# Maxwells equation, Magnetism of matter

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## Guass law for magnetic fields

The simplest magnetic structure that can exist is a magnetic dipole (magnetic monople do not exist).

$$\begin{aligned} \Phi_B &= \oint \vec{B}.d\vec{A} \\ &= 0 \end{aligned} \tag{1}$$

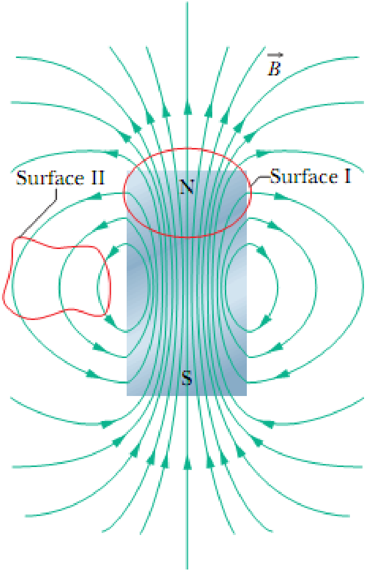


Figure: The field lines for the magnetic field of a short bar magnet. The red curves represent cross sections of closed, three-dimensional Gaussian surfaces.

The net magnetic flux through any closed guassian surface is zero.

## Induced magnetic fields

It has been established earlier that a changing magnetic field induces an electric field(faradays law of induction)

$$\oint \vec{E}.d\vec{A} = -\frac{d\Phi_B}{dt} \tag{2}$$

$\Phi_B$ is the magnetic flux over the gaussian surface so also it can be established that a changing electric field induces a magnetic field. (Maxwells law of induction).

$$\oint \vec{B}.d\vec{S} = -\mu_0\epsilon_0\frac{d\Phi_E}{dt} \tag{3}$$

$\Phi_E$ is the magnetic flux over the gaussian surface Ampere related the magnetic field to the encircled current in the loop as

$$\oint \vec{B}.d\vec{S} = \mu_0 i_{enc} \tag{4}$$

Thus both Maxwell and faradays law can be combined

$$\oint \vec{B}.d\vec{S} = \mu_0\epsilon_0\frac{d\Phi_E}{dt} + \mu_0 i_{enc} \tag{5}$$

when there is no charge in electric field, first term on the right $\Phi_E = 0$ and when there is no current second term $i_{enc} = 0$

## Displacement current

the quantity $\epsilon_0 \frac{d\Phi_E}{dt}$ has essential the dimension of current so it is called displacement current *(though not as displacement)*

$$i_0 = \epsilon_0 \frac{d\Phi_E}{dt} \tag{6}$$

the Maxwell-Faraday relationship can be written as

$$\oint \vec{B}.d\vec{S} = \mu_0 i_d + \mu_0 i_{enc} \tag{7}$$

**Assignment** Given that $q = \epsilon_0 AE$ and $\frac{dq}{dt}$ and that $\Phi_B = \int \vec{B}.d\vec{A} = BA$ show that $i_d = i$

## Maxwells Equations

| | | |
|---|---|---|
| Gauss Law for Electricity | $\oint \vec{B}.d\vec{A} = \frac{q_{enc}}{\epsilon_0}$ | Relates net electric flux to net enclosed charg |
| Gauss Law for Magnetism | $\oint \vec{B}.d\vec{A} = 0$ | Relates net magnetic flux to net enclosed magnetic |
| Faraday's Law | $\oint \vec{B}.d\vec{S} = \frac{\Phi_B}{dt}$ | Relates induced el field to changing r |
| ampere-Maxwell Law | $\oint \vec{B}.d\vec{S} = \mu_0\epsilon_0\frac{\Phi_E}{dt} + \mu_0 i_{enc}$ | Relates induced m to changing electr |

## Earths magnetic field

This can be approximated as being that of the magnetic dipole whose dipole moment makes an angle at $11.5^0$ with Earths rotation axis and with south pole of the dipole in the northern hemisphere. The direction of the local magnetic field at any point on earths surface is given by the field declination and the field inclination

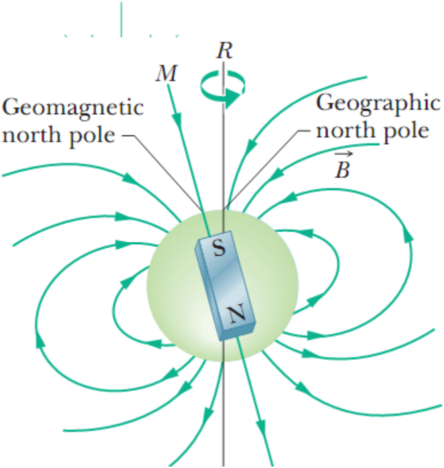


Figure: Earths magnetic field represented as a dipole field.The dipole axis MM makes an angle of 11.5 with Earths rotational axis RR. The south pole of the dipole is in Earths Northern Hemisphere.

