Contents

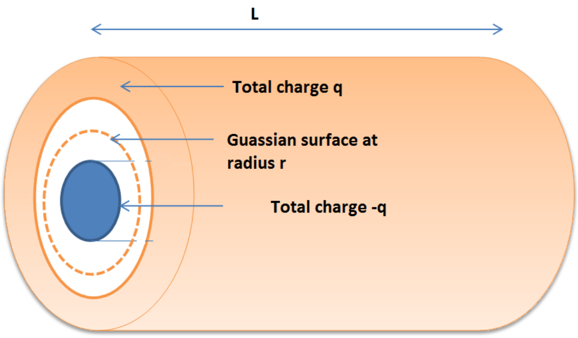
Chapter 1

Contd.

1.1 Capacitors

1.1.1 A Cylindrical Capacitor

A Cylindrical Capacitor Cross section of two coaxial capacitors of length L, smaller radius a and bigger radius b . Assuming $L \gg b$.



Guassian surface is chosen as a cylinder of length L and radius r closed by end caps from eqn 3, we have

$$\begin{aligned} Q &= \epsilon_0 EA \\ &= \epsilon_0 E(2\pi r L) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \textit{(8)} \end{aligned}$$

Where $2\pi r L = A$, the curved surface area of the Guassian surface Note : There is no Flux through end caps. Hence from eqn(3),

$$E = \frac{q}{2\pi\epsilon_0 L} \tag{1.1}$$

Using (9) in (5) in previous class,

$$\begin{aligned} V &= \int_{-}^{+} EdS \\ &= -\frac{q}{2\pi\epsilon_0 L}\int_b^a \left(\frac{dr}{r}\right) \\ &= \frac{q}{2\pi\epsilon_0 L} Ln\left(\frac{b}{a}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \textit{(9)} \end{aligned}$$

From $C = q/v$, we have

$$C = 2\pi\epsilon_0 \frac{L}{Ln(b/a)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \textit{(11)}$$

Thus for a cylindrical capacitor, C depends only on geometrical factors L, b, a.

1.1.2 A Spherical Capacitor

A Spherical Capacitor Figure above also represent cross section of two concentric spherical shells of radii a and b . A sphere of radius r ,concentric with the spherical shells represents the Gaussian surface. This gives

$$\begin{aligned} Q &= \epsilon_0 EA \\ &= \epsilon_0 E(4\pi r^2) \\ & and \\ E &= \frac{1}{4\pi\epsilon_o}\frac{q}{r^2}\text{............................} \qquad \textit{(12)} \end{aligned}$$

substituting this in equation (5) and using dr as dS

$$\begin{aligned} V &= \int_{-}^{+} EdS \\ &= -\frac{q}{4\pi\epsilon_0}\int_b^a \left(\frac{dr}{r^2}\right) \\ &= \frac{q}{4\pi\epsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right) \\ &= \frac{q}{4\pi\epsilon_0}\frac{b-a}{ab}\text{............................} \qquad \textit{(13)} \end{aligned}$$

from eqn (1) and 13

$$C = 4\pi\epsilon\frac{ab}{b-a}\text{............................} \qquad \textit{(14)}$$

for a spherical capacitor

1.1.3 Isolated Sphere

Isolated Sphere Rewriting equation(14) for the concentric spheres

$$C = 4\pi\epsilon_0\frac{a}{1-a/b}\text{............................} \qquad \textit{(15)}$$

for the isolated sphere $b \to \inf$ and $a = R$ (radius of sphere) using these in (15),

