HISTOGRAM OF GRADIENT ORIENTATIONS OF EEG SIGNAL PLOTS FOR BRAIN COMPUTER INTERFACES

por

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EN CUMPLIMIENTO PARCIAL DE LOS
REQUISITOS PARA OPTAR AL GRADO DE
DOCTOR EN INGENIERÍA INFORMATICA

DEL

INSTITUTO TECNOLÓGICO DE BUENOS AIRES
BUENOS AIRES, ARGENTINA
30 DE NOVIEMBRE, 2018

INSTITUTO TECNOLÓGICO DE BUENOS AIRES

DEPARTAMENTO DE DOCTORADO

Los aquí suscriptos certifican que han asistido a la presentación oral de la Tesis **"Histogram of Gradient Orientations of EEG Signal Plots for Brain Computer Interfaces"** cuyo autor es **Rodrigo Ramele** completando parcialmente, los requerimientos exigidos para la obtención del Título de **Doctor en Ingeniería Informatica**.

Fecha: 30 de Noviembre, 2018

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Fecha: **30 de Noviembre, 2018**

Autor: **Rodrigo Ramele**

Título: **Histogram of Gradient Orientations of EEG Signal Plots for Brain Computer Interfaces**

Departamento: **Doctorado**

Título Académico: **Doctor en Ingeniería Informatica**   Convocatoria: **Mes   Año**

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Abstract

Brain Computer Interface (BCI) or Brain Machine Interfaces (BMI), has proved the feasibility of a distinct non-biological communication channel to transmit information from the Central Nervous System (CNS) to a computer device. Promising success has been achieved with invasive BCI, though biocompatibilities issues and the complexity and risks of surgical procedures are the main drive to enhance current non-invasive technologies.

Electroencephalography (EEG) is the most widespread method to gather information from the CNS in a non-invasive way. Clinical EEG has traditionally focused on temporal waveforms, but signal analysis methods which follow this path has been overshadow in BCI research.

This Thesis proposes a method and framework to analyze the waveform, the shape of the EEG signal, using the histogram of gradient orientations, a fruitful technique from Computer Vision which is used to characterize image local features. Inspiration comes from what traditionally electroencephalographers have been doing for almost a century: visually inspecting raw EEG signal plots.

This technique can be outlined in five steps, (1) signal preprocessing, (2) signal segmentation, (3) transformation on a channel by channel basis of each signal segment into a binary image of a signal plot, (4) assignment of keypoint location on a position over the newly created image depending on the physiological phenomena under study and finally (5) the calculation of the histogram of gradient orientations using finite differences from the image around this keypoint. This method generates a feature, a normalized 128-dimension descriptor. This feature is used to compare the signal segments that were used to generate them, hence to analyze the underlying cognitive phenomena.

The validity of the method is verified by studying three cognitive patterns. First, Visual Occipital Alpha Waves are analyzed. An experimental protocol is designed and a dataset is produced using a commercial-grade EEG device. Additionally, the ability of the method to capture oscillatory processes is verified by analyzing a public dataset. Moreover, this methodology is extended to study a related oscillatory process: Motor Imagery Rolandic Mu rhythms. The performance of the method to discriminate right vs left motor imagery

against a public dataset of healthy subjects, is verified. Results are reported.

Finally, the method is modified to capture transient events, particularly the P300 Event Related Potential (ERP). A description on how to extract the ERP from the EEG segment is offered, and a detailed depiction of how to implement a P300-Based BCI Speller application is outlined. Its performance is verified by processing a public dataset of Amiotrophic Lateral Sclerosis (ALS) patients and contrasted against an own dataset produced in-house replicating the same experimental conditions. Results are compared against other methods referenced in the bibliography

The benefits of the approach presented here are twofold, (1) it has a universal applicability because the same basic methodology can be applied to detect different patterns in EEG signals with applications to BCI and (2) it has the potential to foster close collaboration with physicians and electroencephalograph technicians because this direction of work follows the established procedure of the clinical EEG community of analyzing waveforms by their shapes.

Resumen

Las interfaces BCI (Brain Computer Interfaces, interfaces cerebro computadora) o BMI (Brain Machine Interfaces, interfaces cerebro máquina) han surgido como un nuevo canal de comunicación entre el cerebro y las computadoras, máquinas o robots, distinto de los canales biológicos estándar. Se han obtenido resultados prometedores en el empleo de la variante invasiva de BCI pero, además de los problemas de biocompatibilidad, los procedimientos quirúrgicos requeridos son complejos y riesgosos. Estas razones, han impulsado las mejoras de las tecnologías no invasivas.

La electroencefalografía (EEG) es el método más difundido para obtener información del sistema nervioso central de manera no invasiva. La electroencefalografía clínica se ha enfocado tradicionalmente en el estudio de las formas de ondas temporales, pero los métodos de procesamiento de señales que exploren esta metodología han sido ignorados en las investigaciones sobre BCI.

Esta tesis propone un método y un marco para analizar las formas de las señales de EEG utilizando los histogramas de gradientes orientados, una técnica de visión por computadora que es utilizada para identificar y clasificar características locales en regiones de una imagen. Este procedimiento está inspirado en lo que tradicionalmente los técnicos electroencefalógrafos han realizado por casi un siglo: inspeccionar visualmente los registros electroencefalográficos.

El método propuesto puede resumirse en 5 pasos, (1) preprocesamiento de la señal cruda, (2) segmentación de la señal, (3) obtención de una gráfica blanco y negro de la señal canal por canal, (4) asignación de una localización dentro de la imagen para posicionar un parche de un determinado tamaño y escala dependiendo del fenómeno cognitivo en estudio, y (5) cálculo del histograma de los gradientes orientados de la intensidades de los pixeles usando diferencias finitas. Este mecanismo genera un vector de 128 dimensiones, que se utiliza para comparar los segmentos de señales entre sí, y que permite entonces analizar el fenómeno cognitivo subyacente.

La validez del método se verifica estudiando tres patrones cognitivos. Primero se analizan las ondas alfa de la corteza visual occipital sobre dos conjuntos de registros: uno obtenido a partir de la aplicación de un protocolo experimental y mediante la utilización de un

dispositivo electroencefalográfico digital de uso comercial, y otro obtenido de una base de datos pública de registros electroencefalográficos. Segundo, se analiza otro tipo de onda oscilatoria conocida como ritmo Mu correspondiente a la corteza motora que puede ser también activada si el sujeto imagina una actividad motora. Se reporta la efectividad del método para discriminar entre la actividad de la corteza motora derecha e izquierda en base al estudio de otro conjunto de registros públicos de pacientes sanos. Los resultados son reportados y publicados.

Finalmente, el método propuesto se utiliza para estudiar eventos transitorios, particularmente, el potencial evocado P300. La eficiencia del sistema es verificada mediante el procesamiento de un conjunto de registros públicos de pacientes con esclerosis lateral amiotrófica, y corroborada contra un conjunto de registros de sujetos sanos obtenidos de manera experimental, replicando el mismo protocolo. Para ambos conjuntos de registros, se realiza una descripción detallada de cómo extraer este potencial de la señal de EEG, y se implementa un procesador de texto basado en P300 para comparar el desempeño del método propuesto respecto de otros citados en la bibliografía.

Los beneficios de esta propuesta se resumen en, (1) tiene una aplicación potencialmente universal, debido que el mismo tipo de metodología puede ser aplicada para detectar cualquier tipo de patrón obtenido en la señal de EEG con potenciales aplicaciones a BCI, y (2) ofrece la posibilidad de incentivar la colaboración y utilización de estas técnicas en la clínica médica especializada en electroencefalografía ya que esta perspectiva basada en el estudio de las formas de onda de las señales, es un procedimiento conocido y ya establecido por esa comunidad.

Lists of Publications

Conference Proceedings,

- Ramele, R. and Villar A.J. and Santos J.M., "A Brain Computer Interface Classification Method Based on SIFT Descriptors." VI Latin American Congress on Biomedical Engineering CLAIB 2014, Paraná, Argentina 29,30,31 October 2014. Springer International Publishing, 2015.

- Ramele, R. and Villar A.J. and Santos J.M., "A Brain Computer Interface Classification Method Based on Signal Plots." 4th Winter Conference on Brain Computer Interfaces, Yongpyong, Korea, February 2016. IEEE Signal and Processing, 2016.

