09/02/2016

TORSION ~~OF~~ SHAFTS

Torsion is a ~~shifting~~ twisting or turning effect.

Shaft is a bar or rod joining parts in a machine or transmitting power in a material.

They are usually cylindrical in section, solid or hollow. They are made up of mild steel, alloy steel.

Shaft is subjected to
- → Torsional load
- → Bending ✓
- → Axial ✓
- → Combination of above (3) loads

Shafts are designed on the basis ~~of~~ of strength and rigidity.

When a member is subjected to moment ~~about~~ a centroidal axis, ~~XXXXXXX~~

Torque is said to be supplied and the member is said to be in torsion

Rotating shafts are used to transmit energy

Stanchion is column made of steel

12 inches → 1 foot
3 feet → 1 yard
3 inches → 1 yard

~~St~~ 1 inch = 25mm
1 foot = 0.3 m

A member will be under pure torsion when it is subjected to torque only without been associated to any bending moment or axial force.

When a shaft is under the action of pure torsion, its cross section are under pure stresses.

Torsional stress and strain in circular shaft

Assumptions
- → The material of the shaft is homogenous and isotropic
- → Plane cross sections of the circular shaft, ~~the~~ main plane and circular before and after twisting

$$\boxed{\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}}$$

↓ Torsional equation

T = maximum twisting torque (Nm)
J = Polar moment of Inertia (m^4)
τ = Shear stress (N/m^2)
r = radius (m)
G = modulus of rigidity (N/m^2)

θ = angle of twist

l = length of shaft

Assumptions made

~~i) Material~~

→ All diameters of d cross section of the shaft remain straight, with their length unchanged before ad after twist,

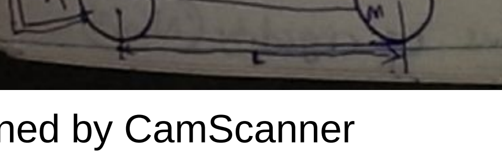
→ Twist is uniform along d length of shaft

→ Stresses induced in shaft due to torsion do not exceed the proportional limit

→ The relative rotation between any two cross sections of the shaft is proportional 2 d distance between them.

Example!

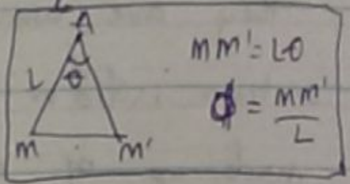
A solid circular shaft of length L and radius r is fixed at one end and subjected to a torque T at the other end as shown in the figure



If a line AM is drawn on the shaft it will be distorted to AM' on the application of the torque, thus cross section will be twisted through angle θ and unsurface by angle ϕ

Here, shear strain ϕ

$$\phi = \frac{MM'}{l}$$



where ϕ = shear strain of element at distance r from the axis

(ϕ is constant for constant T)

$\therefore MM' = l\phi$ ——— ①

Also relative angle of twist.

$\angle OMM' = \theta$

$MM' = r\theta$ — (ii)

equating equation ① and (ii)

$l\phi = r\theta$ ——— (iii)

Modulus of rigidity

$$G = \frac{\text{shear stress}}{\text{shear strain}}$$

$$G = \frac{\tau}{\phi}$$

$\phi = \frac{\tau}{G}$ —— (iv)

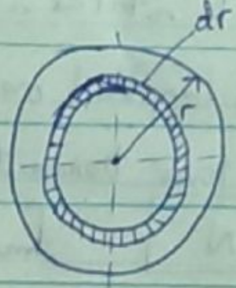
Put equation (iv) into equ (iii)

$l\phi = r\theta$

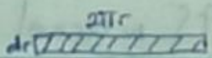
$l\frac{\tau}{G} = r\theta$

$$\boxed{\frac{\tau}{R} = \frac{G\theta}{l}} \text{ —— (v)}$$

Consider an element ring of thickness dr at a radius r and let the shear stress at the radius be τ



The turning force on the elemental ring (F) = $\tau \cdot 2\pi r \cdot dr$



but Torque $\simeq \Sigma$ moment of tangential stress on the element $2\pi r \cdot dr$

$$T = \int \tau (2\pi r \cdot dr) r$$

From equation (v) $\tau = \frac{G\theta}{l} r$

$$T = \int \frac{G\theta}{l} r (2\pi r \cdot dr) r$$

$$T = \frac{G\theta}{l} \int (2\pi r \, dr) r^2$$

where $\int (2\pi r \, dr) r^2$ is the polar second moment of inertia represented by J

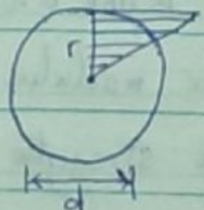
$\therefore T = \frac{G\theta}{l} J$

$\frac{T}{J} = \frac{G\theta}{l}$ —— (vi)

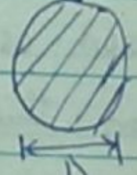
equating to equ v, we have

$$\boxed{\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{l}}$$

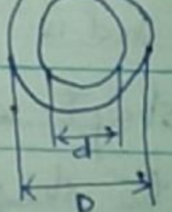
For a given torques shear stress is proportional to radius.



