An Error Correction Model to Brazil's Exchange Rate

Aluno: Fausto Junior

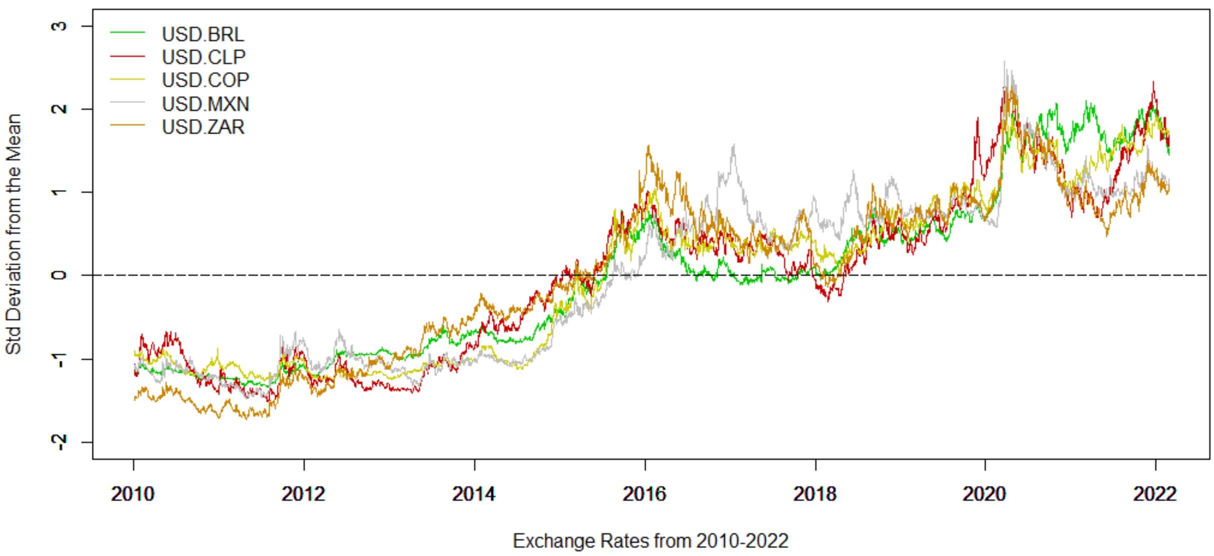
27/02/2022

Summary

The objective of this project is to propose an Error-Correction Model (ECM) to Brazil's Exchange Rate. Initially, I offer a brief summary of the literature about predicting exchange rates, pointing out the traditional predictor and the different models commonly used in the literature. The literature agrees that the best benchmark to behavior of the exchange rates is still a Random Walk, confirming the difficulty to predict its fluctuations using economic models. Using a single-equation linear ECM, I find that Brazil's Exchange Rate cointegrates with its pairs, and the model indicates that the Brazilian Real (BRL) could have been undervalued since the beginning of the pandemic. I also find that the fitted-value for the Brazilian Exchange is nearly a projection of BRL into the first factor of the Factor Model.

Introduction

Since the beginning of the COVID-19 pandemic, the emerging markets suffered a tremendous depreciation of its currencies. In particular, Brazil had one the worst performances compared with its pairs, as the graph bellow shows. This emerging basket started depreciating since 2012 and reached its peak on 2020. It can be easily seen that the green color (BRL) stayed above all the other countries for almost 2 years. That means the Brazilian Real didn't only depreciate against the dollar, but also among emerging markets.



Having the best possible models to predict exchange rates, specially in sharp movement as seen above, is of interest for several market participants. Economists, researchers and Central Banks would be interested in knowing which predictors and models successfully explain exchange rates. Exporters and importers would like to hedge their future financial expenses from exchange fluctuation risks. Policymakers would better off because successful policy depends on good forecasts.

