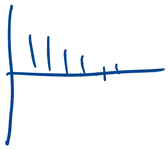
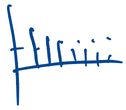
Uji ADF → cek Stasioner!

H_0: Data Tidak stasioner (Memiliki Unit root)

H_1: Data Stasioner (Tidak Memiliki Unit root)

$\boxed{\text{MA} \to \text{ACF Cutoff}}$

$\boxed{\text{AR} \to \text{PAC Cutoff}}$

ACF  PACF 

Cek Cutoff after lag berapa?

ISTUTI MPDW Part 2

Oktober 2024

1. Dalam suatu analisis, disimpulkan bahwa:
   - Uji Dickey-Fuller unit root test untuk deret waktu {Yt} tolak H0 → Y_t → Stasioner
   - ACF untuk deret waktu {Yt} menurun secara eksponensial
   - PACF untuk deret waktu {Yt} signifikan pada lags 1, 2 dan 3 (pada lag lainnya tidak nyata) → AR(3)

   Model manakah yang paling konsisten dengan kesimpulan di atas?

   a. IMA(1,3)
   b. ARI(3,1)
   c. ARIMA(3,0,0)
   d. IMA(2,3)

$\nabla^2 = (1-B)^2$ → $By_t = y_{t-1}$, $B^2 y_t = y_{t-2}$

2. Dalam suatu analisis diperoleh kesimpulan,
   - ACF untuk deret waktu {Yt} menurun $\boxed{\text{sangat lambat sekali}}$
   - PACF untuk deret waktu $\{\nabla^2 Yt\}$ signifikan pada lags 1 dan 2 (tidak nyata pada lainnya)
   - Uji Augmented Dicky Fuller menunjukan tidak tolak H_0 untuk deret waktu {Yt}

   ARIMA (p, d, q)

   Model manakah yang paling konsisten dengan kesimpulan diatas…

   a. IMA(1,1) ×
   b. ARI(2,2)
   c. ARIMA(2,2,2) ×
   d. IMA(2,2) ×

$\nabla^2 Y_t \Rightarrow$ AR(2)

ARIMA (2,2,0)

3. Misakan Y_t mengikuti model AR(2) dengan model $Y_t = \phi_2 \, Y_{t-2} + e_t$. Maka kisaran nilai ϕ_2 agar model stasioner adalah…

   Syarat AR(2) → Stasioner

   a. $\phi_2 > 1$ sehingga $Var(Y_{t-2}) \geq 0$ ×
   b. $\phi_2 < 1$ sehingga $Var(Y_{t-2}) \geq 0$ ×
   c. $-1 \ < \phi_2 < 1$ sehingga $Var(Y_{t-2}) < 0$
   d. $-1 \ < \phi_2 < 1$ sehingga $Var(Y_{t-2}) \geq 0$

$|\phi_2| < 1 \to -1 < \phi_2 < 1$

$\phi_1 + \phi_2 < 1$

$\phi_2 - \phi_1 < 1$

$(1 - B^4)^2$

4. Perhatikan persamaan berikut

$$(1-B)(1-B^4)(1-0.6B)(1-0.8B^4)Z_t = e_t$$

   Manakah model yang merepresentasikan model diatas ?

   a. ARIMA(1,1,1)(1,0,1)[4]
   b. ARIMA(1,0,1)(1,1,1)[4]
   c. ARIMA(1,1,0)(1,0,1)[4]
   d. ARIMA(1,1,0)(1,1,0)[4]

   non musim — musiman

$\underbrace{(1-0{,}8B)}_{\text{MA}}\underbrace{(1-0{,}4B^4)}_{\text{MA musim}}Z_t$

ARIMA (p,d,q)(P,D,Q)

