Theory of Automata

221-6946
BS CS 5E

Homework #1

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Problem#1:

Consider $\Sigma = \{a, b\}$

(a) $L1 = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \Sigma^3$ $|L| = ?$

$L1 = \{\Lambda, a, b, aa, ab, ba, bb,$
$aaa, aab, aba, abb, baa, bab, bba,$
$bbb\}$

$\boxed{|L1| = 15}$ Answer

(b) $L2 = \{w \text{ over } \Sigma \mid |W| > 5 \text{ and } |w| \leq 10 \text{ and } |w| \text{ is odd}\}$ $|L_2| = ?$

$L2 = \Sigma^7 \cup \Sigma^9$

$|W| > 5$ & $|w| \leq 10$
6, 7, 8, 9, 10
$|W|$ is odd
7, 9

there are two elements (a, b) or set of alphabets
so

$2^7 = 128$
$2^9 = 512$

$\boxed{|L2| = 640}$ Answer

(c) $L3 = \{ w \text{ over } \Sigma \mid |W| > 5 \ \& \ |W| \le 10$
$\& \ |W| \text{ is even} \}$

$|L_3| = ?$ $L3 = \Sigma^6 \cup \Sigma^8 \cup \Sigma^{10}$

$|W| > 5 \ \& \ |W| \le 10$

6, 7, 8, 9, 10

$|W|$ is even

6, 8, 10

Since, set of alphabets contains two elements so

$2^6 + 2^8 + 2^{10}$

$= 64 + 256 + 1024$

$\boxed{|L3| = 1344}$ Answer

(d) $L4 = \{ w \text{ over } \Sigma \mid |W| > 5 \ \& \ |W| \le 10 \}$

$|L4| = ?$ $L4 = \Sigma^6 \cup \Sigma^7 \cup \Sigma^8 \cup \Sigma^9 \cup \Sigma^{10}$

$|W| > 5$ and $|W| \le 10$

6, 7, 8, 9, 10

Since set of alphabets consist of two elements so,

$2^6 + 2^7 + 2^8 + 2^9 + 2^{10}$

$64 + 128 + 256 + 512 + 1024$

$\boxed{|L4| = 1984}$ Answer

(e) L

(e) $L_2 \cap L_3$ and $|L_2 \cap L_3| = ?$

$L_2 = \Sigma^7 \cup \Sigma^9$

$L_3 = \Sigma^6 \cup \Sigma^8 \cup \Sigma^{10}$

$\boxed{L_2 \cap L_3 = \{\} \text{ or } \phi}$

$\boxed{|L_2 \cap L_3| = 0}$ Answer

(f) $L_2 \cap L_4$ and $|L_2 \cap L_4| = ?$

$L_2 = \Sigma^7 \cup \Sigma^9$

$L_4 = \Sigma^6 \cup \Sigma^7 \cup \Sigma^8 \cup \Sigma^9 \cup \Sigma^{10}$

$L_2 \cap L_4 = \Sigma^7 \cup \Sigma^9$ $\quad 2^7 + 2^9 = 640$

$\boxed{|L_2 \cap L_4| = 640}$ Answer

(g) $L_3 \cap L_4$ and $|L_3 \cap L_4| = ?$

$L_3 = \Sigma^6 \cup \Sigma^8 \cup \Sigma^{10}$

$L_4 = \Sigma^6 \cup \Sigma^7 \cup \Sigma^8 \cup \Sigma^9 \cup \Sigma^{10}$

$L_3 \cap L_4 = \Sigma^6 \cup \Sigma^8 \cup \Sigma^{10}$ $\quad 2^6 + 2^8 + 2^{10} = 1344$

$\boxed{|L_3 \cap L_4| = 1344}$ Answer

(h) $L_2 - L_4$ and $|L_2 - L_4| = ?$

$L_2 = \cancel{\Sigma^7} \cup \cancel{\Sigma^9}$

$L_4 = \cancel{\Sigma^6} \cup \Sigma^7 \cup \Sigma^8 \cup \Sigma^9$

$\boxed{L_2 - L_4 = \{\}}$ or ϕ

$\boxed{|L_2 - L_4| = 0}$ Answer

(i) $L_3 - L_4$ & $|L_3 - L_4| = ?$

$L_3 = \cancel{\Sigma^6} \cup \cancel{\Sigma^8} \cup \cancel{\Sigma^{10}}$

$L_4 = \Sigma^6 \cup \Sigma^7 \cup \Sigma^8 \cup \Sigma^9 \cup \Sigma^{10}$

$\boxed{L_3 - L_4 = \{\}}$ or ϕ

$\boxed{|L_3 - L_4| = 0}$ Answer

Problem # 2:

$\Sigma = \{0, 1, 2\}$

(a) $L_1 = \{s \text{ over } \Sigma \mid |s| \leq 4$ & sum of digits in s are ≥ 5 and $\leq 10\}$

