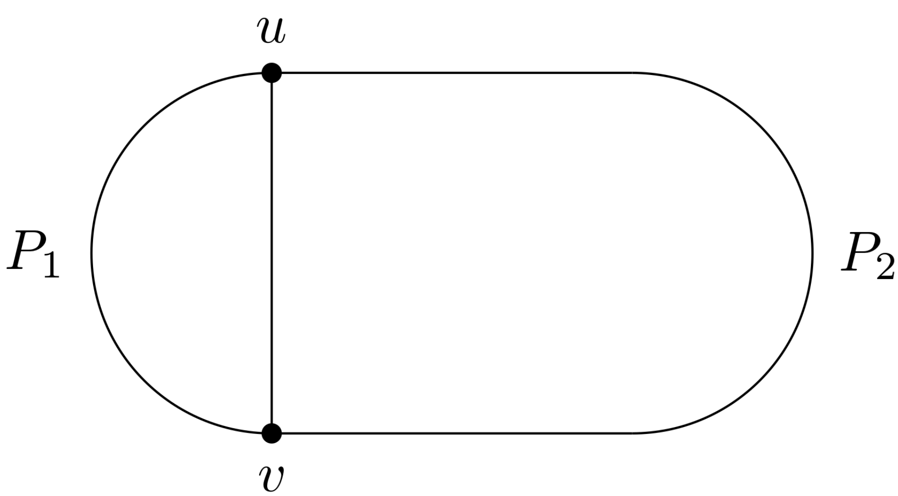
Definition 1. We define a family of graphs we call *bifurcated cycles* and denote as $Q_{m,n}$. As the name suggests, bifurcated cycles are cycles of length $m+n$ with a single chord which divides the cycle into paths P_1 and P_2 of lengths m and n.

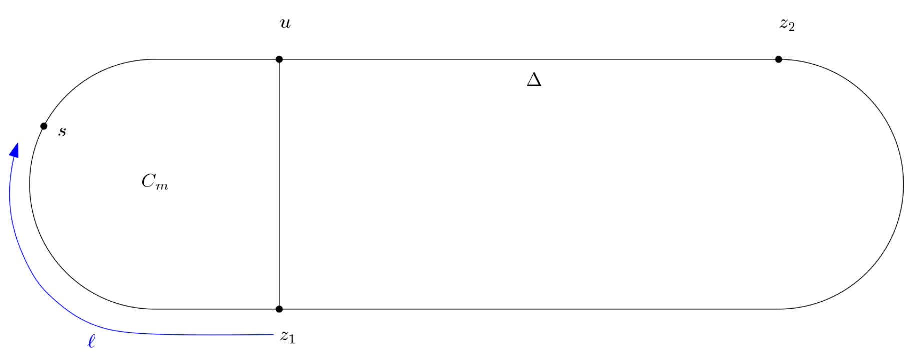


Theorem 1. The Bifurcated cycle $Q_{m,n}$ is 2-zombie win.

Proof. First, we show that a certain game state is a losing position for the survivor. Second, we show how to position the zombies at the start of the game so that – no matter where the survivor starts – a losing position is inevitably reached.

Part 1. Cornering the Survivor on a Cycle

Suppose that the game has reached the following state: the survivor is P_1, the first zombie is on v, and the second zombie is at a distance of Δ from u. Denote the length of the clockwise path from v to s as ℓ. Note that we must have $2 \le \ell \le m-1$, else the survivor is caught.



There are four posible z_2s-paths. We wish to guarantee that z_2 will follow the Δ-path towards u, which translates into the following inequalities:

$$\Delta + (m - \ell) \leq n - \Delta + 1 + m - \ell$$

and

$$\Delta + (m - \ell) \leq n - \Delta + \ell$$

or

$$\Delta + 1 + \ell \leq n - \Delta + 1 + m - \ell$$

and

$$\Delta + 1 + \ell \leq n - \Delta + \ell$$

□