CSE 211: Discrete Mathematics **(Due: 01/11/21)**

Homework #1

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Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- It is not a group homework. Do not share your answers to anyone in any circumstance. Any cheating means at least -100 for both sides.
- Do not take any information from Internet.
- No late homework will be accepted.
- For any questions about the homework, send an email to gizemsungu@gtu.edu.tr
- The homeworks (both latex and pdf files in a zip file) will be submitted into the course page of Teams.
- The latex, pdf and zip files of the homeworks should be saved as "Name_Surname_StudentId".{tex, pdf, zip}.
- If the answers of the homeworks have only calculations without any formula or any explanation -when needed- will get zero.
- Writing the homeworks on Latex is strongly suggested. However, hand-written paper is still accepted **IFF** hand writing of the student is clear and understandable to read, and the paper is well-organized. Otherwise, the assistant cannot grade the student's homework.

Problem 1: Conditional Statements

(6+6+6=18 points)

State the converse, contrapositive, and inverse of each of these conditional statements.

(a) If the education is hybrid, then I will go to the campus.
(Solution)

Converse: I will go to the campus, if the education is hybrid.
Contrapositive: I will not go to the campus, if the education is not hybrid.
Inverse: If the education is not hybrid, then I will not go to the campus.

(b) I sleep late whenever I drink a cup of coffee.
(Solution)

Converse: Whenever I drink a cup of coffee I sleep late.
Contrapositive: Whenever I do not drink a cup of coffee I do not sleep late.
Inverse: I do not sleep late whenever I do not drink a cup of coffee.

(c) If I don't attend the lectures, then I fail from the course.
(Solution)

Converse: I fail from the course, if I do not attend the lectures.
Contrapositive: I do not fail from the course, if I attend the lectures.
Inverse: If I attend the lectures, then I do not fail from the course.

Problem 2: Truth Tables For Logic Operators (5+5+5=15 points)

Construct a truth table for each of the following compound propositions.
(a) (p $\oplus$ $\neg$ q)
(Solution)

p	q	$\neg q$	$p \oplus \neg q$
T	T	F	T
T	F	T	F
F	T	F	F
F	F	T	T

(b) (p $\iff$ q) $\oplus$ ($\neg$ p $\iff$ $\neg$ r)
(Solution)

p	q	r	$\neg r$	$\neg p$	$p \iff q$	$\neg p \iff \neg r$	$(p \iff q) \oplus (\neg p \iff \neg r)$
T	T	T	F	F	T	T	F
T	T	F	T	F	T	F	T
T	F	T	F	F	F	T	T
T	F	F	T	F	F	F	F
F	T	T	F	T	F	F	F
F	T	F	T	T	F	T	T
F	F	T	F	T	T	F	T
F	F	F	T	T	T	T	F

(c) (p $\oplus$ q) $\Rightarrow$ (p $\oplus$ $\neg$ q)
(Solution)

p	q	$\neg q$	$p \oplus q$	$p \oplus \neg q$	$(p \oplus q) \Rightarrow (p \oplus \neg q)$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

Problem 3: Predicates and Quantifiers (21 points)

There are three predicate logic statements which represent English sentences as follows.

- P(x): "x can communicate with people in English."
- Q(x): "x knows two or more programming languages."
- H(x): "x gets a good salary."

Express each of the following sentences in terms of P(x), Q(x), H(x), quantifiers, and logical connectives or vice versa. The domain for quantifiers consists of all developers at the software company.

(a) There is a developer at the software company who can communicate with people in English and who knows two or more programming languages.
(Solution)

$$\exists x(P(x) \wedge Q(x))$$

(b) There is a developer at the software company who can communicate with people in English but who knows only one programming language.
(Solution)

$$\exists x(P(x) \wedge \neg Q(x))$$

(c) Every developer at the software company either can communicate with people in English or knows two or more programming languages.
(Solution)

$$\forall x(P(x) \vee Q(x))$$

(d) No developer at the software company can communicate with people in English or knows two or more programming languages.
(Solution)

$$\neg\forall x(P(x) \vee Q(x))$$

(e) If there is a student at the university who can communicate with people in English and know two or more programming languages, then she/he gets a good salary.
(Solution)

$$\exists x(P(x) \wedge Q(x)) \Rightarrow \forall x H(x)$$

(f) At least two developers get good salaries at the software company.
(Solution)

$$\exists x H(x) >= 2$$

(g) $\neg\forall x(Q(x) \wedge P(x))$
(Solution)
No developer at the software company knows two or more programming language and can communicate people in English.

