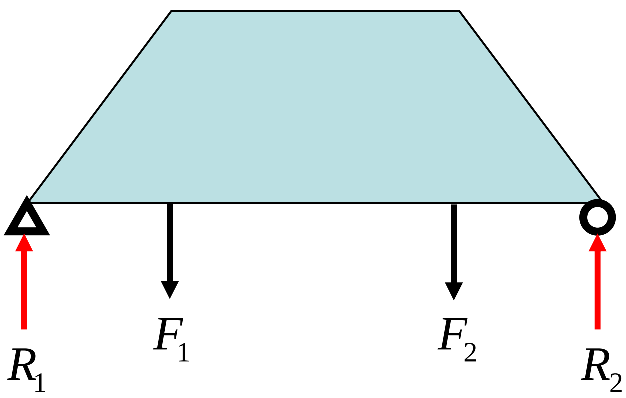
Chapter 7
Trusses, Frames, and Machines

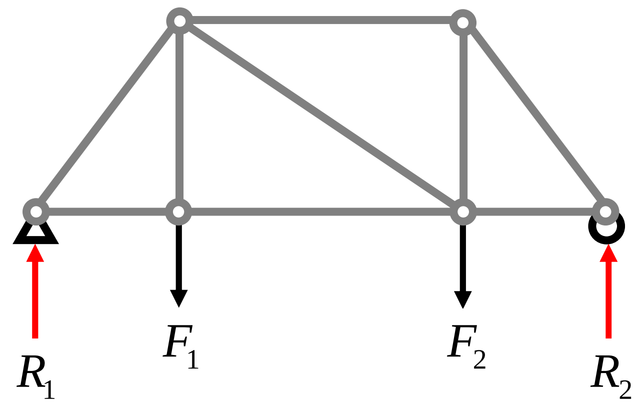
7.2 Plane Trusses

Before this chapter



Determine the reactions, R_1 and R_2, of a rigid body subjected to a pair of forces F_1, and F_2.

In this chapter



Determine the reactions, R_1 and R_2, and the forces in nine rigid members that are joined together with six pin joints, subjected to a pair of forces F_1, and F_2.

7.2 Plane Trusses

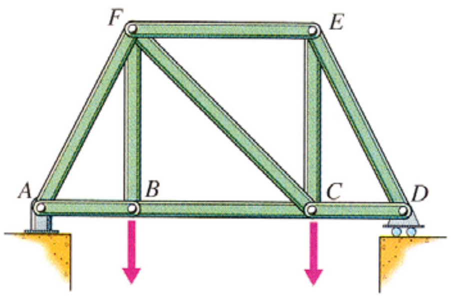
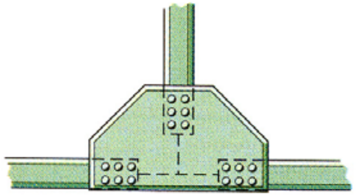


Figure 7-4 Idealized truss consisting of pin-connected, two-force members with all loads applied at the joints.

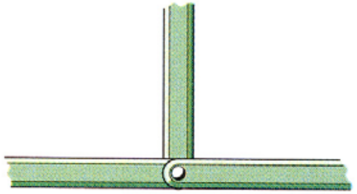
Idealized trusses

1. Members are connected together at their ends only.
2. Members are connected together by frictionless pins.
3. Loads are applied only at the joints. (Thus, all members are two-force members.)
4. Weights of members are neglected.



(a)

An actual riveted truss joint, which transmits both forces and moments among connecting members

