

CSCI 341 Problem Set 7

Beyond CFL and the Nature of Computation

Due Friday, October 31

Don't forget to check the webspace for hints and additional context for each problem!

Problem 1 (Three's a Crowd). Prove that the following language in the alphabet $0, 1$ is not context-free:

$$L = \{w0w0w \mid w \in \{0, 1\}^*\}$$

Solution. □

Problem 2 (5 out of 16). Consider the decision problem

$$D_5 = \{5n \mid n \in \mathbb{N}\} \subseteq \mathbb{N}$$

Given a natural number $n \in \mathbb{N}$, let $\text{hex}(n)$ be the hexadecimal representation of n . Find a regular expression r such that $(\text{hex}, \mathcal{L}(r))$ is a faithful representation of D_5 .

Solution. □

Problem 3 (Composing Representations). In this problem, we are going to show that representations of functions “compose”. We need a bit of notation: given functions $f_1: S_1 \rightarrow S_2$ and $f_2: S_2 \rightarrow S_3$, we are going to define $f_2 \circ f_1: S_1 \rightarrow S_3$ to be the function defined by

$$f_2 \circ f_1(s) = f_2(f_1(s))$$

This function, $f_2 \circ f_1$, is called the *composition* of f_1 and f_2 .

Now on to the problem. Let S_1, S_2, S_3 be sets and let A be an alphabet. Let $\rho_i: S_i \rightarrow A^*$ be a string representation for each $i = 1, 2, 3$, and let $f_1: S_1 \rightarrow S_2$ and $f_2: S_2 \rightarrow S_3$ be functions.

$$S_1 \xrightarrow{f_1} S_2 \xrightarrow{f_2} S_3$$

Given a representation (ρ_1, g_1, ρ_2) of f_1 and another a representation (ρ_2, g_2, ρ_3) of f_2 , prove that $(\rho_1, g_2 \circ g_1, \rho_3)$ is a representation of $f_2 \circ f_1$.

Solution. □

Problem 4 (OR WHAT). Let $\vee: B \times B \rightarrow B$ be the logical “or” function. Find a λ -representation OR of \vee , and evaluate its truth table.

Solution. □

Problem 5 (Multiply by Three). Find a λ -term M_3 that represents multiplication by 3. That is, if $C: \mathbb{N} \rightarrow \lambda\text{Term}$ is the Church-numeral representation of natural numbers, find M_3 such that

$$M_3 C_n \Downarrow C_{3n}$$

Verify that $M_3 C_3 = C_9$ using your definition of M_3 .

Solution. □

Problem 6 ((BONUS) 3 out of 2). Let $D = \{3n \mid n \in \mathbb{N}\}$. Find a language $L \subseteq \{0, 1\}^*$ such that (bin, L) is a faithful representation of D . Find a stack automaton \mathcal{S} with a state x such that $L = \mathcal{L}(\mathcal{S}, x)$.

Solution. □