

# CSCI 341 Problem Set 6

## Stack Automata

Due Friday, October 17

Don't forget to check the webspace for hints and additional context for each problem!

### Pumping Lengths

**Problem 1** (Pop-Push). By induction on the length of the program, prove that every valid stack program is equivalent to either skip or a program of the following form:

$$\text{pop } \sigma_1.\text{pop } \sigma_2 \dots \text{pop } \sigma_n.\text{push } \tau_1.\text{push } \tau_2 \dots \text{push } \tau_m \quad (*)$$

for some  $n, m \in \mathbb{N}$ .

*Solution.*

□

**Problem 2** (2 is better than 1). Consider the language

$$L = \{w \mid w = w^{\text{op}}\}$$

in the alphabet  $A = \{a, b, c\}$ . Design a stack automaton  $\mathcal{S} = (Q, A, \Sigma, \delta, F)$  with a state  $x \in Q$  such that  $L = \mathcal{L}(\mathcal{S}, x)$ .

*Solution.*

□

**Problem 3** (Arithmetic is Not Regular). Prove that the language of arithmetic expressions  $ArExp \subseteq A^*$ , derived from  $E$  in the grammar  $\mathcal{G} = (X, A, R)$  below

$$\begin{aligned} E &\rightarrow N \mid (E + E) \mid (E \times E) \mid (E - E) \mid (E/E) \\ N &\rightarrow 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9 \mid NN \end{aligned}$$

where the alphabet is

$$A = \{ (, ), +, \times, -, /, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 \}$$

is not regular.

*Solution.*

□

### Parse Trees

**Problem 4** (Left on Your Own). Let  $\mathcal{G}$  be a grammar with a variable  $x$ , and let  $w \in A^*$ . Prove that if  $w$  has a derivation from  $x$ , then  $w$  has a left-most derivation from  $x$ .

*Solution.*

□

### Counter Automata

**Problem 5** (Cats > Dogs). Let  $A = \{c, a, t, d, o, g\}$ . Design a counter automaton with a state  $x$  that accepts the language  $L_{cat}$  of all words  $w \in A^*$  such that the string "cat" appears in  $w$  more times than "dog" appears in  $w$ .