The sequence $d_i = d(z_i, s_i)$ of distances between a zombie and a survivor on round i is non increasing.

Proof. We assume Z & S is played on a connected graph $G = (V, E)$.

By induction on the round number i.

On round 1, the zombie starts on $z_1 \in V$. The survivor responds with $s_1 \in V$.

If $z_1 = s_1$, then $d(z_1, s_1) = 0$ and the game is over, so the sequence d_i is finite and non increasing.

Else, $z_1 \neq s_1$ and G connected guarantees the existence of a shortest $z_1 s_1$-path. Say

$$P : z_1 = u_1, u_2, \ldots, u_k = s_1$$

so that $d(z_1, s_1) = k$.

On turn 1, the zombie must move to $z_2 \in N(z_1)$ such that $d(z_2, s_1) = d(z_1, s_1) - 1$. We can suppose that $z_2 = u_2$ is the next vertex along P.

In response, the survivor moves to $s_2 \in N(s_1)$. Now since

$$P' : z_2 = u_2, u_3, \ldots, u_k = s_1, u_{k+1} = s_2$$

is a $z_2 s_2$-path of length k, we have that the shortest path between z_2 and s_2 must be smaller than

$$d(z_2, s_2) \leq d(z_2, s_1) + d(s_1, s_2) = (d(z_1, s_1) - 1) + 1 = d(z_1, s_1)$$

The proof for the induction step is entirely analogous.

$\square$