Implementation of Launch Control on Formula Student monopost

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***Abstract*—This paper deals with the design and implementation of the Launch Control algorithm(Wheel slip control), which is specifically designed for the best possible vehicle acceleration. The algorithm controls the torque applied to each wheel based on the angular velocity of the wheel and the velocity of the vehicle, allowing traction to be optimized for the best acceleration. The algorithm can automatically adapt to changing road conditions and characteristics, ensuring that even in adverse conditions, the best acceleration is achieved. The functionality of the model and design of the algorithm is presented through simulations in the Matlab Simulink environment.**

***Index Terms*—Launch Control, Control system, Formula Student, Vehicle Dynamics, Matlab Simulink, Slip Ratio**

I. Introduction

Formula Student is a motorsport event where teams from universities from all around the world design, build, and race one-seated cars. Each event is divided into different disciplines. One of those disciplines is acceleration. In this discipline, cars accelerate on a straight track of length 75 meters, and the best time wins. In this context, there is a demand for the design of an algorithm for the best acceleration, Launch Control. In the last years, this algorithm was implemented in the car, but with no consideration of load transfer and longitudinal tyre force. The core challenge addressed in this paper is to design a new improved algorithm for Launch Control with consideration of load transfer and longitudinal tyre force.

II. Principle of Launch Control

Launch Control is a term used in motorsport which represents an algorithm for the best acceleration. To achieve the best acceleration by controlling the slip of the tyre. The variable that represents how much your tyre slips is called slip ratio σ_x and it's defined as

$$\sigma_x = 1 - \frac{v}{\omega r_{eff}}. \tag{1}$$

Where:

- v ... Velocity of the vehicle [m/s]
- ω ... Angular velocity of the wheel [rad/s]
- r_{eff} ... Effective radius of the tyre [m]

Usually, slip ratio is represented in %. Next, the important variable is longitudinal tyre force F_x, this force is defined as the friction force from the ground that acts on the tyre [1]. This force is defined as

$$F_x = D_x \sin[C_x \arctan(B_x\sigma_x - E_x(B_x\sigma_x - \arctan(B_x\sigma_x)))] + S_v. \tag{2}$$

Where:

- C_x ... Shape constant for simple slip [-]
- D_x ... Amplitude for simple slip [N]
- E_x ... Curvature constant for simple slip [-]
- S_v ... Sliding in vertical direction for simple sliding [N]

Each of these variables consists of many other variables and they aren't constant, they vary with normal tyre load F_z, camber of the tyre γ, and other parameters [2]. The plot of longitudinal tyre force F_x as a function of slip ratio σ_x and for constant normal tyre load F_z, from equation (2), is in figure 1.

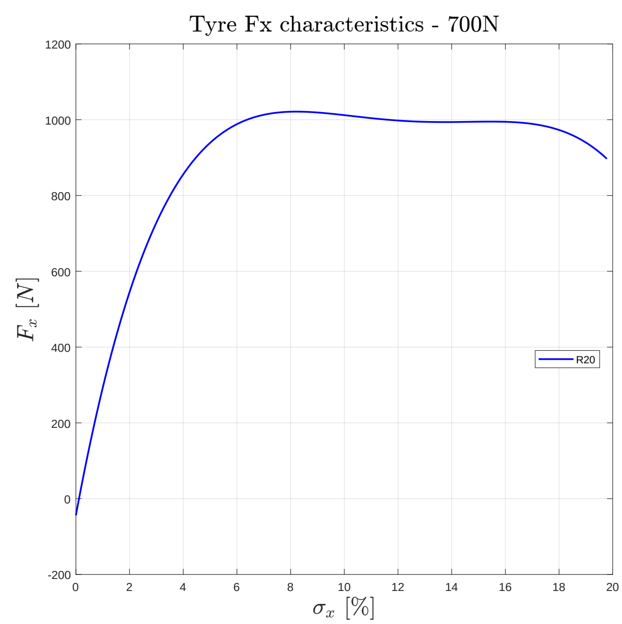

Fig. 1. Graph of $F_x = f(\sigma_x, F_z)$

Hoosier R20, in figure 1, is the name of the tyre and its compound that is being used in Formula Student [3]. The best acceleration is achieved when the highest possible F_x

is transferred to the road. In the figure 1 the highest possible F_x is around 8 - 10%. Our goal for controller design is to achieve a slip ratio σ_x value of around 8 - 10%.