However, it is well known that the exchange rates are very difficult to predict using economic models (Rossi, 2013). The Random Walk Model frequently finds better forecasts then economic models, what is know as "The Meese and Rogoff puzzle". It's called a puzzle because of the following contrasting ideas. If the Efficient Market Hypothesis is correct, the exchange rates should be the market's best guess of the relative value of two currencies based on all available information at that time. In other words, it does not implies that exchange rates should fluctuate randomly around their past values, as the random walk model says.

The next section is entirely based on the work of Rossi (2013). It introduces the most commonly used models and predictor to forecast exchange rates. For a more detailed and critical survey of the literature, see Rossi (2013).

Literature Review

The traditional predictors used for modelling exchange rates are the following:

1. Interest Rate Differentials

   According to Rossi (2013), Fisher (1896) provided a general analysis of how interest rates can be related to expected changes in the relative value of international currencies. It has become know as the Uncovered Interest Rate Parity (UIRP). It simply mean that, by arbitrage and in the absence of transaction costs, the return of a home bond must be (in expectation) equal to the return of a foreign bond return when converted back in home currency. The UIRP equation can be written as:

   $$\mathbb{E}_t(s_{t+h} - s_t) = \alpha + \beta(i_{t+h} - i^*_{t+h})$$

   where s_t = log of the nominal bilateral exchange rate at t, s_{t+h} = log of the nominal exchange rate h-period ahead of t, i_t = domestic interest rate at $t + h$ and i^*_{t+h} = foreign interest at $t + h$.

2. Price and Inflation Differentials

   According to Purchasing Power Parity (PPP), the real price of comparable commodity basket in two countries should be the same. The PPP equation can be written as:

   $$s_t = \alpha + \beta(p_t - p^*_t) + \epsilon_t$$

   where s_t = log of the nominal bilateral exchange rate at t, p_t and p^*_t the log of commodity price index in the home and foreign country, respectively, and $\alpha = 0$ and $\beta = 1$.

3. Money and Output Differentials

   According to the monetary model of exchange rate determination, nominal exchange rate fluctuation should reflect movements in countries' relative money, output, interest rates and prices. This can be written as:

   $$s_t = \eta(i_{t+1} - i^*_{t+1}) - \phi(y_t - y^*_t) + (m_{t+1} - m^*_{t+1})$$

   where m_t = log of nominal money, y_t = log of real output.

4. Portfolio Balance

   Traditional portfolio balance models include a variable of stock balances (b):

$$s_t = \beta_0 + \beta_1(i_t - i_t^* - \mathbb{E}_t(s_{t+1} - s_t)) + b_t - b_t^*$$

Briefly, the empirical evidence of the four predictors described above are not favorable. Rossi (2013) pointed out that the out-of-sample forecast ability of these predictors do not outperform a random walk systematically. These results remained the same to forecasts at short or long horizon.

Some authors, according to Rossi (2012), propose fundamentals based on a Taylor rule for monetary policy. They propose alterations to the Taylor rule in order to explain the behavior of exchange rate. One example of this is the following equation, found in Molodtsova and Papell (2009). Both the in-sample and the out-of-sample empirical evidence are mostly favorable to Taylor-rule fundamentals.

$$\mathbb{E}_t(s_{t+1} - s_t)) = \tilde{\mu} + \tilde{\lambda}(\pi_t - \pi_t^t) + \tilde{\gamma}(y_t^{gap} - y_t^{gap*})$$

According to Rossi (2012), other models consider that not only the current account, but the whole dynamic process of net export, foreign asset holding and return on the portfolio of net foreign assets are important predictors of exchange rates. Thus, some propose net forein asset (NXA) as a potential predictor for future exchange rate fluctuation. The empirical evidence is favorable to NXA. For longer horizons, researchers find contradictory result, but for short horizons the models embedded with NXA can predict exchange rates out-of-sample significantly better than the random walk.

