Discrete Structures Spring 2024 – Week 4

Valid Arguments

Lecture 1

Preliminary

- A compound proposition that is always true, no matter what the truth values of the propositional variables that occur in it, is called a **tautology**.

- A compound proposition that is always false is called a **contradiction**.

- A compound proposition that is neither a tautology nor a contradiction is called a **contingency**

Tautology, Contradiction, Contingency – Examples

• Either it will rain tomorrow, or it will not rain tomorrow – a *tautology*
• It is raining now, and it is not raining now – a *contradiction* • If we go to store, we will buy some apples – a *contingency*

Tautology, Contradiction, Contingency – Using Truth Table

- Either it will rain tomorrow, or it will not rain tomorrow
- Let p denote "it will rain tomorrow"

- $p \vee \neg p$

p	**¬p**	**p v ¬p**
T	F	**T**
F	T	**T**

Tautology, Contradiction, Contingency – Using Truth Table

- It is raining now, and it is not raining now
- Let p denote "it is raining now"
- $p \wedge \neg p$

p	**¬p**	**p ∧ ¬p**
T	F	**F**
F	T	**F**

Tautology, Contradiction, Contingency – Using Truth Table

- If we go to store, we will buy some apples
- Let *p* denote “we go to store”, and *q* denote “we buy apples”
- *p* □ *q*

T	T

p	

T	
F	

F	F

Tautology, Contradiction, Contingency – Exercise

- Construct truth tables to test the following sentences for tautology, contradiction and contingency

1. $\neg (p \wedge \neg p)$
2. $p \vee (q \rightarrow p)$
3. $\neg p \;\square\; p$

Argument

- An argument in propositional logic is a sequence of propositions
- All but the final proposition in the argument are called **premises / hypotheses**
- The final proposition is called the **conclusion**

Argument – Example

- Consider the following argument (sequence of propositions)
- *"If you have a current password, then you can log onto the network."*
- *"You have a current password."*
- *Therefore, "You can log onto the network."*

- To determine if it is a valid argument?

Validity of an Argument

- A **valid argument** is an argument in which the conclusion must be true whenever the premises are true
- An argument is **valid** if and only if it is impossible for all the premises to be true and the conclusion to be false

Argument Form

- Using p to represent "You have a current password" and q to represent "You can log onto the network."
- Then, the argument has the form

- $p \rightarrow q$
- p
- $\therefore q$

- Symbol $\therefore$ denotes "therefore."

Validity of an Argument

- In this example, when both $p \rightarrow q$ and p are true, q must also be true
- This form of argument is **valid** because whenever all its premises are true, the conclusion must also be true

Validity of Argument

P	Q	P □ Q	(P □ Q) ^ P
T	T	T	T
T	F	F	F

F	T	T	F
F	F	T	F

Validity of an Argument

- An argument is **valid** if the truth of all its premises implies that the conclusion is true.
- For p and q being propositional variables, the statement $((p \to q) \wedge p) \to q$ is a **tautology**
- In the case of a valid argument we say the conclusion follows from the premises

Validity of Argument

P	Q	P → Q	(P → Q) ^ P	((P → Q) ^ P) → Q
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Invalid Argument – Example

- If it is raining, then the streets are wet. The streets are wet. Therefore, it is raining.”

Another Example

- Consider the following argument (sequence of propositions)

- *"If it is raining, then the streets are wet"*
- *"The streets are wet"*
- *Therefore, "It is raining"*

- Determine if it is a valid argument?

Argument Form

- Using p to represent "It is raining" and q to represent "Streets are wet” • Then, the argument has the form

- $p \rightarrow q$
- q
- $\therefore p$

Validity of an Argument

P	Q	P □ Q	(P □ Q) ^ Q
T	T	T	T

T	F	F	F
F	T	T	T
F	F	T	F

Validity of Argument – Not a Tautology

P	**Q**	**P □ Q**	**(P □ Q) ^ Q**	**((P □ Q) ^ Q) □ P**
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

Validity of an Argument – Important Note!

- Important to distinguish between the notions of the truth and the validity
- While individual statements / propositions may be either true or false, arguments cannot.
- Similarly, arguments may be described as valid or invalid, but statements / propositions cannot

Validity of an Argument – Important Note!