$(1-0{,}6B)Z_t = Z_t - 0{,}6BZ_t$

$= \underbrace{Z_t - 0{,}6\,Z_{t-1}}_{\text{AR(1)}}$

$(1-0{,}8B^4)Z_t$

$= \underbrace{Z_t - 0{,}8\,Z_{t-4}}_{\text{AR(1)}}$ → Komponen musiman

$(1-B^4)Z_t = Z_t - B^4 Z_t$

$= \underbrace{Z_t - Z_{t-4}}_{\text{differencing musiman}}$

$(1-B)Z_t = \underbrace{Z_t - Z_{t-1}}_{\text{Process differencing}}$

∇Z_t

Model non musiman → ARIMA (1,1,0)

Model musim → (1,1,0)

$Ma(1) : \underbrace{\overbrace{e_t - \theta_1 e_{t-1}}}$

ARMA( )

$\hookrightarrow (Y_t, e_{t-1})$

$\to AR(p) \to \boxed{e_t \quad Y_t}$

Saling bebas

$Y_t = \phi_2 Y_{t-2} + e_t$

$Var(Y_t) = Var(\phi_2 Y_{t-2} + e_t)$

$= Var(\phi_2 Y_{t-2}) + Var(e_t)$

$= \phi_2^2 \; Var(Y_{t-2}) + Var(e_t)$

$= \phi_2^2 \; Var(Y_{t-2}) + \sigma_e^2$

$\boxed{Var(y_t) = Var(y_{t-2})} \to$ Model Stasioner.

Misalkan $Var(Y_t) = Var(Y_{t-2}) = a$

$a = \phi_2^2 \; a + \sigma_e^2$

$a - \phi_2^2 a = \sigma_e^2$

$a(1 - \phi_2^2) = \sigma_e^2$

$a = \dfrac{\sigma_e^2}{1 - \phi_2^2}$

$\boxed{Var(Y_{t-2}) = Var(Y_t) = a}$ → Ingat kembali

$Var(Y_{t-2}) = \dfrac{\sigma_e^2 \; (+)}{\boxed{1 - \phi_2^2} \; (+)}$ → $-1 < \phi_2 < 1$

≥ 0 //

ARMA (1,2)

$(1 - 0{,}8B) Z_t = e_t + 0{,}7 e_{t-1} + 0{,}6 e_{t-2}$

$Z_t - 0{,}8 Z_{t-1} = \ldots$

$Z_t = 0{,}8 Z_{t-1} + e_t + 0{,}7 e_{t-1} + 0{,}6 e_{t-2}$.

$e \sim N(0, \sigma_e^2)$ BSI

$\rho_k \to \dfrac{\gamma_k}{\boxed{\gamma_0}}$

Variance.

$Cov(Y_t, Y_t) = Var(Y_t)$

$= \gamma_0$

$Cov(\underbrace{Y_t, Y_{t-1}}) = E[Y_t, Y_{t-1}]$

$= \gamma_1$

$Cov[Y_t, Y_{t-k}) = \gamma_k$

$Cov(Y_t, e_t) = E(0{,}8 Z_{t-1} + e_t + 0{,}7 e_{t-1} + 0{,}6 e_{t-2}, e_t)$

$-\underbrace{E(Y_t, e_t)} = E(0{,}8 Z_{t-1}, e_t) + E(e_t, e_t) + E(0{,}7 e_{t-1}, e_t) + E(0{,}6 e_{t-2}, e_t)$

$= 0{,}8 \; E(Z_{t-1}, e_t) + \sigma_e^2 + 0{,}7 \; E(e_{t-1}, e_t) + 0{,}6 \; E(e_{t-2}, e_t)$

$= 0 + \sigma_e^2 + 0 + 0$

$= \sigma_e^2$ //

$E(Y_t, e_{t-1}) = E(0{,}8 Z_{t-1} + e_t + 0{,}7 e_{t-1} + 0{,}6 e_{t-2}, e_{t-1}]$

$= E(0{,}8 Z_{t-1}, e_{t-1}) + E(e_t, e_{t-1}) + E(0{,}7 e_{t-1}, e_{t-1}) + E(0{,}6 e_{t-2}, e_{t-1})$