Lastly, commodity prices are also considered a potential macroeconomic fundamental for exchange rates for some authors. The main idea is that exchange rates are endogenously determined in equilibrium together with other macroeconomic variable. So, if it were possible to identify an exogenous commodity price changes, then, at least for small countries where primary commodities constitute a significant share of exports, exchange rates could be predicted based on this exogenous shocks. The empirical evidence is favorable of commodity prices as predictors of exchange rates, at least in-sample forecast.

After describing the common predictors in the literature, the models used also had great effect on the attempt to forecast exchange rates. Models can be divided into three groups: single-equation, multiple equations and panel models. Each model in this group could be either linear or nonlinear, and may or may not allow for cointegration or time variation in the parameters.

1. Single-Equation, Linear Models

$$\mathbb{E}_t(s_{t+h} - s_t) = \beta_0 + \beta_1^{'} f_{t+h}$$

   where s_t = log of the nominal bilateral exchange rate at t, s_{t+h} = log of the nominal exchange rate h-period ahead of t and f_{t+h} = fundamentals used.

   Notice that the actual value of the fundamentals is used in this equation. Some variations of this model use the forecasted fundamentals ($\mathbb{E}_t f_{t+h}$) instead of the actual ones (f_{t+h}). The empirical evidence based on the single-equation linear model is mixed. The contemporaneous fundamental is not successful in predicting exchange rates, however, even the forecasted fundamentals is not clear if really improves the prediction. Rossi (2013) concludes that is the predictor that matters in determining the strength of the predictability rather than the specification of the single-equation linear model.

2. Single-Equation, ECM models

$$\mathbb{E}_t(s_{t+h} - s_t) = \beta_0 + \beta_1(s_t - \gamma^{'} f_t)$$

   with the variable described as previously .

   The Error Correction Model assume that there is a long-run relationship between the level of the exchange rate, s_t, and the level of the fundamentals. Then, to forecast the fluctuation of the exchange rate, it includes a correction term that captures the disequilibrium between the exchange rate and the fundamental levels, $(s_t - \gamma^{'} f_t))$. The empirical evidence is in favor of ECM models at long horizons.

3. Nonlinear Models

   Most of the literature is concerned with linear models. Even though it is a theoretical possibility for predicting exchange rates, the empirical evidence is not favorable. The vast majority of the literature, according to Rossi (2013), finds that nonlinear models forecast poorly.

4. Time-Varying Parameter (TVP) Models

   A special form of nonlinearity may be induced by time-variation in the parameters. The single-equation time varying parameter model can be seen below. The empirical evidence is mixed. Some papers find out-of-sample forecast improvements over the random walk, but in other cases the forecast improvements are unclear.

$$s_t = \beta_t^{'} f_t + u_t$$

$$\beta_t = \Gamma \beta_{t-1} + K + A v_t$$

5. Multivariate Models

   Many types of multivariate models have used in the literature, typically generalizations of the single-equation models discussed above. The most used are VARs, Factor Models and VECMs. For a more detailed analysis, see Rossi (2013).

Among the model specifications considered in the literature, the least successful are nonlinear specifications and the most successful are linear specifications. Among the linear models, the single-equation ECM is one of the most successful at long horizons.

Methodology

The first choice made for the project was which model and predictors to use. Some of the above models would take a lot of time to implement and the data would be difficult to obtain. Being pragmatic, the choice was made in other to finish the project in the due time. However, I was inclined to try a VECM using interest rate differential, net foreign asset and stock market value as fundamentals. The initial simulations using an VECM was also done, but it found some incorrect results. I attribute these results to some error in programming, which I didn't have enough time to correct, and consequently, I stopped the VECM attempt.

Next, I chose to use a single-equation linear ECM model, using as fundamentals the current exchange rate of others emerging markets. The main assumption is that there is a long-run relationship between the level of the exchange rate of a emerging country and its pairs. Emerging countries share many similar characteristics: higher inflation, higher interest rates, higher unemployment rates, greater swap rates, more volatility. These characteristics are not enough to guarantee cointegration between Brazilian Exchange Rates and its pairs. In other to do so, it is done a Phillips-Perron Unit Root Test on the level of exchange rates and in the error of cointegration.

