

# CSCI 341 Problem Set 7

Beyond CFL and the Nature of Computation

Due Friday, October 31

Don't forget to check the webspace for hints and additional context for each problem!

**Problem 1** (Three's a Crowd). Prove that the following language in the alphabet  $0, 1$  is not context-free:

$$L = \{w0w0w \mid w \in \{0, 1\}^*\}$$

*Solution.*

□

**Problem 2** (5 out of 16). Consider the decision problem

$$D_5 = \{5n \mid n \in \mathbb{N}\} \subseteq \mathbb{N}$$

Given a natural number  $n \in \mathbb{N}$ , let  $\text{hex}(n)$  be the hexadecimal representation of  $n$ . Find a regular expression  $r$  such that  $(\text{hex}, \mathcal{L}(r))$  is a faithful representation of  $D_5$ .

*Solution.*

□

**Problem 3** (Composing Representations). In this problem, we are going to show that representations of functions “compose”. We need a bit of notation: given functions  $f_1: S_1 \rightarrow S_2$  and  $f_2: S_2 \rightarrow S_3$ , we are going to define  $f_2 \circ f_1: S_1 \rightarrow S_3$  to be the function defined by

$$f_2 \circ f_1(s) = f_2(f_1(s))$$

This function,  $f_2 \circ f_1$ , is called the composition of  $f_1$  and  $f_2$ .

Now on to the problem. Let  $S_1, S_2, S_3$  be sets and let  $A$  be an alphabet. Let  $\rho_i: S_i \rightarrow A^*$  be a string representation for each  $i = 1, 2, 3$ , and let  $f_1: S_1 \rightarrow S_2$  and  $f_2: S_2 \rightarrow S_3$  be functions.

$$S_1 \xrightarrow{f_1} S_2 \xrightarrow{f_2} S_3$$

Given a representation  $(\rho_1, g_1, \rho_2)$  of  $f_1$  and another a representation  $(\rho_2, g_2, \rho_3)$  of  $f_2$ , prove that  $(\rho_1, g_2 \circ g_1, \rho_3)$  is a representation of  $f_2 \circ f_1$ .

*Solution.*

□

**Problem 4** (OR WHAT). Let  $\vee: B \times B \rightarrow B$  be the logical “or” function. Find a  $\lambda$ -representation OR of  $\vee$ , and evaluate its truth table.

*Solution.*

□

**Problem 5** (Multiply by Three). Find a  $\lambda$ -term  $M_3$  that represents multiplication by 3. That is, if  $C: \mathbb{N} \rightarrow \lambda\text{-Term}$  is the Church-numeral representation of natural numbers, find  $M_3$  such that

$$M_3 C_n \Downarrow C_{3n}$$

Verify that  $M_3 C_3 = C_9$  using your definition of  $M_3$ .

*Solution.*

□

**Problem 6** ((BONUS) 3 out of 2). Let  $D = \{3n \mid n \in \mathbb{N}\}$ . Find a language  $L \subseteq \{0, 1\}^*$  such that  $(\text{bin}, L)$  is a faithful representation of  $D$ . Find a stack automaton  $\mathcal{S}$  with a state  $x$  such that  $L = \mathcal{L}(\mathcal{S}, x)$ .

*Solution.*

□