$= 0{,}8 \; E(Z_{t-1}, e_{t-1}) + 0 + 0{,}7 \; E(e_{t-1}, e_{t-1}) + 0$

$= 0{,}8 \; \sigma_e^2 + 0{,}7 \sigma_e^2$

$= 1{,}5 \; \sigma_e^2$

$E(Y_t, e_{t-2}) = E(0{,}8 Y_{t-1} + e_t + 0{,}7 e_{t-1} + 0{,}6 e_{t-2}, e_{t-2})$

$= E(0{,}8 Y_{t-1}, e_{t-2}) + E(e_t, e_{t-2}) + E(0{,}7 e_{t-1}, e_{t-2}) + E(0{,}6 e_{t-2}, e_{t-2})$

$= 0{,}8 \; E(Y_{t-1}, e_{t-2}) + 0 + 0 + 0{,}6 \; E(e_{t-2}, e_{t-2})$

$= 0{,}8 [1{,}5 \sigma_e^2] + 0 + 0 + 0{,}6 \sigma_e^2$

$= 1{,}8 \sigma_e^2$

ρ_k

$$\gamma_0 = Var(Y_t)$$
$$= E(Y_t, Y_t)$$
$$= E(0{,}8\,Y_{t-1} + e_t + 0{,}7\,e_{t-1} + 0{,}6\,e_{t-2}\ ,\ Y_t)$$
$$= E(0{,}8\,Y_{t-1}, Y_t) + E(e_t, Y_t) + E(0{,}7\,e_{t-1}, Y_t) + E(0{,}6\,e_{t-2}, Y_t)$$
$$= 0{,}8\,\gamma_1 + \sigma_e^2 + 0{,}7\,(1{,}5\,\sigma_e^2) + 0{,}6\,(1{,}8\,\sigma_e^2)$$
$$= 0{,}8\,\gamma_1 + \sigma_e^2 + 2{,}13\,\sigma_e^2$$
$$= \underline{0{,}8\,\gamma_1} + 3{,}13\,\sigma_e^2$$

$$\gamma_1 = Var(Y_t, Y_{t-1})$$
$$= E(0{,}8\,Y_{t-1}, Y_{t-1}) + E(e_t, Y_{t-1}) + E(0{,}7\,e_{t-1}, Y_{t-1}) + E(0{,}6\,e_{t-2}, Y_{t-1})$$
$$= 0{,}8\,\gamma_0 + 0 + 0{,}7\,\sigma_e^2 + 0{,}6\,(1{,}5\,\sigma_e^2)$$
$$= 0{,}8\,\gamma_0 + 1{,}6\,\sigma_e^2$$

γ_0 ? =

$$\gamma_0 = 0{,}8\,(0{,}8\,\gamma_0 + 1{,}6\,\sigma_e^2) + 3{,}13\,\sigma_e^2$$
$$= 0{,}64\,\gamma_0 + 1{,}28\,\sigma_e^2 + 3{,}13\,\sigma_e^2$$
$$\gamma_0 - 0{,}64\,\gamma_0 = 4.41\,\sigma_e^2$$
$$\gamma_0\,(1 - 0{,}64) = 4{,}41\,\sigma_e^2$$
$$\gamma_0 = \frac{4{,}41\,\sigma_e^2}{0{,}36} \quad //$$

$$\gamma_2 = Var(Y_t, Y_{t-2})$$
$$= E(0{,}8\,Y_{t-1}, Y_{t-2}) + E(e_t, Y_{t-2}) + E(0{,}7\,e_{t-1}, Y_{t-2}) + E(0{,}6\,e_{t-2}, Y_{t-2})$$
$$= 0{,}8\,\gamma_1 + 0 + 0 + 0{,}6\,\sigma_e^2$$
$$= 0{,}8\,\gamma_1 + 0{,}6\,\sigma_e^2$$