III. Mathematical model of the vehicle

Our inputs to the model are torques that drive the rear wheels. Those torques are described as

$$T_{Mi} = G_R \cdot c_\phi \cdot I_{com}, \tag{3}$$

where:

- T_{Mi} ... Torque generated by the motor that drives wheel [Nm]
- c_ϕ ... Motor constant [Nm/A]
- I_{com} ... Current command from vehicle control unit [A]
- G_R ... Gear ratio of transmission [-]

Index i describes: $i = RL, RR$ as for Rear Left and Rear Right wheel. Next are our model states, which are slip ratios σ_{xi} of both rear wheels, angular velocities of both rear wheels ω_i, and the velocity of the vehicle v. Slip ratio σ_{xi} is defined as

$$\dot{\sigma}_{xi} = \frac{(\dot{w}_i \cdot r_{eff} - \dot{v})w_i \cdot r_{eff} - \dot{w}_i \cdot r_{eff}(w_i \cdot r_{eff} - v)}{(\omega_i \cdot r_{eff})^2}, \tag{4}$$

where:

- $\dot{w}_i$ ... angular acceleration [rad/s^2]
- $\dot{v}$ ... acceleration of the vehicle [m/s^2]

Angular velocity for rear wheel ω_i is defined as

$$\dot{\omega}_i = \frac{1}{J_W} \cdot (T_{Mi} - r_{eff} \cdot F_{xi}), \tag{5}$$

where:

- J_W ... Moment of inertia [kg m^2]

For vehicle velocity v the equation is defined as

$$\dot{v} = \frac{1}{m} \cdot (\frac{T_{MRL} + T_{MRR}}{r_{eff}} - F_{aero} - F_z \cdot C_R), \tag{6}$$

where:

- m ... Mass of the vehicle with driver [kg]
- F_{aero} ... Aerodynamic drag force [N]
- F_z ... Normal tyre load [N]
- C_R ... Rolling resistance coefficient [-]

F_{aero} and F_z are calculated in the model [1].

IV. Validation of the mathematical model

Validation of the mathematical model ensures that the model accurately represents the real system. The next table describes the values of the mathematical model. Those values were retrieved from the TU Brno Racing team.

TABLE I
Values of parameters for the mathematical model

Variable	Unit
c_ϕ	0.492 Nm/A
J_W	0.126 kgm^2
G_R	12
r_{eff}	0.181 m
m	240 kg
C_R	0.01

The model scheme that was validated is shown in the next figure 2.

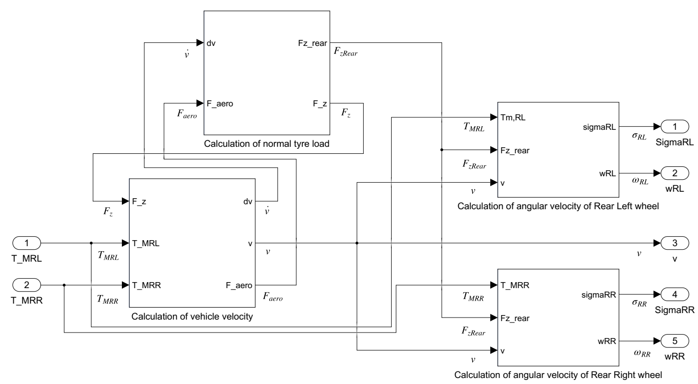

Fig. 2. Validation scheme for model of the vehicle

Next figure 3 shows input torques to the mathematical model. Torques were calculated from equation (3) with measured current I_{com} from the last season's race of the TU Brno racing monopost eD4. These torque values are before multiplication by parameter G_R.

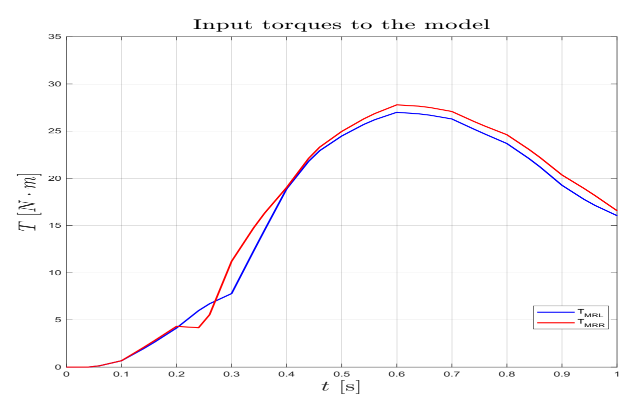

Fig. 3. Input torques to the model

Validation of vehicle velocity is shown in the next figure 4 for data from last season's race of the TU Brno racing monopost ED4.