- It is important to understand that the premises of an argument do not have *actually to be true* in order for the argument to be valid
- It is possible for valid arguments to contain either true or false premises / hypotheses

- An argument is valid if the premises and conclusion are related to each other in the right way so that if the premises *were* true, then the conclusion would have to be true as well

Valid Argument with false premises

- All numbers are positive
- All positive numbers are larger than 5
- Therefore, all numbers are larger than 5

Valid Argument with False Premises

- Consider the following argument
- *“If I teach Discrete Mathematics, I am a Superman”*
- *“I teach Discrete Mathematics”*

- *Therefore, "I am a Superman"*

- A Valid Argument with a False Premise

Sound Argument

- An argument is *sound* if and only if it is both **valid**, and all of its premises are **actually true**.
- Otherwise, a deductive argument is *unsound*.

Validity of an Argument

- The key to showing that an argument in propositional logic is valid is to show that its **argument form** is valid

- Need to know the techniques to show that argument forms are valid Spring

Rules of Inference

Rules of Inference for Propositional Logic

- A truth table can be used to show that an argument form is valid by showing that whenever the premises are true, the conclusion must also be true.
- However, it's a tedious approach.
- If an argument form involves 10 different propositional variables, to use a truth table to show this argument form is valid requires $2^{10} = 1024$

different rows.

Rules of Inference

- **Rules of inference** – simple argument forms
- These rules of inference can be used as building blocks to construct more complicated valid argument forms

Rules of Inference

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$(p \land (p \to q)) \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$(\neg q \land (p \to q)) \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$((p \to q) \land (q \to r)) \to (p \to r)$	Hypothetical syllogism
$p \lor q$ $\neg p$ $\therefore q$	$((p \lor q) \land \neg p) \to q$	Disjunctive syllogism

Rules of Inference (contd.)

p $\therefore p \vee q$	$p \rightarrow (p \vee q)$	Addition
$p \wedge q$ $\therefore p$	$(p \wedge q) \rightarrow p$	Simplification
p q $\therefore p \wedge q$	$((p) \wedge (q)) \rightarrow (p \wedge q)$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$	Resolution

1. Modus Ponens

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore q \end{array}$$

Corresponding Tautology:

$(p \land (p \to q)) \to q$

Example:

Let p be "It is snowing."
Let q be "I will study discrete math."

"If it is snowing, then I will study discrete math."
"It is snowing."

"Therefore , I will study discrete math."

2. Modus Tollens

$$\begin{array}{l} p \to q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

Corresponding Tautology:

$(\neg q \wedge (p \to q)) \to \neg p$

Example:

Let p be "it is snowing."

Let q be "I will study discrete math."

"If it is snowing, then I will study discrete math."

"I will not study discrete math."

"Therefore , it is not snowing."

3. Hypothetical Syllogism

$$\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array}$$

Corresponding Tautology:

$((p \rightarrow q) \land (q \rightarrow r)) \rightarrow (p \rightarrow r)$

Example:

Let *p* be "it snows."

Let *q* be "I will study discrete math."

Let *r* be "I will get an A."

"If it snows, then I will study discrete math."

"If I study discrete math, I will get an A."

"Therefore , If it snows, I will get an A."

4. Disjunctive Syllogism

$$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

Corresponding Tautology:

$(\neg p \wedge (p \vee q)) \rightarrow q$

Example:

Let *p* be "I will study discrete math."

Let *q* be "I will study English literature."

"I will study discrete math or I will study English literature."

"I will not study discrete math."

"Therefore , I will study English literature."

5. Addition

$$\frac{p}{\therefore p \vee q}$$

Corresponding Tautology:

$p \rightarrow (p \vee q)$

Example:

Let p be "I will study discrete math."

Let q be "I will visit Las Vegas."

"I will study discrete math."

"Therefore, I will study discrete math or I will visit Las Vegas."

6. Simplification

$$\frac{p \land q}{\therefore q}$$

Corresponding Tautology:

$(p \land q) \to p$

Example:

Let p be "I will study discrete math."

Let q be "I will study English literature."

"I will study discrete math and English literature"

"Therefore, I will study discrete math."

7. Conjunction

$$\begin{array}{l} p \\ q \\ \hline \therefore p \land q \end{array}$$

Corresponding Tautology:

$((p) \land (q)) \to (p \land q)$

Example:

Let p be "I will study discrete math."

Let q be "I will study English literature."

"I will study discrete math."

"I will study English literature."

"Therefore, I will study discrete math and I will study English literature."