Y_{t-4}

$$\gamma_3 = Var(Y_t, Y_{t-3})$$
$$= E(0{,}8\,Y_{t-1}, Y_{t-3}) + E(e_t, Y_{t-3}) + E(0{,}7\,e_{t-1}, Y_{t-3}) + E(0{,}6\,e_{t-2}, Y_{t-3})$$
$$= 0{,}8\,\gamma_2 + 0 + 0 + 0$$
$$= 0{,}8\,\gamma_2$$

$$\gamma_4 = 0{,}8\,\gamma_3 + 0 + 0 + 0$$

$$\boxed{k > 2} \rightarrow \boxed{\gamma_k = 0{,}8\,\gamma_{k-1}}$$

$$\rho_k = \frac{\gamma_k}{\gamma_0} = \frac{0{,}8\,\gamma_{k-1}}{\gamma_0}$$
$$= 0{,}8\,\rho_{k-1}$$

non musim

MA → musiman

→ musiman

5. Perhatikan persamaan berikut:

non - musiman ← $(1 - B)(1 - B^{12})Z_t = (1 + 0.2B)(1 + 0.8B^{12})e_t$

Manakah model yang merepresentasikan persamaan diatas?

a. ARIMA(1,1,0)(1,0,1)[12]

b. ARIMA(1,1,1)(1,0,1)[12]

c. ARIMA(0,2,1)(1,1,1)[12]

d. ARIMA(0,1,1)(0,1,1)[12]

6. Misalkan Z_t adalah deret waktu yang mengikuti model berikut:

$$(1 - \boxed{0.8B})Z_t = \underbrace{e_t + 0.7e_{t-1} + 0.6\, e_{t-2}}_{MA(2)}$$

Pernyataan yang sesuai adalah

AR(1)

a. $\rho_k = 0.8\,\rho_{k-1}\ for\ k \geq 1$

b. $\rho_k = 0.8\,\rho_{k-1}\ \boxed{for\ k > 2}$

c. $\rho_k = 0.8\,\rho_{k-1}\ + 0.6\,\sigma_e^2$

d. $\rho_k = 0.8\,\rho_{k-1}\ + 0.7\,\sigma_e^2$

ARMA(1)(2)

↳ $k > 2$ ......

$0{,}8\,\rho_{k-1}$

7. Andi memiliki data deret waktu dari *profit* sebuah toko kelontong, pada beberapa periode pengamatan, toko kelontong yang Andi kelola mengalami BEP (*Break Even Point*). Jika Andi melakukan *time series forecasting* terhadap data yang ia miliki. Metriks evaluasi yang dapat digunakan adalah, kecuali?

Profit = 0

a. MSE →

b. RMSE → $\sqrt{MSE}$

c. MAPE

d. MAD

$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$

$MAD = \frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}_i|$

$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{y_i - \hat{y}_i}{y_i}\right|$ 0

8.

Misalkan $\{X_t\}$ adalah suatu deret waktu yang mengikuti proses $AR(3)$ dengan model

$$X_t = 0.4X_{t-1} + 0.2X_{t-2} - 0.5X_{t-3} + \varepsilon_t$$

$E(a_t, y_{t+1})$

AR(3)

Maka pernyataan yang tepat untuk model diatas adalah

a. $\rho_2 = 0.4\rho_2 + 0.2 - 0.5\rho_2$

b. $\rho_2 = 0.4 + 0.2\rho_1 - 0.5\rho_2$

c. $\rho_1 = 0.4\rho_1 + 0.2 - 0.5\rho_1$

d. $\rho_1 = 0.4 + 0.2\rho_1 - 0.5\rho_2$

γ_2, γ_1, γ_0

$\rho_1 = \frac{\gamma_1}{\gamma_0}$

$\rho_2 = \frac{\gamma_2}{\gamma_0}$

$$\gamma_0 = Var(Y_t)$$
$$= E(Y_t, Y_t)$$
$$= E(0{,}4\, Y_{t-1} + 0{,}2\, Y_{t-2} + 0{,}5\, Y_{t-3} + e_t,\ Y_t)$$
$$= E(0{,}4\, Y_{t-1}, Y_t) + E(0{,}2\, Y_{t-2}, Y_t) + E(0{,}5\, Y_{t-3}, Y_t) + \overset{\to 0}{E(e_t, Y_t)}$$
$$= 0{,}4\, \gamma_1 + 0{,}2\, \gamma_2 + 0{,}5\, \gamma_3$$