Peer-reviewed Journals,

- Ramele, R. and Villar A.J. and Santos J.M., "Histogram of Gradient Orientations of Signal Plots applied to P300 Detection", Frontiers in Neuroscience, Special Issue:"Computational Methodologies in Brain Imaging and Connectivity: EEG and MEG Applications" (Under Review)

- Ramele, R. and Villar A.J. and Santos J.M., "EEG Waveform Analysis of P300 ERP with applications to Brain Computer Interfaces", MDPI Brain Sciences Journal, Special Issue:"Brain-Computer Interfaces for Human Augmentation" (Under Review)

Poster Presentations,

- Ramele, R. and Villar A.J. and Santos J.M., Poster Presentation: "Assistive Brain Computer Interfaces", HYPER Workshop 2011, La Alberca, Salamanca, Spain, Sept 18-23, 2011.

- Ramele, R. and Villar A.J. and Santos J.M., Poster Presentation: "Towards a Cognitive Monitoring BCI Application for Assistive Robotics", COGCOMP 2013, Stirling, Scotland, UK, Aug 25-30, 2013.

- Ramele, R. and Villar A.J. and Santos J.M., Poster Presentation: "Histogram of Gradient Orientations of Signal Plots applied to Brain Computer Interfaces", BCI Society Conference "BCIs: Not Getting Lost in Translation", Asilomar, CA, USA, May 21-25, 2018.

Acknowledgements

Es un falacia creer que existe alguna actividad en nuestra vida que la hacemos solos. Todos nosotros tenemos una interminable lista de personas a quienes le debemos agradecimiento, desde el primer segundo respirado hasta el último, y especialmente por aquellos actos ciclópeos que nos demandan todo lo que tenemos.

List of Acronyms

The following abbreviations are used in this thesis:

ALS: Anterior Lateral Sclerosis

EEG: electroencephalography

BCI: Brain Computer Interfaces

BMI: Brain Machine Interfaces

BNCI: Brain-Neural Computer Interfaces

SNR: Signal to Noise Ratio

CNS: Central Nervous System

DC: Direct Current

ERP: Event-Related Potential

P300: Positive deflection at 300 ms

ITR: Information Transfer Rate

BTR: Bit Transfer Rate

SIFT: Scale Invariant Feature Transform

SHCC: Slope Horizontal Chain Code

PE: Permutation Entropy

MP: Matching Pursuit

ICU: Intensive Care Unit

EKG: Electrocardiogram

PAA:Period Amplitude Analysis

SVM:Support Vector Machine

NBNN: Naive Bayes Near Neighbor

SMR: Sensorimotor Rhythms

ERD: Event Related Desynchronization

ERS: Event Related Synchronization

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List of Symbols

N	Length in sample points of the Segment
F_s	Sampling Frequency
w	Signal Segment Size
γ	Signal Amplitude Scale Factor
γ_t	Time Scale Factor
$\mathbf{kp}$	Keypoint
S_t	Horizontal Patch Scale
S_v	Vertical Patch Scale
$\mathbf{S}_x$	Patch Width
$\mathbf{S}_y$	Patch Height
$\mathbf{d}$	Descriptor
Δ_s	Pixels in unit scale
$\mathbf{kp_d}$	Keypoint Density
λ	Signal Span
$\Delta \mu V$	Peak-to-peak Amplitude
W_x	Image Width
H_y	Image Height
bpc	Best Performing Channel
k_a	Intensification Sequences Repetitions
C	Number of available channels

Notation

- X - a multichannel digital signal $X \in \mathbb{R}^{C \times N}$, with N being the length of the digitalized signal in sample points, and C is the number of available channels.

- $\mathbf{x}(n)$ vector column of EEG matrix is a vector for a sample point n in digital time for every available channel.

- $x(n, c)$ - a multichannel digital signal as a scalar time-series for a particular channel c.

- $x(n)$ - a multichannel digital signal as a scalar time-series emphasizing a single-channel processing scheme, for any c.

- $\lfloor \cdot \rfloor$ - Floor operation, rounding of the number argument to the closest smaller integer number.

- $\lceil \cdot \rceil$ - Ceil operation, rounding to the closest bigger integer number.

- $\lfloor \cdot \rceil$ - Rounding operations to the closest number.

- $f = \{f_i\}_1^n$ or $f = \{f_i\}_{i \in J}$ - Concatenation of scalar values to form a multidimensional feature vector $f = \{f_1, f_2, ..., f_n\}$.

Chapter 1

Introduction

The brain is a machine with the sole purpose to respond appropriately to external and internal events, and to spread its own presence into the environment where it belongs [1]. Hence, the brain needs to communicate and it posses mainly two natural ways to do it: hormonal or neuromuscular. When those natural channels are interrupted, they are not available or when it needs to increase or enhance the communication alternatives, a new artificial communication channel which is not based on natural pathways, is needed. It is based, instead, on a new technology feat that decode the information from the CNS and transmit it directly to a computer or machine.

Brain Computer Interface, BCI, is a system that measures brainwaves and converts them into artificial output that replaces, restores, enhances, supplements and improves natural brain output and changes the ongoing interactions between the Central Nervous System and its external or internal environment [136]. Brain Machine Interface (BMI) generally refers to invasive devices. Brain Neural Computer Interfaces (BNCI) may refer to devices that do not exclusively use information from the CNS, they also may use any kind of biological signal that can be harnessed with the purpose of volitionally transmit information. Above all, every kind of BCI system is after all a communication device.

There are five motives behind BCI: the **first** is the aging of societies: estimated for 2025, 800 millions people will be over 65 years old, and 2/3 of them on developing countries [72]. This may lead to an increased tendency to develop diseases that affect motor pathways and require some form of assistance from technology. The **second** reason is the digital world that calls for more methods of interactions. This digital society [35] demands more mechanisms to interpret the surrounding world and to translate human intentions through digital gadgets. Additionally, the advancement of smart wearable devices that can be used

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[1]The sensorimotor Hipothesis [138, 136] and The Extended Mind Thesis [25]

over the skin is also pushing the frontiers to go deeper into the body to find there useful information. The **third** motive is the impulse of neuroscience research and the advances that this discipline is having worldwide. The **fourth** reason is the potentialities of BCI as a clinical tool which can help to diagnose diseases, as aid in the field of neurorehabilitation, or to provide neurofeedback. The **fifth**, final and most important motive, the reason behind Brain Computer Interfaces, is the still unfulfilled societal promise of social inclusion of people with disabilities. It is known that the ability to walk and live independently is a key indicator of psychological and physical health, and we have to do all we can to provide the technological tools to achieve this goal [101, 26, 136, 57].

In line with the aforementioned motives, there are several applications currently under development for BCI. People affected by any kind of neurodegenerative diseases, particularly those affected by advanced stages of Amyotrophic Lateral Sclerosis (ALS) with locked-in syndrome may find in BCIs the only remaining alternative to communicate. Other applications targeted for the general population include alertness monitoring, telepresence, gaming, education, art, human augmentation [139], biometric identification, virtual reality avatar, assistive robotics and education. Novel niches where this new communication channel can be useful are found routinely [83]. In spite of all this hype [41], there is still a long way ahead. This area advanced rapidly but the complexity of brain signals in all their forms is still a big problem to tackle.

Electroencephalography (EEG) is the most widespread device to capture electrical brain information in a non-invasive and portable way, and it is the most used device in BCI research and applications. The clinical and historical tactic to analyze EEG signals were based on detecting visual patterns out of the EEG trace or polygraph[116]: multichannel signals were extracted and continuously plotted over a piece of paper. Electroencephalographers or Electroencephalography technician have decoded and detected patterns along the signals by visually inspecting them [110]. Nowadays clinical EEG still entails a visually interpreted test [116].

In contrast, automatic processing, or quantitative EEG, was based first on analog electronic devices and later on computerized digital processing methods [60]. They implemented mathematically and algorithmically complex procedures to decode the information with good results [139]. The best materialization of the automatic processing of EEG signals rests precisely in the BCI discipline, where around 71.2% is based on noninvasive EEG [47].

Hence, the traditional strategy of analyzing the electroencephalography by signal shapes

on plots, was mainly overshadow in BCI research, and the waveform of the EEG was replaced by procedures that were difficult to link to existing clinical EEG knowledge.

On the other hand, the study of biological visual sensory system provided insights and models that are very useful to understand brain functions. Additionally, they serve as inspiration to develop Computer Vision algorithms that aimed to reproduce a similar level of accuracy as those obtained by biological beings, including humans. The Histogram of Gradient Orientations is one successful method from Computer Vision useful to image recognition that aims to mimetically reproduce how the visual cortex discriminate shapes.

This thesis tries to unravel the following question: is it possible to analyze and discriminate electroencephalographic signals by automatic processing the shape of the waveforms using the Histogram of Gradient Orientations ?