Problem 4: Mathematical Induction (18 points)

Prove that $2 + 2 \cdot 7 + 2 \cdot 7^2 + \ldots + 2 \cdot 7^n = \frac{7^{n+1}-1}{3}$ whenever n is a nonnegative integer.
(Solution)
BasisStep Let n=1 be, then

$$\frac{7^2-1}{3} = 16, 2 + 2 \cdot 7^1 = 16$$

two equations are equal each other so we show that the basis step is correct.
InductiveStep Let n=k, then we accept that

$$(2 + 2 \cdot 7^1 + ... + 2 \cdot 7^k) = a = (\frac{7^{k+1}-1}{3}) = a$$

.
Let n=k+1, then we have to prove that

$$(2 + 2 \cdot 7 + ... + 2 \cdot 7^k) = a + 2 \cdot 7^{k+1} = \frac{7^{k+2}-1}{3}$$

$$\frac{7^{k+1}-1}{3} + 2 \cdot 7^{k+1} = \frac{7^{k+2}-1}{3} ====> \frac{7^{k+2}-1}{3} - \frac{7^{k+1}-1}{3} = 2 \cdot 7^{k+1}$$

Note that :

$$7^{k+2} = 7 \cdot 7^{k+1} and t = 7^{k+1}$$

$$\frac{7t-1}{3} - \frac{t-1}{3} = 2t ===> \frac{7t-t-1+1}{3} = 2t ===> \frac{6t}{3} = 2t ==> 2t = 2t ==> 2 \cdot 7^{k+1} = 2 \cdot 7^{k+1}$$

Problem 5: Mathematical Induction (18 points)

Prove that n^2 - 1 is divisible by 8 whenever n is an odd positive integer.
(Solution)
BasisStep Let n=3, then 9-1 = 8, 8/8=1 . So we know that the basic step is correct.
InductiveStep Let n =2k+1, then we accept that

$$2k + 1^2 - 1 ==> divides 8$$

Let n=2k+3, then we have to prove that

$$2k + 3^2 - 1 ==> divides 8$$

$$2k + 1^2 - 1 = 4k^2 + 4k + 1 = 4k^2 + 4k = 4 \cdot (k^2 + k) = 4k(k + 1) ==> divides 8$$

$$2k + 3^2 - 1 = 4k^2 + 12k + 9 - 1 = 4k^2 + 12k + 8 = 4 \cdot (k^2 + 3k + 2) = 4 \cdot (k + 2) \cdot (k + 1) ==> divides 8$$

Problem Problem 6: Logical Statements (10 points)

Let p and q be the statements as follows.

- **p:** It is sunny.
- **q:** The flowers are blooming.

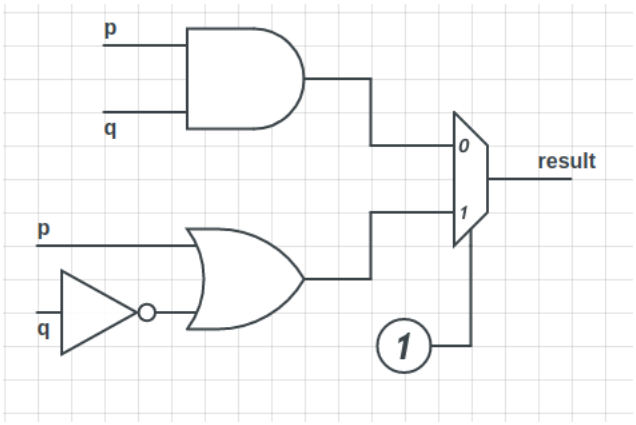

Figure 1: Combinational Circuit

In Figure 1, the two statements are used as input. The circuit has 3 gates as AND, OR and NOT operators. It has also a 2x1 multiplexer[1] which provides to select one of the two options.
(a) Write the sentence that "result" output has.
(Solution)
p $\vee$ $\neg$ q (It is sunny or the flowers are not blooming.)
(b) Convert Figure 1 to an algorithm which you can write in any programming language that you prefer (including pseudocode).
(Solution)
I wrote the code as c++ code

```
#include <iostream>

using namespace std;

bool m(bool multiplexer)
{
    bool p, q;
    if(multiplexer == 0)
        return p && q;
    else if(multiplexer == 1)
        return p || !q;
}

int main(){
    bool multiplexer = true; //because comes 1.
    bool result = m(multiplexer);
    cout << "The result is " << result << endl;
}
```

[1] https://www.geeksforgeeks.org/multiplexers-in-digital-logic/