$$\gamma_1 = E(Y_t, Y_{t-1})$$
$$= E(0{,}4\, Y_{t-1}, Y_{t-1}) + E(0{,}2\, Y_{t-2}, Y_{t-1}) - E(0{,}5\, Y_{t-3}, Y_{t-1})$$
$$= 0{,}4\, \gamma_0 + 0{,}2\, \gamma_1 - 0{,}5\, \gamma_2$$

$$\gamma_2 = E(Y_t, Y_{t-2})$$
$$= E(0{,}4\, Y_{t-1}, Y_{t-2}) + E(0{,}2\, Y_{t-2}, Y_{t-2}) - E(0{,}5\, Y_{t-3}, Y_{t-2})$$
$$= 0{,}4\, \gamma_1 + 0{,}2\, \gamma_0 - 0{,}5\, \gamma_1$$
$$\gamma_2 = 0{,}9\, \gamma_1 + 0{,}2\, \gamma_0$$

$$\rho_1 = \frac{\gamma_1}{\gamma_0} = \frac{0{,}4\gamma_0 + 0{,}2\gamma_1 - 0{,}5\gamma_2}{\gamma_0} = \frac{0{,}4\cancel{\gamma_0}}{\cancel{\gamma_0}} + \frac{0{,}2\gamma_1}{\gamma_0} + \frac{0{,}5\gamma_2}{\gamma_0}$$
$$= 0{,}4 + 0{,}2\rho_1 - 0{,}5\rho_2$$

$$\rho_2 = \frac{\gamma_2}{\gamma_0} = \frac{0{,}9\gamma_1 + 0{,}2\gamma_0}{\gamma_0} = 0{,}9\rho_1 + 0{,}2$$

5. Perhatikan persamaan berikut:

$$(1-B)(1-B^{12})Z_t = (1+0.2B)(1+0.8B^{12})e_t$$

Manakah model yang merepresentasikan persamaan diatas?

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6. Misalkan Z_t adalah deret waktu yang mengikuti model berikut:

$$(1-0.8B)Z_t = e_t + 0.7e_{t-1} + 0.6\, e_{t-2}$$

Pernyataan yang sesuai adalah

a. $\rho_k = 0.8\,\rho_{k-1}\ for\ k \geq 1$

b. $\rho_k = 0.8\,\rho_{k-1}\ for\ k > 2$

c. $\rho_k = 0.8\,\rho_{k-1}\ + 0.6\,\sigma_e^2$

d. $\rho_k = 0.8\,\rho_{k-1}\ + 0.7\,\sigma_e^2$

7. Andi memiliki data deret waktu dari *profit* sebuah toko kelontong, pada beberapa periode pengamatan, toko kelontong yang Andi kelola mengalami BEP (*Break Even Point)*. Jika Andi melakukan *time series forecasting* terhadap data yang ia miliki. Metriks evaluasi yang dapat digunakan adalah, kecuali?

a. MSE

b. RMSE

c. MAPE

d. MAD

8.

Misalkan $\{X_t\}$ adalah suatu deret waktu yang mengikuti proses $AR(3)$ dengan model

$$X_t = 0.4X_{t-1} + 0.2X_{t-2} - 0.5X_{t-3} + \varepsilon_t$$

Maka pernyataan yang tepat untuk model diatas adalah

a. $\rho_2 = 0.4\rho_2 + 0.2 - 0.5\rho_2$

b. $\rho_2 = 0.4 + 0.2\rho_1 - 0.5\rho_2$

c. $\rho_1 = 0.4\rho_1 + 0.2 - 0.5\rho_1$

d. $\rho_1 = 0.4 + 0.2\rho_1 - 0.5\rho_2$