To do that, this thesis unfolds as follows: Chapter 2 gives details of what is a Brain Computer Interface and the particularities of the first window of the electric mind: the EEG. It also covers the state of the art in the methods that explore the waveform automatically. Chapter 3 provides an overview on the procedure to construct a plot representing the signal. Chapter 4 is the core of this thesis and describes the Histogram of Gradient Orientations and how it can be used to process one-dimensional signals. Next, results and experimental procedures are described to analyze EEG signals and implement BCI paradigms: Alpha Waves are covered in Chapter 5 and Motor Imagery in Chapter 6. The P300 Wave is studied in Chapter 7. Future Work and Conclusions are addressed in Chapter 8.

1.1   Significance

This thesis propose

- A procedure to construct analyzable 2D-images based on one-dimensional signals.

- A mapping procedure to link time-series characteristics based on feature of the 2D-image representation.

- A feature extraction method for EEG signals that can be used objectively to construct a representation of the waveform.

- A classification algorithm that can be used effectively with these features.

1.2 Summary

- What is this all about?: a method to analyze EEG signals based on extracting local features from their 2D image plot representation.

- What will be found in this thesis?: a point of view that emphasizes the importance of providing mechanisms that help to understand signals based on how they look like on plots.

- Does it work?: It works when the waveform contains the discriminating information. If a person is able to discriminate the signals, this method would also do that.

- Can it be used?: Yes, it can. The developed software is open-source and it can be used out-of-the-box. It is particular useful when an intelligible automatic classification procedure is required.

Chapter 2

Interface between the Computer and the Brain

> ...the brain is not a passive decoder of information but a dynamic and distributed modeler of a reality...
>
> ————————————————————
>
> Nicolelis

With Vidal's work in 1970s, Brain-Computer Interfaces started as a technological amusement, and it steadily moved toward a mature and highly researched area of work. Outstanding success has been achieved with invasive BCI, i.e. with surgically implanted electrodes. Success stories has been made public like Braingate's implant on Jan Scheuermann, Cathy Hutchinson and Dennis Degray [92]. Other works include the total reproduction of arm movement [54], the restoration of reaching and grasping movements through a brain-controlled muscle stimulation device on a person with tetraplegia [2] and the remote control of a manipulator by a macaque using brainwave information [135] albeit of persistent biocompatibilities issues and the pervasive complexity and risks of surgical procedures. One noteworthy aspect of this novel communication channel is the ability to transmit information from the central nervous system to a computer device and from there use that information to control a wheelchair [19], as input to a speller application [48], in a Virtual Reality environment [75] or as aiding tool in a rehabilitation procedure [65]. Other novel applications include the real-time control of flight simulators [87] and the implementation of neuroadaptive interfaces where the computer detects the correctedness of a given command based on brainwave analysis [141]. Overall, the holly grail of BCI is to implement a new complete and alternative pathway to restore lost locomotion [136].
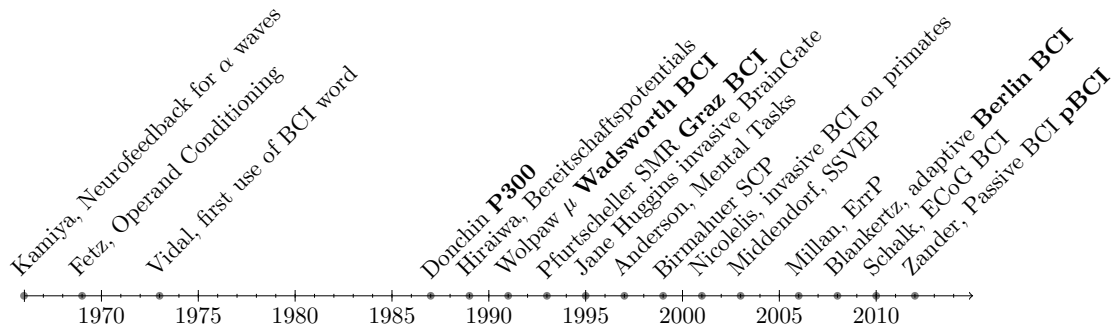
This graphic shows a brief chronology of the main events in BCI history, starting from the early works on Neurofeedback in the 70s and walking through the different paradigms. In recent years, this discipline has gained mainstream public awareness with worldwide challenge competitions like Cybathlon [104, 88] and even been broadcasted during the inauguration ceremony of the 2014 Soccer World Cup [89]. New developments are approaching the out-of-the-lab high-bar and they are starting to be used in real world environments [47, 56]. Moreover, BCI research had rampantly been advanced accomplishing a BCI Society, a BCI Journal, BCI Award, annual conference meetings, practical applications, myriads of startups companies and even included in the Gartner Hype Cycle [41].

From its root as assistive technology it has now expanded to include several application niches like temporal induced disability, neuroergonomy, early detection of human error, affective computing, biometric authentication, teleprescence (improvement of haptic interface), cyberinfrastructure and assistive robotics. Intensive Care Units (ICU) and Disorders of Consciousness (DoC) [6] (detection of remaining brain activity in comatose patients) are recent disciplines where BCI is showing tremendous prospects and possible applications.

Their adoption as a clinical tool is still years ahead. Stroke rehabilitation is the only area where clinical trials for BCI are being conducted. It is understood that the neurofeedback provided by a BCI interface improves the prognosis of motor rehabilitation [5].

BCI Definition (circa 2018)

Definition 2.0.1. *A system that measures central nervous system activity and converts it into artificial output that replaces, restores, enhances, supplements, or improves natural CNS output and thereby changes the ongoing interactions between the CNS and its external and internal environment [136].*

Hybrid BCI, or Brain Neural Computer Interface, are BCI devices that use not only signals from the CNS, they utilize any kind of available biosignal that can be volitionally

modulated to transmit information (this is called dependant BCI). When the pace of the BCI is regulated by external stimulus it is called synchronous and when the user choose their own pace to transmit information, it is often called asynchronous or self-paced BCI.

Recent years have seen an incredible advance of passive BCI, pBCI [140]. The original definition of BCI did not include passive modalities but per definition 2.0.1 it is now part of this discipline. The important aspect is that passive technologies do not entail necessary the volitional requirement to transmit information. EEG-based passive BCI is a promising and advancing area of research and of commercial applications.

Despite all this, its primary objective, its core motive of moving into real applications for disabled people has yet to come [17, 64, 3]. They still lack the necessary robustness, and its performance is well behind any other method of human computer interaction, including any kind of detection of residual muscular movement [26]. Among the many and current challenges of BCI [17] one which is still perennial is precisely their inability to be used and applied outside the BNCI community and specifically in clinical context.

Quoting experts in the field,

"We yet have an impractical and inaccessible exotica for very specific user groups" *(Allison 2010)*

"Effectiveness of non-invasive BCI systems remain limited..." (Wolpaw 2011)

"...to ponder if BCIs are really promising and helpful, or if they are simple a passing rod, reinforced by their sci-fi side..." (Lotte 2016)

The feasibility of the system has been proved but there are several challenges in BCI that need to be tackled. They can be summarizing as increasing the ITR, the pervasive low signal-to-noise ratio of brainwaves, particularly of noninvasive signals [73], the reliability of the system, its portability, and the usability of the system [133], and at the same time decreasing the setup, the training and,calibration time and the subject's inter/intra variability. The search for practical, relevant, and invariant *features* that convey good-enough information about the underlying cognitive process is still a goal to be achieved [94]. Ethical aspects of BCI [139] must also be considered and handled: cybersecurity threats and privacy concerns, agency and identity issues that might be occurring by deleterious plasticity with BCI users and the strict peg to the *Primun non nocere* [1] mandate.

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[1] *First, do not harm*, in reference to the Hippocratic Oath
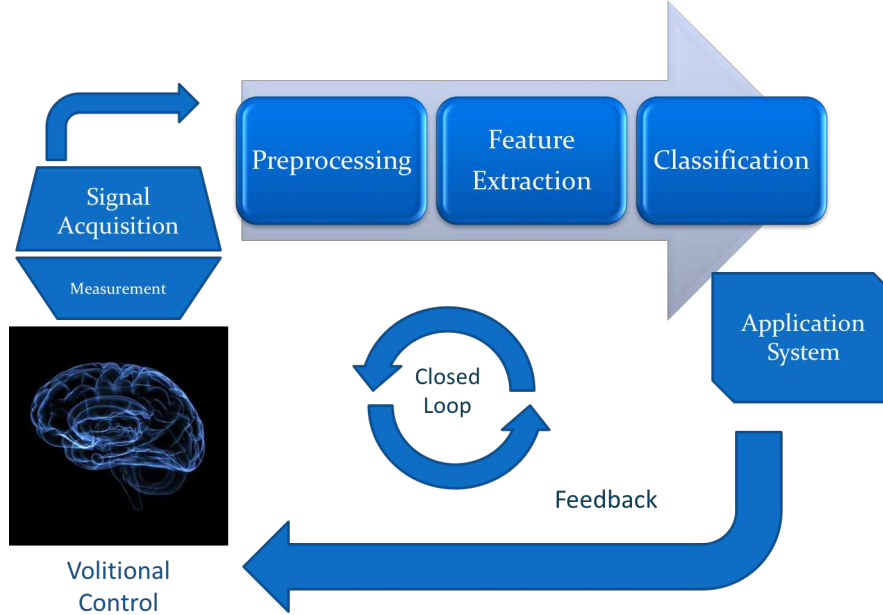
Figure 2.1: General components of a BCI system.