Then, it is also done some visual qualification of the result. Even if the model is too simplistic, the interpretation of the graphs are straightforward since the time series exhibit a similar behavior during the last 22 years of data, which is described next session. In order to strengthen the intuition, I also proposed a principal component analysis using the emerging market basket as a factor.

All the computational tools and methodologies will be available at: https://github.com/faustojrbr/Project-Exchange. The code will also be available for further general use.

Data

This project started with a much more ambitious goal. First, I obtained the data for the exchange rates of Brazil (USD.BRL), Chile (USD.CLP), Colombia (USD.COP), Mexico (USD.MXN) and South Africa (USD.ZAR). These data were obtained at Bloomberg, date 24/02/2022, and downloaded locally in my machine. I first downloaded the data at various frequencies (daily, monthly and quarterly).

Then, I downloaded the NXA, interest differencials and the main stock market index for these country. The initial simulations using an VECM was also done, but as I said before, it found some incorrect results. Since then, I decided to use a ECM model using only the levels of the exchange rates of the countries on a daily frequency and the whole sample of 22 years.

Preliminary Results

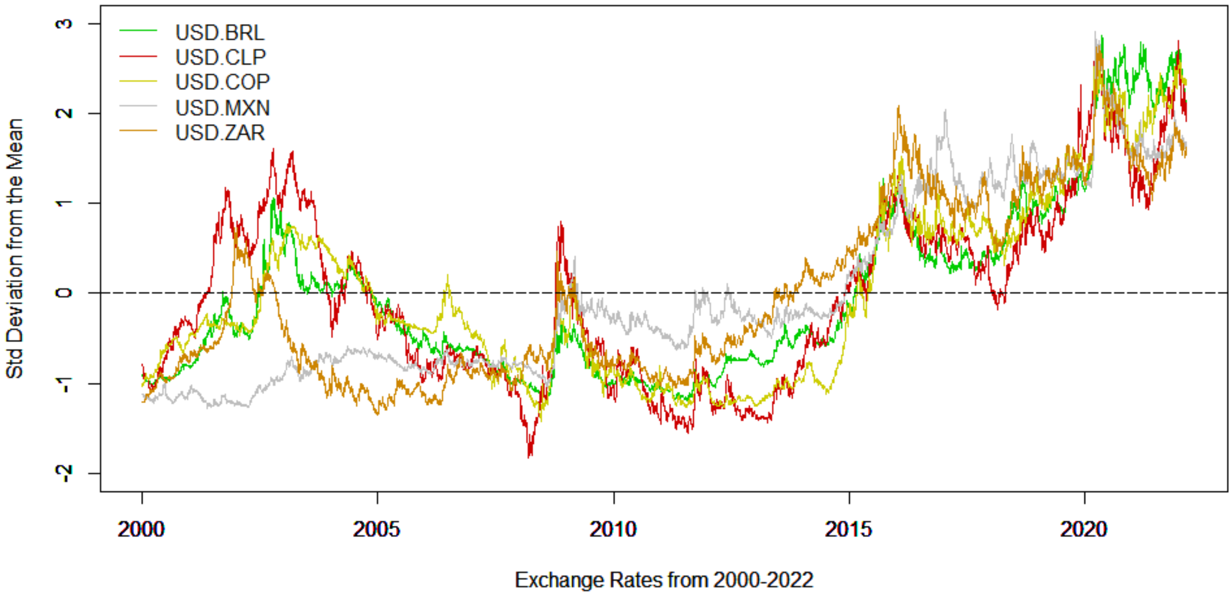
The ECM model would only make sense if the series were non-stationary and integrable. First, it was done a Phillips-Perron Unit Root Test in order to check whether the series were stationary or not.

