

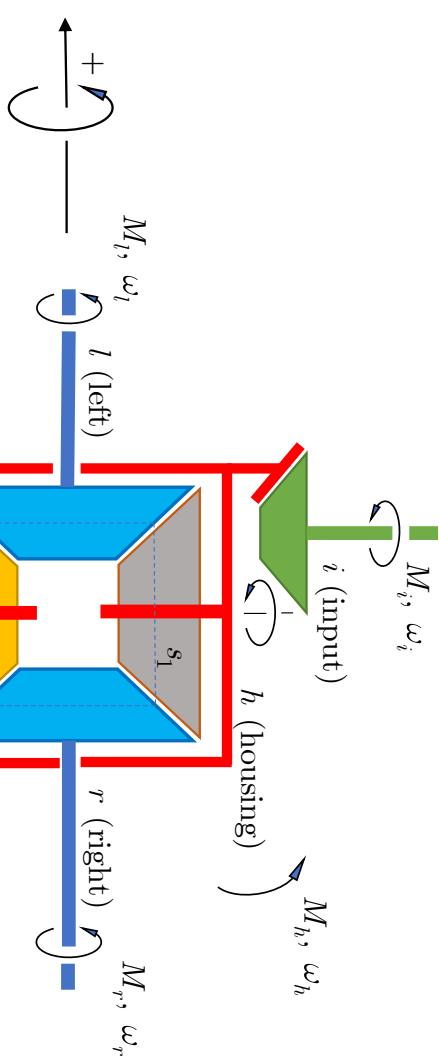
Schema cineto-statico del differenziale aperto (open) – 1/2

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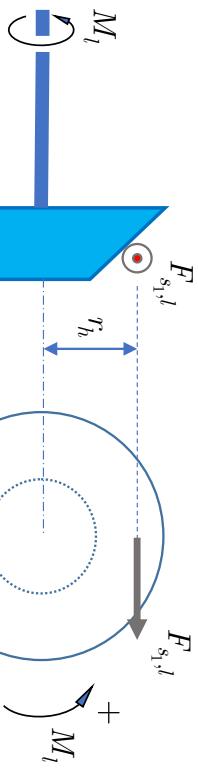
Equazioni cinematiche di congruenza (da f. di Willis)

$$\tau_{r,l}^0 = \frac{\omega_r^0}{\omega_l^0}$$

$$\tau_{r,l}^0 = \frac{\omega_r^0 \omega_s^0}{\omega_s^0 \omega_l^0} = -\frac{z_s z_l}{z_r z_s} = -\frac{z_l}{z_r} = -1 \quad \square \quad \boxed{\omega_h = \frac{\omega_l + \omega_r}{2}}$$

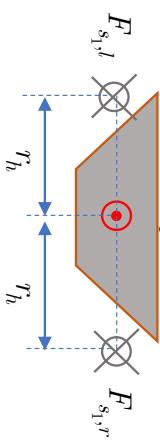


Schema di carico del solare l (left)



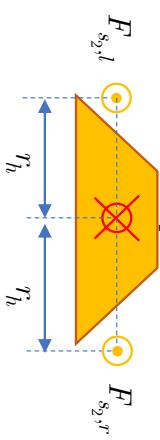
$$M_l - F_{s_1,l} r_h - F_{s_1,r} r_h = 0$$

Schema di carico del satellite s1



$$+ \cancel{\otimes} \quad F_{s_1,l} + F_{s_1,r} - F_{h_1} = 0$$

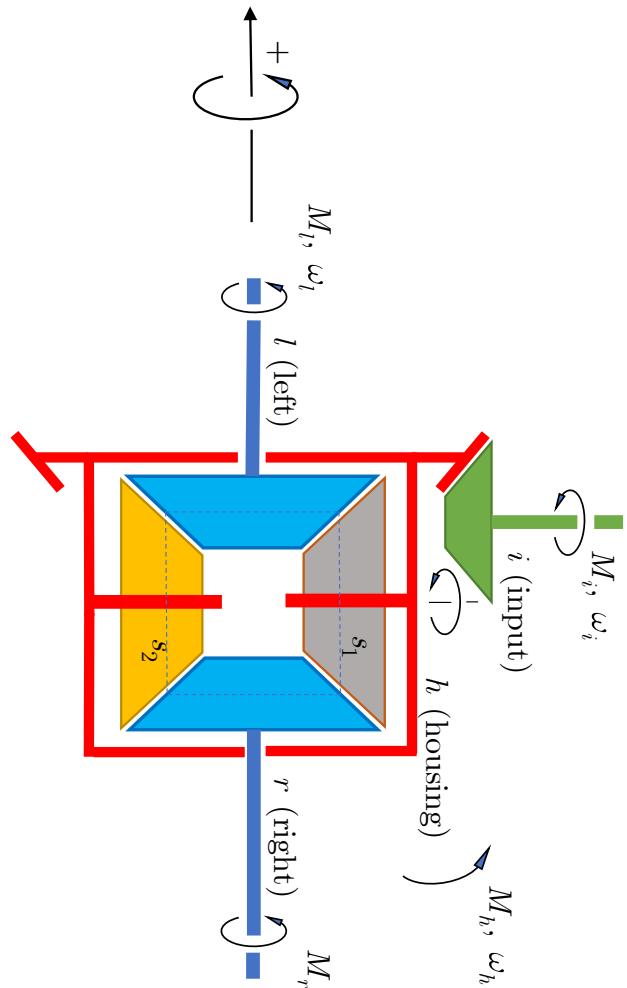
Schema di carico del satellite s2



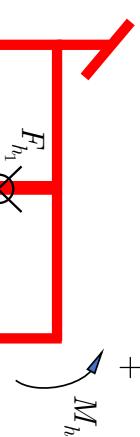
$$+ \cancel{\otimes} \quad -F_{s_2,l} - F_{s_2,r} + F_{h_2} = 0$$

Schema cineto-statico del differenziale aperto (open) – 2/2

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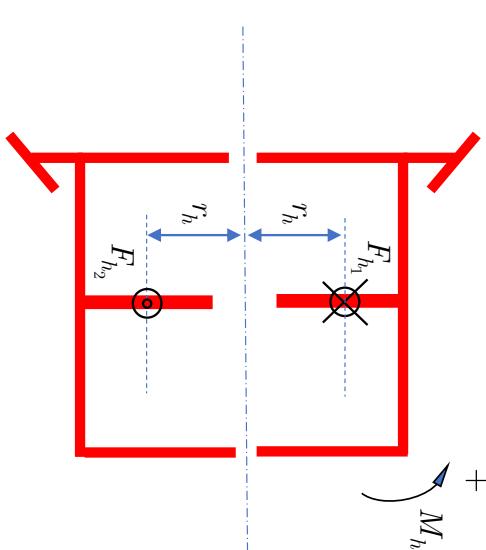
Schema di carico del portatreno h (housing)



$$F_{h_1} - F_{h_2} = 0$$

$$M_{h_1} + F_{h_2}r_h + F_{h_1}r_h = 0$$

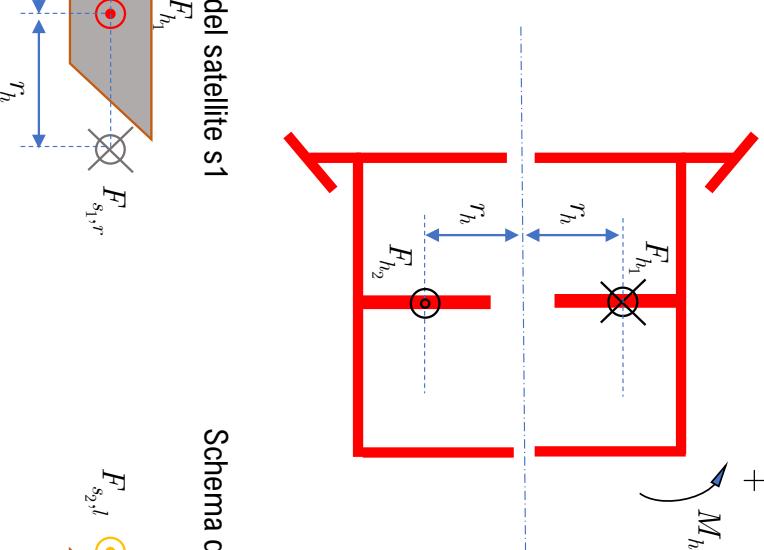
Schema di carico del satellite s1



$$+ \cancel{\otimes} \quad F_{s_1,l} + F_{s_1,r} - F_{h_1} = 0$$

$$F_{s_2,r} - F_{s_1,r} \quad r_h = 0$$

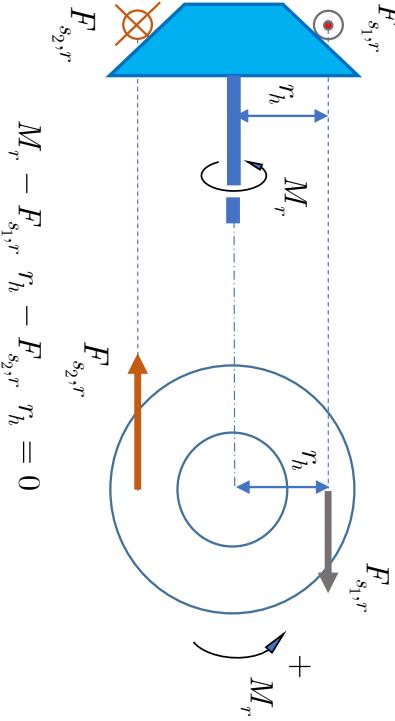
Schema di carico del satellite s2



$$+ \cancel{\otimes} \quad -F_{s_2,l} - F_{s_2,r} + F_{h_2} = 0$$

$$F_{s_1,l}r_h - F_{s_2,r}r_h = 0$$

Schema di carico del solare r (right)

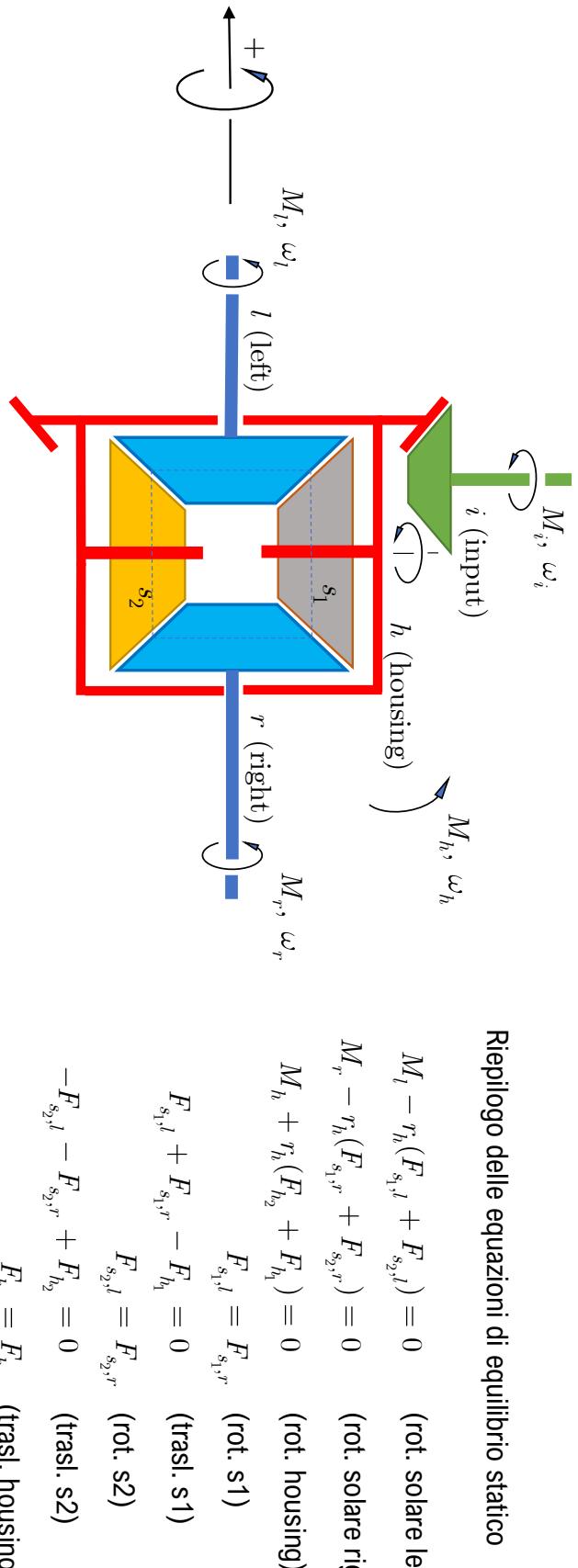


$$F_{s_2,r} \quad r_h = 0$$

$$M_r - F_{s_1,r} \quad r_h = F_{s_2,r} \quad r_h = 0$$

Schema cineto-statico del differenziale aperto (open) – equilibrio

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Equazioni: 8; Parametro noto: 1: M_h ; Incognite: 8: $F_{s_1,l}; F_{s_2,l}; F_{s_1,r}; F_{s_2,r}; F_{h_1}; M_l; M_r; F_{h_1}; F_{h_2}$

Soluzione (variabili interessanti)

$$\left. \begin{array}{l} M_l = -\frac{M_h}{2} \\ M_r = -\frac{M_h}{2} \end{array} \right\} \text{(azioni che ruote fanno sui semi-assi)} \quad \Rightarrow \quad \left. \begin{array}{l} M_{w,l} = -M_l = \frac{M_h}{2} \\ M_{w,r} = -M_r = \frac{M_h}{2} \end{array} \right\}$$

$$F_{s_1,l} = -\frac{M_h}{4r_h} \quad M_h > 0; \quad F_{s_1,l} < 0 \quad (\text{opposto al disegno})$$

Azione dai semi-assi alle ruote

$$\left. \begin{array}{l} M_{w,l} = -M_l = \frac{M_h}{2} \\ M_{w,r} = -M_r = \frac{M_h}{2} \end{array} \right\}$$

Schema cineto-statico del differenziale aperto (open) – forze a terra

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Condizione Power On

$$M_h > 0$$

$$F_r^{(x)} = \frac{M_h}{2r}$$

$$M_r = -\frac{M_h}{2}$$

r (right)

Condizione Power Off

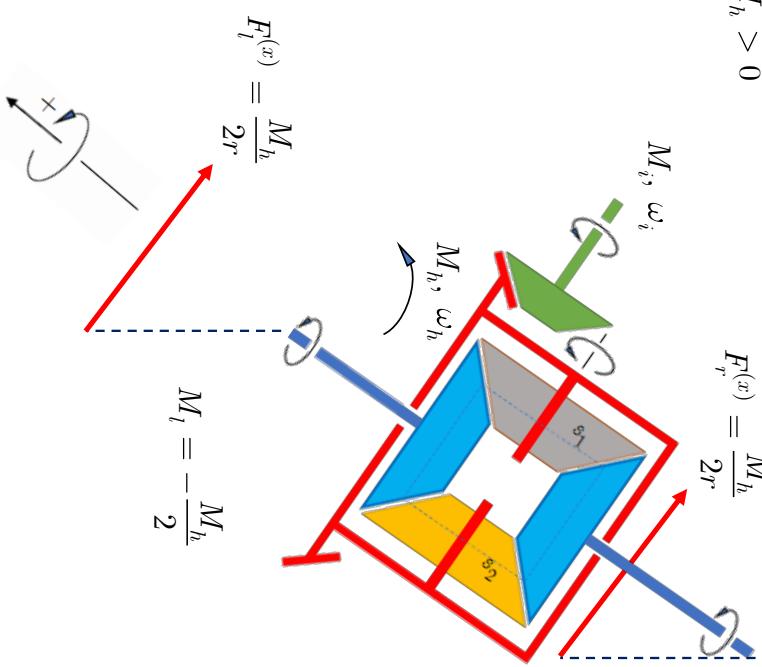
$$M_h < 0$$

$$M_i, \omega_i$$

$$F_r^{(x)} = \frac{|M_h|}{2r}$$

$$M_r = -\frac{|M_h|}{2}$$

r (right)

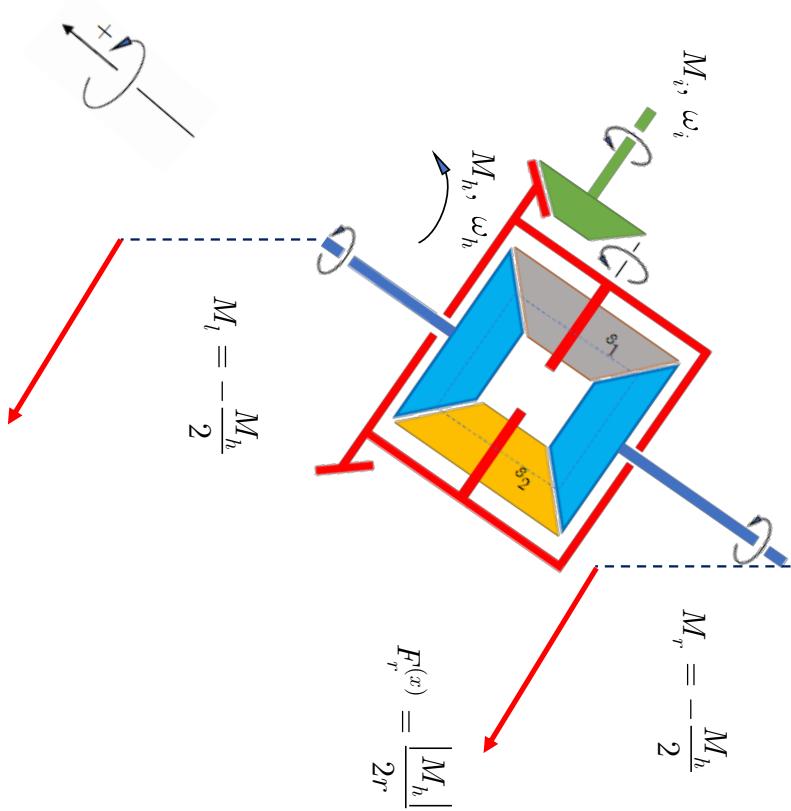


Azione dalle ruote ai semi-assi

$$F_l^{(x)} = \frac{M_h}{2r}$$

$$M_l = -\frac{M_h}{2}$$

$$M_r = -\frac{M_h}{2}$$



Azione dai semi-assi alle ruote

$$M_{w,l} = -M_l = \frac{M_h}{2}$$

$$M_{w,r} = -M_r = \frac{M_h}{2}$$

$$F_l^{(x)} = \frac{|M_h|}{2r}$$

Schema cineto-statico di differenziale LSD – torque sensitive – 1/2

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precarico costante

Equazioni cinematiche di congruenza (da f. di Willis)

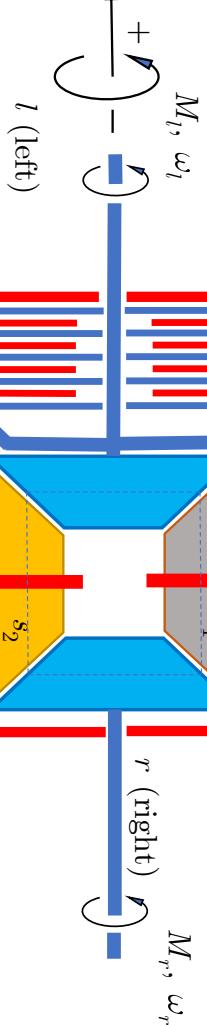
dischi di frizione

$$\tau_{r,l}^0 = \frac{\omega_r^0}{\omega_l^0}$$

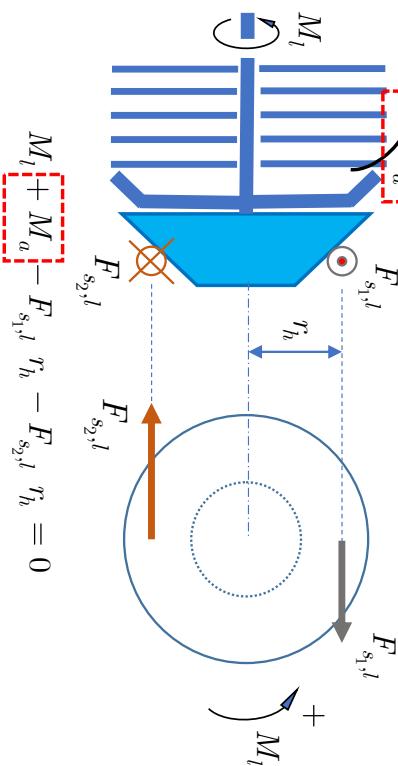
$$\tau_{r,l}^0 = \frac{\omega_r^0 \omega_s^0}{\omega_l^0 \omega_s^0} = -\frac{z_s z_l}{z_r z_s} = -\frac{z_l}{z_r} = -1 \quad \square \quad \boxed{\omega_h = \frac{\omega_l + \omega_r}{2}}$$

$$F_{s_1,l} r_h - F_{s_1,r} r_h = 0$$

$$F_{s_2,l} r_h - F_{s_2,r} r_h = 0$$



Schema di carico del solare l (left)

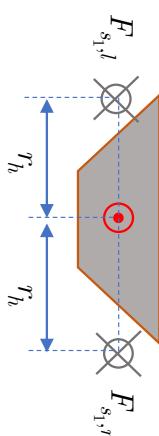


$$+ \otimes \quad F_{s_1,l} + F_{s_1,r} - F_{h_1} = 0$$

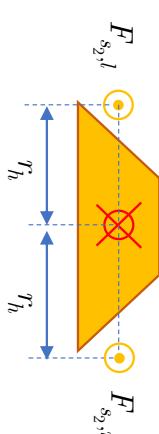
$$F_{s_2,l} - F_{s_2,r} = 0$$

$$M_l + M_a - F_{s_1,l} r_h - F_{s_2,l} r_h = 0$$

Schema di carico del satellite s1



Schema di carico del satellite s2

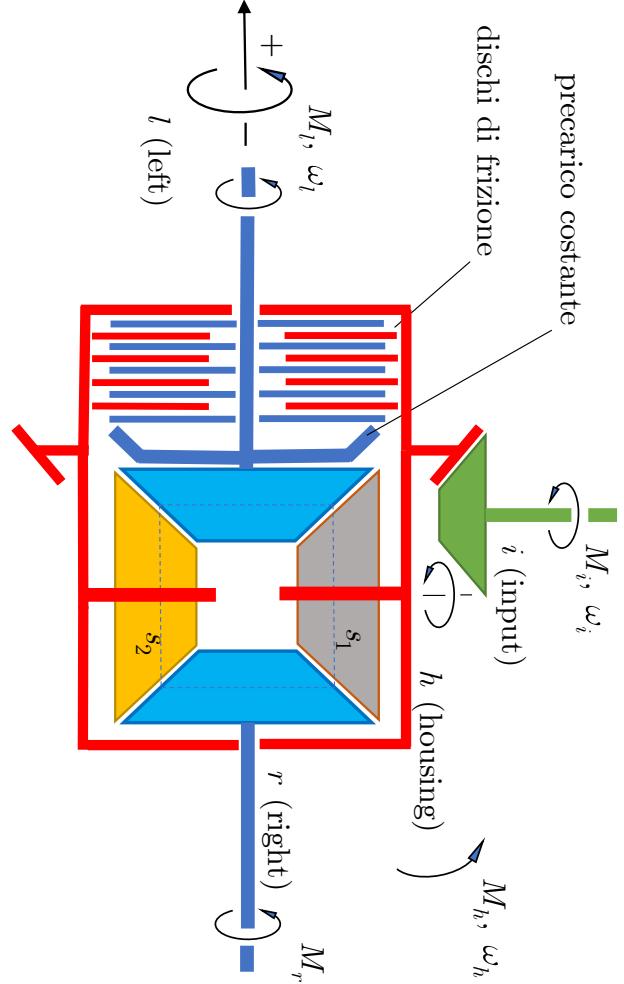


$$+ \otimes \quad -F_{s_2,l} - F_{s_2,r} + F_{h_2} = 0$$

$$F_{s_2,l} r_h - F_{s_2,r} r_h = 0$$

Schema cineto-statico di differenziale LSD – torque sensitive – 2/2

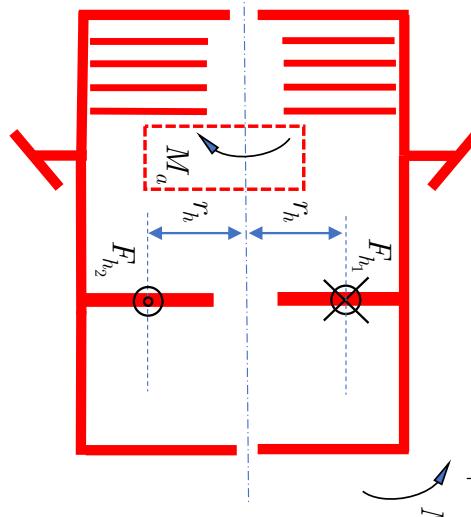
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Schema di carico del portatreno h (housing)

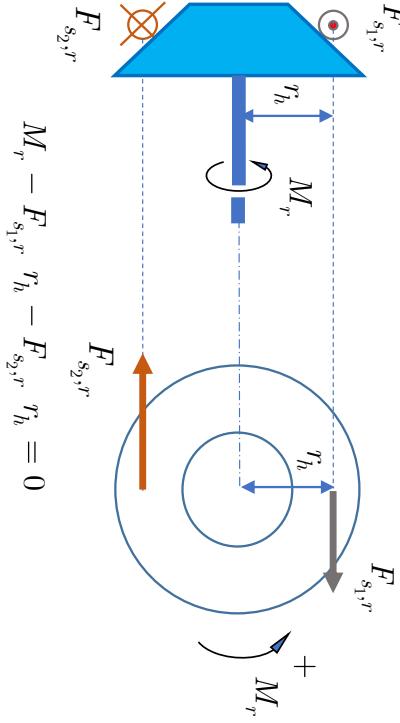
$$+ \otimes F_{h_1} - F_{h_2} = 0$$

$$M_h - M_a + F_{h_2} r_h + F_{h_1} r_h = 0$$



Schema di carico del satellite s1

Schema di carico del satellite s2



Schema di carico del satellite s1

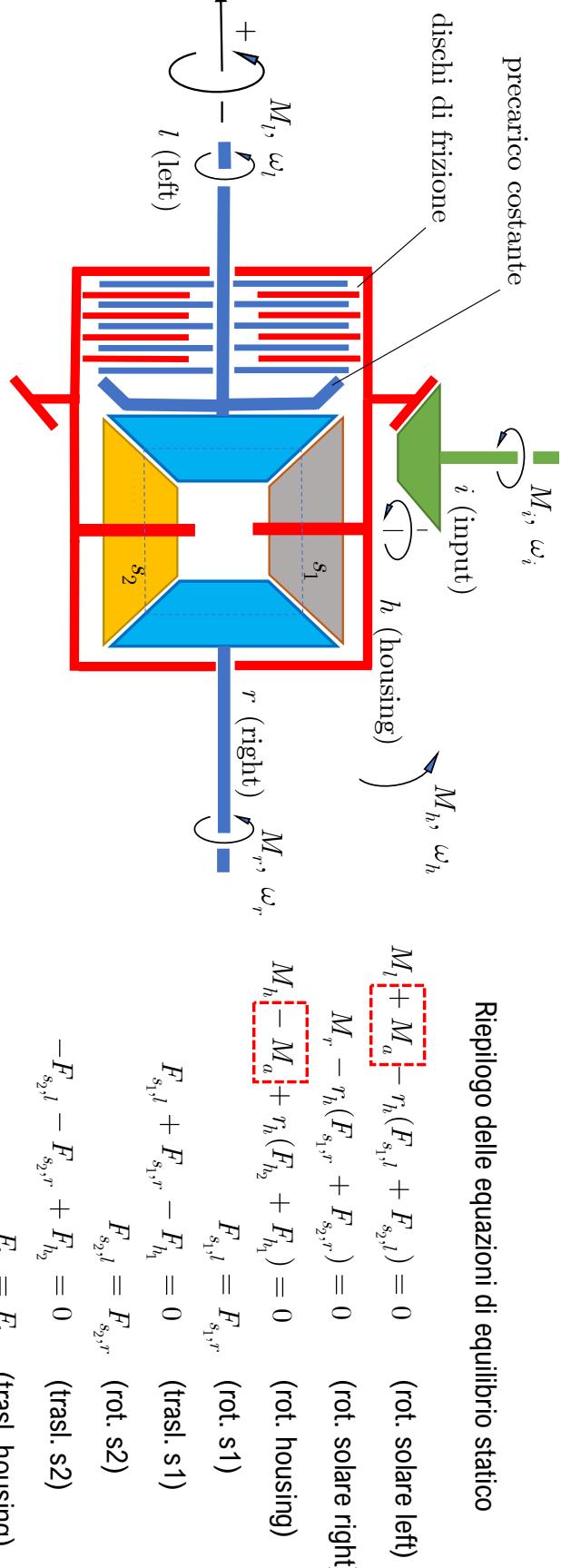
$$+ \otimes F_{s1,l} + F_{s1,r} - F_{h_1} = 0$$

$$+ \otimes -F_{s2,l} - F_{s2,r} + F_{h_2} = 0$$

$$M_r - F_{s1,r}, \quad r_h - F_{s2,r}, \quad r_h = 0$$



Schema cineto-statico di differenziale LSD – torque sensitive – equilibrio



Equazioni: 8; Param. noto: 1: M_h ; Param. distribuzione M_a ; Incognite: 8: $F_{s_1,l}; F_{s_2,l}; F_{s_1,r}; F_{s_2,r}; M_i; M_r; F_{h_1}; F_{h_2}$

Soluzione (variabili interessanti)

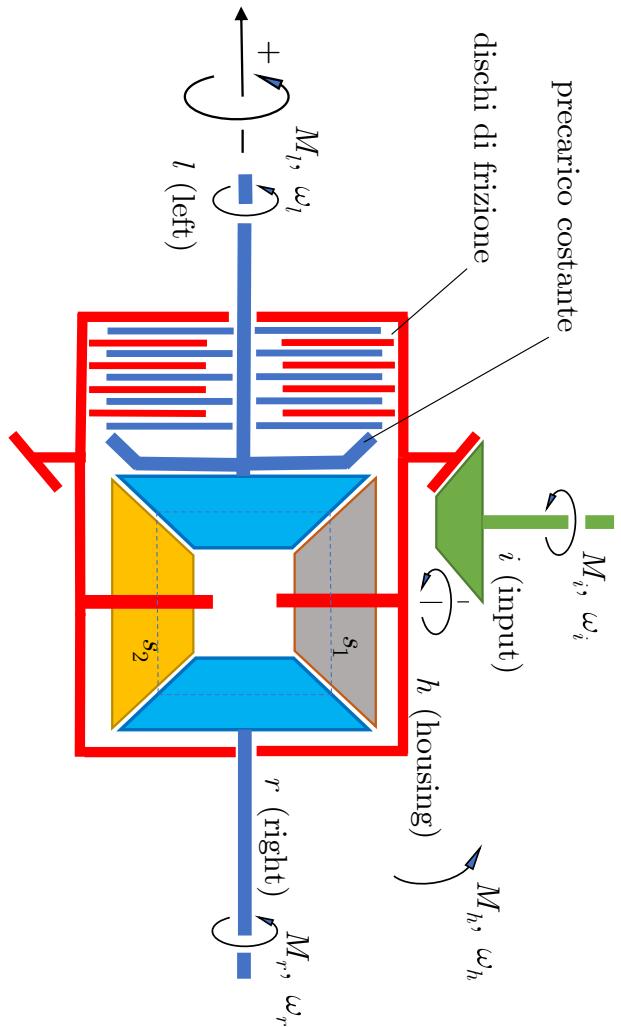
$$\left. \begin{aligned} M_l &= -\frac{(M_h + M_a)}{2} \\ M_r &= -\frac{(M_h - M_a)}{2} \\ F_{s_1,l} &= -\frac{(M_h - M_a)}{4r_h} \end{aligned} \right\} \text{(azioni che ruote fanno sui semi-assi)} \quad \rightarrow$$

Azione dai semi-assi alle ruote

$$\left. \begin{aligned} M_{w,l} &= -M_l = \frac{(M_h + M_a)}{2} = \frac{M_h}{2} + \frac{M_a}{2} \\ M_{w,r} &= -M_r = \frac{(M_h - M_a)}{2} = \frac{M_h}{2} - \frac{M_a}{2} \end{aligned} \right\} \rightarrow M_{w,l} - M_{w,r} = \Delta M_{l,r} = M_a$$

Schema cineto-statico di differenziale LSD – osservazioni

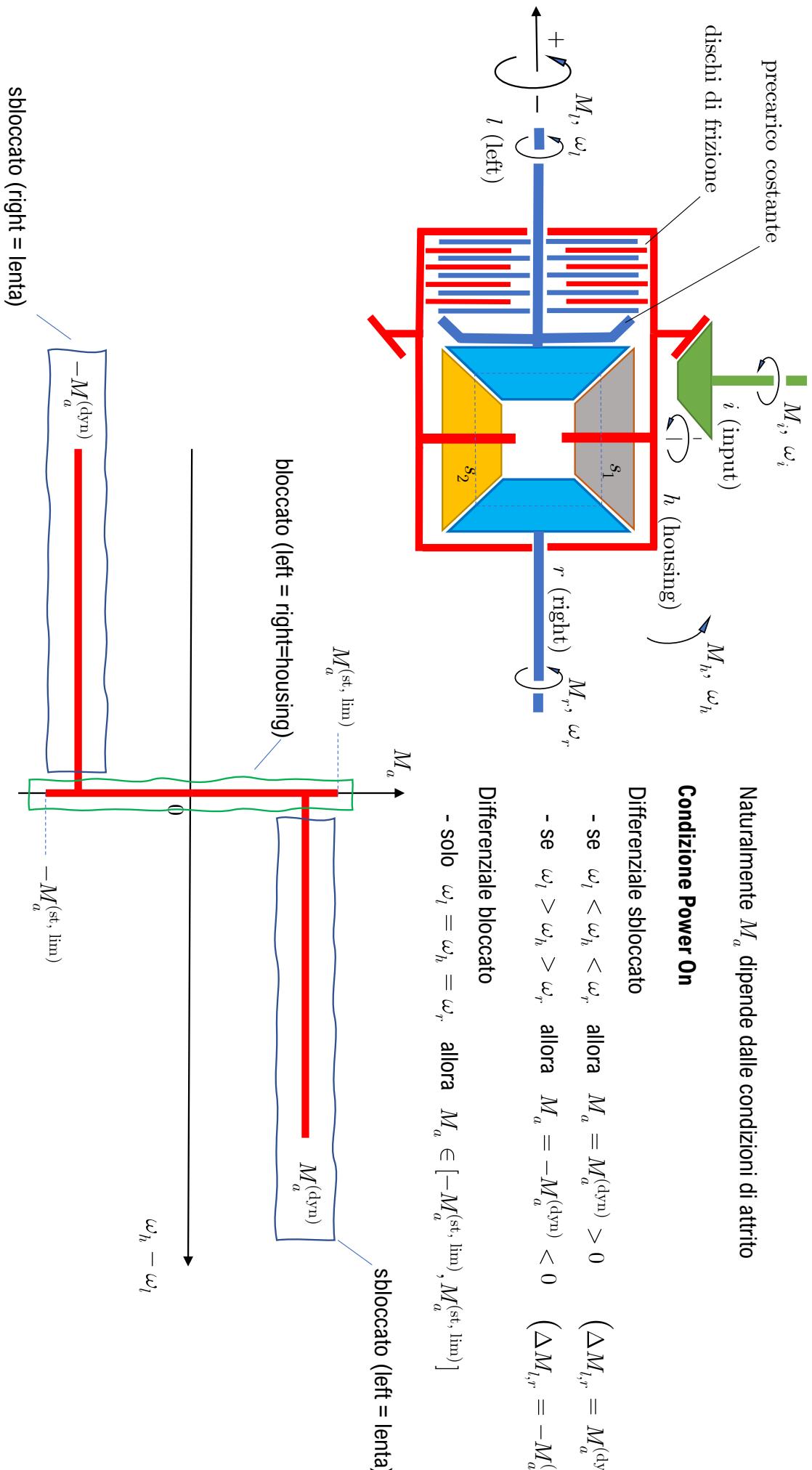
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Osservazioni

- M_a rappresenta la coppia necessaria per sbloccare solare left da housing
- la medesima risulta la differenza di coppia trasmessa fra ruota l e r
- $M_{w,l} - M_{w,r} = \Delta M_{l,r} = M_a$
- tale M_a può essere costante oppure dipendere da M_h a seconda del tipo di differenziale. Questo distingue diverse tipologie di differenziali come i geared differentials (Torsen) ed i clutch-patch differentials (Salisbury e Trac-loc). Si potrà definire:
- $M_a = M_a(M_h)$ genericamente (comodo per un Torsen)
- $M_a^{(st,lim)} = M_a^{(st,lim)}(M_h)$ facendo riferimento a differenziali con frizioni con carico costante o precarico + dipendenza da M_h secondo varie leggi

Schema cineto-statico di differenziale LSD – momento di attrito



Funzionamento differenziale LSD – torque sensitive – sbloccato power on

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Condizione Power On

$M_h > 0$

Ruota lenta: left

$M_a = M_a^{(\text{dyn})} > 0;$
 $(\omega_l < \omega_h < \omega_r)$

$F_l^{(x)} = \frac{1}{r} \left[\frac{M_h}{2} + \frac{M_a}{2} \right]$

$M_l = -\frac{M_h}{2} - \frac{M_a}{2}$

$M_r = -\frac{M_h}{2} + \frac{M_a}{2}$

M_h, ω_h

$F_r^{(x)} = \frac{1}{r} \left[\frac{M_h}{2} - \frac{M_a}{2} \right]$

$M_w,l - M_w,r = \Delta M_{l,r} = M_a > 0$

Condizione Power On

$M_h > 0$

Ruota lenta: right

$M_a = -M_a^{(\text{dyn})} < 0;$
 $(\omega_l > \omega_h > \omega_r)$

$F_l^{(x)} = \frac{1}{r} \left[\frac{M_h}{2} + \frac{M_a}{2} \right]$

$M_l = -\frac{M_h}{2} - \frac{M_a}{2}$

$M_r = -\frac{M_h}{2} + \frac{M_a}{2}$

M_h, ω_h

$F_r^{(x)} = \frac{1}{r} \left[\frac{M_h}{2} - \frac{M_a}{2} \right]$

$M_w,l - M_w,r = \Delta M_{l,r} = M_a < 0$

Azione dalle ruote ai semi-assi

$$M_l = -\frac{M_h}{2} - \frac{M_a}{2}$$

Azione dai semi-assi alle ruote

$$M_{w,l} = -M_l = \frac{M_h}{2} + \frac{M_a}{2}$$

Funzionamento differenziale LSD – torque sensitive – sbloccato power off

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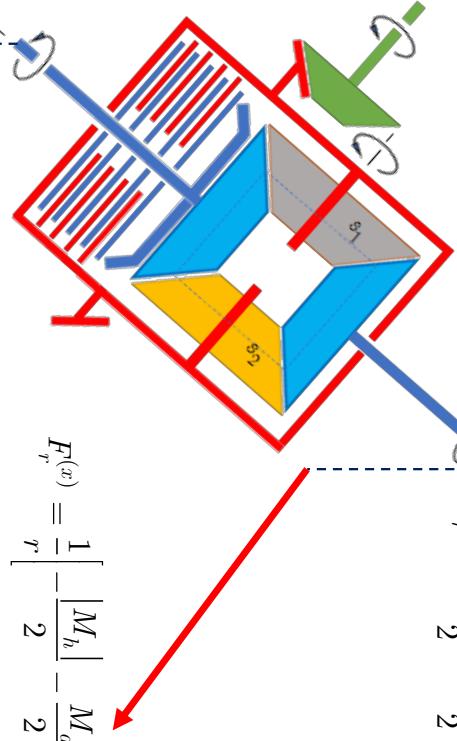
Condizione Power Off

$$M_h < 0$$

Ruota lenta: left

$$M_a = M_a^{(\text{dyn})} > 0; \\ (\omega_l < \omega_h < \omega_r)$$

$$M_h, \omega_h \\ M_r = -\frac{M_h}{2} + \frac{M_a}{2}$$



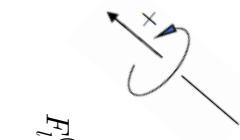
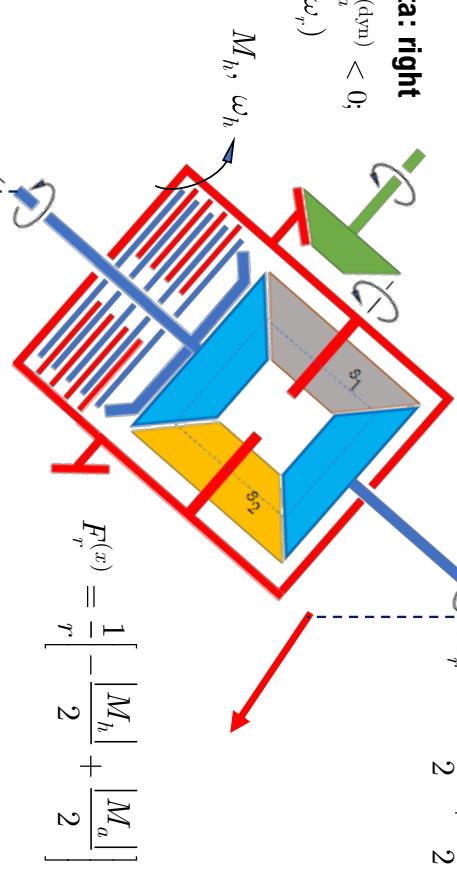
Condizione Power Off

$$M_h < 0$$

Ruota lenta: right

$$M_a = -M_a^{(\text{dyn})} < 0; \\ (\omega_l > \omega_h > \omega_r)$$

$$M_h, \omega_h \\ M_r = -\frac{M_h}{2} + \frac{M_a}{2}$$



$$F_l^{(x)} = \frac{1}{r} \left[-\frac{|M_h|}{2} - \frac{|M_a|}{2} \right]$$

Azione dalle ruote ai semi-assi

$$M_l = -\frac{M_h}{2} - \frac{M_a}{2}$$

$$M_{w,l} - M_{w,r} = \Delta M_{l,r} = M_a > 0$$



Azione dai semi-assi alle ruote

$$F_l^{(x)} = \frac{1}{r} \left[-\frac{|M_h|}{2} - \frac{|M_a|}{2} \right] \\ M_l = -\frac{M_h}{2} - \frac{M_a}{2}$$

$$M_{w,l} - M_{w,r} = \Delta M_{l,r} = M_a < 0$$

$$F_r^{(x)} = \frac{1}{r} \left[-\frac{|M_h|}{2} - \frac{|M_a|}{2} \right] \\ M_r = -\frac{M_h}{2} + \frac{M_a}{2}$$

Azione dalle ruote ai semi-assi

$$M_l = -\frac{M_h}{2} - \frac{M_a}{2} \\ M_r = -\frac{M_h}{2} + \frac{M_a}{2}$$

Funzionamento differenziale LSD – torque sensitive – bloccato power on

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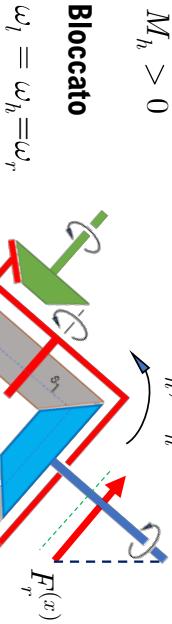
Condizione Power On

$$M_h > 0$$

Bloccato

$$\omega_l = \omega_h = \omega_r$$

$$M_a > 0$$



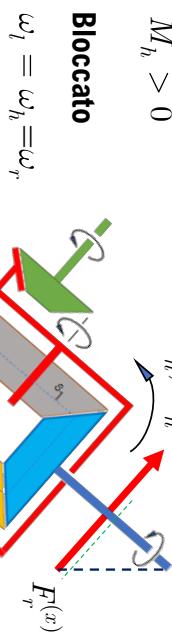
Condizione Power On

$$M_h > 0$$

Bloccato

$$\omega_l = \omega_h = \omega_r$$

$$M_a = 0$$



Condizione Power On

$$M_h > 0$$

Bloccato

$$\omega_l = \omega_h = \omega_r$$

$$M_a < 0$$



Zona in stiction up

$$F_l^{(x)} = \frac{1}{r} \left[\frac{M_h}{2} + \frac{M_a}{2} \right]$$

$$F_r^{(x)} > F_r^{(s)}$$



Zona zero attrito

$$F_l^{(x)} = F_r^{(x)}$$



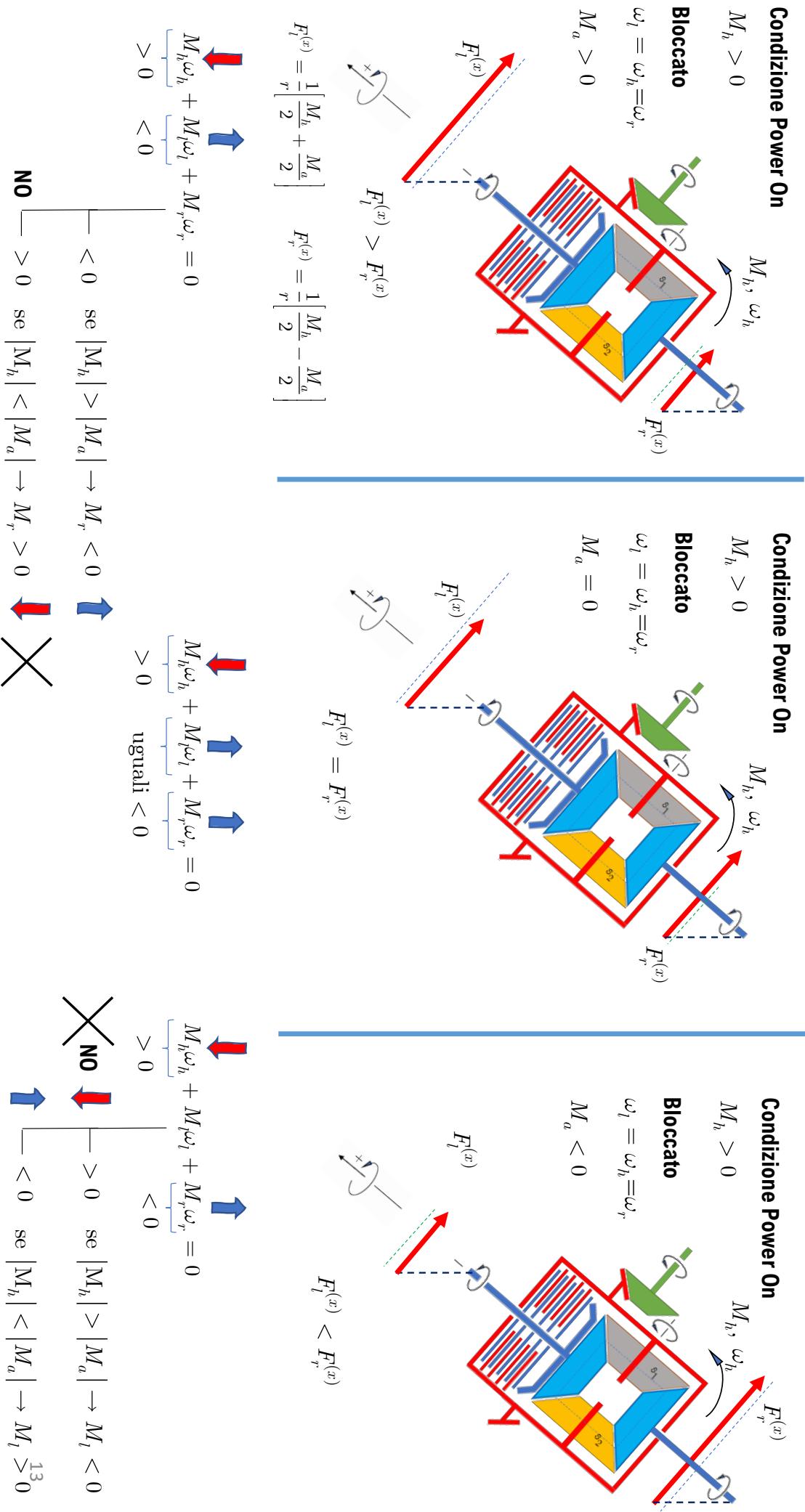
Zona in stiction down

$$F_l^{(x)} < F_r^{(x)}$$



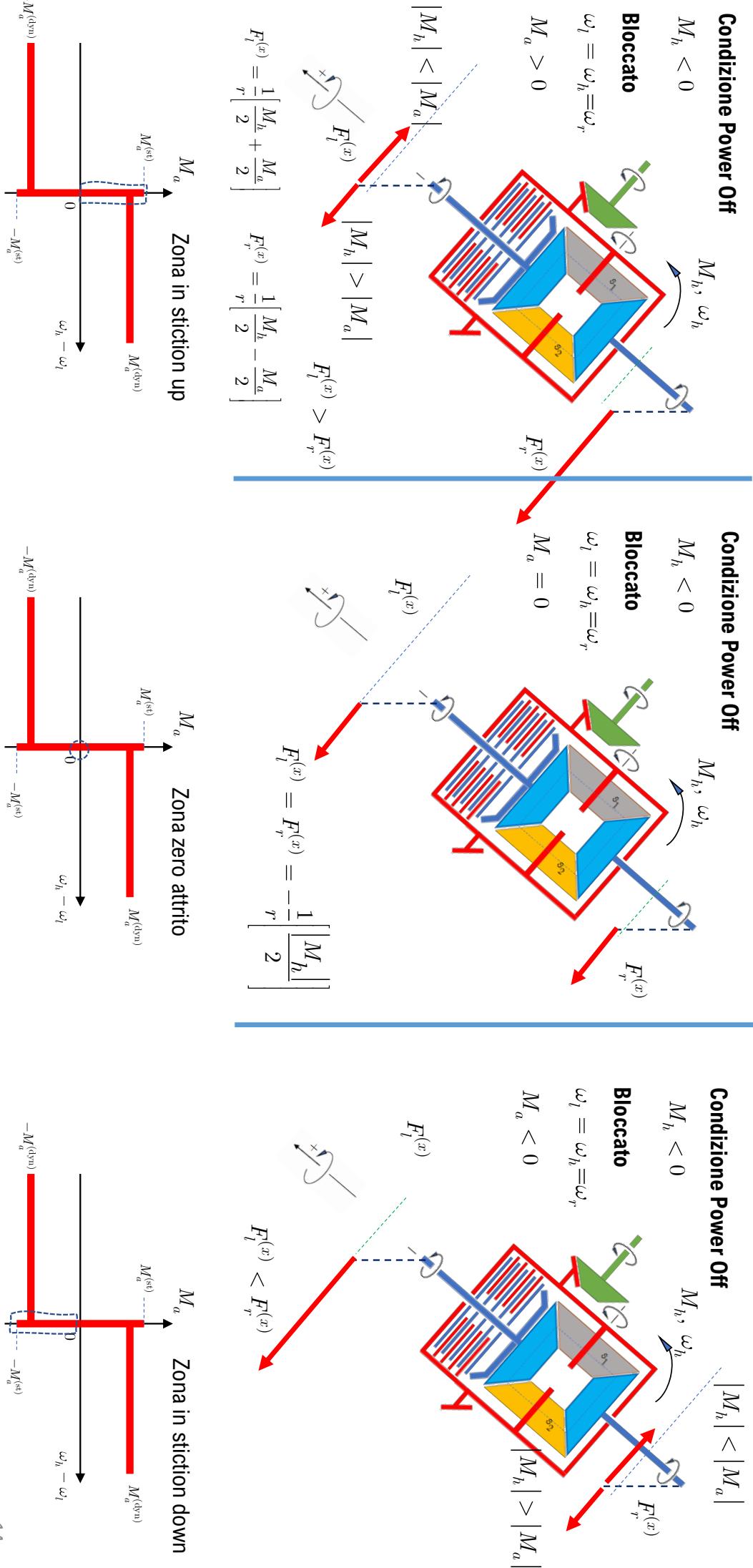
Funzionamento differenziale LSD – torque sensitive – bloccato power on bil. potenza

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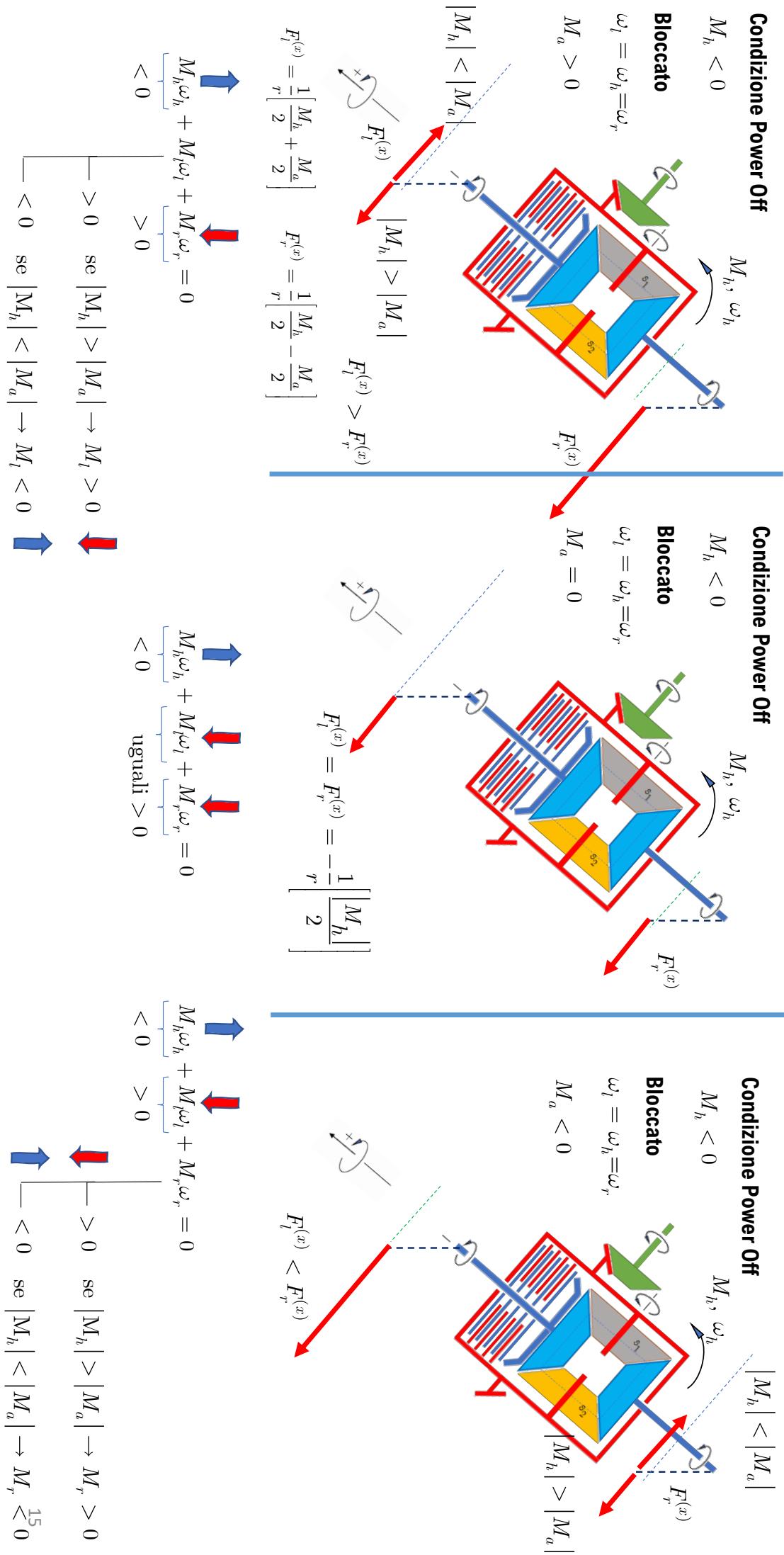
Funzionamento differenziale LSD – torque sensitive – bloccato power off

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Funzionamento differenziale LSD – torque sensitive – **bloccato power off** bil. potenza

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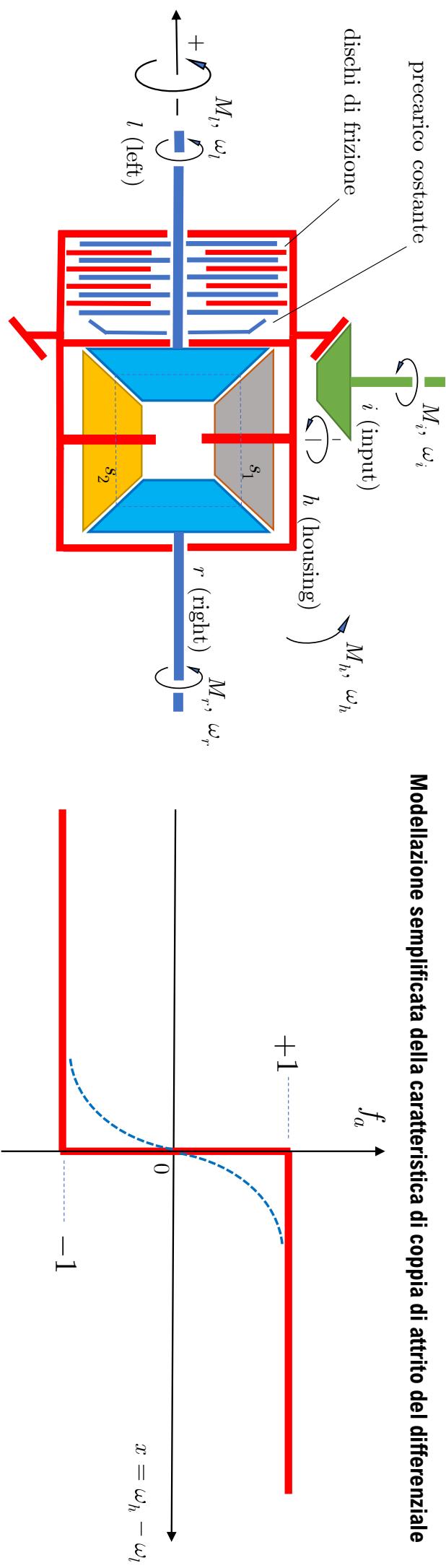


Schema cineto-statico di differenziale LSD – torque sensitive – stick-slip rilassato

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precarico costante

Modellazione semplificata della caratteristica di coppia di attrito del differenziale



Possibili scelte di funzioni di rilassamento del gradino (-1, +1) a $x=0$

$$f_{a1}(x) = \tanh(\chi x) = \tanh\left(\frac{1}{\epsilon}x\right) = \frac{1 - e^{-2\chi x}}{1 + e^{-2\chi x}}; f_{a1}'(x) = \chi(1 - f_{a1}(x)^2)$$

$$f_{a1}(0) = 0; f'_{a1}(0) = \chi$$

$$M_a^{(\text{st, lim})} = M_a^{(\text{dyn})} = \mu_s \bar{P} r_e;$$

$$f_{a2}(x) = \frac{\arctan(\chi x)}{(\pi / 2)} = \frac{\arctan(\frac{1}{\epsilon}x)}{(\pi / 2)}; f_{a2}'(x) = \frac{2\chi}{\pi(1 + \chi^2 x^2)}$$

$$f_{a2}(0) = 0; f'_{a2}(0) = \frac{2\chi}{\pi}$$

χ (grande); ϵ (piccolo);

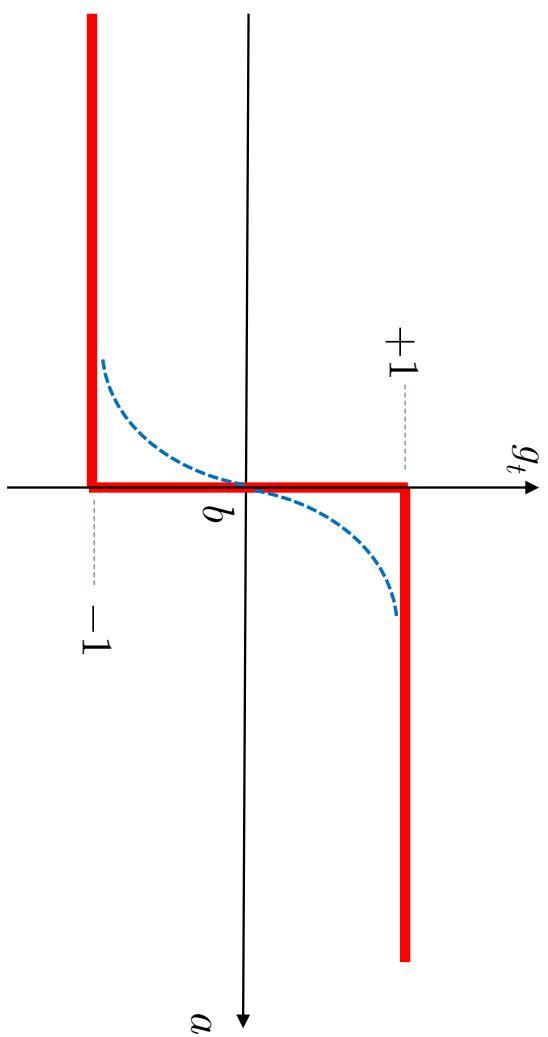
Modello rilassato (il differenziale non è mai veramente bloccato)

$$M_a(x) = M_a^{(\text{dyn})} f_a(x) \quad x \leftarrow \omega_h - \omega_l$$

$$f_{a2}(0) = 0; f'_{a2}(0) = \frac{2\chi}{\pi}$$

Rendere continua (e derivabile) una funzione discontinua (rilassandola)

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Funzione «greater than» rilassata

$$g_t(a, b) = \tanh[\chi(a - b)] \quad \begin{aligned} 0 &\leftarrow a = b \\ -1 &\leftarrow a < b \end{aligned}$$

Funzione «lower than» rilassata

$$l_t(a, b) = -\tanh[\chi(a - b)] \quad \begin{aligned} -1 &\leftarrow a > b \\ 0 &\leftarrow a = b \\ +1 &\leftarrow a < b \end{aligned}$$

Funzione «if-else» è una condizione if else «rilassata»

$$\text{if-else}(d, f_{\text{true}}, f_{\text{false}}) = \left(\frac{f_{\text{true}}}{2}\right)(1 + d) + \left(\frac{f_{\text{false}}}{2}\right)(1 - d)$$

Funzione originale

Funzione originale

$$f = \text{if}(a > b, f_{\text{true}}, f_{\text{false}})$$

Funzione originale

$$f = \text{if}(a > c, f_c, \text{if}(a < b, f_b, f_m))$$

$$M_a = \begin{cases} -M_a^{(\text{dyn})}, \text{ se } M_a^{(st)} < -M_a^{(\text{st, lim})} \\ +M_a^{(\text{dyn})}, \text{ se } M_a^{(st)} > +M_a^{(\text{st, lim})} \\ M_a^{(st)}, \text{ altrimenti} \end{cases}$$

Funzione rilassata

$$f = \text{if-else}(g_t(a, b), f_{\text{true}}, f_{\text{false}})$$

$$f = \text{if-else}(g_t(a, c), f_c, \text{if-else}(l_t(a, b), f_b, f_m))$$

$$M_a^{(\text{low})} = \text{if-else}(l_t(M_a^{(st)}, -M_a^{(st, \text{lim})}), -M_a^{(\text{dyn})}, M_a^{(st)})$$

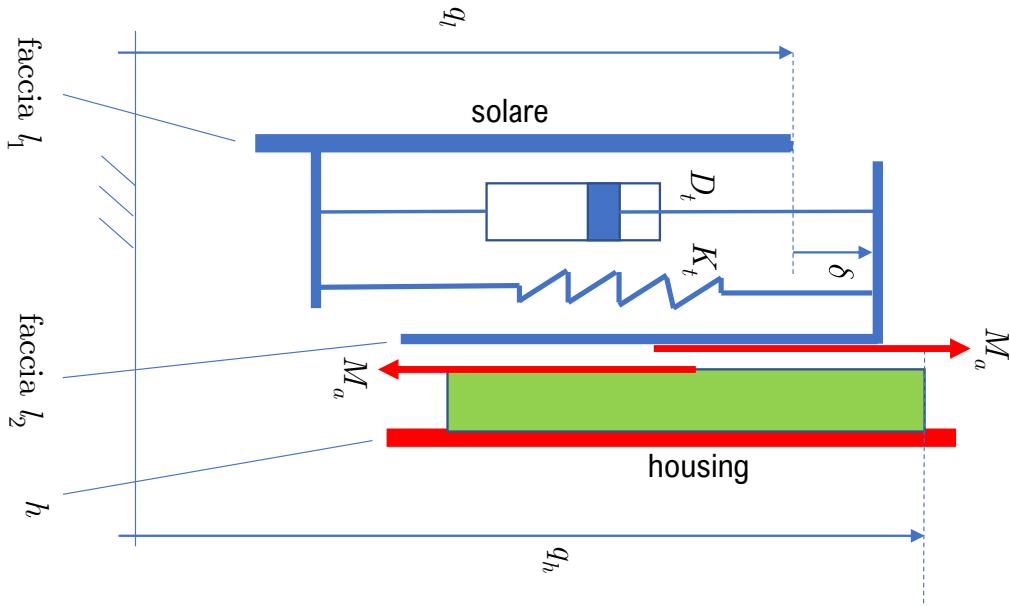
Funzione rilassata

$$M_a = \text{if-else}(g_t(M_a^{(st)}, M_a^{(st, \text{lim})}), M_a^{(\text{dyn})}, M_a^{(\text{low})})$$



$$\frac{d\delta(t)}{dt} = \frac{1}{D_t} [M_a - K_t \delta(t)] \quad M_a^{(st)} = K_t \delta + D_t (\omega_h - \omega_l) \mathbb{1}_8$$

Schema di riferimento per modello stick-slip con stato interno



Definizioni: $\omega_l = \omega_{l1} = \dot{q}_l$, velocità angolare faccia 1 solare left
 $\omega_{l_2} = \dot{q}_l + \dot{\delta}$, velocità angolare faccia 2 solare left
 $\omega_h = \dot{q}_h$, velocità angolare housing

$$\begin{aligned}\Delta\omega_{h,l_1} &= \omega_h - \omega_{l_1} = \dot{q}_h - \dot{q}_l, \text{ velocità angolare relativa housing, faccia 1 solare left} \\ \Delta\omega_{h,l_2} &= \omega_h - \omega_{l_2} = \dot{q}_h - \dot{q}_l - \dot{\delta}, \text{ velocità angolare relativa housing, faccia 2 solare left} \\ &= \omega_h - \omega_l - \dot{\delta}\end{aligned}$$

$$\Delta\omega_{h,l_1} = \omega_h - \omega_{l_1} = \dot{q}_h - \dot{q}_l, \text{ velocità angolare relativa housing, faccia 1 solare left}$$

$$\text{Condizione cinematica di stiction: } \Delta\omega_{h,l_2} = 0 \iff \omega_h - \omega_l - \dot{\delta}_{\text{st}} = 0 \iff \dot{\delta}_{\text{st}} = \omega_h - \omega_l$$

$$\text{Momento trasmesso da } h \text{ a } l_2 \text{ (e quindi poi a } l_1) \text{ in stiction: } M_a^{(\text{st})} = K_t \delta + D_t \dot{\delta}_{\text{st}} = K_t \delta + D_t (\omega_h - \omega_l)$$

Momento trasmesso da h a l_2 (e quindi poi a l_1) in condizione generica: $M_a = K_t \delta + D_t \dot{\delta}$ ($\dot{\delta}$ generico)

ad es. $\Delta\omega_{h,l_2} > 0 \rightarrow \omega_h - \omega_l - \dot{\delta} > 0 \rightarrow \dot{\delta} < \omega_h - \omega_l = \dot{\delta}_{\text{st}}$ es. con $\dot{\delta}_{\text{st}} = 0$, $\dot{\delta} < 0$ (molla si scocca)

$$\Delta\omega_{h,l_2} < 0 \rightarrow \omega_h - \omega_l - \dot{\delta} < 0 \rightarrow \dot{\delta} > \omega_h - \omega_l = \dot{\delta}_{\text{st}} \text{ es. con } \dot{\delta}_{\text{st}} = 0, \dot{\delta} > 0 \text{ (molla si allunga)}$$

Evoluzione dello stato interno definita da eq.ne diff. in condizione generica.

$$\frac{d\delta(t)}{dt} = \frac{1}{D_t} [M_a - K_t \dot{\delta}(t)]$$

dove:

$$M_a^{(\text{st})} = K_t \delta + D_t (\omega_h - \omega_l)$$

con: $M_a = \begin{cases} -M_a^{(\text{dyn})}, & \text{se } M_a^{(\text{st})} < -M_a^{(\text{st, lim})} \text{ (satura in basso)} \\ +M_a^{(\text{dyn})}, & \text{se } M_a^{(\text{st})} > +M_a^{(\text{st, lim})} \text{ (satura in alto)} \\ M_a^{(\text{st})}, & \text{altrimenti (è in stiction)} \end{cases}$

$$M_a^{(\text{dyn})} = \mu_d \bar{P}_e;$$

Analisi energetica di differenziali LSD – analisi valida solo nell'**ipotesi di funzionamento da sbloccato**^{Gabiccini 2022}

Equazioni di congruenza (f. di Willis): $\omega_h = \frac{\omega_l + \omega_r}{2} \longrightarrow \omega_l - \omega_h = -(\omega_r - \omega_h)$

Equilibrio alla rotazione: $M_l + M_r + M_h = 0 \longrightarrow M_h = -(M_l + M_r)$

Bilancio di potenza: $M_h\omega_h + M_l\omega_l + M_r\omega_r - W_d = 0 \longrightarrow M_l(\omega_l - \omega_h) + M_r(\omega_r - \omega_h) = W_d$

$W_d > 0$, potenza dissipata

Power on $M_h > 0$, $\mathbf{I = lenta}$

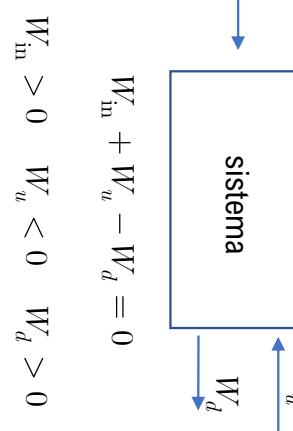
$$\left. \begin{array}{l} M_l(\omega_l - \omega_h) + M_r(\omega_r - \omega_h) = W_d \\ < 0 \quad < 0 \quad < 0 \quad > 0 \quad > 0 \\ > 0 \quad > 0 \quad > 0 \quad > 0 \end{array} \right\} \text{Potenza fluisce da l a r}$$

$$W_{in} \quad W_u \quad W_d$$

Power off $M_h < 0$, $\mathbf{I = lenta}$

$$\left. \begin{array}{l} M_l(\omega_l - \omega_h) + M_r(\omega_r - \omega_h) = W_d \\ > 0 \quad < 0 \quad > 0 \quad > 0 \quad > 0 \\ < 0 \quad > 0 \quad < 0 \quad > 0 \end{array} \right\} \text{Potenza fluisce da r a l}$$

$$W_{in} \quad W_u \quad W_d$$



Power on $M_h > 0$, $\mathbf{r = lento}$

$$\left. \begin{array}{l} M_l(\omega_l - \omega_h) + M_r(\omega_r - \omega_h) = W_d \\ < 0 \quad > 0 \quad < 0 \quad < 0 \quad > 0 \\ < 0 \quad < 0 \quad > 0 \quad > 0 \end{array} \right\} \text{Potenza fluisce da r a l}$$

$$W_{in} \quad W_u \quad W_d$$

Power off $M_h < 0$, $\mathbf{r = lento}$

$$\left. \begin{array}{l} M_l(\omega_l - \omega_h) + M_r(\omega_r - \omega_h) = W_d \\ > 0 \quad > 0 \quad > 0 \quad < 0 \quad > 0 \\ > 0 \quad > 0 \quad < 0 \quad > 0 \end{array} \right\} \text{Potenza fluisce da r a l}$$

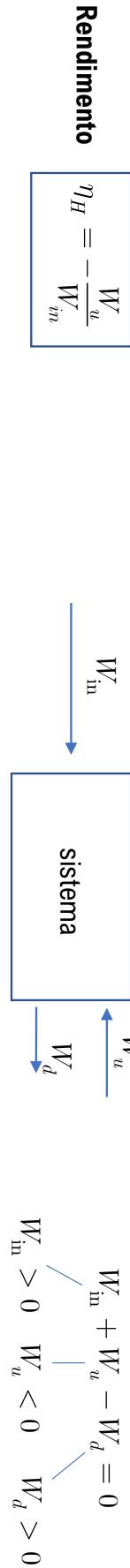
$$W_{in} \quad W_u \quad W_d$$

$$\left. \begin{array}{l} M_l(\omega_l - \omega_h) + M_r(\omega_r - \omega_h) = W_d \\ < 0 \quad > 0 \quad < 0 \quad < 0 \quad > 0 \\ < 0 \quad < 0 \quad > 0 \quad > 0 \end{array} \right\} \text{Potenza fluisce da r a l}$$

$$W_{in} \quad W_u \quad W_d$$

Analisi energetica di differenziali LSD – analisi valida solo nell'**ipotesi di funzionamento da sbloccato**

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Power On $M_h > 0$, \mathbf{r} = lenta - **A**

$$\eta_H = -\frac{W_u}{W_{in}} = -\frac{M_r(\omega_r - \omega_h)}{M_l(\omega_l - \omega_h)} = \frac{M_r}{M_l}$$

$$M_l < M_r < 0 < M_h$$

Power Off $M_h < 0$, \mathbf{r} = lenta - **C**

$$\eta_H = -\frac{W_u}{W_{in}} = -\frac{M_l(\omega_l - \omega_h)}{M_r(\omega_r - \omega_h)} = \frac{M_l}{M_r}$$

$$M_h < 0 < M_l < M_r$$

Power On $M_h > 0$, \mathbf{r} = lenta - **B**

$$\eta_H = -\frac{W_u}{W_{in}} = -\frac{M_l(\omega_l - \omega_h)}{M_r(\omega_r - \omega_h)} = \frac{M_l}{M_r}$$

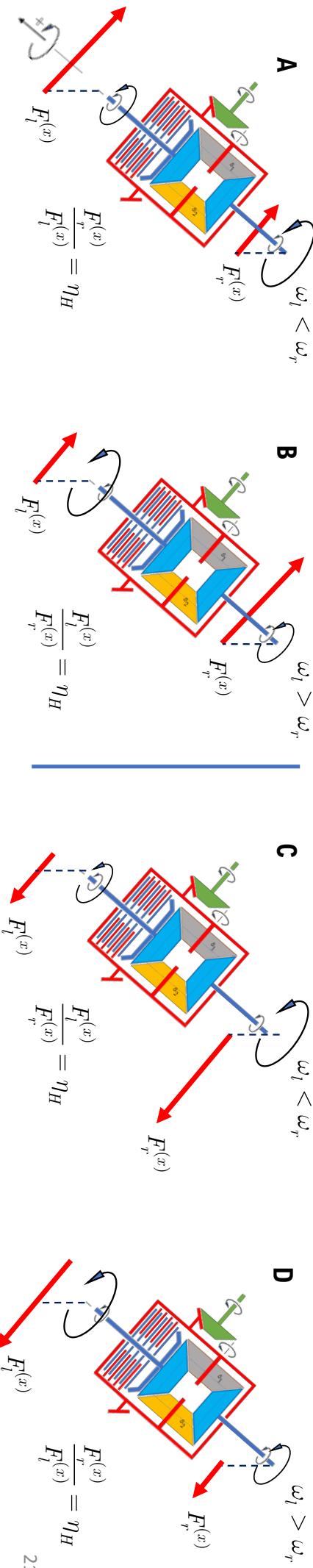
$$M_r < M_l < 0 < M_h$$

Power Off $M_h < 0$, \mathbf{r} = lenta - **D**

$$\eta_H = -\frac{W_u}{W_{in}} = -\frac{M_r(\omega_r - \omega_h)}{M_l(\omega_l - \omega_h)} = \frac{M_r}{M_l}$$

$$M_h < 0 < M_l < M_r$$

Il funzionamento da bloccato non può essere analizzato mediante queste formule, poiché derivate nell'ipotesi che esista uno scorrimento. E' evidente che in Power On così come in Power Off i momenti che ruota applica all'assale (ovvero strada applica alla ruota) M_l, M_r debbono essere concordi, poiché il loro rapporto è il rendimento che è $\eta_H > 0$. Stessa cosa (essere concordi) vale per le forze che strada applica alle ruote $F_l^{(x)}, F_r^{(x)}$.



Ipotesi di funzionamento da sbloccato del differenziale LSD

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$$\text{Torque Bias Ratio (TBR)} \quad \text{TBR} = \frac{1}{\eta_H} \geq 1$$

Power On $M_h > 0$, \mathbf{r} = lenta - A

$$\frac{M_{\text{slow}}}{M_{\text{fast}}} = \frac{M_l}{M_r} = \text{TBR}$$

$$M_l < M_r < 0 < M_h$$

Power On $M_h > 0$, \mathbf{r} = lenta - B

$$\frac{M_{\text{slow}}}{M_{\text{fast}}} = \frac{M_r}{M_l} = \text{TBR}$$

$$M_r < M_l < 0 < M_h$$

Se si considerano le relazioni che coinvolgono M_{slow} , M_{fast} queste sono l'una la reciproca dell'altra fra il funzionamento in Power On e Power Off

$$\eta_H > 0$$

Power Off $M_h < 0$, \mathbf{r} = lenta - C

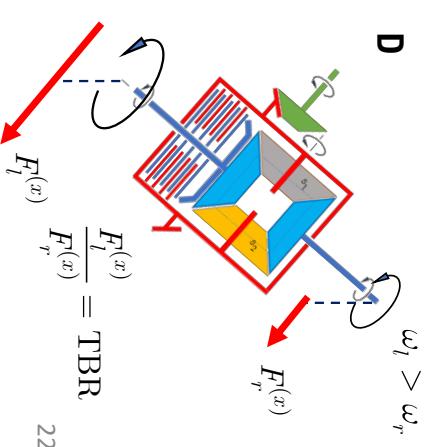
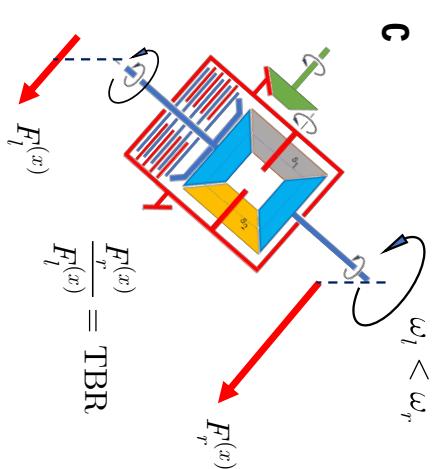
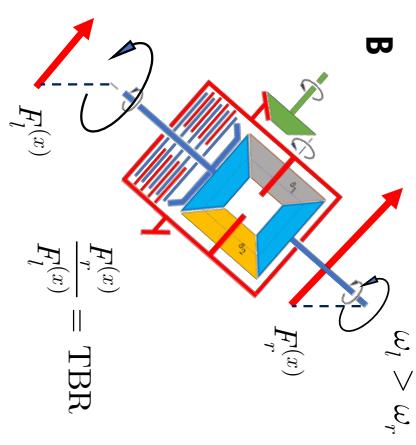
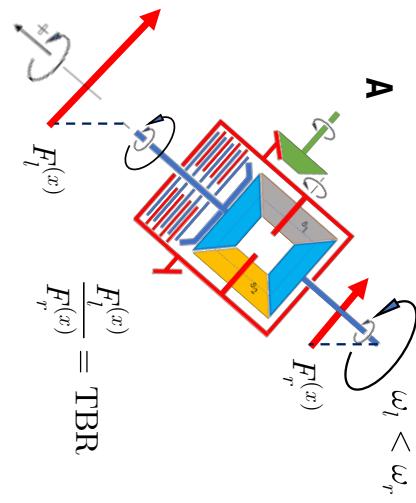
$$\frac{M_{\text{fast}}}{M_{\text{slow}}} = \frac{M_r}{M_l} = \text{TBR}$$

$$M_h < 0 < M_r < M_l$$

Power Off $M_h < 0$, \mathbf{r} = lenta - D

$$\frac{M_{\text{fast}}}{M_{\text{slow}}} = \frac{M_l}{M_r} = \text{TBR}$$

$$F_l^{(x)}, F_r^{(x)}$$



Ipotesi di funzionamento da sbloccato del differenziale LSD – Locking coefficient

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$$\text{Locking coefficient} \quad \varepsilon_h = \frac{1 - \eta_h}{1 + \eta_h} = \frac{\text{TBR} - 1}{\text{TBR} + 1}$$

$$\text{Rendimento} \quad \eta_H = \frac{1 - \varepsilon_h}{1 + \varepsilon_h}$$

$$\text{Dalla definizione di rendimento} \quad \eta_H = \frac{|M_h| - \Delta M}{|M_h| + \Delta M}$$

$$\text{in cui compare } \Delta M \quad \frac{1 - \varepsilon_h}{1 + \varepsilon_h} = \frac{|M_h| - \Delta M}{|M_h| + \Delta M}$$

$$\Delta M = \varepsilon_h |M_h| = f(|M_h|)$$

In Torsen $\eta_H = \text{cost}$, $\varepsilon_h = \text{cost}$ $\Delta M = M_a = \varepsilon_h |M_h|$

$$\Delta M = 0, \text{ se } M_h = 0$$

Sensibilità lineare nella coppia M_h

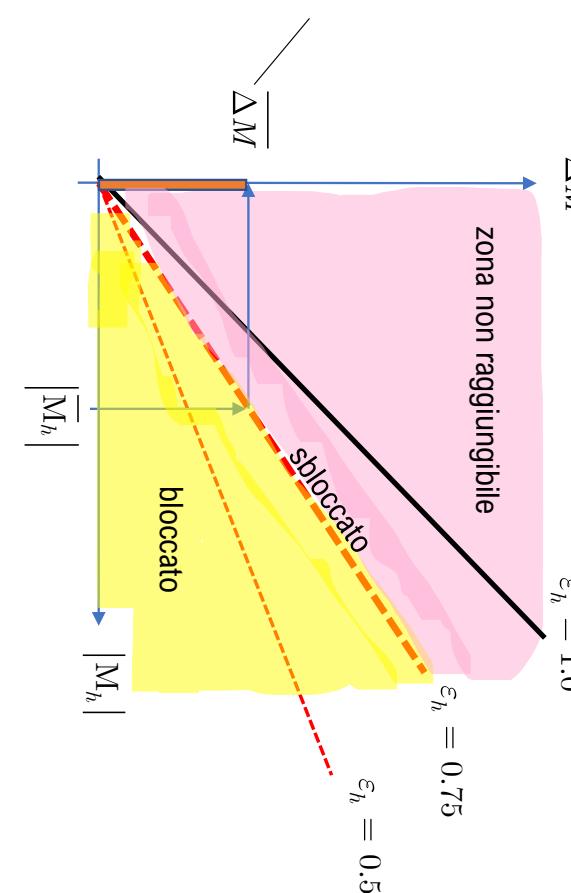
housing worm gears

A questa delta coppia il differenziale funziona da sbloccato e valgono formule

Power On $\frac{M_{\text{slow}}}{M_{\text{fast}}} = \text{TBR}$

Power Off $\frac{M_{\text{fast}}}{M_{\text{slow}}} = \text{TBR}$

questa è da intendersi come la coppia di sbloccaggio



Ipotesi di funzionamento da sbloccato del differenziale LSD – Locking coefficient

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$$\text{Locking coefficient} \quad \varepsilon_h = \frac{1 - \eta_h}{1 + \eta_h} = \frac{\text{TBR} - 1}{\text{TBR} + 1}$$

$$\text{Rendimento} \quad \eta_H = \frac{1 - \varepsilon_h}{1 + \varepsilon_h}$$

$$\text{Dalla definizione di rendimento} \quad \eta_H = \frac{|M_h| - \Delta M}{|M_h| + \Delta M}$$

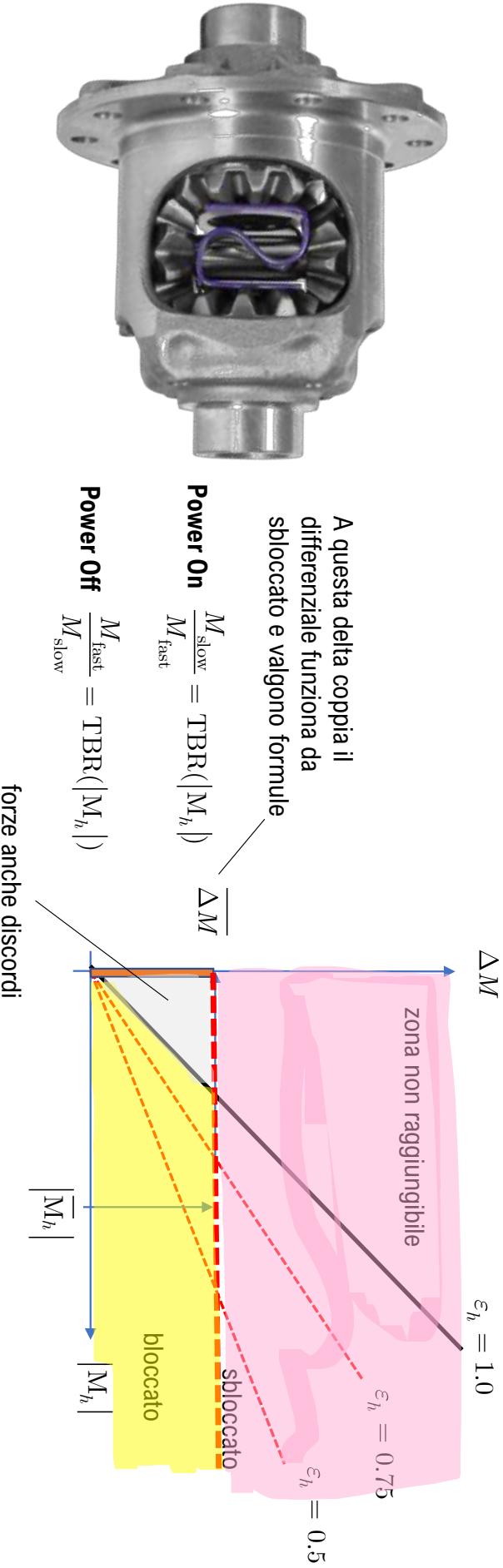
$$\text{in cui compare } \Delta M$$

$$\frac{1 - \varepsilon_h}{1 + \varepsilon_h} = \frac{|M_h| - \Delta M}{|M_h| + \Delta M}$$

$$\Delta M = \varepsilon_h |M_h| = f(|M_h|)$$

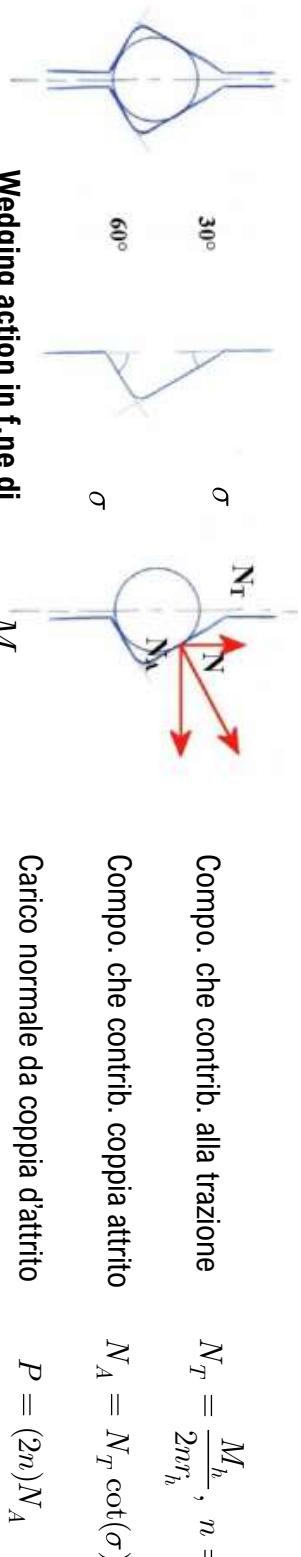
In Clutch-Pack $\Delta M = M_a = \text{cost}$, indipendentemente da M_h

questa è da intendersi come la coppia di sbloccaggio



Ipotesi di funzionamento da sbloccato del differenziale LSD – Locking coefficient

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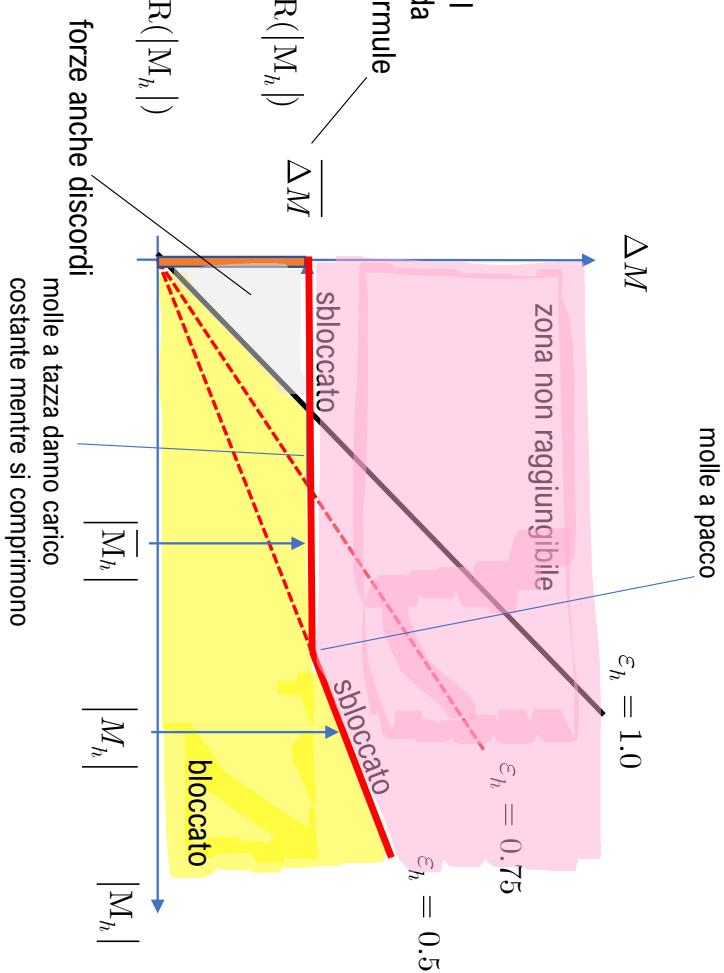
In Salisbury Clutch Pack

$\Delta M = M_a = \text{variabile e dipendente da } M_h$

A questa delta coppia il differenziale funziona da sbloccato e valgono formule

$$\text{Power On} \quad \frac{M_{\text{slow}}}{M_{\text{fast}}} = \text{TBR}(|M_h|) \quad \frac{\Delta M}{\Delta M}$$

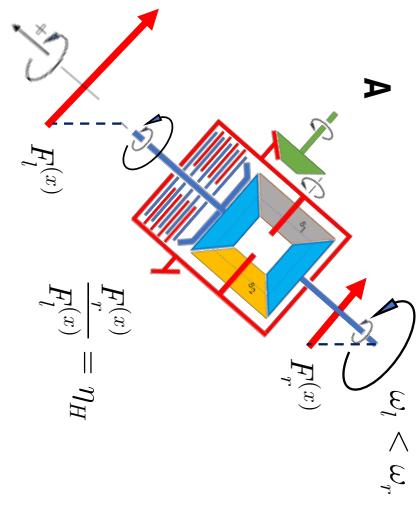
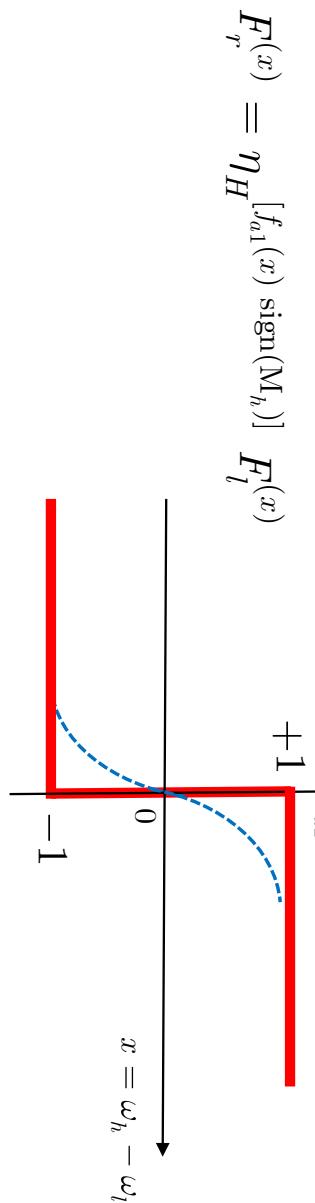
$$\text{Power Off} \quad \frac{M_{\text{fast}}}{M_{\text{slow}}} = \text{TBR}(|M_h|)$$



Analisi energetica di differenziali LSD – modello semplice rilassamento stick-slip

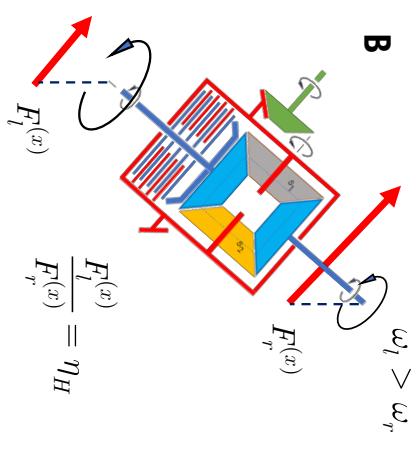
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Nell'ipotesi di voler costruire un modellino di differenziale con rendimento costante η_H (quindi valido per un Torsen ma non generale) e che «recuperi in qualche modo» la condizione di differenziale bloccato mediante uno stratagemma, si può usare la seguente equazione costitutiva:



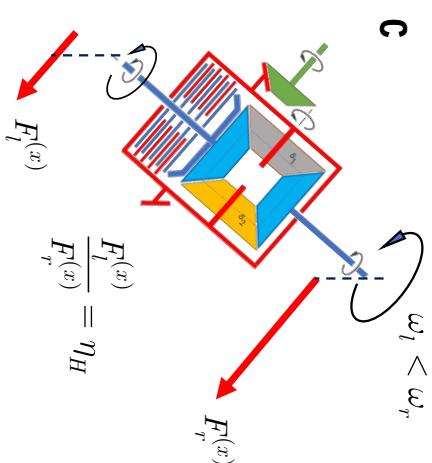
$$\mathbf{A:} \quad x = \omega_h - \omega_l > 0; \quad \text{sign}(M_h) > 0;$$

$$F_{(x)}^t = U_+^{H_t} F_{(x)}^t U_+$$



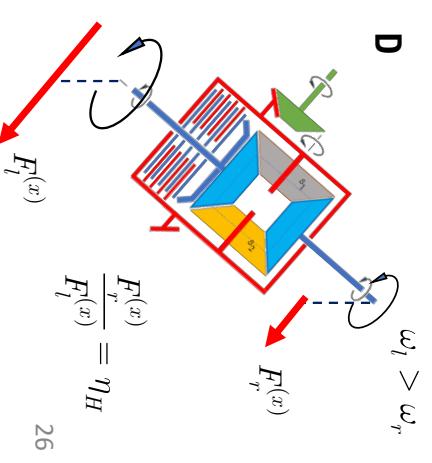
$$\mathbf{B:} \quad x = \omega_h - \omega_l < 0; \quad \text{sign}(M_h) >$$

$$H_U = \frac{1}{l} F_{(x)}^l H_U = F_{(x)}^{l-1} H_U$$



$$\mathbf{C}: x \equiv \omega_h = \omega_l > 0; \text{ sign}(M_h) < 0;$$

$$F_{(x)}^l \frac{H_U}{1} = {}_{(x)}^l F_{[-]} H_U = {}_{(x)}^l F$$



$$x \equiv \omega_h = \omega_l < 0; \text{ sign}(\mathrm{M}_h) < 0;$$

$$F_{(x)}^l H_U = F_{(x+1)}^l H_U =$$