CSCI 341 Problem Set 7

From Double Pumping to the λ-calculus

Due Friday, October 31

Don't forget to check the webspace for hints and additional context for each problem!

Problem 1 (Three's a Crowd). Prove that the following language in the alphabet $0, 1$ is not context-free:

$$L = \{w0w0w \mid w \in \{0,1\}^*\}$$

Solution. □

Problem 2 (5 out of 16). Consider the decision problem

$$D_5 = \{5n \mid n \in \mathbb{N}\} \subseteq \mathbb{N}$$

Given a natural number $n \in \mathbb{N}$, let $\mathsf{hex}(n)$ be the hexidecimal representation of n. Find a regular expression r such that $(\mathsf{hex}, \mathcal{L}(r))$ is a faithful representation of D_5.

Solution. □

Problem 3 (Composing Representations). In this problem, we are going to show that representations of functions "compose". We need a bit of notation: given functions $f_1 \colon S_1 \to S_2$ and $f_2 \colon S_2 \to S_3$, we are going to define $f_2 \circ f_1 \colon S_1 \to S_3$ to be the function defined by

$$f_2 \circ f_1(s) = f_2(f_1(s))$$

This function, $f_2 \circ f_1$, is called the ¡i¿composition of f_1 and f_2¡/i¿.

¡p¿¡/p¿ Now on to the problem. Let S_1, S_2, S_3 be sets and let A be an alphabet. Let $\rho_i \colon S_i \to A^*$ be a string representation for each $i = 1, 2, 3$, and let $f_1 \colon S_1 \to S_2$ and $f_2 \colon S_2 \to S_3$ be functions.

$$S_1 \xrightarrow{f_1} S_2 \xrightarrow{f_2} S_3$$

Given a representation (ρ_1, g_1, ρ_2) of f_1 and a representation (ρ_2, g_2, ρ_3) of f_2, prove that $(\rho_1, g_2 \circ g_1, \rho_3)$ is a representation of $f_2 \circ f_1$.

Solution. □

Problem 4 (OR WHAT). Let $\vee \colon B \times B \to B$ be the logical "or" function. Find a λ-representation OR of $\vee$, and evaluate its truth table.

Solution. □

Problem 5 (Multipy by Three). Find a λ-term M_3 that represents ¡i¿multiplication by 3¡/i¿. That is, if $\#_{Ch} \colon \mathbb{N} \to \lambda \mathit{Term}$ is the Church-numeral representation of natural numbers,

$$\mathsf{M}_3\mathsf{C}_\mathsf{n} \Downarrow \mathsf{C}_{3\mathsf{n}}$$

Verify that $\mathsf{M}_3\mathsf{C}_3 = \mathsf{C}_9$ using your definition of M_3.

Solution. □