BLG 354E - Signals and Systems for Comp. Eng.

Final Project

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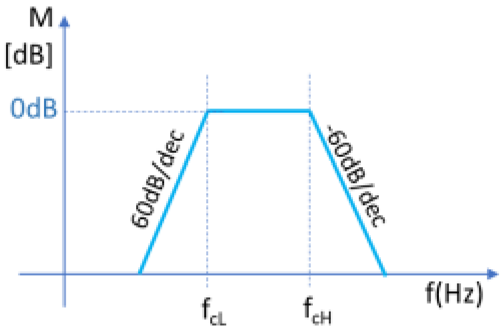
a)



According to our project booklet, since the slope in the given figure in above is 60 dB/Decade and -60 dB/Decade, this Band Pass Filter (BPF) is third-order. So firstly, we should use transformation function of third order BPF :

$$\text{Transfer function of } 3^{rd} \text{ order BPF} = \frac{s^3}{f_{cL}^3\left(1+\frac{s}{f_{cH}^{3}}\right)\left(1+\frac{s}{f_{cL}}\right)^3}$$

As you can see if we use transfer function we are in s domain, but in order to find H(z) we should change s domain to z domain. And in order to do that we should do bilinear transform. I found the bilinear transform with the help of Python since the resulting numbers will be large because the transfer function is a little bit complicated. After finding the function in z domain, I multiply both numerator and denominator with z^{-6} to find a function closer to the format specified for H(z) in our lecture slides. You can see the result on the next page :

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$$
H(z)=\frac{8T^3f_{ch}^3-24T^3f_{ch}^3z^{-2}+24T^3f_{ch}^3z^{-4}-8T^3f_{ch}^3z^{-6}}{\begin{aligned}
&z^{-6}\big(T^6f_{ch}^3f_{cl}^3-6T^5f_{ch}^3f_{cl}^2-6T^5f_{ch}^2f_{cl}^3+\\
&\quad+12T^4f_{ch}^3f_{cl}+36T^4f_{ch}^2f_{cl}^2+12T^4f_{ch}f_{cl}^3-\\
&\quad-8T^3f_{ch}^3-72T^3f_{ch}^2f_{cl}-72T^3f_{ch}f_{cl}^2-\\
&\quad-8T^3f_{cl}^3+48T^2f_{ch}^2+144T^2f_{ch}f_{cl}+\\
&\quad+48T^2f_{cl}^2-96Tf_{ch}-96Tf_{cl}+64\big)+\\
&+z^{-5}\big(6T^6f_{ch}^3f_{cl}^3-24T^5f_{ch}^3f_{cl}^2-24T^5f_{ch}^2f_{cl}^3+\\
&\quad+24T^4f_{ch}^3f_{cl}+72T^4f_{ch}^2f_{cl}^2+24T^4f_{ch}f_{cl}^3-\\
&\quad-96T^2f_{ch}^2-288T^2f_{ch}f_{cl}-96T^2f_{cl}^2+384Tf_{ch}+\\
&\quad+384Tf_{cl}-384\big)+\\
&+z^{-4}\big(15T^6f_{ch}^3f_{cl}^3-30T^5f_{ch}^3f_{cl}^2-30T^5f_{ch}^2f_{cl}^3-\\
&\quad-12T^4f_{ch}^3f_{cl}-36T^4f_{ch}^2f_{cl}^2-12T^4f_{ch}f_{cl}^3+\\
&\quad+24T^3f_{ch}^3+216T^3f_{ch}^2f_{cl}+216T^3f_{ch}f_{cl}^2+\\
&\quad+24T^3f_{cl}^3-48T^2f_{ch}^2-144T^2f_{ch}f_{cl}-\\
&\quad-48T^2f_{cl}^2-480Tf_{ch}-480Tf_{cl}+960\big)+\\
&+z^{-3}\big(20T^6f_{ch}^3f_{cl}^3-48T^4f_{ch}^3f_{cl}-144T^4f_{ch}^2f_{cl}^2-\\
&\quad-48T^4f_{ch}f_{cl}^3+192T^2f_{ch}^2+576T^2f_{ch}f_{cl}+\\
&\quad+192T^2f_{cl}^2-1280\big)+\\
&+z^{-2}\big(15T^6f_{ch}^3f_{cl}^3+30T^5f_{ch}^3f_{cl}^2+30T^5f_{ch}^2f_{cl}^3-\\
&\quad-12T^4f_{ch}^3f_{cl}-36T^4f_{ch}^2f_{cl}^2-12T^4f_{ch}f_{cl}^3-\\
&\quad-24T^3f_{ch}^3-216T^3f_{ch}^2f_{cl}-216T^3f_{ch}f_{cl}^2-\\
&\quad-24T^3f_{cl}^3-48T^2f_{ch}^2-144T^2f_{ch}f_{cl}-48T^2f_{cl}^2+\\
&\quad+480Tf_{ch}+480Tf_{cl}+960\big)+\\
&+z^{-1}\big(6T^6f_{ch}^3f_{cl}^3+24T^5f_{ch}^3f_{cl}^2+24T^5f_{ch}^2f_{cl}^3+\\
&\quad+24T^4f_{ch}^3f_{cl}+72T^4f_{ch}^2f_{cl}^2+24T^4f_{ch}f_{cl}^3-\\
&\quad-96T^2f_{ch}^2-288T^2f_{ch}f_{cl}-96T^2f_{cl}^2-384Tf_{ch}-\\
&\quad-384Tf_{cl}-384\big)+\\
&+z^{0}\big(T^6f_{ch}^3f_{cl}^3+6T^5f_{ch}^3f_{cl}^2+6T^5f_{ch}^2f_{cl}^3+12T^4f_{ch}^3f_{cl}+\\
&\quad+36T^4f_{ch}^2f_{cl}^2+12T^4f_{ch}f_{cl}^3+8T^3f_{ch}^3+\\
&\quad+72T^3f_{ch}^2f_{cl}+72T^3f_{ch}f_{cl}^2+8T^3f_{cl}^3+\\
&\quad+48T^2f_{ch}^2+144T^2f_{ch}f_{cl}+48T^2f_{cl}^2+\\
&\quad+96Tf_{ch}+96Tf_{cl}+64\big)
\end{aligned}}
$$

