

# CSCI 341 Problem Set 6

## Stack Automata

Due Friday, October 17

Don't forget to check the webspace for hints and additional context for each problem!

**Problem 1** (Pop-Push). By induction on the length of the program, prove that every valid stack program is equivalent to either `skip` or a program of the following form:

$$\text{pop } \sigma_1.\text{pop } \sigma_2 \dots \text{pop } \sigma_n.\text{push } \tau_1.\text{push } \tau_2 \dots \text{push } \tau_m \quad (*)$$

for some  $n, m \in \mathbb{N}$ .

*Solution.* □

**Problem 2** (2 is better than 1). Consider the language

$$L = \{w \mid w = w^{\text{op}}\}$$

in the alphabet  $A = \{a, b, c\}$ . Design a stack automaton  $\mathcal{S} = (Q, A, \Sigma, \delta, F)$  with a state  $x \in Q$  such that  $L = \mathcal{L}(\mathcal{S}, x)$ .

*Solution.* □

**Problem 3** (Algorithmic Balancing). Let  $L$  be the set of balanced strings of brackets, where the bracket symbols are  $A = \{(\,), [, ], \{, \}\}$ , from Balancing Act. In that problem, you designed a grammar with a variable that derives  $L$ .

- (1) Use the Grammar-to-Automaton construction to design a stack automaton with a state that accepts  $L$ .
- (2) Can you think of a smaller stack automaton with a state that accepts  $L$ ?

*Solution.* □

**Problem 4** (Basic Stack Programs Unite). Fix a set of stack symbols  $\Sigma$ . Let  $p_1, p_2, \dots, p_n \in \{\text{push } \sigma, \text{pop } \sigma \mid \sigma \in \Sigma\}$ , i.e., let them be basic stack programs. Assume that  $p_1 \dots p_i = \text{skip}$  if and only if  $i = 0$  or  $i = n$ . Now prove that

- (1)  $p_1 p_n = \text{skip}$
- (2)  $p_2 \dots p_{n-1} = \text{skip}$

*Solution.* □

## Counter Automata

**Problem 5** (Cats > Dogs). Let  $A = \{c, a, t, d, o, g\}$ . Design a counter automaton with a state  $x$  that accepts the language  $L_{\text{cat}}$  of all words  $w \in A^*$  such that the string "cat" appears in  $w$  more times than "dog" appears in  $w$ .