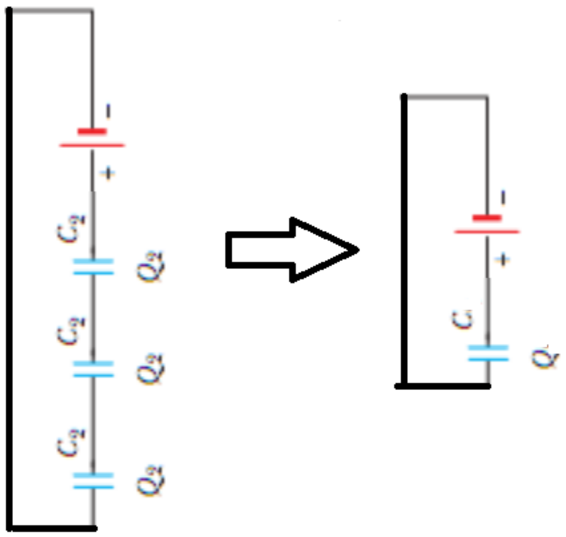
$$C = 4\pi\epsilon_0 R\text{............................} \qquad \textit{(16)}$$

(for an isolated sphere)

1.2 Connection of Capacitors

1.2.1 Capacitors in Parallel

Capacitors in Parallel Parallel capacitors can be combined to give an equivalent capacitor



In parallel V is same for all the capacitors total charge q stored on the capacitors is the sum of the charges

$$q = q_1 + q_2 + q_3$$

but $q_1 = C_1V$,$q_2 = C_2V$,$q_3 = C_3V$.

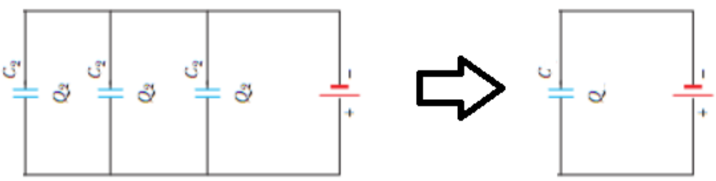
$$\begin{aligned} q &= (C_1 + C_2 + C_3)V \\ C_{eq} &= \frac{q}{V} \\ &= C_1 + C_2 + C_3(inparallel) \end{aligned}$$

for n capacitors

$$C_{eq} = \sum_{j=1}^{n} C_j \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (17)$$

1.2.2 Capacitors in Series

Capacitors in Series Here charges are identical i.e. $= q$ Sum of the p.d.



across all the capacitor = applied Voltage

$$V = V_1 + V_2 + V_3$$

but $V_1 = q/C_1$, $V_2 = q/C_2$, $V_3 = q/C_3$.

$$\begin{aligned} V &= V_1 + V_2 + V_3 \\ &= q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) \\ & therefore \\ C_{eq} &= q/V \\ \frac{1}{C_{eq}} &= \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) \end{aligned}$$

for n capacitors

$$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j} \dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (18)$$

1.3 Energy stored in an electric Field

Energy stored in an electric Field Work required to charge a capacitor is stored as the electric potential energy U in the electric field between the plates.

$$\begin{aligned} dW &= V'dq' \\ &= \frac{q'}{C}dq' \\ therefore & \\ W &= \int dW \\ &= \frac{1}{C}\int_0^q q'dq' \\ &= q^2/(2C) \end{aligned}$$

thus

$$\begin{aligned} &= \frac{q^2}{2C} \\ &= \frac{1}{2}CV^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \textit{(19)} \end{aligned}$$

1.3.1 Energy Density

This is the potentiam energy per unit volume between the plates.

$$\begin{aligned} u &= U/(Ad) \\ &= \frac{CV^2}{2Ad} \end{aligned}$$

using $C = \epsilon_0 A/d$

$$\begin{aligned} u &= \frac{1}{2}\epsilon_0 \left(\frac{V}{d}\right)^2 \\ &= \frac{1}{2}\epsilon_0 E^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \textit{(20)} \end{aligned}$$

1.4 Capacitor and dielectrics

Capacitor with a dielectric A dielectric is an insulating material. If the space between the plates of a capacitor is completely filled with a dielectric, the capacitance C is increased by a factor κ called the dielectric constant of the material.

$$C = \kappa C_{air}$$

where C_{air} is the capacitance with air filling the plates

1.4.1 Dielectrics and Gauss' Law

Dielectrics and Gauss' Law The presence of the dielectric weakens the original field and gauss' law becomes

$$\epsilon_0 \oint \kappa \vec{E}.d\vec{S} = q \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \textit{(21)}$$

where κ is the dielectric constant.

