

CSCI 341 Problem Set 10

Complexity

Due Tuesday, December 2

Don't forget to check the webspace for hints and additional context for each problem!

Problem 1 (Composing Tractable Problems). Let $f: S_1 \rightarrow S_2$ and $g: S_2 \rightarrow S_3$ be functions between sets S_1, S_2, S_3 . Prove that if the problems of computing f and g are both tractable, then the problem of computing $g \circ f$ is tractable.

Solution.

□

Problem 2 (Composing Reductions Again). Let $L_1, L_2, L_3 \subseteq A^*$ be languages. Let $r_1: L_1 \leq L_2$ and $r_2: L_2 \leq L_3$. Prove that if r_1 and r_2 are computable in polynomial time, then $r_2 \circ r_1: L_1 \leq L_3$ is a polynomial time reduction. Conclude that if $L_3 \in P$, then $L_1 \in P$.

Solution.

□

Problem 3 (Empty Language Problem for Automata). Recall that a state $x \in Q$ of an automaton $\mathcal{A} = (Q, A, \delta, F)$ accepts a word $w = a_1 \cdots a_n$ if there is a path $x \xrightarrow{a_1} x_1 \xrightarrow{a_2} \cdots \xrightarrow{a_n} x_n$ and $x_n \in F$, and that $\mathcal{L}(\mathcal{A}, x)$ is the set of all words accepted by the state x in \mathcal{A} . Show that deciding whether a state accepts the empty language, i.e., $\mathcal{L}(\mathcal{A}, x) = \{\}$, is in NP by finding a polynomial time reduction to *Rch* above.

Solution.

□

Problem 4 (Hardness by Reduction). Let $\text{Fam} \subseteq 2^{A^*}$ be a family of languages, and let $L \subseteq A^*$ be Fam-hard. Suppose that we had another language $W \subseteq A^*$ and a polynomial time reduction $r: L \leq W$. Show that W is also Fam-hard.

Solution.

□