# Analysis and Design of Model Based Fault Diagnosis Systems for Large Scale Models

### Exercises to the interactive session

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The exercises in the interactive session are based on a two-tank system. The system is presented in Section 1 and a model of the system is given in Section 2. Then Section 3 includes the exercises. To be able to solve the exercises the fault diagnosis toolbox at faultdiagnosistoolbox.github.io need to be downloaded and installed. The installation procedure is simple; download the ziparchive, unpack, and add the folder to your Matlab search path. Furthermore the following two matlab-files are needed

- skeleton.m
- WaterTankModel.m

Some code is given in skeleton.m and you are supposed to complete the solutions in this file. In WaterTankModel.m the model of the water tank system is defined.

## 1 The water tank process

The water tank system is illustrated in Figure 1, with an input signal u and four outputs  $y_1, \ldots, y_4$ . Water is pumped into the upper tank (tank 1) and the control signal u is controlled with respect to the water level in the upper tank. The tanks have level measurements  $y_1$  and  $y_2$ , and flow measurements  $y_3$  and  $y_4$ . Table 1 summarizes the faults that are considered in the system.

	V
$\overline{F_a}$	Actuator fault in the pump.
$F_{h1}$	Fault in sensor 1 measuring the water level $h_1$ in the upper tank, tank 1.
$F_{h2}$	Fault in sensor 2 measuring the water level $h_2$ in the lower tank, tank 2.
$F_{f1}$	Fault in sensor 3 measuring the flow $W_1$ between tank 1 and tank 2.
$F_{l2}$	Leakage between sensor 3 and tank 2.
$F_{l3}$	Leakage between tank 2 and sensor 4.
$F_{c1}$	Partial obstruction (clogging) in the pipe between tank 1 and tank 2.

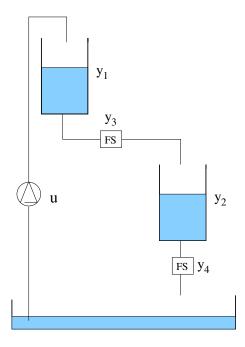


Figure 1: A schematic setup of two coupled water tanks. Each tank also has a level sensor  $(y_1 \text{ and } y_2)$ . The flow out of each tank is measured with a flow sensor (FS)  $(y_3 \text{ and } y_4)$ .

### 2 A model of the water tank system

A model for the water tank system is obtained from first principles as

$$A_1 \cdot dh_1 = W_{in} - W_1 \tag{1}$$

$$A_2 \cdot dh_2 = (1 - f_{l2})W_1 - W_2 \tag{2}$$

$$W_{in} = c_3 u + f_a \tag{3}$$

$$W_1 = (1 - f_{c1})c_1\sqrt{h_1} (4)$$

$$W_2 = c_2 \sqrt{h_2} \tag{5}$$

$$W_s = (1 - f_{l3})W_2 (6)$$

$$y_1 = h_1 + f_{h1} (7)$$

$$y_2 = h_2 + f_{h2} (8)$$

$$y_3 = W_1 + f_{f1} (9)$$

$$y_4 = W_s \tag{10}$$

$$dh_1 = \frac{d}{dt}h_1 \tag{11}$$

$$dh_2 = \frac{d}{dt}h_2 \tag{12}$$

where Table 2 describes the variables and parameters in the model. The different fault modes introduced in the system (see Table 1) is modeled as signals or deviations in constant parameters, e.g.,  $F_a$  denotes fault mode and  $f_a$  denotes the modeled fault where  $f_i = 0$  means that the corresponding fault is not present.

Table 2: Variables and parameters in the water tank model.

Parameters		
$A_i$	Bottom area of tank $i$ .	
$c_i$	Geometrical constants.	
Unknown variables		
$h_i$	Water level in $tank i$ .	
$W_{in}$	Water flow into tank 1.	
$W_i$	Water flow out from tank $i$ .	
$W_s$	Water flow through sensor $y_4$ .	

#### 3 Exercises

Different versions of the model (1)-(12) will be used. In Exercises 1-4 we will assume that sensor  $y_1$  is not available, in Exercise 5 no sensors is assumed to be installed, and in Exercise 6 that sensor  $y_3$  is not available. All versions of the model are loaded by running the script WaterTankModel.m.