Σ^0 λ ×

Σ^1 0, 1, 2 ×

Σ^2 00 02 12 ×
01 20 21
10 22
11

Σ^3 000 (×) 002 (×)
001 020
010 022
⋮

Working:

	string	s	
	λ		
Σ^1	0	0	✓✓
	1	1	
	2	2	✓
Σ^2	00	0	✓✓
	01	1	
	10	1	
	11	2	✓
	02	2	✓
	20	2	✓
	22	4	✓
	12	3	✓
	21	3	✓
Σ^3	000	0	✓✓
	001	1	
	010	1	
	011	2	✓
	100	1	
	101	2	✓
	110	2	✓
	111	3	✓
	002	2	✓
	020	2	✓
	022	4	✓
	200	2	✓
	202	4	✓
	220	4	✓
	$\boxed{222}$	$\boxed{6}$	✓✓
	112	4	✓
	121	4	✓
	$\boxed{122}$	$\boxed{5}$	
	211	4	✓

string	s	
$\boxed{212}$	$\boxed{5}$	
$\boxed{221}$	$\boxed{5}$	
012	3	✓
201	3	✓
120	3	✓
021	3	✓
102	3	✓
210	3	✓

Total = 4

Σ^4 valid strings

2222	8
2012	5
2201	5
2120	5
2021	5
2102	5
2210	5

1212	6
1221	6
2112	6
2022	6
1112	6
~~1121~~	6
1211	6
2111	6
1122	6

2211	6
2121	6
1222	7
2122	7
2212	7
2221	7

$\Sigma^4 = 3^4 = 81$ combinations

0000	0	1011	3	2020	4
0001	1	1012	4	2100	3
0002	2	1020	3	2101	4
0010	1	1021	4	2102	5
0011	2	1022	5	2110	4
0012	3	1100	2	2111	5
0020	2	1101	3	2112	6
0021	3	1102	4	2120	5
0022	4	1110	3	2121	6
0100	1	1111	4	2122	7
0101	2	1112	5	2200	4
0102	3	1120	4	2201	5
0110	2	1121	5	2202	6
0111	3	1122	6	2210	5
0112	4	1200	3	2211	6
0120	3	1201	4	2212	7
0121	4	1202	5	2220	6
0122	5	1210	4	2221	7
0200	2	1211	5	2222	8
0201	3	1212	6		
0202	4	1220	5		
0210	3	1221	6		
0211	4	1222	7		
0212	5	2000	2		
0220	4	2001	3		
0221	5	2002	4		
0222	6	2010	3		
1000	1	2011	4		
1001	2	2012	5		
1002	3	2022	6		
1010	2	2021	5		

Total = 31

There are total 19 digits whose sum of digits in
S is 5

$S = 11$ when sum $= 6$
$S = 4$ when sum $= 7$
$S = 1$ when sum $= 8$

So, $\boxed{|L_1| = 35}$ Answer

(b) $L_2 = \{ s \text{ over } \Sigma \mid |s| \le 4$ and sum of digits is divisible by $2 \}$

0
2
11
02
20
22
000
011
101
110
002
020
022
200
202
220
222
112
121

00
211
0000
0002
⋮

check working on previous page marked with red pen

There are total 62 words in L_2

$\boxed{|L_2| = 62}$ Answer

(c) $L3 = \{ s$ over $\Sigma \mid |s| \leq 4$ and sum of digits in s is divisible by $3 \}$

$\boxed{|L3| = 40}$ Answer

There are total 40 words divisible by 3.

working shown with black pen

(d) $L4 = \{ s$ over $\Sigma \mid |s| \leq 4$ and sum of digits in s is div by 2 OR div by $3 \}$

So, there are total 87 words

$\boxed{|L4| = 87}$ Answer

(e) $L5 = \{ s$ over $\Sigma \mid |s| \leq 4$ and sum of digits in s is div by 2 AND by $3 \}$

$\boxed{|L5| = 15}$ Answer

(f) $L6 = \{ s$ over $\Sigma \mid |s| \leq 4$ and sum in s is div by 2 but NOT by $3 \}$

$\boxed{|L6| = 47}$ Answer

(g) $L7 = \{ s$ over $\Sigma \mid |s| \leq 4$ and sum of digits in s is div by 3 but NOT by 2 $\}$

$\boxed{|L7| = 25}$ Answer

(h) $L8 = \{ s$ over $\Sigma - \{2\} \mid |s| \leq 4$ and sum in s is div by 3 $\}$

$\boxed{|L8| = 9}$ Answer

represented by dot (•) in working.

(i) $L6 = \{ s$ over $\Sigma - \{2\} \mid |s| \leq 4$ and sum in s is div by 2 $\}$

$\boxed{|L6| = 15}$ Answer.

underlined