Looking at the p-value, we can see that all the series for the countries are non-stationary, i.e, they have unit root. The only exception is the ECT, which will be explanied latter. The same result can be easily seen by the above graph of the exchange rate's levels for the whole sample (next page). A bold black line was put around zero to give us some perspective. Notice that none of time series had a behavior similar to a stationary process around zero. It give us further evidence that these time series are not stationary. They had persistent year overvalued (2006,2007) and undervalued (2019-2022).

Since our series not stationary, it is possible to deal with the ECM model and its results. If our main assumption is correct, that the time series for these emerging markets share some stochastic behavior for exchange rates,

Phillips-Perron Unit Root Test		
Variable	Dickey-Fuller Z(alpha)	p-value
Brazil Exchange Rate (BRL)	-3.3801	0.9171
Chilean Peso Rate (CLP)	-5.5785	0.7987
Colombian Peso Rate (COP)	-3.5146	0.9111
Mexican Peso Rate (MXN)	-18.558	0.0934
South African Rand Rate (ZAR)	-9.6244	0.5731
Error Correction (ECT)	-36.017	0.0100

Table 1: Table to test captions and labels.



it is expected a similarity between the error correction and a stationary process. This is also shown in the Cointegration Error graph below.

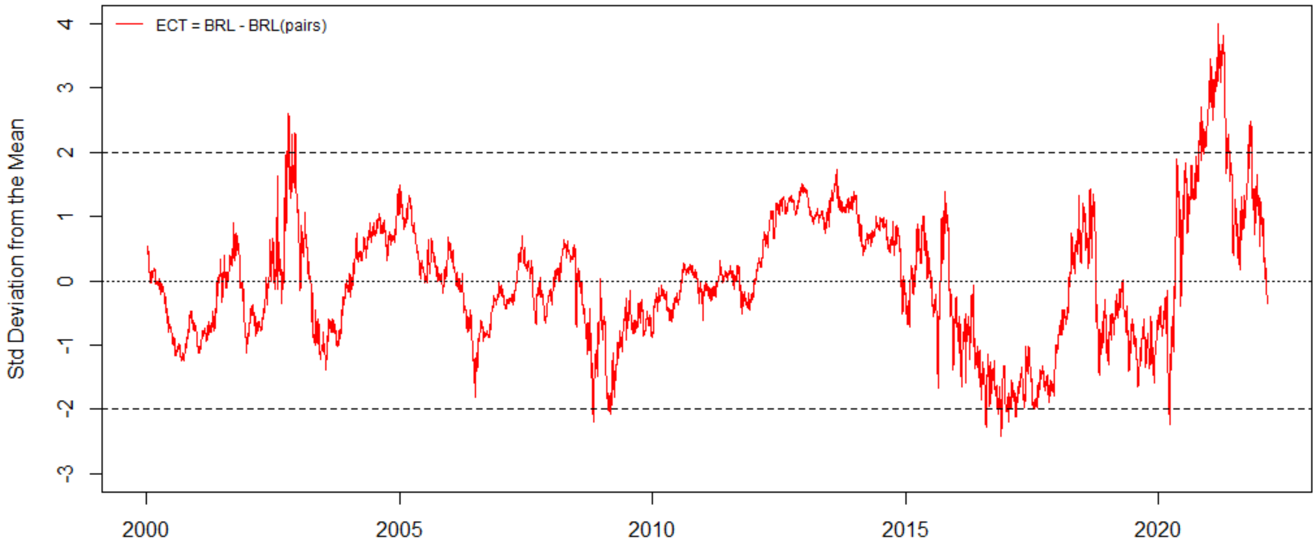
The interpretation of the graph above should be done carefully. First, let's clarify what the meaning of Error Correction (ECT) is. Our models is the following:

$$\mathbb{E}_t(s_{t+h} - s_t) = \beta_0 + \beta_1(s_t - \gamma' f_t)$$

As previously said, the fundamentals, f_t, will be levels of exchange rates of the others countries. The ECT can be described as the residuals of the following regression:

$$USD.BRL = \beta_0 + \beta_1 USD.CLP + \beta_2 USD.COP + \beta_3 USD.MXN + \beta_4 USD.ZAR$$

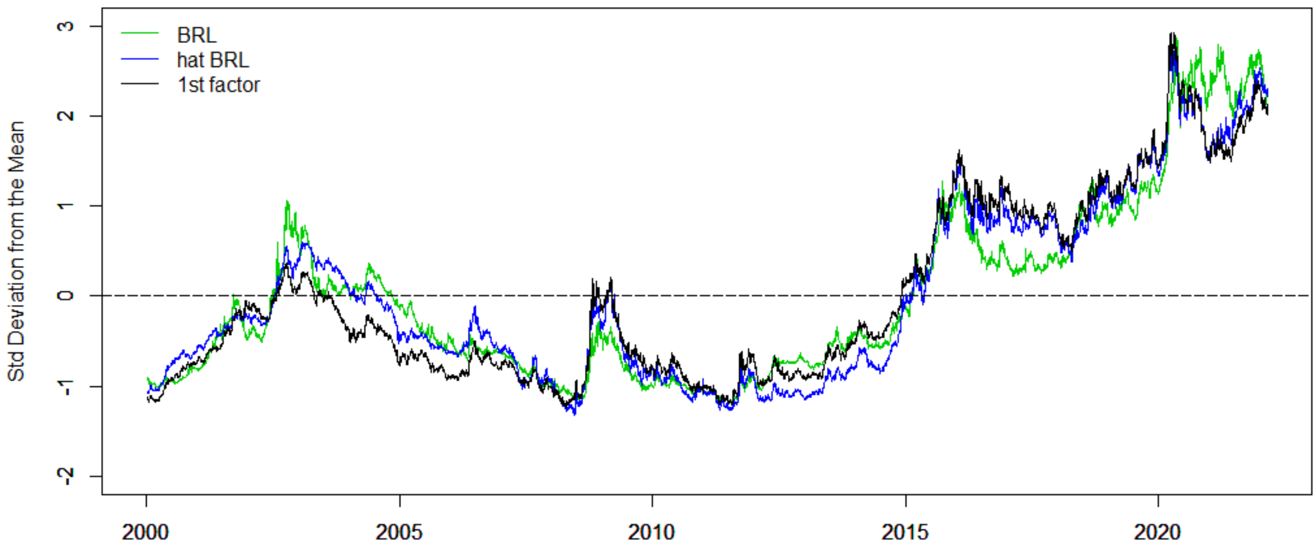
Naively, it could be wrongly said that the BRL was undervalued against the dollar in 2021, when the ECT hit 4 std. deviation. As the last equation expresses, this is completely mistaken. Even if the level of every currency was measured against the dollar, the model express the BRL as a function of the other currency pairs.



Cointegration Error of the ECM model

So, during the pandemic, the BRL depreciated not only against the dollar, but against the basket composed of CLP, COP, MXN and ZAR. In other words, the ECT is the idiosyncratic behavior of the BRL against this basket.

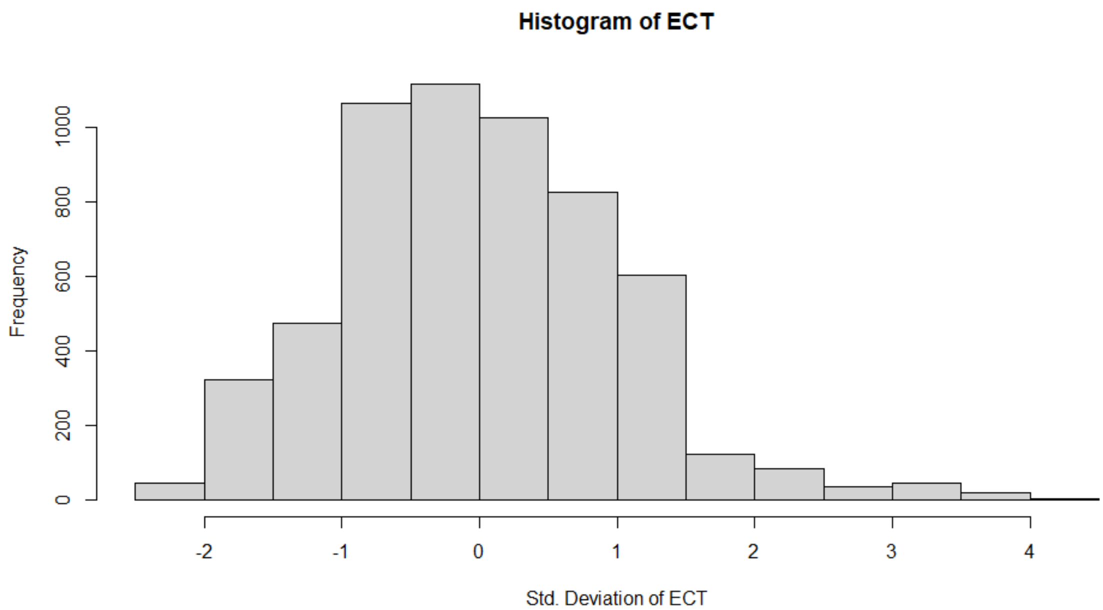
Another way to look this would be through a principal component analysis. The basic idea is to reduce the dimensionality of the emerging market basket ($k = 4$) while retaining as much as possible of the variation present in the data set. The graph below shows the BRL and the fitted value of the BRL (hat BRL) from the ECM model before. We can see that "hat BRL" is almost a projection of BRL into the first factor of the component analysis.



Principal Component Analysis and Fitted Value of ECM model

This model only makes sense because the ECT showed a stationary behavior. This can be seen loosely looking at the Cointegration Error Graph. It has a clear path around the zero during a period almost 22 years long, with the extreme exception during the pandemic. Specially after 2020, the interpretation has to take account for another fact. BRL clearly seemed to detach from the fundamentals that makes it behave like the emergent basket. There are at least two possibilities to account for this, which are necessary to understand the undervaluation of BRL.

These two possibilities are: Brazil had a idiosyncratic shock that will put BRL in a different equilibrium, or BRL could have a huge appreciation in the following moments. It is obvious it could also be a combination of both situations. Brazil had a lot of political noise during the beginning of the pandemic, but it also had one of the best vaccination rate in the world. There were a lot of speculation around the fiscal situation, and part of the market was convinced the government would have a deficit fiscal result. The ECM doesn't give an answer to this question, however, it suggest the BRL could have a lot of room for appreciation if it were to converge to the emerging basket equilibrium. But notice that this interpretation is a bit superficial, since the model doesn't include the changes in the economic fundamentals. For example, if we look at the histogram of ECT, we can see that the ETC of BRL has more frequency in depreciation. What is behind this shape is that countries have different risks, and our model didn't have a measure for risk. This is only one of the fundamentals that this model doesn't have.



Code Efficiency

(INCOMPLETE)

As my first goal, I tried to programming the code without using econometric packages. The first version of this project took at least 53.2 seconds to process. This minimum was reached when I closed all programs and cleaned up the memory and processor from other tasks.

After I started using specific libraries and function already programmed for regression, residuals, fitted value function, and etc, the processing time dropped to only 0.4123 seconds. Not only the performance increased, but the code looked a lot cleaner than what is was.

References

Aloosh, Arash and Bekaert, Geert, Currency Factors (January 31, 2021). Management Science forthcoming, Available at SSRN: https://ssrn.com/abstract=3022623 or http://dx.doi.org/10.2139/ssrn.3022623

Engel, Charles M. and Mark, Nelson Chung and West, Kenneth D., Factor Model Forecasts of Exchange Rates (September 2012). NBER Working Paper No. w18382, Available at SSRN: https://ssrn.com/abstract=2147093

Rossi, Barbara. "Exchange Rate Predictability." Journal of Economic Literature, vol. 51, no. 4, American Economic Association, 2013, pp. 1063–119.