#### Exercise 1.

- a) Run WaterTankModel.m that loads the different versions of the DAE-model (1)-(12). Here the model model should be used where sensor  $y_1$  has been removed.
- b) Use the command Lint to see if the model is well defined. What is the degree of redundancy of the model?
- c) Plot the structure of the model with unknown variables, known variables, and faults using the command PlotModel.
- d) Plot the structure of the unknown variables with the equivalence classes and faults using the command PlotDM.
- e) Compute the structural isolability of the model with mixed and integral causality using the command IsolabilityAnalysis.

**Exercise 2.** Rewrite the model used in Exercise 1 in state-space form and compute the structural isolability of this model. The model without faults can be written as

$$\dot{h}_1 = \frac{1}{A_1} \left( c_3 u - c 1 \sqrt{h_1} \right) \tag{13}$$

$$\dot{h}_2 = \frac{1}{A_2} \left( c_1 \sqrt{h_1} - c_2 \sqrt{h_2} \right) \tag{14}$$

$$y_2 = h_2 \tag{15}$$

$$y_3 = c_1 \sqrt{h_1} \tag{16}$$

$$y_4 = c_2 \sqrt{h_2} \tag{17}$$

- a) The state-space model for the fault free case is given in the script WaterTankModel.m. Extend this model with the faults and create a new DiagnosisModel object model\_statespace.
- b) Compare the structural isolability of the two analytically equivalent models and discuss the results.

Exercise 3. Continue with the model in Exercise 1.

- a) Compute all MSO sets and all MTES sets in the model using MSO and MTES respectively. Compare the difference.
- b) Check observability and index of the MSO sets with the commands IsObservable and IsLowIndex respectively.

- c) Select a minimum cardinality set of low-index MSO sets or MTES sets to achieve maximum single fault isolation under integral causality by using TestSelection.
- d) Compute the fault signature matrix of the selected tests with FSM and compute the isolability of these tests with the command IsolabilityAnalysisArrs.

#### Exercise 4.

a) Generate code for the selected tests. The tests should either be algebraic or sequential residual generators with integral causality.

As a help consider first the following example. The code for generating a sequential residual generator for the MSO set  $\{(2), (5), (6), (8), (9), (11)\}$  is

```
mso1 = [2 5 6 8 9 11];
model.MSOCausalitySweep( mso1 )
red1 = mso1(5);
M01 = setdiff(mso1,red1);
Gamma1 = model.Matching( M01 );
model.SeqResGen( Gamma1, red1, 'ResGen1' );
```

Here the 5:th equation in the MSO set, i.e., equation (9), is selected as the redundant equation and the rest of the equations, in the code stored in MO1, are used to solve for all unknown variables. To choose redundant equation consider the following properties:

- i) The causality obtained by looking at the result of MSOCausalitySweep.
- ii) Try to avoid selecting a differential constraint as the redundant equation.

The resulting code of the residual generator is stored in ResGen2.m.

b) Run and evaluate the response of the residuals for different fault scenarios using the provided simulation environment. Try at least the modes NF,  $F_{c1}$ ,  $F_a$ , and  $F_{h2}$ , which are prepared in the code, and compare the response of the residuals with the fault signature matrix computed in Exercise 3d).

Exercise 5. Consider the model model\_nosens in this exercise. It describes the water tank system without any sensors.

- a) Check that the model is exactly determined.
- b) Specify that  $h_1, h_2, W_{in}, W_1$ , and  $W_s$  are the possible sensor locations by using PossibleSensorLocations.
- c) Compute all minimal sensor sets with maximum possible single fault isolability given that added sensors cannot fail with SensorPlacementIsolability.
- d) Assume that all sensors have faults by using SensorLocationsWithFaults.
- e) Compute all minimal sensor sets with maximum possible single fault isolability given sensors can fail. Compare the results with the solutions obtained in Exercise 5c).

**Exercise 6.** In this exercise we consider the water tank system without sensor  $y_3$ . The model is defined in WaterTankModel.m with the name model\_no\_y3.

- a) Compute the structural isolability of the model with mixed and integral causality.
- b) Design a diagnostic system with minimum number of sequential residual generators providing maximum possible isolability under integral causality.
- c) To the diagnostic system developed in Exercise 6b) add a minimum number of sequential residual generators with arbitrary differential causality in order to obtain maximum single fault isolability.
- d) Evaluate the diagnostic system by comparing the fault response in simulations with the fault signature matrix.