09/02/2016

TORSION ~~OF~~ SHAFTS

Torsion is a ~~shifting~~ twisting or turning effect.

Shaft is a bar or rod joining parts in a machine or transmitting power in a material.

They are usually cylindrical in section, solid or hollow. They are made up of mild steel, alloy steel.

Shaft is subjected to

→ Torsional load

→ Bending

→ Axial

→ Combination of above (3) loads

Shafts are designed on the basis of strength and rigidity.

When a member is subjected to moment ~~about~~ a centroidal axis ~~XXXXXXX~~

Torque is said to be supplied and the member is said to be in torsion

Rotating shafts are used to transmit energy

Stanction is column made of steel

12 inches → 1 foot
3 feet → 1 yard
3 inches → 1 yard

1 inch = 25mm
1 foot = 0.3 m

A member will be under pure torsion when it is subjected to torque only without been associated to any bending moment or axial force.

When a shaft is under the action of pure torsion, its cross section are under pure ~~stresses~~.

Torsional stress and strain in circular shaft

Assumptions

→ The material of the shaft is homogenous and isotropic

→ Plane cross sections of the circular shaft, ~~the~~ main plane and circular before and after twisting

$$\boxed{\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}}$$

Torsional equation

T = Maximum twisting torque (Nm)

J = Polar moment of Inertia (m^4)

τ = Shear stress (N/m^2)

r = radius (m)

G = modulus of rigidity (N/m^2)

θ = angle of twist

l = length of shaft

Assumptions made

~~i) Material~~

→ All diameters of the cross section of the shaft remain straight, with their length unchanged before and after twist.

→ Twist is uniform along the length of shaft

→ Stresses induced in shaft due to torsion do not exceed the proportional limit

→ The relative rotation between any two cross sections of the shaft is proportional to the distance between them.

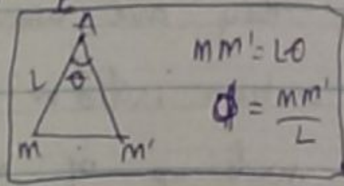
Example!

A solid circular shaft of length L and radius r is fixed at one end and subjected to a torque T at the other end as shown in the figure

If a line AM is drawn on the shaft it will be distorted to AM' on the application of the torque, thus cross section will be twisted through angle θ and unsurface by angle ϕ

Here, shear strain ϕ

$$\phi = \frac{MM'}{l}$$



where ϕ = shear strain of element at distance r from the axis

(ϕ is constant for constant T)

$\therefore MM' = l\phi$ —— (i)

Also relative angle of twist

$\angle OMM' = \theta$

$MM' = r\theta$ — (ii)

equating equation (i) and (ii)

$l\phi = r\theta$ —— (iii)

Modulus of rigidity

$$G = \frac{\text{shear stress}}{\text{shear strain}}$$

$$G = \frac{\tau}{\phi}$$

$$\phi = \frac{\tau}{G} \text{ ——— (iv)}$$

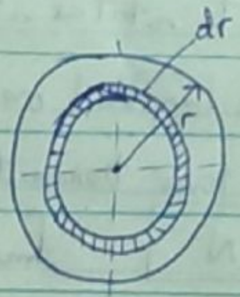
Put equation (iv) into equ (iii)

$$l\phi = r\theta$$

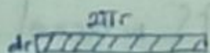
$$l\frac{\tau}{G} = r\theta$$

$$\boxed{\frac{\tau}{R} = \frac{G\theta}{l}} \text{ ——— (v)}$$

Consider an element ring of thickness dr at a radius r and let the shear stress at the radius be τ



The turning force on the elemental ring $(F) = \tau \cdot 2\pi r \cdot dr$



but Torque $\simeq \Sigma$ moment of tangential stress on the element $2\pi r \cdot dr$

$$T = \int \tau (2\pi r \cdot dr)\, r$$

From equation (v) $\tau = \frac{G\theta}{l} r$

$$T = \int \frac{G\theta}{l} r\,(2\pi r \cdot dr)\, r$$

$$T = \frac{G\theta}{l} \int (2\pi r \cdot dr)\, r^2$$

where $\int (2\pi r\, dr)\, r^2$ is the polar second moment of inertia represented by J

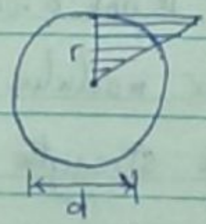
$$\therefore T = \frac{G\theta}{l} J$$

$$\frac{T}{J} = \frac{G\theta}{l} \text{ ——— (vi)}$$

equating to equ v, we have

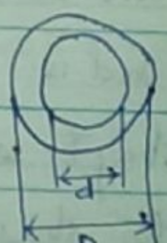
$$\boxed{\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{l}}$$

For a given torques shear stress is proportional to radius.



$$I = \frac{\pi D^4}{64}$$

$$J = \frac{\pi D^4}{32}$$



$$J = \frac{\pi D^4}{32} - \frac{\pi d^4}{32}$$

$$= \frac{\pi}{32}(D^4 - d^4)$$

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}$$

Torsional stiffness

$$K = \frac{T}{\theta} = \frac{GJ}{L}$$

<u>Solid shape</u>

$$J = \frac{\pi D^4}{32} \text{ for solid circular shape}$$

for hollow shape

$$\frac{\pi D^4}{32} - \frac{\pi d^4}{32}$$

GJ = Torsional rigidity

<u>N.B:- for a given shaft</u>, J and r are constant and J/r is not a constant and is known as polar modulus of shaft section thus the strength of the shaft in twisting.

$$K = \text{torque per radian} = \frac{T}{\theta} = \frac{GJ}{L}$$

Torsional rigidity is the torque that produces a twist of 1 radian in a shaft of unit length.

