IE 306 / HOMEWORK 3 / FATMANUR YAMAN / 2019402204 / 05.12.2022

Question 1

① a) $f(x) = \begin{cases} e^{2x}, & -\infty < x \leq 0 \\ e^{-2x}, & 0 < x < \infty \end{cases}$ $\quad F(x) = \int f(x)\,dx$

$$F(x) = \begin{cases} \frac{1}{2}e^{2x} + c_1 & -\infty, x \leq 0 \\ -\frac{1}{2}e^{-2x} + c_2 & 0 < x < \infty \end{cases}$$

$F(-\infty) = c_1 = 0$

$F(0) = \frac{1}{2} = -\frac{1}{2} + c_2$

$c_2 = 1$

$$= \begin{cases} \frac{1}{2}e^{2x} & -\infty < x \leq 0 \\ -\frac{1}{2}e^{-2x} + 1 & 0 < x < \infty \end{cases}$$

$R = F(x)$

$R = \frac{1}{2}e^{2x}$

$x = \frac{\ln 2R}{2}$

$0 < R \leq \frac{1}{2}$

$R = F(x)$

$R = -\frac{1}{2}e^{-2x} + 1$

$x = \frac{\ln(2-2R)}{-2}$

$\frac{1}{2} < R < 1$

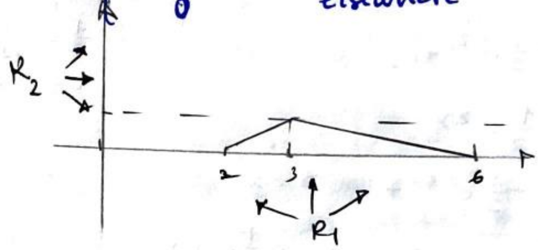
b) $0 < R_1 = 0.27 \leq \frac{1}{2} \rightarrow x = \frac{\ln(2\cdot 0.27)}{2} = -0.3081$

$\frac{1}{2} < R_2 = 0.81 < 1 \rightarrow x = \frac{\ln(2 - 2\cdot 0.81)}{-2} = 0.4838$

Question 2

2) a)

$$f(x) = \begin{cases} \frac{x}{2} - 1 & 2 \le x < 3 \\ 1 - \frac{x}{6} & 3 < x \le 6 \\ 0 & \text{elsewhere} \end{cases}$$



1 - $\max\{f(x)\} = \frac{1}{2}$ & $\frac{f(x)}{1/2} \le 1$

2 - $X = 2 + 4R$

3 - Obtain R_1 & R_2

4 - If $2 \le 2 + 4R_1 < 3$

If $R_2 \le \frac{f(2+4R_1)}{\max f(x)}$

$\le 4R_1$

Generate $X = 2 + 4R_1$

Else, reject R_1 & R_2, try again.

If $3 < 2 + 4R_1 \le 6$

If $R_2 \le \frac{f(2+4R_1)}{\max f(x)}$

$\le \frac{4}{3}(1 - R_1)$

Generate $X = 2 + 4R_1$

Else, reject R_1 & R_2, try again.

b)

(1) $R_1 = 0.02$ & $R_2 = 0.36$

$2 \le 2 + 4R_1 < 3$ ✓

$R_2 > 4R_1 \longrightarrow$ Reject R_1 & R_2 try again.

(2) $R_1 = 0.55$ & $R_2 = 0.15$

$3 < 2 + 4R_1 \le 6$ ✓

$R_2 \le \frac{4}{3}(1 - R_1)$ ✓ $\longrightarrow$ Generate $X = 2 + 4R_1$

$X = \underline{\underline{4.2}}$

(3) $R_1 = 0.88$ & $R_2 = 0.73$

$3 < 2 + 4R_1 \le 6$ ✓

$R_2 > \frac{4}{3}(1 - R_1) \longrightarrow$ Reject R_1 & R_2 try again

(4) $R_1 = 0.21$ & $R_2 = 0.1$

$2 \le 2 + 4R_1 < 3$ ✓

$R_2 \le 4R_1$ ✓ $\longrightarrow$ Generate $X = 2 + 4R_1$

$= 2.84$

Question 3

(3) binomial (n,p) → number of successes in a given number of trials.

geometric (p) → number of trials till the first success.

