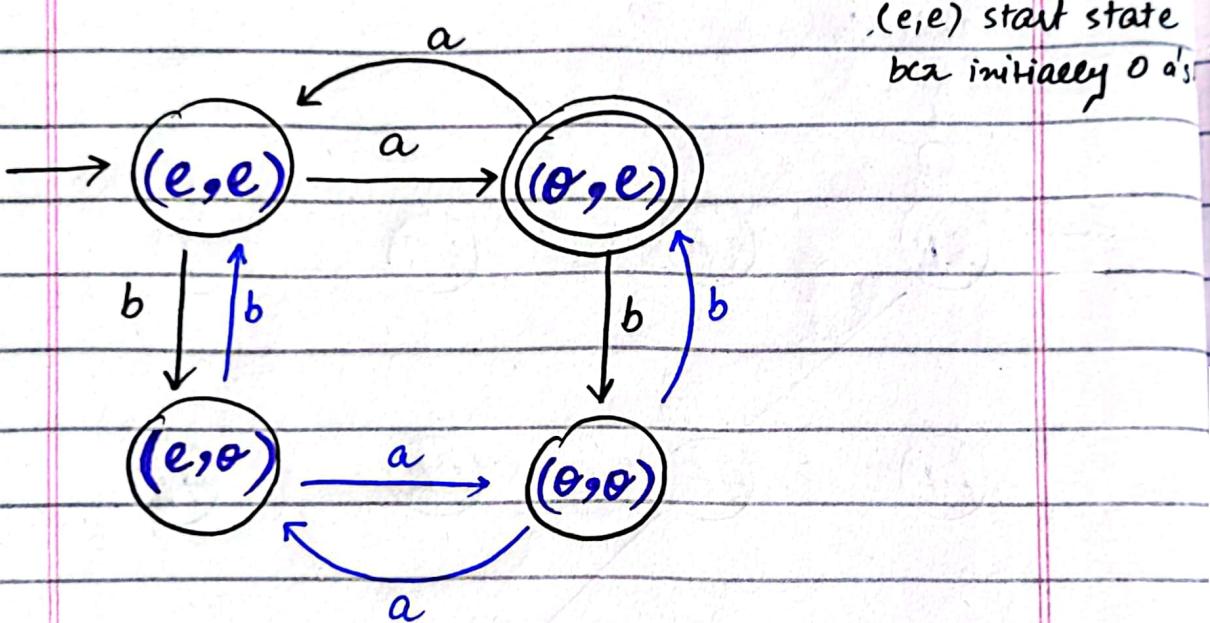


Q16-

$L = \{w \in \{a,b\}^* \mid w \text{ contain odd number of } a's \text{ and even number of } b's\}$