1.5 Current and Resistance

Current and Resistance

The discussions on charges at rest is referred to as electrostatics while chaarges in motion imply electric currents

the steady state flow of charge dq in time dt through a hypothetical plane deffines the current i i.e.

$$i = \frac{dq}{dt} \dots\dots\dots\dots\dots\dots \quad (1)$$

in a time interval 0 to t, the charge q that passes is given by

$$\begin{aligned} q &= \int dq \\ &= \int_0^t idt \dots\dots\dots\dots\dots\dots \quad (2) \end{aligned}$$

S.I. unit for current is Ampere $\rightarrow$ 1A = 1 coulomb/sec. Charge is conserved. hence the splitting of an initial current i_0 to two currents i_1 and i_2 requires that

$$i_0 = i_1 + i_2 \dots\dots\dots\dots\dots\dots \quad (3)$$

1.5.1 Direction of current

Direction of current A current arrow is drawn in the direction whic positive charge carriers would move.

1.5.2 Current Density

Current Density this is the amount of current flowing through a unit surface and denoted as J

$$J = \frac{i}{A} \dots\dots\dots\dots\dots\dots \quad (4)$$

S.I. unit is $Ampere/squareMeter\ A/m^2$ current density is greater in narrower conductors.

1.5.3 Drift Speed

Drift Speed when an electric field is applied to a conductor, the conduction electrons move with a drift speed v_d in a direction opposite to that of the applied field. If a wire of length L and cross-sectional area A has n charge carriers per unit volume, then the total charge is given by

$$q = (nAL)e$$

if the carriers all move along the wire with speed v_d in time interval t, then

$$t = \frac{L}{v_d}$$

but

$$\begin{aligned} i &= \frac{q}{t} \\ &= \frac{nALe}{L/v_d} \\ &= nAev_d \end{aligned} \tag{1.2}$$

therefore

$$\begin{aligned} v_d &= \frac{i}{nAe} \\ &= \frac{J}{ne} \end{aligned}$$

thus the drift speed is related to the current density by

$$\vec{j} = (ne)\vec{v_d}............................ \quad \textit{(5)}$$

1.5.4 Resistance and Resistivity

Resistance and Resistivity

The electrical resistance R between any two points of a conductor is determined by applying a potential difference V between the points and measuring the resulting current

$$R = \frac{V}{i}............................ \quad \textit{(6)}$$

S. I. unit is the ohm(Ω): $1\Omega = 1V/a$ A conductor which provides specified resistance $\rightarrow$ resistor. current symbol 

1.5.5 Resistivity ρ

of a material is the ratio of the electric field at a point to the current density at that point.

$$\rho = \frac{E}{J} \text{..........................} \quad (7)$$

S. I. unit $= \frac{V/m}{A/m^2} = \frac{Vm}{A} = \Omega m(ohm - meter)$
in vector form

$$\vec{E} = \rho\vec{J} \text{..........................} \quad (8)$$

(valid for isotropic materials i.e. electrical properties are the same in all directions)

1.5.6 The conductivity σ

of a material is the reciprocal of its resistivity

$$\sigma = \frac{1}{\rho}$$

S. I. unit : $(\Omega m^{-1})mho$ hence we can write

$$\vec{J} = \sigma\vec{E} \text{..........................} \quad (9)$$

Note: resistance is a property of an object while resistivity is a property of a material.

1.5.7 Ohm's Law

Ohm's Law A device obeys ohm's law if current through the device is always directly proportional to the applied potential difference.

1.5.8 Power in Electric circuits

Power in Electric circuits The Rate of energy transfer in an electrical device across which a potential V is maintained is given by

$$P = iV \text{..........................} \quad (10)$$

In a Resistor, elctrical potential is lost as thermal energy (dissipation)at the rate

$$\begin{aligned} P &= i^2R \text{..........................} \quad (11) \\ &= V^2/R \end{aligned}$$