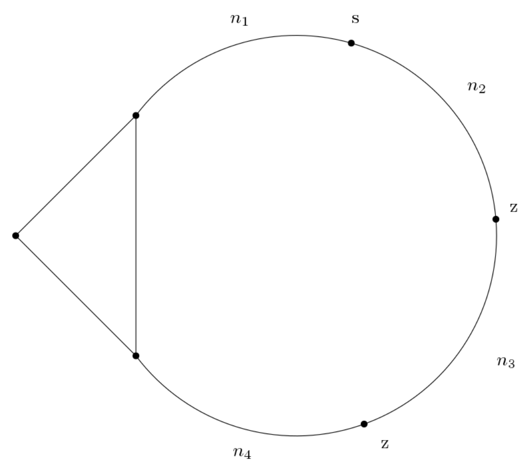
1. Proof that Bifurcated cycle $Q_{2,n}$ is 2-zombie win

   Given the following configuration for the zombie and survivor start positions:



   Assuming

$$
\begin{aligned}
n_1 + n_2 + n_3 + n_4 &= n \\
n_1 &\geq 0 \\
n_2 &\geq 2 \\
n_3 &\geq 1 \\
n_4 &\geq 0 \\
n_1 + n_4 &\geq 1 \qquad \text{and} \\
n_i &\in \mathbb{Z}^+
\end{aligned}
$$

   Explanation: we must have $n_2 \geq 2$ and $n_1 + n_4 \geq 1$ or the survivor starts adjacent to a zombie and loses immediately.

   Now, we see that the survivor wins by running counter-clockwise if

$$
\begin{aligned}
n_2 &< n_1 + n_3 + n_4 + 1 \qquad \text{and} \\
n_2 + n_3 &< n_1 + n_4 + 1
\end{aligned}
$$

   or by running clockwise if

$$
\begin{aligned}
n_2 &> n_1 + n_3 + n_4 + 1 \qquad \text{and} \\
n_2 + n_3 &> n_1 + n_4 + 1
\end{aligned}
$$

   If either of these conditions are met, the zombies have no choice but to follow the shortest path to the survivor around a cycle, which leads to a survivor win. In fact, these conditions are necessary (?) for the survivor to have a chance.

Now, we can guarantee that these conditions are violated by ensuring that the two zombies are positioned on opposite sides of the cycle. That is, by making $n_1 + n_2 + n_4 = n_3 + 1$ if n is even, or $n_1 + n_2 + n_4 = n_3$ if n is odd (?).

If $n_1 + n_2 + n_4 + 1 = n_3$ then

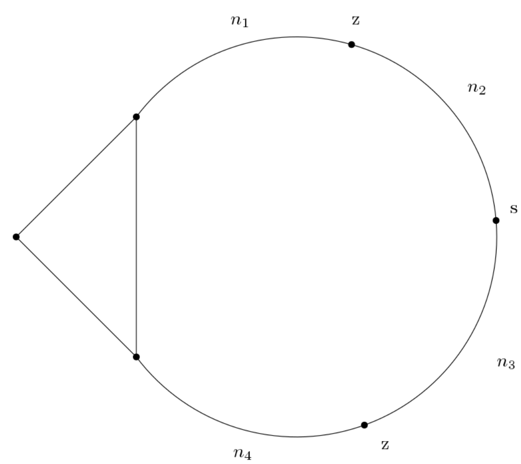
$$\begin{aligned} n_2 + (n_1 + n_2 + n_4 + 1) &< n_1 + n_4 + 1 \\ 2n_2 &< 0 \end{aligned}$$

Which contradicts our assumption that $n_2 \geq 2$. Similarly, if $n_1 + n_2 + n_4 = n_3$ then

$$\begin{aligned} n_2 + (n_1 + n_2 + n_4) &< n_1 + n_4 + 1 \\ 2n_2 &< 1 \end{aligned}$$

Again a contradiction. This shows that there is a zombie start which makes it impossible for the survivor to win in this configuration.

Suppose then that the players are positioned as follows:



Assuming

$$\begin{aligned} n_1 + n_2 + n_3 + n_4 &= n \\ n_1 &\geq 0 \\ n_2 &\geq 2 \\ n_3 &\geq 2 \\ n_4 &\geq 0 \\ n_i \in \mathbb{Z}^+ \end{aligned}$$

Explanation: we must have $n_2 \geq 2$ and $n_3 \geq 2$ or the survivor starts adjacent to a zombie and loses immediately.

Now, we see that the survivor wins by running counter-clockwise if

$$\begin{aligned} n_1 + n_3 + n_4 &< n_2 && \text{and} \\ n_3 &< n_1 + n_2 + n_4 + 1 \end{aligned}$$

or by running clockwise if

$$\begin{aligned} n_1 + n_3 + n_4 &> n_2 && \text{and} \\ n_3 &> n_1 + n_2 + n_4 + 1 \end{aligned}$$