$L = \{a, abb, \overset{\text{aa}}{aaaabb}, abbbb, \dots\}$

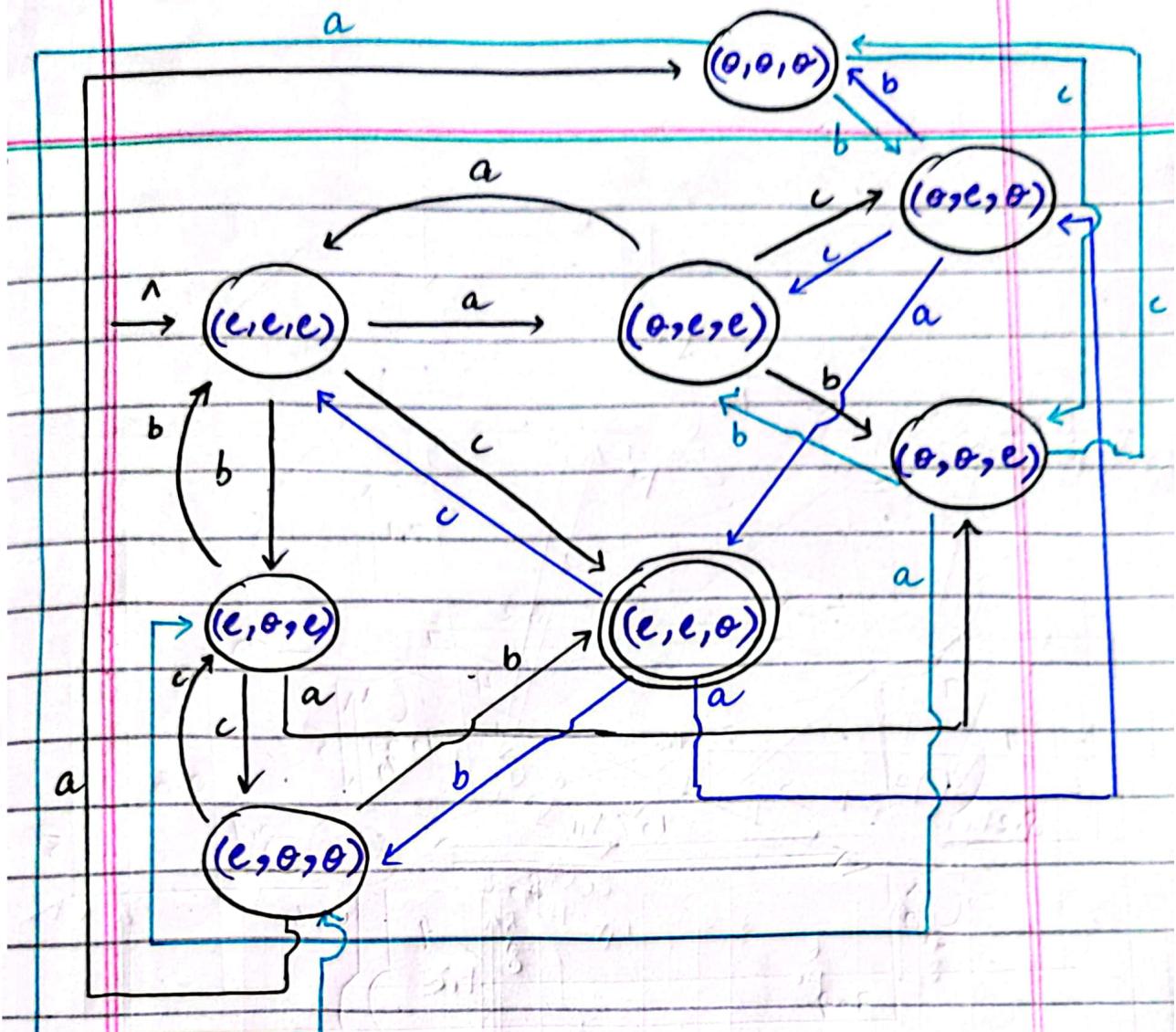
we can track parity of number of
a's and b's



Q17-

$L = \{w \in \{a,b,c\}^* \mid n_a(w) \text{ and } n_b(w) \text{ are even but } n_c(w) \text{ are odd}\}$

$L = \{c, ccc, cccc, aabbc, \dots\}$



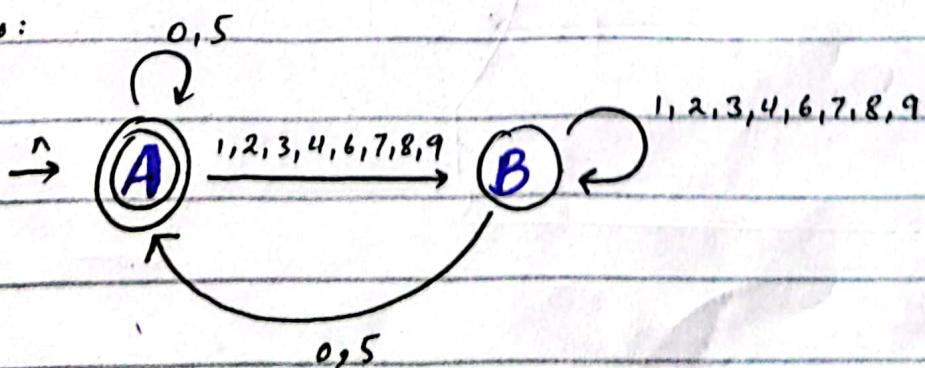
Q15

$L = \{w \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}^* \mid w \text{ is divisible by 5 and 6}\}$