2.1 Brain Computer Interface Model and Architecture

The draft architecture of a BCI system can be summarized in Figure 2.1. A volitional control, a will to transmit information, is exerted by a user. A brain signal acquisition device captures her/his signals using a measurement modality. This module obtains the brainwaves and the information is digitalized and transmited to a computer device. Signal preprocessing is applied to eliminate nuisances and artifacts and to enhance the Signal to Noise Ratio (SNR), or to apply spatial or frequency filters. In the next step, a *feature* is carefully constructed in order to differentiate at least between two different mental states. Finally a classification step is applied to derive the actual information bit out of the system. An application system uses this information to affect some external device. By visual or any other sensory means, the feedback is fed back to the user and a loop is finally closed.

The central point of this system is called the *Brain Machine Dilemma* [136]. The underlying idea is that the BCI system adapts to the user's thinking patterns but at the same time the brain is adapting to what the system is doing, and changing their own signals in the process. This is the reason why it is often called, a *co-adaptive system*, where two different intelligent devices, one biological and the other electronic, try to adapt to each other.

Let X be a multichannel digital signal $X \in \mathbb{R}^{C \times N}$, with N being the length of the

digitalized signal in sample points, and C is the number of available channels. This signal matrix is

$$X = \begin{bmatrix} X(1,1) & \cdots & X(n,1) & \cdots & X(N,1) \\ \vdots & & \vdots & & \vdots \\ X(1,c) & \cdots & X(n,c) & \cdots & X(N,c) \\ \vdots & & \vdots & & \vdots \\ X(1,C) & \cdots & X(n,C) & \cdots & X(N,C) \end{bmatrix}. \tag{2.1}$$

The n-column $\mathbf{x}(n)$ of this matrix is a vector for a sample point n in digital time for every available channel. Additionally, $x(n,c)$ is a c-row with the multichannel signal as a scalar time-series for a particular channel c. When the particular channel is not important, the notation $x(n)$ is using to emphasize a single-channel processing scheme.

The basic model of any BCI is to take this multichannel digital signal $\mathbf{x}(n)$, and transform it to an output control signal $y(n)$ which can be a scalar or binary function. The BCI system can be modeled as the transformation T, which operates on the equation

$$y(n) = T\left[\mathbf{x}(n)\right]. \tag{2.2}$$

What a BCI system must do, is to take at least a single bit of information out of $y(n)$ and use that information to derive some action.

2.2   Signal Processing

From this signal processing point of view, BCIs are:

- Causal: $y(n) = T\left[\mathbf{x}(m)\right]$, where $m \leq n$. The action of a BCI system depends on the history of the captured brainwaves.

- Dynamic: $y(n) = T\left[\mathbf{x}(m), \dot{\mathbf{x}}(m), \ddot{\mathbf{x}}(m), \cdots\right]$. A BCI system is dynamic, where the output function do not depend only on the current value being observed, it does depend on its dynamic interactions.

- Time invariant: $y(n) = T\left[\mathbf{x}(n)\right] \Rightarrow y(n-p) = T\left[\mathbf{x}(n-p)\right]$. The output of a BCI system does not depend on the particular time frame where it is being used. However, adaptive BCI, which do adapt to the user behavior, are time variant.

- Nonlinear: a system is linear when $T[a_1\mathbf{x}(n) + a_2\mathbf{x}(n)] = a_1 T[\mathbf{x}(n)] + a_2 T[\mathbf{x}(n)]$. Due to brainwave complexity, BCI systems are not linear.

- Multirate or broadband [81]: The energy of brainwave spectrum is not confined to a certain band, and almost all frequency channels may convey some information.

There are several filters that can be applied to the system to eliminate artifacts, enhance the signal, and to ease the detection of the discriminative information.

Static filters like square or logartihmic, were traditionally used in analog signal processing and are currently already embedded in the measuring device. Wiener and Kallman filters are usually applied to invasive techniques [52]. The filter, particularly when it is linear, can be viewed as the matrix M in:

$$y(n) = MT[\mathbf{x}(n)] \tag{2.3}$$

Spatial filters are carefully adapted to the arrangement of sensors around or within the head and they emphasize the spatial structure of the information that is being captured. The head is divided in anatomical regions and electrode locations around the head are arranged according to neuroanatomical planes or axes (Figure 2.2).

Spectral filters, on the other hand, do consider brainwaves as a digital signals, and they perform different transformations based on the spectral information contained within the signal $\mathbf{x}(n)$. They can be combined and aggregated creating *filter banks* to enhance signal quality.

(a) Neuronal Planes

(b) Neuroanatomical regions of the brain.
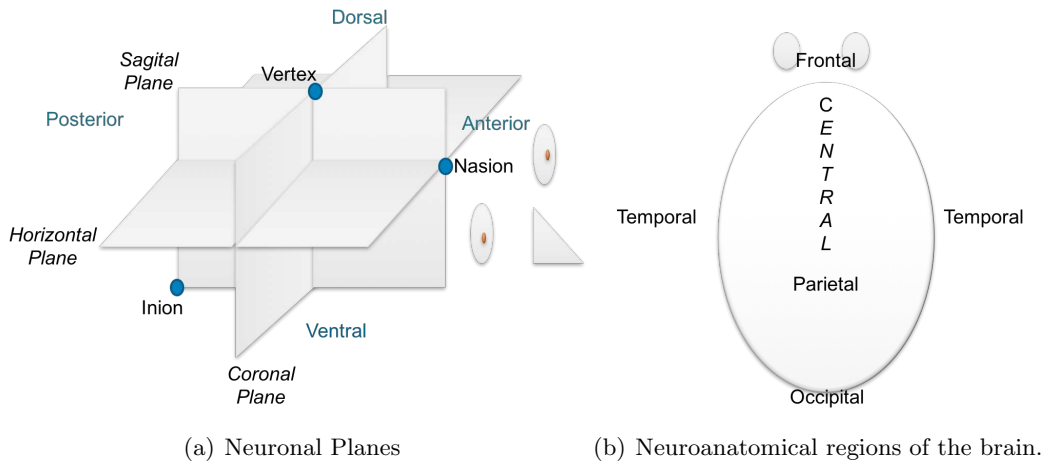
Figure 2.2: Neuronal Planes regularly used in neuroscience research. In BCI they are used to understand electrode location and spatial filters.

2.3 The Forward and Inverse Model

Brainwaves are obtained via sensors. Each one of them captures only a part or a version of the information. However, whatever is actually happening inside the brain is recovered indirectly from the *sensor space*. From there, the information can be traced back to the real landscape where the information source is located, inside the *source space*. This is a regular problem found in engineering and it is not different in BCI. *Calculating* the signal on each a sensor from a projection of a known source of information from within the head is called *The Forward Problem*[93, 136] and doing the opposite, *estimating* the contributions of different sources to whatever activity is found on sensors is called *The Inverse Problem*. Although the latter is a complex ill-posed problem, it is more relevant in BCI because it allows to determine source origins that can be mapped more directly to cognitive activities.

Particularly for noninvasive electrophysiological modalities, an additional problem makes things harder. Due to its electromagnetic properties, the brain acts like conductive gel, and any signal that is generated inside the brain is irradiated to every direction and it can influence every sensor regardless of its position. This is called *Volume conduction* [83, 18] and can be visualized in Figure 2.3.

2.4 Brain Signals Measuring Techniques

The measuring technique determines the most important taxonomic differentiation in BCI, according to the methodology that is applied to extract the information from the CNS. All of them have been used so far for BCI applications.

1. EEG Electroencephalography: it is based on the electrical voltage detected by electrodes at the scalp. It is explained in detail in Section 2.5.

2. ECoG Electrocorticography: the electrodes are located bellow the skull and above the cortex, on the exposed region of the brain. Thus, a craniotomy is required. It offers a very high temporal resolution, broader bandwidth and much better spatial resolution than EEG. This modality has allowed very good performance in complex BCI schemes like speech synthesis from direct neural signals [53].