8. Resolution

$$\begin{array}{l} \neg p \lor r \\ p \lor q \\ \hline \therefore q \lor r \end{array}$$

Corresponding Tautology:

$((\neg p \lor r) \land (p \lor q)) \to (q \lor r)$

Example:

Let *p* be "I will study discrete math."

Let *r* be "I will study English literature."

Let q be "I will study databases."

"I will not study discrete math or I will study English literature."

"I will study discrete math or I will study databases."

"Therefore, I will study databases or I will study English literature."

Using the Rules of Inference to Build Valid Arguments

- A valid argument is a sequence of statements. Each statement is either a premise or follows from previous statements by rules of inference. • The last statement is called conclusion.
- A valid argument takes the following form:
- S1
- S2
- .
- .
- Sn
- ∴C

Valid Arguments – Example I

- From the single proposition $p \wedge (p \to q)$
- Show that q is a conclusion
- **Solution**:

Step	**Reason**
1. $p \wedge (p \to q)$	Premise
2. p	Simplification using (1)
3. $p \to q$	Simplification using (1)
4. q	Modus Ponens using (2) and (3)

Valid Arguments – Example II

- With the following hypotheses:
- *"It is not sunny this afternoon and it is colder than yesterday."*
- *"We will go swimming only if it is sunny."*
- *"If we do not go swimming, then we will take a canoe trip."*
- *"If we take a canoe trip, then we will be home by sunset."*

- Using the inference rules, construct a valid argument for the conclusion: • *"We will be home by sunset."*

Valid Arguments – Example II

- **Solution:**
- Choose propositional variables
- **p**: "It is sunny this afternoon." **r**: "We will go swimming." **t**: "We will be home by sunset." **q**: "It is colder than yesterday." **s**: "We will take a canoe trip."
- Translation into propositional logic:

Hypotheses: $\neg p \land q$, $r \to p$, $\neg r \to s$, $s \to t$
Conclusion: t

Valid Arguments – Example II

- Construct the Valid Argument

Step	**Reason**
1. $\neg p \wedge q$	Premise
2. $\neg p$	Simplification using (1)
3. $r \rightarrow p$	Premise
4. $\neg r$	Modus tollens using (2) and (3)
5. $\neg r \rightarrow s$	Premise
6. s	Modus ponens using (4) and (5)
7. $s \rightarrow t$	Premise
8. t	Modus ponens using (6) and (7)

Exercises on Rules of Inference

Lecture 3

What Rule of Inference is used in the Argument?

- Alice is a mathematics major.
- Therefore, Alice is either a mathematics major or a computer science major.

What Rule of Inference is used in the Argument?

- Jerry is a mathematics major and a computer science major.
- Therefore, Jerry is a mathematics major.

What Rule of Inference is used in the Argument?

- If it is rainy, then the pool will be closed.

- It is rainy.
- Therefore, the pool is closed.

What Rule of Inference is used in the Argument?

- If it snows today, the university will close.
- The university is not closed today.
- Therefore, it did not snow today.

What Rule of Inference is used in the

Argument?

- If I go swimming, then I will stay in the sun too long.
- If I stay in the sun too long, then I will sunburn.
- Therefore, if I go swimming, then I will sunburn.

Using Rules of Inferences

- Use rules of inference to show that the hypotheses:
- *"If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on,"*
- *"If the sailing race is held, then the trophy will be awarded," and*
- *"The trophy was not awarded"*
-
- Imply the conclusion *"It rained."*

Using Rules of Inferences

- Use rules of inference to show that the hypotheses
- *"John works hard,"*
- *"If John works hard, then he is a dull boy," and*
- *"If John is a dull boy, then he will not get the job"*

- Imply the conclusion *"John will not get the job."*

Using Rules of Inferences

- Show that the premises
- *"If you send me an e-mail message, then I will finish writing the program,"* •
"If you do not send me an e-mail message, then I will go to sleep early," and •
"If I go to sleep early, then I will wake up feeling refreshed"

- Lead to the conclusion *"If I do not finish writing the program, then I will wake up feeling refreshed."*

Using Rules of Inference

- From the given premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion

- *"I am either clever or lucky."*
- *"I am not lucky."*
- *"If I am lucky, then I will win the lottery."*

Using Rules of Inference

- From the given premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion

- *"If I eat spicy foods, then I have strange dreams."*
- *"I have strange dreams if there is thunder while I sleep."*
- *"I did not have strange dreams."*