Timing the Core Equity Factor

Important remarks:

- This work is an **INVIVIDUAL WORK**
- Deadline: TbD
- A complete .zip file must be sent by email at guillaume.monarcha.univ@gmail.com before this deadline, with the following content:
    - The working code (Jupyter notebook)
    - The report in pdf format
    - Any other additional files used to conduct this work.
- Format and content of the report
    - A maximum of 3 pages
    - A clear description and justification of the methodological options employed to conduct this work.
    - A clear presentation of the results.
    - No need to incorporate any pieces of code in the report.

Dataset: DATA.xlsx

Content: Monthly returns of European stocks and of the Eurostoxx 50 index (the market benchmark), from June 2001 to November 2024.

Guidelines

PART 1

1. Through principal component analysis of the European equity returns, extract the first latent factor (principal component) F_1 of the European equity market, i.e. the core factor that drives the returns of European equities.

   We define the core equity factor as the first principal component, rescaled to the same volatility as that of the benchmark.

   Note: if the eigenvector of the first principal component mainly contents negative values, then consider $-F_1$ instead of F_1, to obtain a positive correlation between this first factor and the returns of individual stocks.

2. Estimate the exposures $\hat{b}_{1,i}$ of each stock *i* to the core equity factor from the following linear model:

$$r_{i,t} = \alpha_i + \sum_{k=1}^{K} b_{i,1} F_{1,t} + \varepsilon_t$$

3. Compute the weights of the equity portfolio designed to replicate the core equity factor, defined as:

$$Argmin_{w_k}\left(w_1{}'\hat{\Omega}w_1\right)$$

subject to:

$$\sum_{i=1}^{N} w_{1,i} = 1$$

$$w_{1,i} \geq 0$$

$$\sum_{i=1}^{N} w_{1,i}\hat{b}_{i,1} = 1$$

with:

- $\hat{\Omega}$ the sample covariance matrix of the stock returns,
- $\hat{b}_{i,1}$ the estimated sensitivity of stock i to the 1st core equity factor,
- w_1 the optimal weight vector of the replicating portfolio for the 1st core equity factor.

PART 2

4. Estimate the alpha of this portfolio against the market benchmark.

5. Assess the impact of estimation errors in the covariance matrix $\widehat{\Omega}$ on the alpha of the replicating portfolio. Compute the 95% confidence interval of the estimated alpha.

PART 3

6. From the price index of the estimated core portfolio ($I_{1,t}$), estimate the global trend and its slope from a local linear trend model of the form:

$$I_{1,t} = T_{1,t} + \varepsilon_t$$

$$T_{1,t} = T_{1,t-1} + S_{1,t-1} + u_t$$

$$S_{1,t} = S_{1,t-1} + v_t$$

where $T_{1,t}$ is the trend component of the price index, $S_{1,t}$ is the slope of the trend, ε_t the measurement error, and u_t and v_t the state innovation errors associated with the trend and the slope respectively.

Note 1 : that the variance terms should respect the following relationships: $\sigma_u^2 = \frac{1}{2}\sigma_\varepsilon^2$ *and* $\sigma_v^2 = \frac{1}{2}\sigma_u^2$.

Note 2 : we assume that the weights of the core portfolio are constant over the period.

7. Retrieve the track record of an investment strategy that follows the investment rules described below:

If $S_{1,t-1} > 0$, then $R_{s,t} = R_{c,t}$ (long the core portfolio)

If $S_{1,t-1} \leq 0$, then $R_{s,t} = \frac{3\%}{12}$ (invested in cash, assuming a 3% constant risk free rate)

8. Test if the Sharpe ratio of this long/flat investment strategy is due to luck.