3. MEG Magnetoencephalography: when active neurons generate electric currents, minuscule magnetic fields are generated. It is considered complementary to EEG and ECoG, due to the fact that it is sensitive to neurons firing parallel to the scalp,
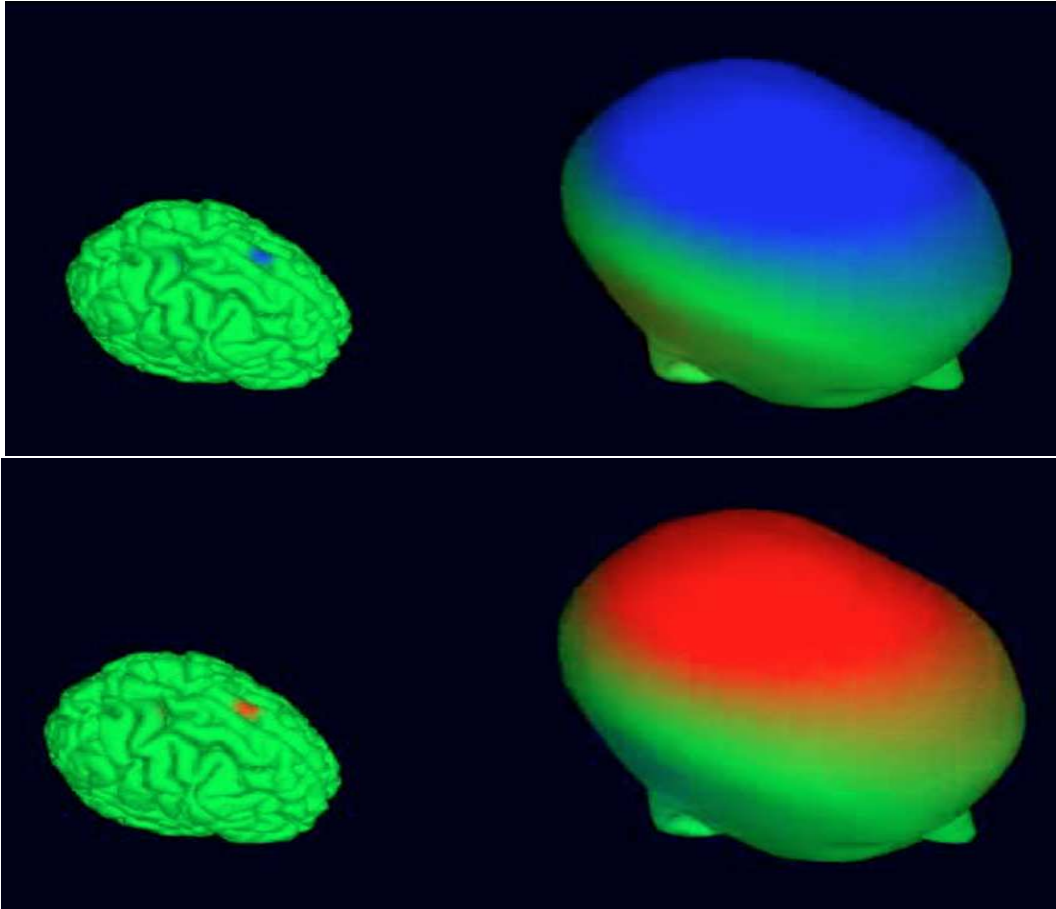
Figure 2.3: A source signal with positive/negative polarity is generated in a very specific region of the brain but due to volume conduction their influence affects a widespread area of the scalp where sensors are located (Image of the brain from Swartz Center for Computational Neuroscience).

which are hard to detect in EEG and ECoG. Although MEG equipment is bulky and room-size, recent advances [13] are aiming to develop portable and wearable versions.

4. PET Positron Emission Tomography: this radio nuclear measuring device, use a tracer molecule like fludeoxyglucose, which emits positrons. Positrons interacts with biological tissue generating photons in exact opposite directions. The tracer is spread around the body and the brain, and its concentration is higher in those areas where active neuron firing is being conducted which requires more glucose [82].

5. fMRI functional Magnetic Resonance Imaging: this noninvasive, non-portable measuring technique, measures the so-called BOLD response: the Blood Oxygen Level-Dependent contrast. This is based on the principle that firing neurons generates an imbalance of oxyhemoglobin and deoxyhemoglobin which can be detected on the magnetic resonator, with a very high spatial accuracy.

6. fNIRS functional Near Infra Red Spectroscopy: it also measures the concentration changes of oxy/deoxy-hemoglobin using light pulses of near-infrared wavelengths. The different types of hemoglobin molecules absorb these light frequencies at different rates. It is an indirect measure of brain activity. Although it provides very good spatial localization, temporal localization is hindered by the hemodynamic response [82].

7. INR Intracortical Neuron Recordings: Electrodes, tetrodes, or Multielectrode arrays MEAs (i.e. Utah Array) [2] can be implanted inside the brain. Often called iEEG, intracraneal EEG, it is designed to detect Local Field Potentials LFPs or even single-unit recording [18].

ECoG and INR are invasive technologies that require a neurosurgery or craniotomy. The implantation of electrodes is performed inside the skull for the former, and inside the brain for the latter. The remaining measuring techniques are external or noninvasive.

2.5   Electroencephalography

Above all, electroencephalography, is the most widespread method to gather information from the CNS in a non-invasive way. It is of particular interest in BCI mainly because of its non-invasiveness, its optimal time resolution and acceptable spatial resolution. Moreover, it is portable, cheap, wearable and can be more easily integrated into fashionable designs aimed for real users, which prefer cap-like devices [57].
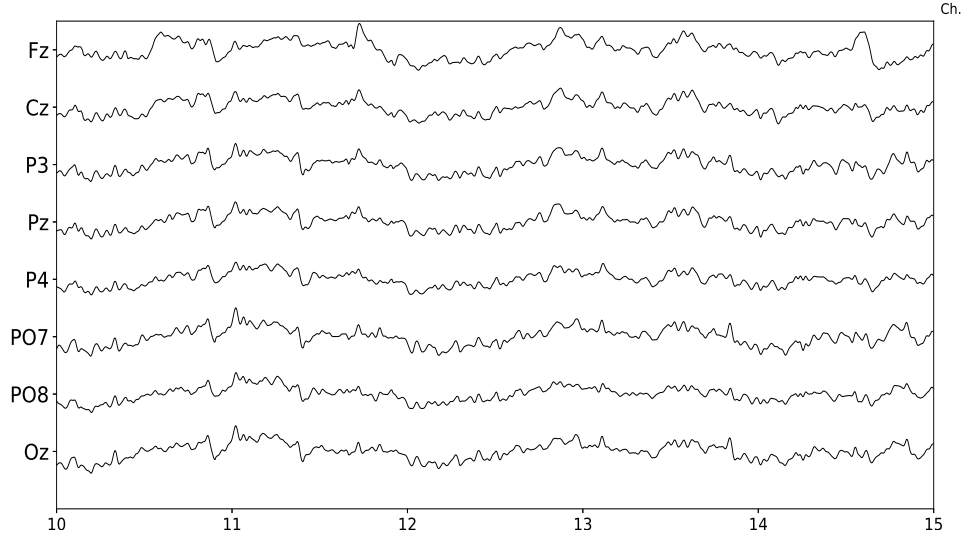
Figure 2.4: Sample EEG signal obtained from (g.Nautilus, g.Tec, Austria). Time axis is in seconds and five seconds are displayed. The eight channels provided by this device are shown.

The electroencephalography consists on the measurement of small variations of electrical voltage over the scalp. This represents the summed activity of post-synaptic potentials PSPs of pyramidal neurons located perpendicular to the scalp [83]. Only one percent of synchronized activity of pyramidal neurons are stronger that the remaining desynchronized neurons [110] and explain ninety-nine percent of the signals obtained from EEG. This technique is one of the most widespread used methods to capture brain signals and was initially developed by Hans Berger in 1924 and has been extensively used for decades to diagnose neural diseases and other medical conditions. Figure 2.4 shows a sample EEG signal trace obtained with a digital and wearable EEG device.

The first characterization that Dr. Berger detected was the Visual Cortical Alpha Wave, the *Berger Rythm* [60]. He understood that the amplitude and shape of this rhythm was coherently associated to a cognitive action (eyes closing). We should ask ourselves if the research advancement that came after that discovery would have happened if it weren't so evident that the shape alteration was due to a very simple and verifiable cognitive process.

The EEG signal is a highly complex multi-channel time-series. It can be modeled as a linear stochastic process with great similarities to noise [120]. It is measured in microvolts, and those slightly variations are contaminated with heavy endogenous artifacts and exogenous spurious signals.