And according to our lecture slide,

$$\mathrm{H(z)} = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + \dots\dots + b_m z^{-m}}{a_0 + a_1 z^{-1} + \dots\dots + a_n z^{-n}}$$

And from here if we take look to the function which I got in the previous page,

$b_0 = 8T^3 f_{cH}^3$

$b_1 = 0$

$b_2 = -24T^3 f_{cH}^3$

$b_3 = 0$

$b_4 = 24T^3 f_{cH}^3$

$b_5 = 0$

$b_6 = -8T^3 f_{cH}^3$

$a_0 =$

$$(T^6 f_{ch}^3 f_{cl}^3 + 6T^5 f_{ch}^3 f_{cl}^2 + 6T^5 f_{ch}^2 f_{cl}^3 + 12T^4 f_{ch}^3 f_{cl} + 36T^4 f_{ch}^2 f_{cl}^2 + 12T^4 f_{ch} f_{cl}^3 + 8T^3 f_{ch}^3 + 72T^3 f_{ch}^2 f_{cl} + 72T^3 f_{ch} f_{cl}^2 + 8T^3 f_{cl}^3 + 48T^2 f_{ch}^2 + 144T^2 f_{ch} f_{cl} + 48T^2 f_{cl}^2 + 96T f_{ch} + 96T f_{cl} + 64)$$

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$a_1 =$

$$(6T^6 f_{ch}^3 f_{cl}^3 + 24T^5 f_{ch}^3 f_{cl}^2 + 24T^5 f_{ch}^2 f_{cl}^3 + 24T^4 f_{ch}^3 f_{cl} + 72T^4 f_{ch}^2 f_{cl}^2 + 24T^4 f_{ch} f_{cl}^3 - 96T^2 f_{ch}^2 - 288T^2 f_{ch} f_{cl} - 96T^2 f_{cl}^2 - 384T f_{ch} - 384T f_{cl} - 384) +$$