## Spin Magnetic Dipole Moment

An electron has an intrinsic angular momentum called spin angular momentum( or spin) S , with which an intrinsic spin magnetic dipole moment is associated

$$\vec{\mu_S} = -\frac{e}{m}\vec{S} \tag{8}$$

$e$ = electric charge, $\mu_s$ = spin magnetic dipole moment, $m$ = mass of electron, $\vec{S}$ = spin angular momentum Spin $\vec{S}$ cannot be measured directly,but its component along any axis can be measured. Assuming that the measurement is along the z-axis of a coordinated system,the component Sz can have only the values given by

$$S_z = m_s\frac{h}{2\pi} \tag{9}$$

where $h = 6.626 \times 10^{-34} Js$ is the Planck's constant Similarly the electrons spin magnetic dipole moment Us cannot be measured directly. Along a z-axis.the component is

$$\begin{aligned}\mu_{s,z} &= \pm\frac{eh}{4\pi m} \\ &= \pm\mu_B\end{aligned}$$

where $\mu_B$ is the bohr magneton.

$$\begin{aligned}\mu_B &= \frac{eh}{4\pi m} \\ &= 9.27 \times 10^{-24} J/T \end{aligned} \tag{10}$$

The potential energy U associated with the orientation of the spin magnetic dipole moment in an external field text is

$$\begin{aligned}U &= -\vec{\mu_S}.\vec{B_{ext}} \\ &= -mu_{s,z} B_{ext}\end{aligned} \tag{11}$$

## Orbital Magnetic Dipole Moment

An electron in an atom has an additional angular momentum called its orbital angular momentum called its orbital angular moment lorb,with which an orbital magnetic dipole moment $\vec{\mu_{orb}}$ is associated

$$\vec{\mu_{orb}} = -\frac{e}{2h}\vec{L_{orb}} \tag{12}$$

orbital angular momentum is quantised and can have only values given by

$$\vec{L_{orb,z}} = m_l \frac{h}{2\pi} \tag{13}$$

for $m_l = 0, \pm 1, \pm 2...$ Thus,the magnitude of the orbital angular momentum is

$$\begin{aligned} \mu_{orb} &= -m_l \frac{eh}{4\pi m} \\ &= -m_l \mu_B \end{aligned} \tag{14}$$

the potential energy U associated with the orientation of the orbital magnetic dipole moment in an external magnetic field $\vec{B_{ext}}$ is

$$\begin{aligned} \mu &= \vec{\mu_{orb}}.\vec{B_{ext}} \\ &= \mu_{orb,z} B_{ext} \end{aligned} \tag{15}$$

**Magnetic Materials**

**Diamagnetism**

Diamagnetic materials do not exhibit magnetism until they are placed in an external magnetic field $B_{ext}$.they then develop a magnetic dipole moment directed opposite $B_{ext}$.if the field is non-uniform, the diamagnetic material is repelled from regions of greater magnetic field.This property is called diamagnetism.

**Paramagnetism**

In a paramagnetic material each atom has a permanent magnetic dipole moment U, but the dipole moments are randomly oriented and the material as a whole lacks a magnetic field,However, an external magnetic field $B_{ext}$,can partially align.The atomic dipole moments to give the material a net magnetic dipole moment in the directions of $B_{ext}$. If $B_{ext}$ is nonuniform, the material is attracted to regions of greater magnetic field. these properties are called paramagnetism. The alignment of the atomic dipole moments increases with an increases in $B_{ext}$ and decreases with an increase in temperature T. the extent to which a sample of volume V is magnetised is given by its magnetization M,whose magnitude is

$$M = \frac{Measuredmagneticmoment}{V} \tag{16}$$

Complete alignment of all N atomic magnetic dipole in a sample called saturation of the sample corresponds to maximum magnetization value

$$M_{max} = \frac{N\mu}{V} \tag{17}$$

for low values of the ratio $B_{ext}/T$ we have the approximation

$$M = C\frac{B_{ext}}{T} \tag{18}$$

where C is called the Curie constant

## Ferromagnetism

In the absence of an external magnetic field,some of the electrons in a ferromagnetic material have their magnetic dipole moments aligned by means of a quantum physical interaction called exchange coupling,producing regions(domains) within the materials with strong magnetic dipole moments.An external field $B_{ext}$ can align the magnetic dipole moments of those regions,producing a strong net magnetic dipole moment for the material as a whole in the direction of $B_{ext}$.This net magnetic dipole moment can partially persist when field $B_{ext}$ is removed,if $B_{ext}$ is non-uniform,the ferromagnetic material is attracted to regions of greater magnetic field.These properties are called ferromagnetism exchange coupling disappears when a samples temperature exceeds its curie temperature.