The device that captures these small variations in current potentials over the scalp is called
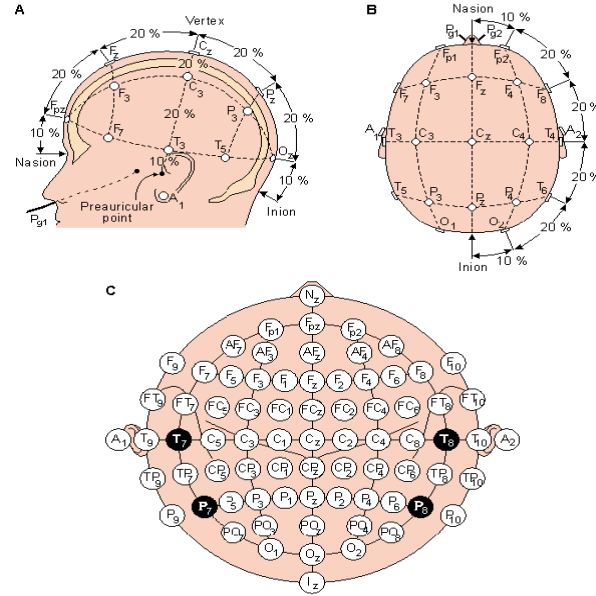
Figure 2.5: International 10-20 system that standardize electrode locations over the scalp.

Figure 2.6: Digital and wearable electroencephalographs.

the electroencephalograph (Figure 2.6). Electrodes are located in predetermined positions over the head, usually embedded in saline solutions to facilitate the electrophysiological interface and are connected to a differential amplifier with a high gain which allowed the measurement of tiny signals. Although initially analog devices were developed and used, nowadays digital versions connected directly to a computer are pervasive. A detailed explanation on the particularities and modeling of EEG can be obtained from [58], and a description of its electrophysiological aspects from [49]. Further details are covered in Chapter 4.

2.6 EEG Signals

Overall, EEG signals can be described by their phase, amplitude, frequency and *waveform*. The following components regularly characterize EEG signals:

- Artifacts: These are signal sources which are not generated from the CNS, but can be detected from the EEG signal. They are called endogeneous or physiological when they are generated from a biological source like face muscles, ocular movements, etc., and exogeneous or non-physiological when they have an external electromagnetic source like line induced currents or electromagnetic noise [134]. Ambulatory studies or out-of-the lab studies introduces artifacts that are derived from the person movement, from any kind of muscular electrical stimulator for rehabilitation treatments or from other devices in hybrid, or multi-modal BCIs.

- Non-Stationarity: the statistical parameters that describe the EEG as a random process are not conserved through time, i.e. its mean and variance, and any other higher-order moments are not time-invariant [60].

- DC drift and trending: in EEG jargon, which is derived from concepts of electrical amplifiers theory, Direct Current (DC) refers to very low frequency components of the EEG signal which varies around a common center, usually the zero value. DC drift means that this center value drifts in time. Although sometimes considered as a nuisance that needs to get rid of, it is known that very important cognitive phenomena like slow cortical potentials or slow activity transients in infants do affect the drift and can be used to understand some particular brain functioning [128, 110].

- Basal EEG activity: the EEG is the compound summation of myriads of electrical sources from the CNS. These sources generate a baseline EEG which shows continuous activity with a small or null relation with any concurrent cognitive activity or task.

- Intra-subject and Inter-subject variability: electroencephalographic signals vary greatly from person to person. Additionally, EEG can be affected by the person's behavior like sleep hygiene, caffeine intake, smoking habit or alcohol intake previously to the signal measuring procedure [38].

Regarding how the EEG activity can be related to an external stimulus that is affecting the subject, it can be considered as

- Spontaneous: activity related to basal EEG, arising spontaneously, or self-regulated by the person.

- Evoked: activity that can be detected synchronously after some specific amount of time from the onset of the stimulus. This is usually referred as time-locked. In contrast to the previous one, it is often called induced activity.

Additionally, according to the existence of a repeated rhythm, the EEG activity can be understood as

- Rhythmic: EEG activity consisting of waves of approximately constant frequency. It is often abbreviated RA (regular rythmic activity). They are loosely classified by their frequencies, and their naming convention was derived from the original naming used by Hans Berger himself:

    - Delta (0-4 Hz)

    - Theta (4-8 Hz)

    - Alpha Waves (10 Hz)

    - Sigma (12-16 Hz)

    - Beta (12-30 Hz)

    - Gamma (30-100 Hz)

    - Omega (60-120 Hz)

    - Rho (250 Hz) hippocampal

    - Sigma Thalamocortical burst (600 Hz) [39].

  The last three are hardly encountered in conventional EEG [128].

- Arrhythmic: EEG activity in which no stable rhythms are present.

- Dysrhythmic: Rhythms and/or patterns of EEG activity that characteristically appear in patient groups and rarely seen in healthy subjects.

The number of electrodes and their positions over the scalp determines a spatial structure: signal elements can be generalized, focal or lateralized, depending on in which channel (i.e. electrode) they are found.

2.7   BCI EEG Paradigms

BCI Paradigms are referred to noninvasive EEG-based BCI configurations that are used
to transmit volitional information. The popularity of EEG in BCI Research influenced the
adoption of conventional paradigms exclusively for noninvasive BCI. Their chronology can
be found at the beginning of the Chapter. They can be roughly [22] described as:

1. Steady State Evoked Potentials: the basis for this paradigm is that when a subject
   attends certain stimulus, the dominant frequency component contained in the stimulus
   source can be found in the brain waves. When stimulus sources are light, this is called
   SS Visual EP and the signals are prominent in occipital regions. A similar process can
   be obtained with auditory stimulus, in which case they are called SS Auditory EPs.
   Finally, this can be extended to somatosensory stimulation (i.e. tactile) and it is called
   SS Somatosensory EP. By using different stimulus sources with different frequencies,
   the one that is **selectively attended** by a subject can be inferred based on the main
   frequency component found on the EEG trace [82].

2. Bereitschaftspotentials, Readiness Potential or Movement-Related (Cortical) Potentials:
   these signals are slow and low frequency $[0.05, 3]$ Hz cortical potential that can appear
   when a subject is just about to engage in a movement related activity. They are used
   in BCI as triggering markers, or to identify movement-related EEG potentials [113].

3. Motor Imagery ERD/ERS: the motor imagery, i.e. the mental visualization of
   movement without actually performing it, triggers a neurophysiological response
   which is very similar to the one obtained when the movement is physically performed.
   Frequencies on the α range of EEG are desynchronized prior to movement imagery,
   and synchronized afterwards. At the same time, β frequencies are resynchronized and
   increase in power after motor imagery. A subject can learn to think about moving a
   feet or moving a limb, and transfer a bit from this thinking patterns. This paradigm
   requires intensive training from subjects. This is further explained in Chapter 6.

4. P300: the positive deflection at 300 ms is activated by a cognitive experiment called
   the oddball paradigm and can be used to detect which symbol a subject is paying
   attention on a flickering matrix. By exploiting this information, a speller application
   can be implemented. This important signal is explained in details in Chapter 7.

5. Mental Tasks: mentally rotating 3D objects, or calculating arithmetic operations are used to generate signals that can be detected and utilized to transfer a bit [114].

6. Slow Cortical Potentials: these are very slow shifts in the electrical activity found in the cortex, of a very low frequency. They can be modulated by **operand conditioning** protocols [136].

7. Error Potentials: when a person recognizes that an error was committed during a task, a recognizable signal called ErrP can be detected along the EEG trace, time-locked to the onset when the error information is fed back to the person. This very important potential is used in BCI applications to enhance the identification of false positives and to improve the overall interaction between the subject and the computer [30].

8. Visual Spatial Covert Attention: oscillatory activity in the α band of EEG can be modulated by changes in visual covert attention. Visual covert attention is the ability to focus attention on objects on the peripheral vision. Humans can voluntarily focus attention to locations in visual space without moving their eyes. This voluntary control is reflected in changes on Visual Occipital Alpha Waves [62]. Alpha Waves are further detailed in Chapter 5.

These paradigms have been exploited in the most popular BCI configurations, the Wadsworth BCI, the Graz BCI, the Berlin BCI and the Tübingen BCI. These platforms introduced pragmatic enhancements to use these paradigms to implement more practical devices [83, 110, 11, 95, 84, 130].

2.8   State of the Art of BCI Algorithms for EEG processing

According to the general layout of a BCI system, Figure 2.1, specific algorithms or techniques are required for both the feature extraction and classification step. The most relevant features used nowadays in BCI are:

- Time points: the sequence of time series, often, concatenated in time or space.