h.o.: Divisible by 5

h.o. = {0, 5, 00, 05, 55, 15, 10, ...} ends at 0 or 5

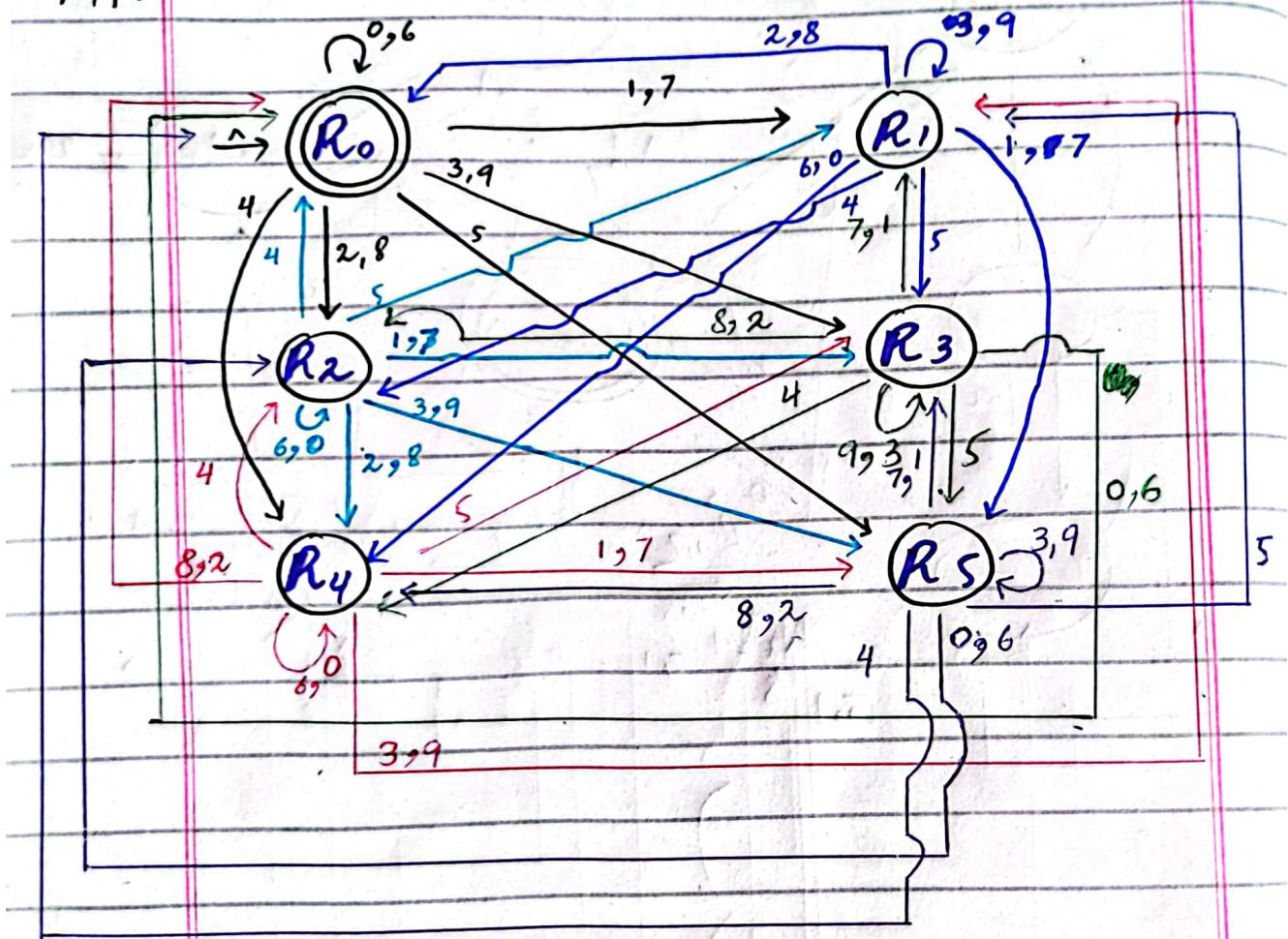
M.o.:



L_1 : Divisible by 6

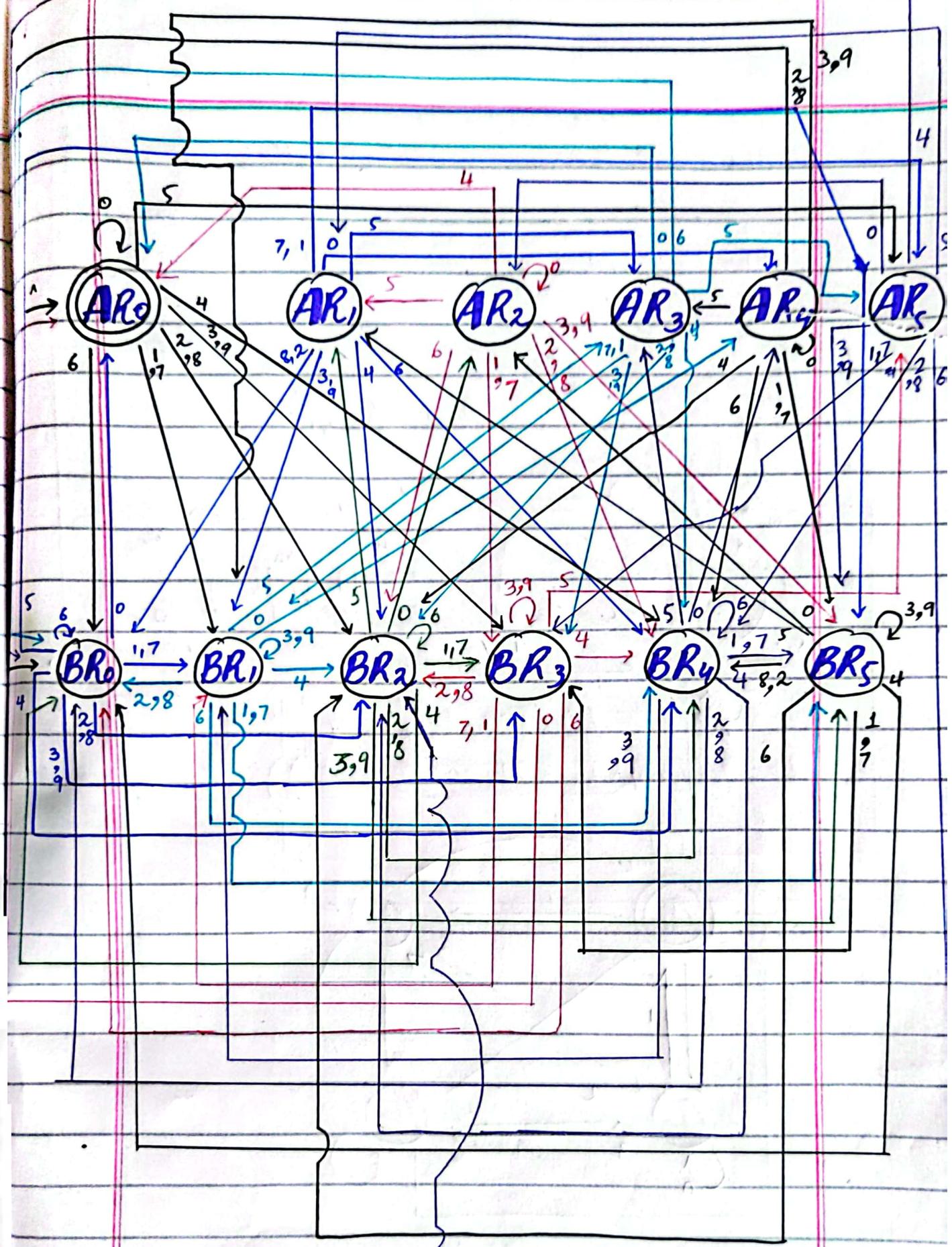
$$L_1 = \{0, 6, 100, 06, 66, 12, \dots\}$$

M_1 :



$M_0 \cap M_1$:

$$\{A, B\} \times \{R_0, R_1, R_2, R_3, R_4, R_5\}$$

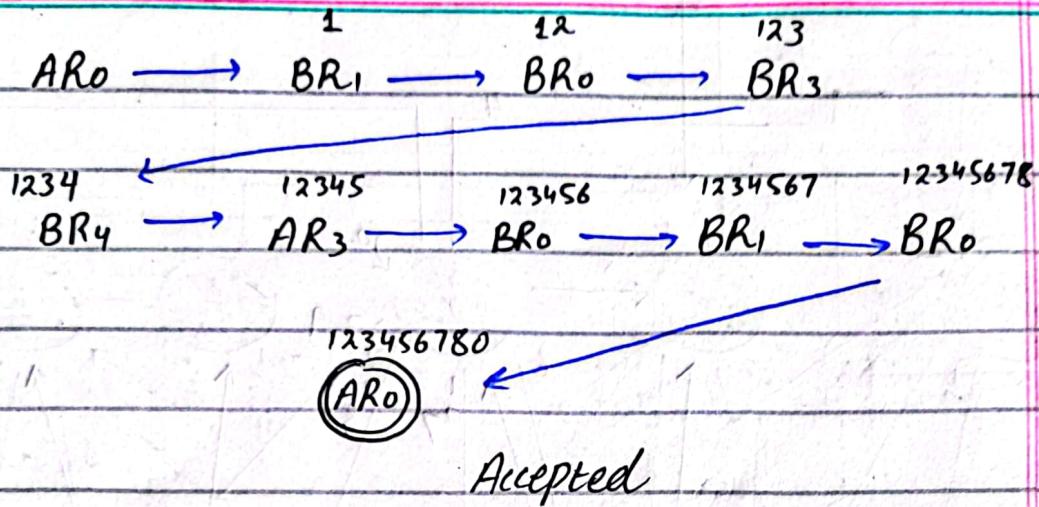


Test:

123456780 divisible by 5 and 6

$$15 = 24691356$$

$$16 = 20576180$$



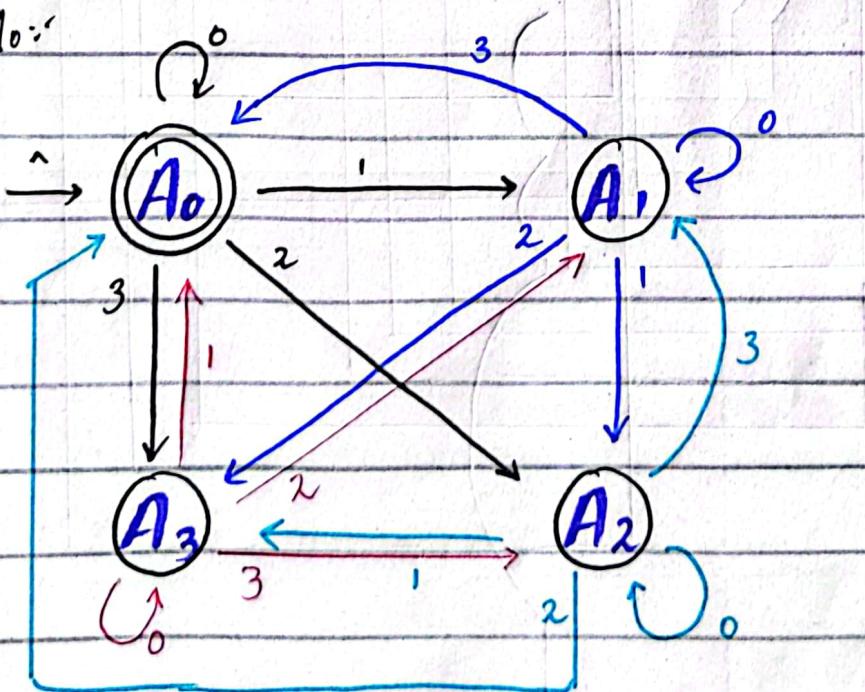
Q14-

$L = \{ w \in \{0, 1, 2, 3\}^* \mid \text{sum of digits in } w \text{ are either divisible by 4 or } 6 \}$

