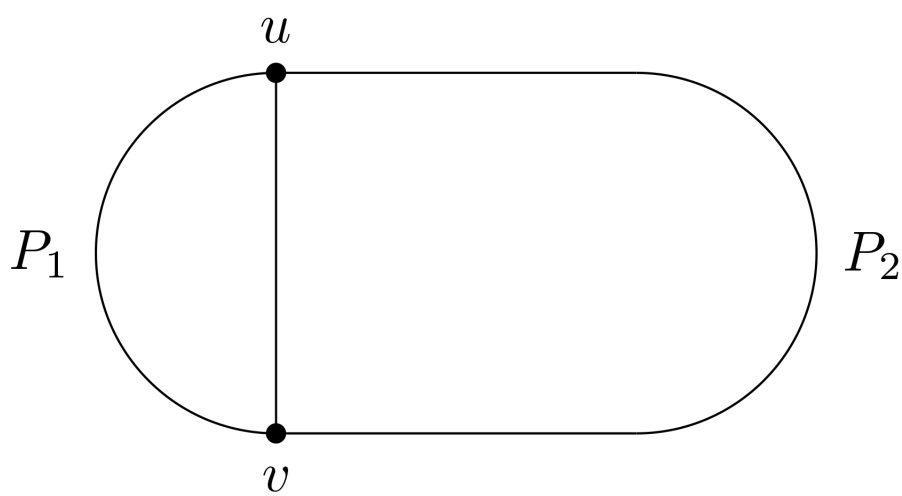
Definition 1. We define a family of graphs we call *bifurcated cycles* and denote as $Q_{m,n}$. As the name suggests, bifurcated cycles are cycles of length $m + n$ with a single chord which divides the cycle into paths P_1 and P_2 of lengths m and n.

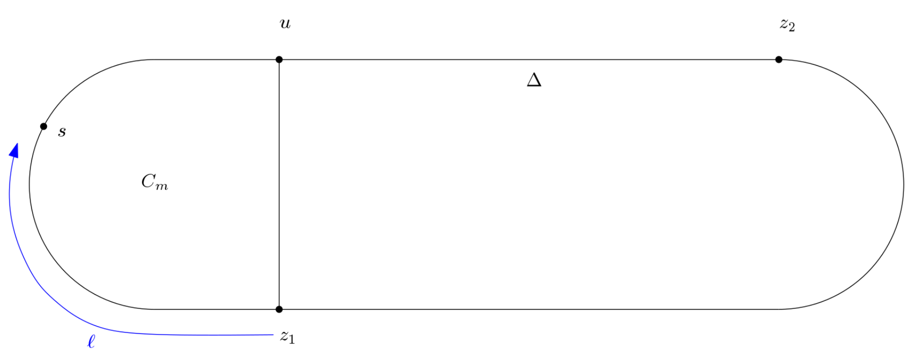


Theorem 1. The Bifurcated cycle $Q_{m,n}$ is 2-zombie win.

Proof. First, we show that a certain game state is a losing position for the survivor. Second, we show how to position the zombies at the start of the game so that – no matter where the survivor starts – a losing position is inevitably reached.

Part 1. Cornering the Survivor on a Cycle

Suppose that the game has reached the following state: the survivor is P_1, the first zombie is on v, and the second zombie is at a distance of Δ from u. Denote the length of the clockwise path from v to s as ℓ. Note that we must have $2 \leq \ell \leq m - 1$, else the survivor is caught.



A. z_1 goes clockwise if

$$l \leq 1 + m - \ell$$
$$4 \leq 2\ell \leq m + 1$$

B. z_2 goes counter-clockwise if

$$1 + m - \ell \leq \ell$$
$$m + 1 \leq 2\ell \leq 2m - 2$$

Assume first that z_1 goes clockwise.

There are four posible $z_2 s$-paths. We wish to guarantee that z_2 will follow the Δ-path towards u, which translates into the following inequalities:

$$\Delta + (m - \ell) \leq n - \Delta + 1 + m - \ell$$

and

$$\Delta + (m - \ell) \leq n - \Delta + \ell$$

or

$$\Delta + 1 + \ell \leq n - \Delta + 1 + m - \ell$$

and

$$\Delta + 1 + \ell \leq n - \Delta + \ell$$

which can be simplified to

$$2\Delta \leq n + 1$$

and

$$2\Delta \leq n - m + 2\ell \leq n + 1$$

or

$$2\Delta \leq n + m - 2\ell$$
$$2\Delta \leq n - 1 \leq n + m - 2\ell$$

So we must have

$$2\Delta \leq n - m + 2\ell$$

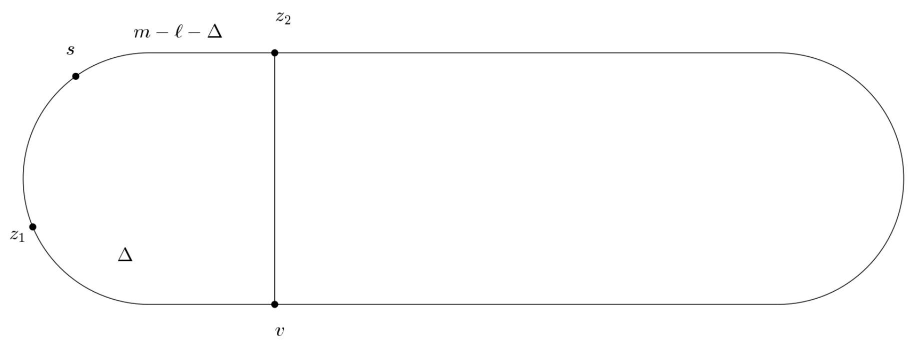
or

$$2\Delta \leq n - 1$$

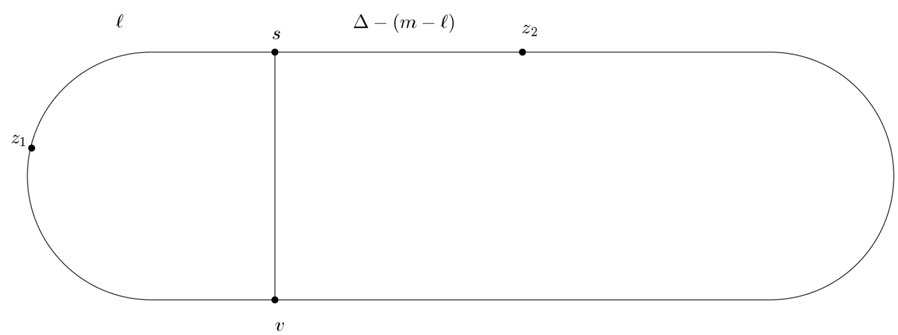
i.e., for z_2 to behave we need:

$$2\Delta \leq \max\{n-m+2\ell, n-1\}$$

Now, after Δ turns we have either



or



$$\begin{aligned}
\Delta-(m-\ell) &\leq 1 \\
\Delta &\leq m-\ell+1 \\
1+\Delta+\ell &\geq m-\ell-\Delta \\
2\Delta &\geq m-2\ell-1 \\
m-2\ell-1 \leq 2\Delta &\leq 2m-2\ell+2 \\
m-4-1 \leq 2\Delta &\leq ??
\end{aligned}$$

$\square$