$a_2 =$

$$(15T^6 f_{ch}^3 f_{cl}^3 + 30T^5 f_{ch}^3 f_{cl}^2 + 30T^5 f_{ch}^2 f_{cl}^3 - 12T^4 f_{ch}^3 f_{cl} - 36T^4 f_{ch}^2 f_{cl}^2 - 12T^4 f_{ch} f_{cl}^3 - 24T^3 f_{ch}^3 - 216T^3 f_{ch}^2 f_{cl} - 216T^3 f_{ch} f_{cl}^2 - 24T^3 f_{cl}^3 - 48T^2 f_{ch}^2 - 144T^2 f_{ch} f_{cl} - 48T^2 f_{cl}^2 + 480T f_{ch} + 480T f_{cl} + 960) +$$

$a_3 =$

$$\left(20T^6 f_{ch}^3 f_{cl}^3 - 48T^4 f_{ch}^3 f_{cl} - 144T^4 f_{ch}^2 f_{cl}^2 - 48T^4 f_{ch} f_{cl}^3 + 192T^2 f_{ch}^2 + 576T^2 f_{ch} f_{cl} + 192T^2 f_{cl}^2 - 1280\right) +$$

$a_4=$

$$\left(15T^6 f_{ch}^3 f_{cl}^3 - 30T^5 f_{ch}^3 f_{cl}^2 - 30T^5 f_{ch}^2 f_{cl}^3 - 12T^4 f_{ch}^3 f_{cl} - 36T^4 f_{ch}^2 f_{cl}^2 - 12T^4 f_{ch} f_{cl}^3 + 24T^3 f_{ch}^3 + 216T^3 f_{ch}^2 f_{cl} + 216T^3 f_{ch} f_{cl}^2 + 24T^3 f_{cl}^3 - 48T^2 f_{ch}^2 - 144T^2 f_{ch} f_{cl} - 48T^2 f_{cl}^2 - 480T f_{ch} - 480T f_{cl} + 960\right) +$$

$a_5=$

$$(6T^6 f_{ch}^3 f_{cl}^3 - 24T^5 f_{ch}^3 f_{cl}^2 - 24T^5 f_{ch}^2 f_{cl}^3 + 24T^4 f_{ch}^3 f_{cl} + 72T^4 f_{ch}^2 f_{cl}^2 + 24T^4 f_{ch} f_{cl}^3 - 96T^2 f_{ch}^2 - 288T^2 f_{ch} f_{cl} - 96T^2 f_{cl}^2 + 384T f_{ch} + 384T f_{cl} - 384) +$$

$a_6 =$

$$(T^6 f_{ch}^3 f_{cl}^3 - 6T^5 f_{ch}^3 f_{cl}^2 - 6T^5 f_{ch}^2 f_{cl}^3 + 12T^4 f_{ch}^3 f_{cl} + 36T^4 f_{ch}^2 f_{cl}^2 + 12T^4 f_{ch} f_{cl}^3 - 8T^3 f_{ch}^3 - 72T^3 f_{ch}^2 f_{cl} - 72T^3 f_{ch} f_{cl}^2 - 8T^3 f_{cl}^3 + 48T^2 f_{ch}^2 + 144T^2 f_{ch} f_{cl} + 48T^2 f_{cl}^2 - 96T f_{ch} - 96T f_{cl} + 64) +$$

And in order to get pseudo code correctly, I multiply both numerator and denominator with A(z) and I get the Y(z) and A(z) correctly according to lecture slide :

$$H(z) = \frac{b_0 + b_1 z^{-1} + \dots + b_m z^{-m}}{a_0 + a_1 z^{-1} + \dots + a_n z^{-n}} \cdot \frac{A(z)}{A(z)} = \frac{Y(z)}{X(z)}$$

$$Y(z) = \frac{1}{a_0}(b_0 + b_1 z^{-1} \dots + b_m z^{-m})\, A(z)$$

$$A(z) = \frac{1}{a_0} X(z) - \frac{1}{a_0}(a_1 z^{-1} + a_2 z^{-2} + \dots + a_n z^{-n})\, A(z)$$

So I put all these values to the Python code and get the pseudo code:

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```
import numpy as np
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T=1/44100  # since sampling rate = 44100 Hz, then T= 1/44100

#decleration of b_0-b_6 values as in the report
b_0=8*T**3*f_ch**3
b_1=0
b_2=-24*T**3*f_ch**3
b_3=0
b_4=24*T**3*f_ch**3
b_5=0
b_6=-8*T**3*f_ch**3

#decleration of a_0-a_6 values as in the report
a_0=T**6*f_ch**3*f_cl**3 + 6*T**5*f_ch**3*f_cl**2 + 6*T**5*f_ch**2*f_cl**3 + 12*T**4*f_ch**3*f_cl + 36*T**4*f_ch**2*f_cl**2 + 12
a_1=6*T**6*f_ch**3*f_cl**3 + 24*T**5*f_ch**3*f_cl**2 + 24*T**5*f_ch**2*f_cl**3 + 24*T**4*f_ch**3*f_cl + 72*T**4*f_ch**2*f_cl**2
a_2=15*T**6*f_ch**3*f_cl**3 + 30*T**5*f_ch**3*f_cl**2 + 30*T**5*f_ch**2*f_cl**3 - 12*T**4*f_ch**3*f_cl - 36*T**4*f_ch**2*f_cl**2
a_3= 20*T**6*f_ch**3*f_cl**3 - 48*T**4*f_ch**3*f_cl - 144*T**4*f_ch**2*f_cl**2 - 48*T**4*f_ch*f_cl**3 + 192*T**2*f_ch**2 + 576*T
a_4=15*T**6*f_ch**3*f_cl**3 - 30*T**5*f_ch**3*f_cl**2 - 30*T**5*f_ch**2*f_cl**3 - 12*T**4*f_ch**3*f_cl - 36*T**4*f_ch**2*f_cl**2
a_5=6*T**6*f_ch**3*f_cl**3 - 24*T**5*f_ch**3*f_cl**2 - 24*T**5*f_ch**2*f_cl**3 + 24*T**4*f_ch**3*f_cl + 72*T**4*f_ch**2*f_cl**2
a_6=T**6*f_ch**3*f_cl**3 - 6*T**5*f_ch**3*f_cl**2 - 6*T**5*f_ch**2*f_cl**3 + 12*T**4*f_ch**3*f_cl + 36*T**4*f_ch**2*f_cl**2 + 12

#decleration of X array, Y array and output arrays
Xarr= []
Yarr=[]

# pseudo code takes input as array since when we read .wav files they will become array
def pseudo(array):
    #initially B,C,D,E,F,G = 0
    B=0
    C=0
    D=0
    E=0
    F=0
    G=0
    for X in array: #filtering every data point in array
        Xarr.append(X)
        #formulas of A(z) and Y(z)
        A=((1/a_0)*X) - ((1/a_0)*(a_1*B + a_2*C + a_3*D + a_4*E + a_5*F + a_6*G))
        Y=(1/a_0)*(b_0*A + b_1*B + b_2*C + b_3*D + b_4*E + b_5*F + b_6*G)
        Yarr.append(Y) # X is input and Y is output

