CSCI 341 Problem Set 10

Computability

Due Tuesday, December 2

Don't forget to check the webspace for hints and additional context for each problem!

Problem 1 (Composing Tractible Problems)**.** Let $f\colon S_1 \to S_2$ and $g\colon S_2 \to S_3$ be functions between sets S_1, S_2, S_3. Prove that if the problems of computing f and g are both tractible, then the problem of computing $g \circ f$ is tractible.

Solution. □

Problem 2 (Composing Reductions Again)**.** Let $L_1, L_2, L_3 \subseteq A^*$ be languages. Let $r_1\colon L_1 \preceq L_2$ and $r_2\colon L_2 \preceq L_3$. Prove that if r_1 and r_2 are computable in polynomial time, then $r_2 \circ r_1\colon L_1 \preceq L_3$ is a polynomial time reduction. Conclude that if $L_3 \in \mathsf{P}$, then $L_1 \in \mathsf{P}$.

Solution. □

Problem 3 (Empty Language Problem for Automata)**.** Recall that a state $x \in Q$ of an automaton $\mathcal{A} = (Q, A, \delta, F)$ accepts a word $w = a_1 \cdots a_n$ if there is a path $x \xrightarrow{a_1} x_1 \xrightarrow{a_2} \cdots \xrightarrow{a_n} x_n$ and $x_n \in F$, and that $\mathcal{L}(\mathcal{A}, x)$ is the set of all words accepted by the state x in $\mathcal{A}$. Show that deciding whether a state accepts the empty language, i.e., $\mathcal{L}(\mathcal{A}, x) = \{\}$, is in NP by finding a polynomial time reduction to Rch.

Solution. □

Problem 4 (Hardness by Reduction)**.** Prove the Hardness by Reduction theorem.