- Keep generating geometric RV.'s.
- Keep summing them until their sum is greater than n.
- Then, the # of such geometric RV's is equal to $X_{binomial}$.
- Number of geometric RV's indicates the successes in n trial → X binomial.

a)

Geometric Distribution CDF

$$F(x) = 1-(1-p)^{x+1}$$

$$F(x-1) = 1-(1-p)^{x}$$

$$1-(1-p)^{x} < R \leq 1-(1-p)^{x+1}$$

$$x = \left\lceil \frac{\ln R}{\ln(1-p)} - 1 \right\rceil \;//$$

$$\sum_{i=1}^{a} \left(\frac{\ln R_i}{\ln(1-p)} - 1 \right) \leq n$$

$$\sum_{i=1}^{a} \frac{\ln R_i}{\ln(1-p)} \leq n+a$$

$$\frac{1}{\ln(1-p)} \sum_{i=1}^{a} \ln R_i \leq n+a$$

$$\ln\left(\prod_{i=1}^{a} R_i \right) \leq (n+a)\ln(1-p)$$

$$\boxed{\prod_{i=1}^{a} R_i < (1-p)^{n+a}} \;//$$

b)

(1) Compute $(1-p)^{n+a}$ & set $a=0$, $Q=1$.

(2) Obtain a RN & $Q \leftarrow Q \cdot RN$

(3) Generate $X=RN$ if $(1-p)^{n+a} > Q$

Else $a \leftarrow a+1$ & compute $(1-p)^{n+a}$

Binomial $(8, 0.2)$

(1) $(1-0.2)^{8+0} = 0.168$ & $a=0$, $Q=1$

(2) $R_1 = 0.02$ & $Q = 1 \cdot 0.02 = 0.02$.

(3) $0.168 > 0.02$ → Generate $X=a=0$ //

(1) $(1-0.2)^{8+0} = 0.168$ & $a=0$, $Q=1$.

(2) $R_2 = 0.36$ & $Q = 1 \cdot 0{,}36 = 0{,}36$

(3) $0.168 < 0.36$ → Reject, $a \leftarrow a+1$, $(1-0.2)^{8+1} = 0.134$

(2) $R_3 = 0.55$ & $Q = 0.36 \cdot 0.55 = 0.198$

(3) $0.134 < 0.198$ → Reject, $a \leftarrow a+1$, $(1-0.2)^{8+2} = 0.107$

(2) $R_4 = 0.15$ & $Q = 0.198 \cdot 0.15 = 0.0297$

(3) $0.107 > 0.0297$ → Generate $X=a=2$ //

Question 4

a)

```
In [1]: import numpy as np
        import seaborn as sns

In [2]: normal_dist  = []
        for i in range(500):
            normal_dist.append(5*sum(np.random.uniform(0,1,12))+20) #sums the batches of uniform dist random variates in size 12

In [3]: sns.histplot(normal_dist, kde = True)

Out[3]: <AxesSubplot:ylabel='Count'>
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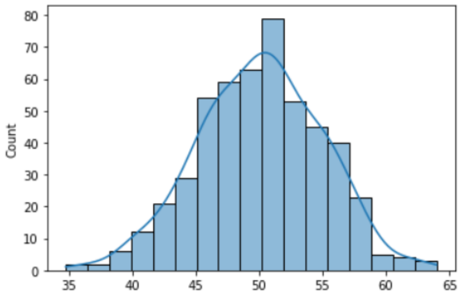

b) When the values are plotted, the shape of the frequency histogram looks like normal distribution. In addition to this, the Chi-Square Test proves that the distribution is normal. The d.o.f is 18 and the critical value is 25.989 with the significance level of 0.9. The calculated chi-square value is too high from the critical value. Thus, the null hypothesis cannot be rejected. The detailed calculations of the Chi-Square Test is provided in the Python code.