Toward Flexible Scheduling of Real-Time Control Tasks: Reviewing Basic Control Models

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Abstract. We review state-space control models in order to identify timing properties that can favour flexible scheduling of real-time control tasks. First, from the state-space model of a linear time-invariant discrete-time control system with time delay, we derive a new model that involves computing the control signal with a predicted state vector at the actuation instant. Second, by allowing irregular sampling instants, we show that the new model only forces a single synchronization point (actuation instant) while having a feasible implementation. This augurs better schedulability for a set of real-time control tasks, and can provide robustness against scheduling induced jitters.

1 Introduction

The state space model for a linear time-invariant discrete-time control system with time delay [1] implies a synchronous timing at the sampling and actuation instants that can be perfectly met by a single periodic real-time control task. Assuming that sampling and actuation occurs at the beginning and at the end of each task execution [2], the timing of the control task execution corresponds to the timing of the model if task period and deadline are set equal to the sampling period and time delay of the model. However, in a multitasking real-time control system, the tight specification of these timing constraints for control tasks impairs schedulability in the general case [3].

Relaxing this specification by setting the deadline greater than the time delay introduces jitters in control tasks executions that violate the synchronous timing of the model, causing control performance degradation [2]. To maintain synchronism, abstract computational models that force two synchronization points (sampling and actuation) for each control task have been developed ([4] or [5]).

Such computational models introduce three main drawbacks. First, they impose an artificially longer time delay in the closed loop system. Second, they constrain system schedulability by requiring two synchronization points. Third, if state feedback control is used, they involve computing the control signal considering a state vector that becomes outdated at the actuation instant.

The contribution of this paper is to derive a novel state-space model for real-time control tasks aimed at solving the above identified deficiencies. First, we

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derive a new model, *prediction-based model*, that involves computing the control signal using the updated (predicted) state vector at the actuation instant, eliminating the possibly unknown and varying time delay between sampling and actuation. Second, by allowing in the new model irregular sampling instants, we show that the new model only forces a single synchronization point (actuation instant) while having a feasible implementation. This offers flexibility for real-time control tasks scheduling, which can improve schedulability and can provide robustness against jitters.

2 Standard Model

Consider the state space model of a linear time-invariant continuous-time system with time delay τ

$$\begin{aligned} \tfrac{dx(t)}{dt} &= Ax(t) + Bu(t-\tau) \\ y(t) &= Cx(t). \end{aligned} \tag{1}$$

where $x(t)$ is the plant state, $u(t)$ and $y(t)$ are the input and output of the plant, and $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ and $C \in \mathbb{R}^{p\times n}$ are the system, input and output matrices respectively. The time delay can model an input/output latency that appears due to the computation of the control algorithm or due to the insertion of a network within a control loop. For periodic sampling, with sampling period h, where $\tau \le h$, we obtain the standard discrete time model

$$\begin{aligned} x_{k+1} &= \Phi(h)x_k + \Phi(h-\tau)\Gamma(\tau)u_{k-1} + \Gamma(h-\tau)u_k \\ y_k &= Cx_k, \end{aligned} \tag{2}$$

where matrices $\Phi(t)$, $\Gamma(t)$ are obtained using the following

$$\Phi(t) = e^{At}, \quad \Gamma(t) = \int_0^t e^{As}B ds. \tag{3}$$

Note that eq. (2) slightly differs from conventional notation. The purpose of the new notation is to explicitly indicate dependencies on h and τ. An state space model of the system (2) is given by

$$\begin{bmatrix} x_{k+1} \\ z_{k+1} \end{bmatrix} = \begin{bmatrix} \Phi(h) & Phi(h-\tau)\Gamma(\tau) \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_k \\ z_k \end{bmatrix} + \begin{bmatrix} \Gamma(h-\tau) \\ I \end{bmatrix} u_k \tag{4}$$

where $z_k \in \mathbb{R}^{m\times 1}$ are the values that represent the past control signals. For closed loop operation, the control signal will be

$$u_k = \begin{bmatrix} L_1 \; L_2 \end{bmatrix} \begin{bmatrix} x_k \\ z_k \end{bmatrix} = L_1 x_k + L_2 z_k \quad \text{with} \quad L_1 \in \mathbb{R}^{1\times n}, \quad L_2 \in \mathbb{R}^{1\times m}. \tag{5}$$

Remark 1. The closed loop model given by (4) and (5) is based on two synchronization points, the sampling and actuation instants. At time t_k the k^{th} sample[1] (x_k) is taken, and at time $t_{k+\tau}$ the k^{th} control signal (u_k) is sent out. The sampling period h is defined from t_k to t_{k+1}, and the time delay τ from t_k to $t_{k+\tau}$.

[1] Sample is used to refer to the full state vector, regardless of whether it has been sampled or observed.