- Band Power: frequency based features.

- Complexity: based on complexity measurements like entropy, or fractal.

- Statistical: Auto-regressive parameters or covariances matrix.

The most successfull used and verified classification methods for BCI [74] can be described as linear versions of machine learning tools. Particularly, Support Vector Machines (SVM), Linear Discriminant Analysis (LDA) and its variant Stepwise Linear Discriminant Analysis (SWLDA). SWLDA is relevant for two reasons: the first is that the stepwise identification of features improves the selection criteria and also the spatial filter that this procedure encompass. Additionally, from a pragmatic perspective, this method is included in the popular BCI2000 [108] package and is the default option for the identification of event related potentials. Spatial filters have also being incorporated and have shown substantial improvements in classification accuracies: the canonical Common Spatial Patterns CSP for the identification of Motor Imagery as well as the xDAWN algorithm for P300 identification.

In recent years (circa 2018) classification accuracies in BCI have improved but the focus was not centered on any particular classification algorithm. Instead, current contributions concentrate their efferts on how these algoritms are used [73]. Recent works can be described as:

- Ensemble classifiers: SVM ensembles [98] and variants of random forest [117]. Features are segmented and divided and the forest performs a classification step on aggregated parts, maximizing classification accuracies.

- Cross-paradigm BCI: the use of a reinforced signal with ErrP feedback or the use of SSVEP in combination with P300 detection [73].

- Adaptive classifiers: the parameters of the classifiers are adapted continuously and online, adapting to the natural variation of the EEG signals [73].

- Transfer learning: transfer the calibration information obtained by users to new subjects. This aims to ease the issue of the intra-subject variability in BCI, and to reduce the set-up and calibration times of a BCI system [142].

- Rienmann geometry classifiers: the EEG stream is directly mapped onto a geometrical space equipped with a suitable Riemannian metric. Hence, further data manipulation is carried out following principles of Riemannian geometry which yields very good results in terms of accuracies [142].

- Tensor-based BCI: the EEG data is viewed as a multidimensional matrix, a tensor. The BCI operation is considered as an optimization problem that can be solved with sparsity and nonnegativity constraints [132, 24].

- Deep learning: deep learning is a very successful technique driven by the increased computational power of current computer devices. Deep learning techniques have also being applied to BCI applications [123].

2.9   EEG Waveform Analysis

This section describes the depictions that are used to describe EEG signals waveforms and the automatic procedures that were developed with this purpose.

2.9.1   EEG Waveform Characterization

The shape of the signal, the waveform, can be defined as the graphed line that represents the signal's amplitude plotted against time. It can also be called EEG biomarker, EEG pattern, motifs, signal shape, signal form and a morphological signal [60].

The signal context is crucial for waveform characterization, both in a spatial and in a temporal domain [60]. Depending on the context, some specific waveform can be considered as noise while in other cases is precisely the element which has a cognitive functional implication.

A waveform can have a characteristic shape, a rising or falling phase, a pronounced plateau or it may be composed of ripples and wiggles. In order to describe them, they are characterized by its amplitude, the arch, whether they have (non)sinusoidal shape, by the presence of an oscillation or imitating a sawtooth (e.g. Motor Cortical Beta Oscillations). The characterization by their sharpness is also common, particularly in Epilepsy, and they can also be identified by their resemblance to spikes (e.g. Spike-wave discharge).

Other depictions may include, subjective definitions of sharper, arch comb or wicket shape, rectangular, containing a decay phase or voltage rise, peaks and troughs, short term voltage change around each extrema in the raw trace. Derived ratios and indexes can be used as well, like peak and trough sharpness ratio, symmetry between rise and decay phase and slope ratio (steepness of the rise period to that of the adjacent decay period). For instance, wording like "Central trough is sharper and more negative that the adjacent troughs" are common in the literature.

Other regular characterizations which are based on shape features may include:

- Attenuation: Also called suppression or depression. Reduction of amplitude of EEG activity resulting from decreased voltage. When activity is attenuated by stimulation,

it is said to have been "blocked" or to show "blocking".

- Hypersynchrony: Seen as an increase in voltage and regularity of rhythmic activity, or within the alpha, beta, or theta range. The term suggest an increase in the number of neural elements contributing to the rhythm and a synchronization of neurons with similar firing patterns [18].

- Paroxysmal: Activity that emerges from background with a rapid onset, reaching (usually) quite high voltage and ending with an abrupt return to lower voltage activity.

- Monomorphic: Distinct EEG activity appearing to be composed of one dominant activity.

- Polymorphic: Distinct EEG activity composed of multiple frequencies that combine to form a complex waveform.

- Transient. An isolated wave or pattern that is distinctly different from background activity.

The conventional clinical procedure consists in analyzing the paper strip that is generated by the plot of the signal obtained from the device. Expert technician and physicians analyze visually the plots looking for specific patterns that may give a hint of the underlying cognitive process or pathology. Atlases and guidelines were created in order to help in the recognition of these complex patterns. Even video-electroencephalography scalp recordings are routinely used as a diagnostic tools [42] . The clinical EEG research has also focused on temporal waveforms, and a whole branch of electrophenomenology has arisen around EEG *graphoelements* [110].

Sleep research has been studied in this way by performing Polysomnographic recordings (PSG) [105], where the different sleep stages are evaluated by visually marking waveforms or graphoelements in long-running electroencephalographic recordings, looking for patterns based on standardized guidelines. Visual characterization includes the identification or classification of certain waveform components, or transient events, based on a subjective characterization (e.g. positive or negative peak polarity) or the location within the strip. It is regular to establish an amplitude difference between different waveforms from which a relation between them is established and a structured index are created (e.g. sleep K-Complex is well characterized based on rates between positive vs. negative amplitude) [124].

Other relevant EEG patterns for sleep stage scoring are alpha, theta, and delta waves, sleep spindles, polysplindles, Vertex Sharp Waves (VSW), and sawtooth waves (REM Sleep).

Moreover, EEG data acquisition is a key procedure during the assessment of patients with focal epilepsy for potential seizure surgery, where the source of the seizure activity must be reliably identified. The onset of the epileptic seizure is defined as the first electrical change seen in the EEG rhythm which can be visually identified from the context and it is verified against any clinical sign indicating seizure onset. The Interictal Epileptiform Discharges (IEDs) are visually identified from the paper strip, and they are also named according to their shape: spike, spike and wave or sharp-wave discharges[16].

2.9.2   EEG Waveform Analysis Algorithms

Shape or waveform analysis methods are considered as nonparametric methods. They explore signal's time-domain metrics or even derive more complex indexes or features from it [121].

One of the earliest approach to automatically process EEG data is the Peak Picking method. Although of limited usability, this procedure has been used to determine latency of transient events in EEG [61, 143]. Straightforward in its implementation, it consists in selecting a simple component based on the expected location of its more prominent deflection [90]. Evoked Potentials (EPs) and Event Related Potentials (ERPs) are transient component that may arise as a brain response to an external visual, tactile or auditory stimulus. The P300 signal that is used for some BCI Spellers is the prototypical Event Related Potential. Particularly, EPs are regularly used to assess auditory response in infants. ERPs are characterized by their most prominent peaks, where the name of many of the EEG features evoke directly a peak within the component, e.g. P300 or P3a, P3b or N100. This leads to a natural procedure to classify them visually by selecting appropriate peaks and matching their positions and amplitudes in an orderly manner. The letter provides the polarity (Positive or Negative) and the numbering shows the time referencing the stimulus onset, or the ordinal position of each peak (first, second, etc). Finally, the trailing letter is added to describe different variants of components that initially were considered the same.

A related method is used in [4] where the area under the curve of the EEG is sumarized to derive a feature. This was used in the seminal work of Farwell and Donchin on P300 [37, 136]. Additionally, a logarithmic graph of the peak-to-peak amplitude which is called amplitude integrated EEG (aEEG) [112] is utilized nowadays in Neonatal Intensive Care Units.

Other works explored the idea to extend human capacities analyzing EEG waveforms [67] where a feature from the amplitude and frequency of its signal and its derivative in time-domain is used. Moreover, alternative schemes explored the use of Mathematical Morphology, where the time-domain structure of contractions and dilations are studied [137]. The Merging of Increasing and Decreasing Sequences (MIDS) [144] provides a filter or heterogeneous downsampling scheme which is based on the waveform structure, similar to what is provided in Local Binary Patterns (1-D LNBP, 1D-LBP and LBP) algorithms [59] . Finally the proposals of Burch, Fujimori, Uchida and the Period Amplitude Analysis (PAA) algorithm are few of the earliest proposals where the idea of capturing the shape of the signal were established [125]. Three algorithms are explained in detail in the following section.