        G=F #G=A.(z**(-6)) = F.(z**(-1))
        F=E #F=A.(z**(-5)) = E.(z**(-1))
        E=D #E=A.(z**(-4)) = D.(z**(-1))
        D=C #D=A.(z**(-3)) = C.(z**(-1))
        C=B #C=A.(z**(-2)) = B.(z**(-1))
        B=A #B=A.(z**(-1))
    return Yarr # return output array
```
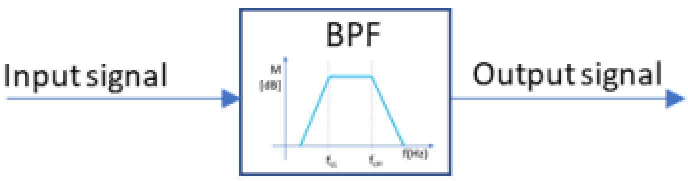

I added comment lines to make the code more understandable. You can find the pseudo code as a separate file among the submitted files.

b)



In option b), firstly I read the Africa.wav and WinnerTakesAll.wav files:
samplerate, data_Africa = scipy.io.wavfile.read('Africa.wav')
samplerate, data_Winner_T_All = scipy.io.wavfile.read('WinnerTakesAll.wav')

And then I get the outputs data_Africa and data_Winner_T_All as arrays. After that, like the figure in above I send these arrays separately to the pseudo code which I implemented in option a).
I added comment lines to make the code more understandable. You can find the pseudo code as a separate file among the submitted files.

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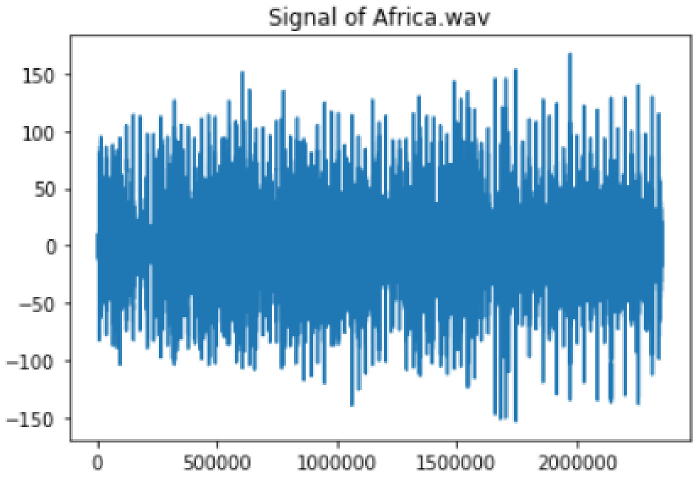
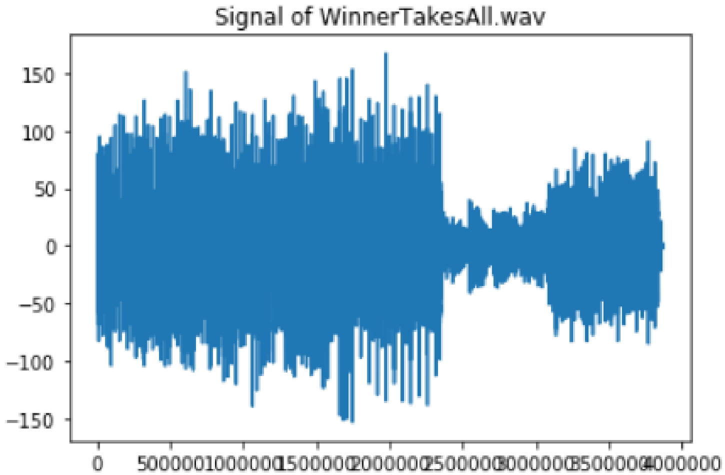
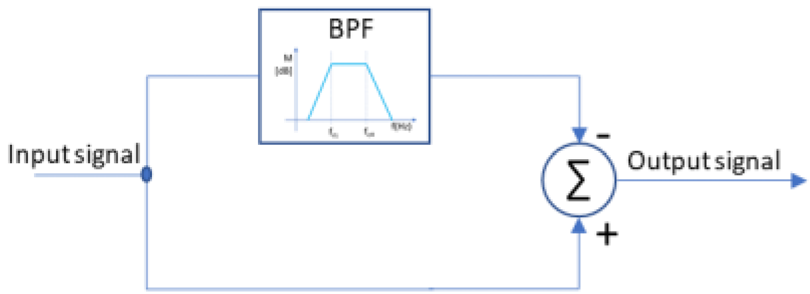
I plot two graphs for each signal and my outputs are:





c)



In option c, like the option b) firstly I read the Africa.wav and WinnerTakesAll.wav files:

samplerate, data_Africa = scipy.io.wavfile.read('Africa.wav')
samplerate, data_Winner_T_All = scipy.io.wavfile.read('WinnerTakesAll.wav')

And then I get the outputs data_Africa and data_Winner_T_All as arrays. And like the figure in above, in order to get output signal, I subtract filtered signal

from the original input signal and plot the graph of output.
I added comment lines to make the code more understandable. You can find the pseudo code as a separate file among the submitted files.
I plot two graphs for each signal and my outputs are: