CSCI 341 Workshop 5

Stack Automata

November 5, 2025

Problem 1. Show that the language $L_{=} = \{0^n 1^n \mid n \in \mathbb{N}\}$ is decidable.

Problem 2. We are going to show that the language

$$L_{11?} = \{\lfloor \mathcal{T} \rfloor * x \mid x \text{ halts on input } \texttt{11} \text{ in the Turing machine } \mathcal{T}\}$$

is undecidable as follows:

(1) Write a program `rep_11` that clears the tape and then prints `11` to the tape (i.e., if $\mathcal{C}$ is the Turing machine containing the state `rep_11`, then $\mathcal{C}_{\texttt{rep\_11}}(w) = \texttt{11}$ for any $w \in A^*$).

(2) As an example of what you'll do next: in Problem 1, you designed a Turing machine $\mathcal{T}$ with a state x such that $\mathcal{T}_x(0^n 1^n) = 1$ and otherwise $\mathcal{T}_x(w) = 0$. Write a program `rep_11_equal` that (1) clears the tape, (2) writes `11` to the tape, then (3) runs $\mathcal{T}$ starting from x.

(3) Now describe a Turing machine $\mathcal{W}$ with a state y that incorporates the code for `rep_11` into the encoding of any other Turing program, in the following way: if $\mathcal{T}$ is a Turing machine with a state x, then $\mathcal{W}_y(\lfloor \mathcal{T} \rfloor * x)$ is an encoding of the Turing program that (1) clears the tape, (2) writes `11` to the tape, then (3) runs $\mathcal{T}$ starting from x.

(4) We are now in a position to reduce L_{Halt} to $L_{11?}$.
Suppose that the Turing program x_{11} in the the Turing machine $\mathcal{E}$ is a decider for the language $L_{11?}$. Now we define the Turing machine $\mathcal{H}$ and its program x_{hlt} as follows: given a Turing machine $\mathcal{T}$ with state x and an input word $w \in \{0,1\}^*$, running x_{hlt} starting with the input string $\lfloor \mathcal{T} \rfloor * x * w$

   (a) runs $\mathcal{W}_y(\lfloor \mathcal{T} \rfloor * x * w)$ and writes its contents to the tape