$I = \frac{\pi D^4}{64}$



$J = \frac{\pi D^4}{32}$



$J = \frac{\pi D^4}{32} - \frac{\pi d^4}{32}$

$= \frac{\pi}{32}(D^4 - d^4)$

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}$$

Torsional stiffness

$$K = \frac{T}{\theta} = \frac{GJ}{L}$$

Solid shape

$$J = \frac{\pi D^4}{32}$$ for solid circular shape

For hollow shape

$$\frac{\pi D^4}{32} - \frac{\pi d^4}{32}$$

GJ = Torsional rigidity

N.B: For a grew shaft, J and r are constant and J/r is not a constant and is known as polar modulus of shaft section thus the strength of the shaft in twisting.

$$K = \text{torque per radian} = \frac{T}{\theta} = \frac{GJ}{L}$$

Torsional rigidity is the torque that produces a twist of 1 radian in a shaft of unit length.

Power transmission.

Shafts are often used to transmi power from an engine to axle, wheel i.e. from the driving motor to a machine tool or from a turbin to an electric motor.

∴ the power transmitted through the shaft can be collected using the following equations,

Power transmitted = Torque × Angle turned per unit time

If a shaft rotates at N (r.p.m) and the torque applied on the shaft (i) T (Nm), power transmitted P = T×2π

$$\boxed{P = T \times 2\pi N}$$ T_{mean}

Example

A hollow shaft 150 external and 75 internal diameter and length 3m is subjected to a torque which produces maximum turning stress of 75 N/mm². Determine the ang of twist, take $G = 2500 Nm^{-2}$

Soln

$\theta = ?$, $G = 2500 N/m^2$

$\tau = 75 Nmm^{-2}$, $l = 3m$, $J = \frac{\pi}{32}(150^4 - 75^4)$

=

$1_{ft} \rightarrow 3m \div 10$

$$\frac{\tau}{r} = \frac{G\theta}{l}$$

$$\theta = \frac{\tau l}{Gr} = \frac{75 Nmm^{-2} \times 3000 mm}{2500 Nmm^{-2} \times 75 mm}$$

$$=$$

2) What can be the length of a 5mm diameter wire so that it can be twisted through one complete revolution without exceeding a shearing strees of 42MN

Soln

$D = 0.005m$, $\theta = 2\pi rad$

18/02/2016

A solid, ~~shaft~~ steel shaft has to transmit 75KW at 200 rpm. Taking allowable shear stress at 70MN/m², find suitable diameter for the shaft if the maximum torque transmit on each revolution exceeds the mean by 30%

$P = 75000 W$ $\quad D = ?$

$\theta = \frac{200 rpm}{60} = 3.33 rps$ $\quad T_{max} = 1.3 T_{mean}$

$(100 + 30 = 130\%)$

$$T_{mean} = \frac{P}{2\pi N}$$

$T_{max} = (100 + 30) = 130\%$

$T_{max} = \frac{130}{100} = 1.3 T_{mean}$

$$= \frac{75 \times 10^3}{2 \times \pi \times 3.33}$$

$$\frac{T_{max}}{J} = \frac{\tau}{r} \Rightarrow J = \frac{T_{max} r}{\tau}$$

but $T_{max} = 1.3 T_{mean} = 1.3 \times 3581 Nm$

$= 4655.3 Nm$

for Circular shaft $J = \frac{\pi D^4}{32}$

$$\frac{\pi D^4}{32} = \frac{4655.3 \times (D/2)}{70 \times 10^6} = 3.33 \times 10^{-5} D$$

$D^3 = 3.39 \times 10^{-4}$

$\Rightarrow D = 0.0697m$

A solid shaft of 60mm diameter is running at 160rpm. Find power in Kilowatt which the shaft can transmit if permissible shear stress is 80mN/m² and maximum torque is likely to exceed the mean by 20%

Soln

$D = 60mm = 0.06m$

$Speed(N) = 160 rpm = \frac{160}{60} rps$

Shear stress $\tau = 80MN/m^2 = 80 \times 10^6 N/m^2$

Power $P = T_{mean} \times 2\pi N$

but $T_{max} = 1.2 \times T_{mean}$

$$\frac{T_{max}}{J} = \frac{\tau}{r}$$

$$T_{max} = \frac{18 \times 10^6 \times (0.06)^3}{16} = 3342.96$$

$$T_{mean} = \frac{T_{max}}{1.2}$$

$$P = T_{mean} \times 2\pi N$$

17/02/2016

1) Determine the suitable diameter of a circular shaft requred to transmit 80.2KN at 180rpm. The shear stress of the shaft is not to exceed 70MN/m² and the maximum torque exceed the main by 40%. Cal. the angle of twist in a length of 2m (D = 0.0757m, θ = 0.0413°)

2) A solid shaft of 60mm diameter is running at 3rps. If the permissible shear stress is 80MN/m² and the maximum torque is likely to exceed the mean by 10%. Find the power in KW which [illegible]haft can transmit.

③ The hollow shaft 120mm external and 60mm internal diameters and leng 3m is subjected to a torque which produced a maximum stress of 70N/m. Determine

(i) torsional stiffness (ii) torsional rigidity of the shafts.

④ Determine the length of a 5mm diameter wire so that it can be twisted through one complete revolution without exceeding a shearing stress of 50MN/m². Take G = 30MN/m²

⑤ A solid shaft of 60mm diameter is running at 180rpm. Find Power KW which the shaft can transmit. If the permissible shear stress is 50MN/m² and maximum torque is likely to exceed the main by 2%