$L_0:$ divisible by 4

$L_0 = \{ 0, 00, 22, 13, 31, 000, 022, 202, 220, \dots \}$

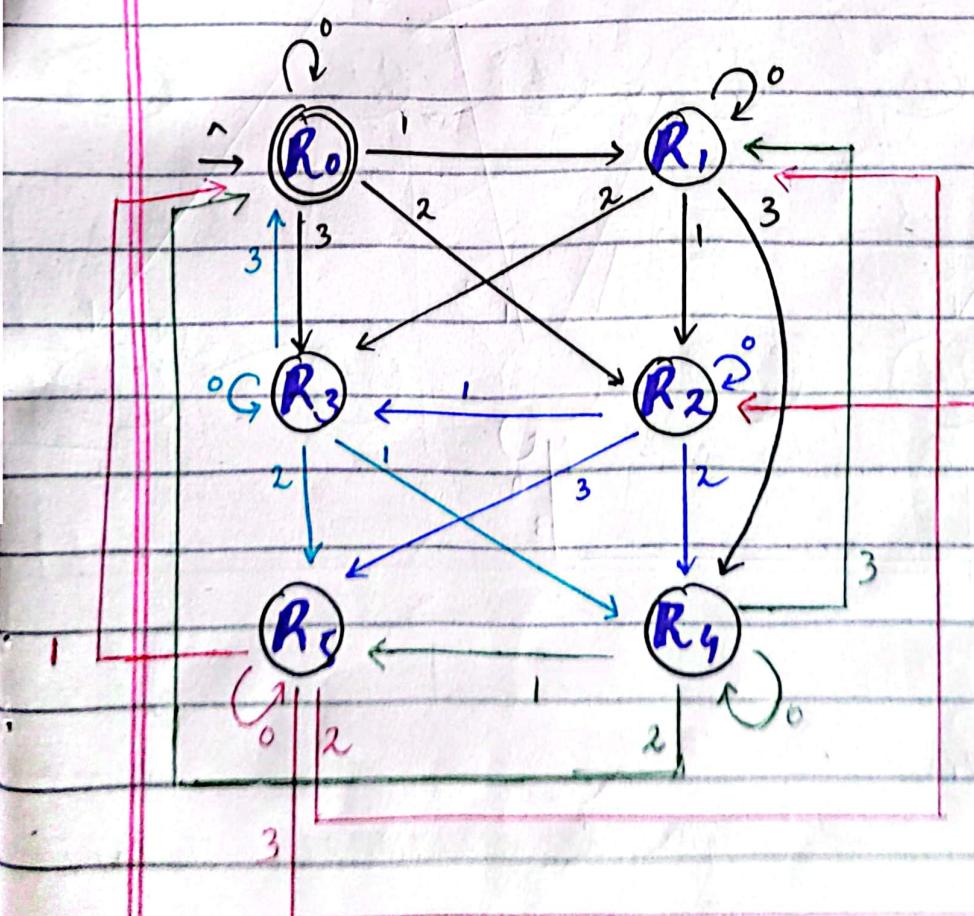
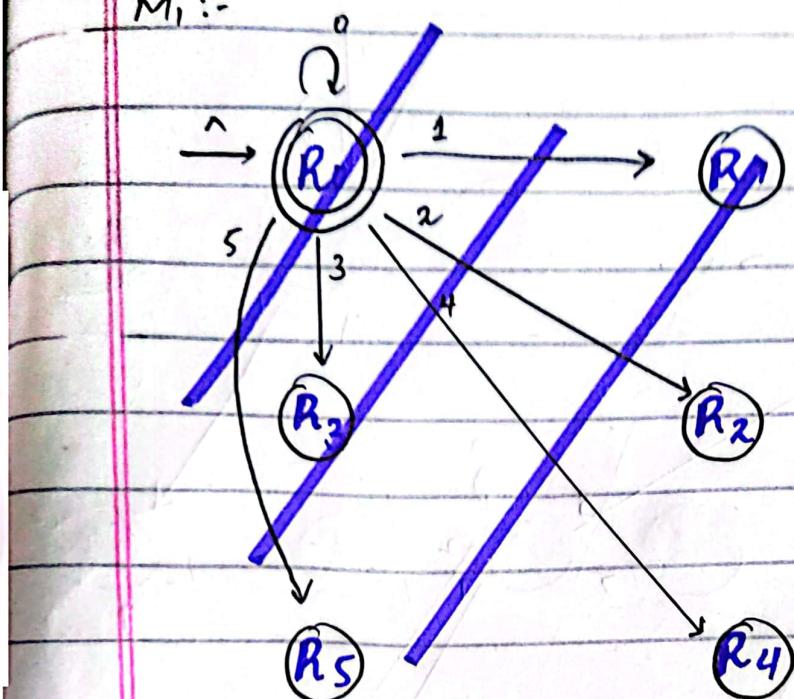
$M_0:$



L₁: Divisible by 6

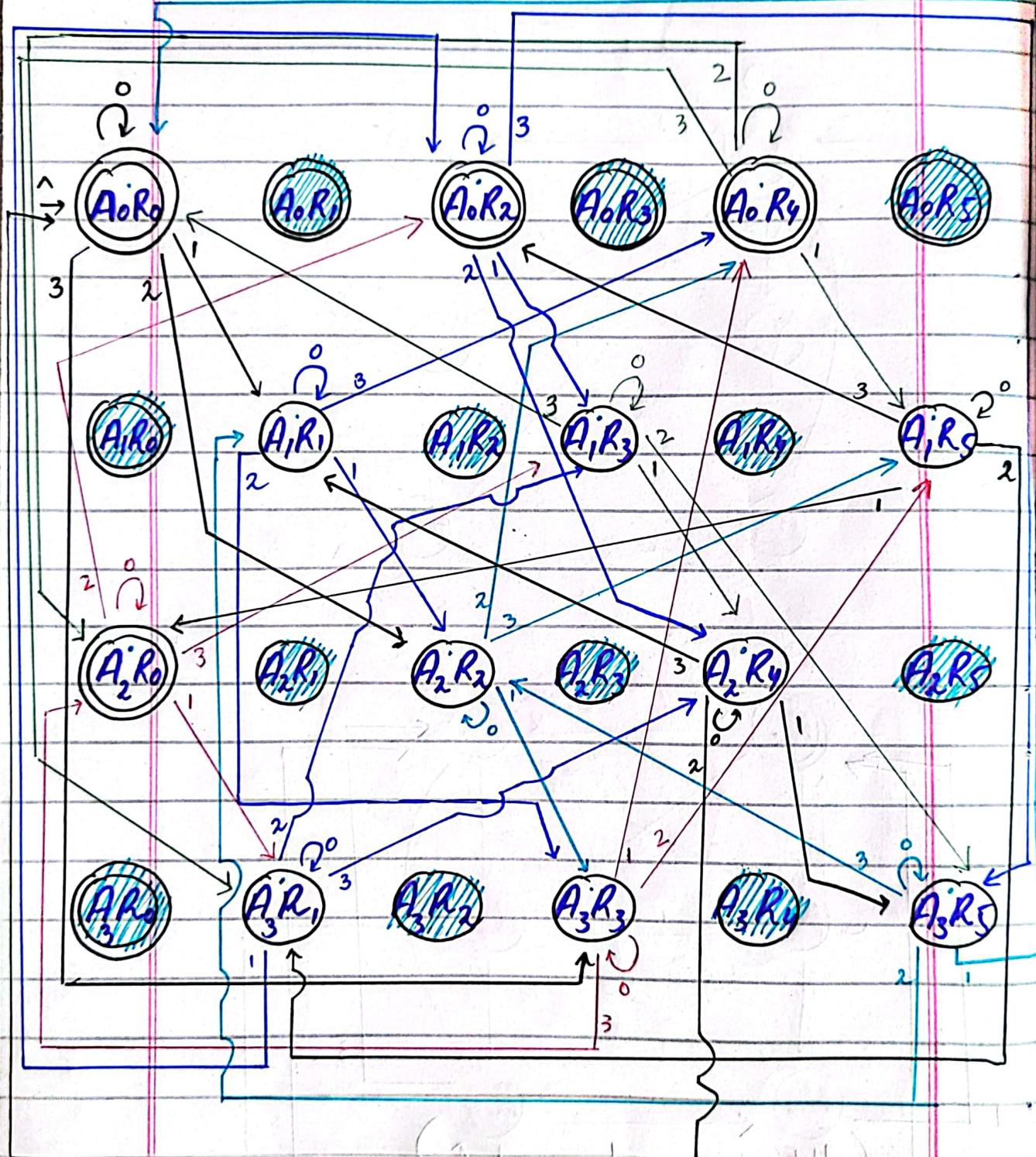
$$L_1 = \{ 0, 00, 33, \cancel{24}, \cancel{42}, \cancel{75}, 000, 222, 321, 123 \dots \}$$

M₁ :-



Mo U MI :-

$$= \{A_0, A_1, A_2, A_3\} \times \{R_0, R_1, R_2, R_3, R_4, R_5\}$$



12 states

Test :- sum of digits in 1122113122 is 16 divisible by 4

Accepted at $(A_0 R_4)$

and 11221131 accepted at

$\stackrel{\wedge}{(A_0 R_0)}$

Divisible by both 4 & 6