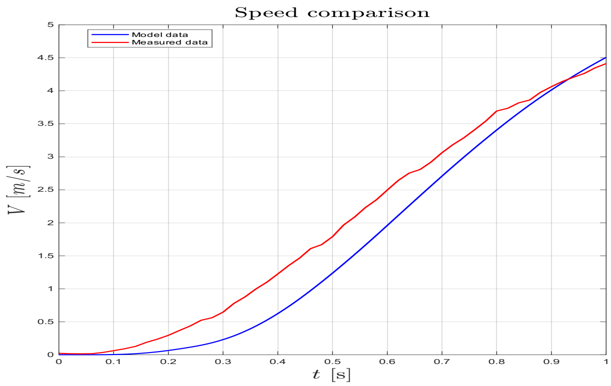

Fig. 4. Validation for vehicle velocity

From figure 4, the model data integrates slower than the measured data. Validation of wheel velocity of the rear left tyre is shown in the next figure 5 for data from last season's race of the TU Brno racing monopost ED4.

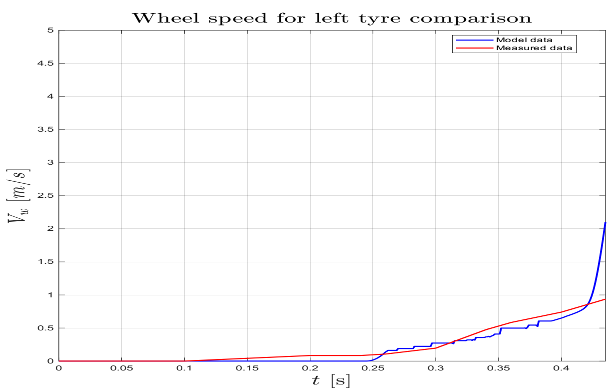

Fig. 5. Validation of wheel velocity of the rear left tyre

Difference in measured and modelled plots from figure 5 is due to the fact that r_{eff} is set as constant, but it changes with normal tyre load by 10-15%. The model also doesn't include longitudinal tyre force for front tyres that don't drive the vehicle. Especially in plot 5, model data copies measured data till 0.43 seconds, when the modelled wheel velocity becomes bigger than measured data.

V. State-space model

State-space model is a mathematical representation of a dynamic system that describes how its internal state evolves over time. State-space model of our model, as was said in chapter III, will have 5 states ω_{RL} , ω_{RR}, σ_{xRL}, σ_{xRR} and v. Let the state model of the system be of the form

$$\dot{x} = Ax + Bu \tag{7}$$

$$y = Cx + Du. \tag{8}$$

where x represents states of the model, u represents inputs of the model, and y represents outputs of the model. For our model matrices A, B, C and D are

$$A = \begin{bmatrix} 0 & 0 & A_1 & 0 & A_3 \\ 0 & 0 & 0 & A_2 & A_4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.00415v \end{bmatrix} \tag{9}$$

$$B = \begin{bmatrix} \frac{496.8v - 1.35w_{RR}}{w_{RR}^2} & \frac{-1.35}{w_{RL}} \\ \frac{-1.35}{w_{RL}} & \frac{496.8v - 1.35w_{RL}}{w_{RL}^2} \\ 95.5 & 0 \\ 0 & 95.5 \\ 0.26 & 0.26 \end{bmatrix} \tag{10}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{11}$$

$$D = 0. \tag{12}$$

Where

- $A_1 = \frac{15.9vF_{xRR} + 993T_{RR}w_{RR} + 1.35T_{RL}w_{RR} - 0.01v^2 w_{RR}}{w_{RR}^3}$
- $A_2 = \frac{15.9vF_{xRL} + 993T_{RL}w_{RR} + 1.35T_{RR}w_{RL} - 0.01v^2 w_{RL}}{w_{RL}^3}$
- $A_3 = \frac{496.8T_{RR} - 7.965F_{xRR} + 0.021Vw_{RR}}{w_{RR}^2}$
- $A_4 = \frac{496.8T_{RL} - 7.965F_{xRL} + 0.021Vw_{RL}}{w_{RL}^2}$

Transfer functions are retrieved from the following equation [4]

$$G = C(sI - A)^{-1}B + D. \tag{13}$$

The transfer functions that will be used in controller design are $G[1,1]$ and $G[2,2]$, because $G_{11}(s) = \frac{\sigma_{RR}(s)}{T_{RR}(s)}$ and $G_{22}(s) = \frac{\sigma_{RL}(s)}{T_{RR}(s)}$. For $G_{11}(s)$, the transfer function is defined as

$$G_{sRR}(s) = G_{11}(s) = \frac{s^2 \cdot S_{RR2} + s \cdot S_{RR1} + S_{RR0}}{s^2 \cdot w_{RR}^3 \cdot (s + 0.00415v)}. \tag{14}$$