Power transmission.

Shafts are often used to transmit power from an engine to axle, wheel i.e. from the driving motor to a machine tool or from a turbine to an electric motor.

$\therefore$ the power transmitted through the shaft can be collected using the following equations,

$$\text{Power transmitted} = \text{Torque} \times \frac{\text{Angle turned}}{\text{per unit time}}$$

If a shaft rotates at N (r.p.m) and the torque applied on the shaft (i) T (Nm), power transmitted $P = T \times 2\pi N$

$$\boxed{P = T \times 2\pi N}$$

T_{mean}

Example

A hollow shaft 150 external and 75 internal diameter and length 3m is subjected to a torque which produces maximum turning stress of 75 N/mm^2. Determine the angle of twist, take $G = 2500 \text{ Nm}^{-2}$

<u>Soln</u>

$\theta = ?$, $G = 2500 \text{ N/m}^2$

$\tau = 75 \text{ Nmm}^{-2}$, $L = 3m$, $J = \frac{\pi}{32}(150^4 - 75^4)$

=

$1_{ft} \rightarrow 3m \div 10$

$$\frac{\tau}{r} = \frac{G\theta}{L}$$

$$\theta = \frac{\tau L}{Gr} = \frac{75 Nmm^{-2} \times 3000mm}{2500 Nmm^{-2} \times 75 mm}$$

$=$

2) What can be the length of a 5mm diameter wire so that it can be twisted through one complete revolution without exceeding a shearing stress of 42MN

Soln

$D = 0.005m$, $\theta = 2\pi rad$

18/02/2016

A solid ~~shaft~~ steel shaft has to transmit 75KW at 200 rpm. Taking allowable shear stress at $70 MN/m^2$, find suitable diameter for the shaft if the maximum torque transmit on each revolution exceeds the mean by 30%

$P = 75000 W$ $\quad D = ?$

$\theta = \frac{200 rpm}{60} = 3.33 rps$ $\quad T_{max} = 1.3 T_{mean}$

$(100 + 30 = 130\%)$

$$T_{mean} = \frac{P}{2\pi N}$$

$T_{max} = (100+30) = 130\%$

$T_{max} = \frac{130}{100} = 1.3 T_{mean}$

$$= \frac{75 \times 10^3}{2 \times \pi \times 3.33}$$

$$\frac{T_{max}}{J} = \frac{\tau}{r} \Rightarrow J = \frac{T_{max}\, r}{\tau}$$

but $T_{max} = 1.3\, T_{mean} = 1.3 \times 3587 Nm$

$= 4655.3 Nm$

for circular shaft $J = \frac{\pi D^4}{32}$

$$\frac{\pi D^4}{32} = \frac{4655.3 \times (D/2)}{70 \times 10^6} = 3.33 \times 10^{-5} D$$

$D^3 = 3.39 \times 10^{-4}$

$\Rightarrow D = 0.0697m$

A solid shaft of 60mm diameter is running at 160 rpm. Find power in Kilowatt which the shaft can transmit if permissible shear stress is $80 MN/m^2$ and maximum torque is likely to exceed the mean by 20%

Soln

$D = 60mm = 0.06m$

$\text{Speed}(N) = 160 rpm = \frac{160}{60} rps$

$$\frac{T_{max}}{J} = \frac{\tau}{r}$$

$$T_{max} = \frac{18 \times 10^6 \times (0.06)^3}{16} = 3342.96$$

$$T_{mean} = \frac{T_{max}}{1.2}$$

$$P = T_{mean} \times 2\pi N$$

17/02/2016

1) Determine the suitable diameter of a circular shaft required to transmit 80.2KW at 180rpm. The shear stress of the shaft is not to exceed $70 MN/m^2$ and the maximum torque exceed the main by 40%. Cal. the angle of twist in a length of 2m ($D = 0.0757m$, $\theta = 0.0413°$)

2) A solid shaft of 60mm diameter is running at 3rps. If the permissible shear stress is $80 MN/m^2$ and the maximum torque is likely to exceed the mean by 10%. Find the power in KW which shaft can transmit.

... torque which produced a maximum stress of $70N/m^2$. Determine (i) torsional stiffness (ii) torsional rigidity of the shafts.

4) Determine the length of a 5mm diameter wire so that it can be twisted through one complete revolution without exceeding a shearing stress of $50 MN/m^2$. Take $G = 30 MN/m^2$

5) A solid shaft of 60mm diameter is running at 180rpm. Find Power in KW which the shaft can transmit. The permissible shear stress is $50 MN/m^2$ and maximum torque is likely to exceed the main by 2%