2.9.3   Waveform-based Feature Extraction Algorithms

The method presented in this Thesis generates a feature that can be classified. Likewise, the following methods provide a feature that can be used as a template, whereas all of them are based on metrics extracted from the shape of the signal. The signal $x(n)$ a single-channel EEG time series EEG out of $x(n, c)$ for a given fixed channel c. The notation $f = \{f_i\}_1^n$ or $f = \{f_i\}_{i \in J}$ will be used to describe the concatenation of scalar values to form a multidimensional feature vector $f = \{f_1, f_2, ..., f_n\}$. These features can be used to create dictionaries or template databases. These templates provide the basis for the pattern matching algorithm and offline classification.

Matching Pursuit - MP 1 and MP 2

Pursuit algorithms refer, in their many variants, as blind source separation [132] techniques that assume the EEG signal as a linear combination of different sparse sources extracted from a template's dictionaries. Matching Pursuit *MP* [79], the most representative of these algorithms, is a greedy variant that decomposes a signal into a linear combination of waveforms, called atoms, that are well localized in time and frequency [21]. Given a signal, this optimization technique, tries to find the indexes of m atoms and their weights (contributions) that minimize,

$$\varepsilon = \left\| x(n) - \sum_{i=1}^{m} w_i g_i(n) \right\| \tag{2.4}$$

which is the error between the signal and its approximation constructed by the weighted w_i

atoms g_i, and calculating the euclidean norm $\|\cdot\|_2$. The algorithm goes by first setting the approximating signal $\tilde{x}_0$ as the original signal itself,

$$\tilde{x}_0(n) = x(n) \tag{2.5}$$

and setting the iterative counter s as 1. Hence, it searches recurrently the best template out of the dictionary that matches current approximation.

$$g_s = \arg\max_{g_i} \left| \sum_{n=1}^{N} \tilde{x}_{s-1}(n) \; g_i(n) \right| \tag{2.6}$$

where g_i are all the available scaled, translated and modulated atoms from the dictionary. The operation $|\cdot|$ corresponds to the absolute value of the inner product. This step determines the atom selection process, and their contribution is calculated based on

$$w_s = \frac{\sum_{n=1}^{N} \tilde{x}_{s-1}(n) \; g_s(n)}{\|g_s\|^2} \tag{2.7}$$

with s representing the index of the selected atom g_s and $\|\cdot\|$ its euclidean norm. Finally the contribution of each atom is subtracted from the next approximation [27, 107, 79]

$$\tilde{x}_s(n) = \tilde{x}_{s-1}(n) - w_s g_s(n) \tag{2.8}$$

The stopping criteria can be established based on a limiting threshold on Equation 2.4 or based on a predetermined number of steps and selected atoms. Two variants of this algorithm are evaluated. In *MP 1* the dictionary is constructed with the normalized templates directly extracted from the real signal segments which is a straightforward implementation of the pattern matching technique. In *MP 2* the coefficients of Daubechies least-asymetric wavelet with 2 vanishing moments atoms are used to construct the dictionary [129]. For the first version, the matching against the template is evaluated according to Equation 2.4 directly, whereas for the latter each feature is crafted by decomposing the signal in its coefficients and building, an eventually sparse, vector with them:

$$f = \left\{ w_i \right\}_1^m \tag{2.9}$$

where D is the size of the dictionary.

Permutation Entropy - PE

Bond and Pompe Permutation Entropy has been extensively used in EEG processing, with applications on anesthesia, sleep Stage evaluation and increasingly for Epilepsy pre-ictal detection [8]. This method generates a code based on the orderly arrangement of sequential samples, and then derives a metric which is based on the number of times each sequence is found along the signal. This numeric value can be calculated as information entropy [85]. Let's consider a signal on a window of length W represented by the sample points

$$(x_1, x_2, ..., x_W) \tag{2.10}$$

and resampled by τ intervals, starting from the sampling point n, doing

$$(x_n, x_{n+\tau}, x_{n+2\tau}..., x_{n+(m-1)\tau}). \tag{2.11}$$

This sequence is of order m, which is the number of sample points used to derive the ordinal element called π. There are $m!$ ways in which this sequence can be orderly arranged, according to the position that each sample point holds within the sequence in a strict order relationship. For example if $m = 3$, and the first sample point is the bigger, the second is the smaller and the third one is in the middle, the ordinal element π corresponds to $(3, 1, 2)$. Thus, along the signal window there can be at most k different ordinal (and overlapping) elements π_s

$$(\pi_1, \pi_2, ..., \pi_k) \tag{2.12}$$

with $k = W - (m-1)\tau$. The probability density function *pdf* for all the available permutations of order m should be $\mathbf{p} = (p_1, p_2, ..., p_{m!})$ with $\sum_{i=1}^{m!} p_i = 1$.

Hence, the time series window is mapped to a new set of k ordinal elements, and the *pdf* can be calculated by the empirical permutation entropy,

$$p_i = \frac{1}{k} \sum_{s=1}^{k} [\pi_s = \pi_i] \tag{2.13}$$

with $1 \leq i \leq m!$. The Iverson Bracket $[\cdot]$ resolves to 1 when their logical proposition argument is true, 0 otherwise. Therefore, for each i only those ordinal elements π_s that were effectively found along the signal are counted to estimate p_i, and zero elsewhere. The

empirical permutation entropy can be calculated from the histogram as,

$$H(\mathbf{p}) = \sum_{i=1}^{m!} p_i \ log\frac{1}{p_i}.$$ (2.14)

The implemented code was derived from [126], and the model description from [9]. This procedure produces a scalar number for the given signal window of size W. To derive a feature, a sliding window procedure must be implemented to cover an entire segment of length N. Thus, the length of the feature is $N - (W + \tau(m - 1))$.

$$f = \left\{ H(\mathbf{p})_u \right\}_{W+\tau m}^{N}.$$ (2.15)

with u varying on a sample by sample basis along the signal, starting from the specified index.

Slope Horizontal Chain Code - SHCC

This algorithm [4] proceeds by generating a coding scheme from a sequence of sample points. This encoding is based on the angle between the horizontal line on a 2D-plane and any segment produced by two consecutive sample points, regarding them as coordinates on that plane.

A signal of length N, can be represented by a list of ordered-pairs e,

$$e = [(x, y)_1, (x, y)_2, ..., (x, y)_N]$$ (2.16)

and it can be divided into G different blocks. These blocks are obtained by resampling the original signal from the index

$$G = \lfloor n + (m\Delta) + 0.5 \rfloor$$ (2.17)

with n being the original sampling index on $1 \leq n \leq N$ and $\lfloor \cdot \rfloor$ being the floor operation, i.e. rounding of the number argument to the closest smaller integer number. On the other hand, Δ can be obtained by

$$\Delta = \left\lceil \frac{N}{G+1} \right\rceil$$ (2.18)

with $G < N$ and using instead $\lceil \cdot \rceil$ as the ceil operation, the rounding to the closest bigger integer number. Lastly, the value m can be derived from

$$m = sign\left(\frac{N-1}{\Delta}\right)\left\lfloor\left|\frac{N-1}{\Delta}\right|\right\rfloor. \tag{2.19}$$

This resampling produces a new sequence of values,

$$e' = \left[(x',y')_1, ..., (x',y')_s, ..., (x',y')_G\right]. \tag{2.20}$$

The next step is the normalization of each ordered-pair as vectors $\mathbf{x'} = (x'_1, ..., x'_G)$ and $\mathbf{y'} = (y'_1, ..., y'_G)$ according to

$$\hat{\mathbf{x}} = \frac{\mathbf{x'} - \min(\mathbf{x'})\mathbf{1}}{\max(\mathbf{x'}) - \min(\mathbf{x'})} \tag{2.21}$$

$$\hat{\mathbf{y}} = \frac{\mathbf{y'} - \min(\mathbf{y'})\mathbf{1}}{\max(\mathbf{y'}) - \min(\mathbf{y'})} \tag{2.22}$$

with $\mathbf{1}$ being the vector of length G with all their components equal to 1. Hence, the scalar components $\hat{x}_s$ of $\hat{\mathbf{x}}$, and $\hat{y}_s$ of $\hat{\mathbf{y}}$, with s varying between 1 and G are effectively normalized to $\hat{x}_s, \hat{y}_s \in [0,1]$.

Finally, the feature is constructed by calculating the point-to-point slope against the horizontal plane,

$$f = \left\{\frac{\hat{y}_s - \hat{y}_{s-1}}{\hat{x}_s - \hat{x}_{s-1}}\right\}_2^G \tag{2.23}$$