For $G_{22}(s)$, the transfer function is defined as

$$G_{sRL}(s) = G_{22}(s) = \frac{s^2 \cdot S_{RL2} + s \cdot S_{RL1} + S_{RL0}}{s^2 \cdot w_{RR}^3 \cdot (s + 0.00415v)}. \tag{15}$$

Where:

- $S_{RR0} = -0.2021vw_{RR} + 6.30157F_{xRR}v^2$
- $S_{RR1} = 2.06172v^2 w_{RR} + 12.8229T_{RL}w_{RR}$
- $S_{RR2} = 496.800v \cdot w_{RR} - 1.350w_{RR}^2$
- $S_{RL0} = -0.20213vw_{RL} + 6.30157F_{xRL}v^2$
- $S_{RL1} = 2.06172v^2 w_{RL} + 12.8229T_{RR}w_{RR}$
- $S_{RL2} = 496.800v \cdot w_{RL} - 1.350w_{RL}^2$

Our system is a 3rd order with a double integrator, and the pole is determined by the velocity of the car. For the calculation of the F_x, the look-up table (LUT) is being used. Velocity of the vehicle v and angular velocities of ω_i will be measured through sensors of the vehicle.

VI. Algorithm structure and compensator design

The limit of the torque of both motors is set to $T_{MAX} = 348$ Nm for each motor. The maximum difference between torques ΔT_M is also set. This limitation is due to the safety of the driver. If the controller sent commands, which were $T_{MRL} >> T_{MRR}$ or $T_{MRR} >> T_{MRL}$, this would result in the car turning right or left on a straight line. The value of this limit is set to $\Delta T_M = 25$ Nm. This value can be changed by small amounts to suit different drivers. To prevent any hazardous states of σ_{xi}, which can happen when $\omega_i = 0$. This can happen in 2 scenarios:

1) $\omega_i = 0$ and $v = 0$, then $\sigma_{xi} = 0$
2) $\omega_i = 0$ and $v > 0$, then $\sigma_{xi} = 1 - \frac{v}{\omega_i r_{eff} + 0.01v}$

Second scenario can happen only during braking, but this case in code is implemented as an edge case.

To regulate slip ratios of both rear wheels, a PI controller is employed for each wheel. The parameters of the PI controller, including the time constant T_i and gain K. T_i is adjusted as functions of v to compensate for the specified pole of transfer functions. The controller design is described by equation(16).

$$G_{Ri}(s) = \frac{K \cdot (T_i s + 1)}{T_i s} \tag{16}$$

$$T_i = \frac{1}{0.00415v}. \tag{17}$$

Static gain of the controller is chosen through optimizing the value of the quadratic integral criterion [5]

$$J_K = \int_0^\infty e(t)^2 \, dt. \tag{18}$$

The flowchart of the design for the algorithm is in the next figure 6

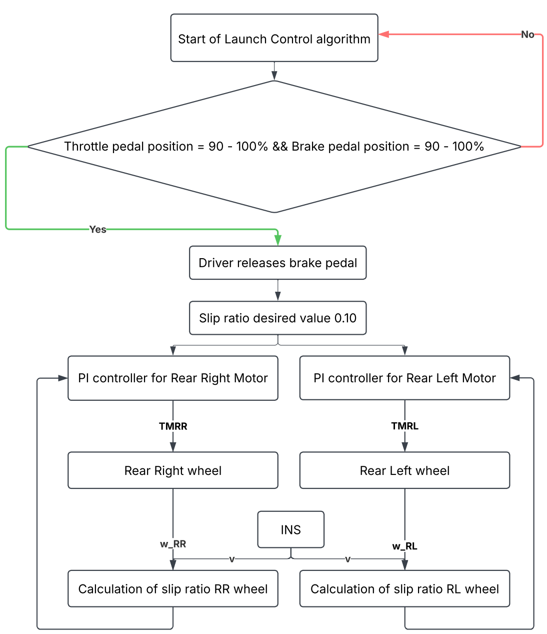

Fig. 6. The flowchart of the design for the Launch Control algorithm

In figure 6 the INS stands for Inertial Navigation System, which measures the velocity of the vehicle v.

VII. Conclusion

Algorithm for Launch Control adeptly utilizes the tyre longitudinal force F_x as LUT due to the challenging calculation of this force. Calculations of each state through the mathematical model are successfully validated through Simulink simulation of the Launch Control and set safety features of the algorithm-designated controller. The next steps are simulation via Simulink with the help of IPG Carmaker software. IPG Carmaker software is used for the development of its own tracks and driver parametrization, with the implemented Formula Student model of the vehicle. The last step is the implementation of this algorithm to the physical car, which is one of the goals for the season 2025 of team TU BRNO Racing.

Further improvements of this algorithm are in implementing full traction control for other disciplines, not for only acceleration.

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