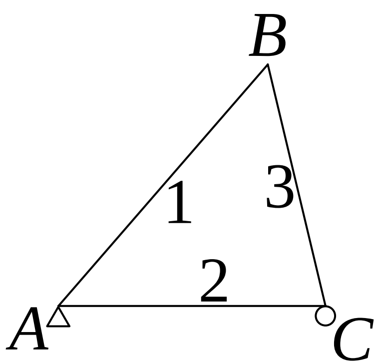


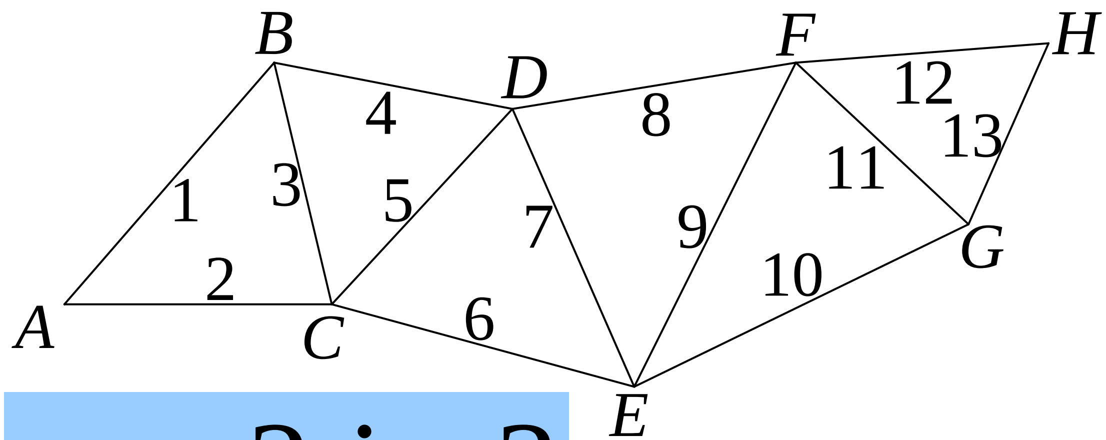
(b)

An idealized frictionless pin connection, which transmits forces among connecting members, but not moments. This assumption can be justified so long as the members are long.

7.2 Plane Trusses

A triangle is the building block of all plane trusses





$$m = 2j - 3$$

m = NUMBER OF MEMBERS; j = NUMBER OF JOINTS

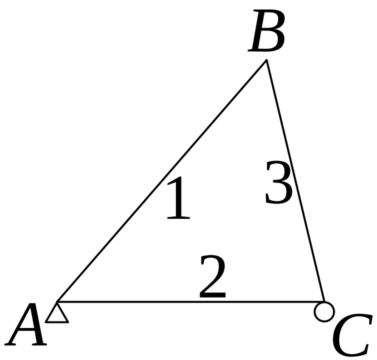
Total unknowns: m (one for each member) + 3 (3 support reactions).

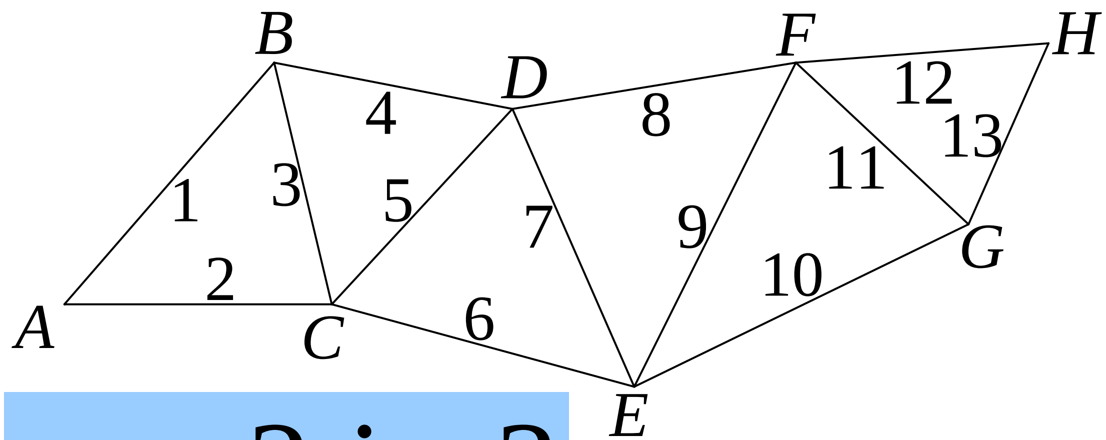
Each joint yields two equations ($\Sigma F_x = 0$, $\Sigma F_y = 0$)

Is a truss always "stable" and "solvable" when m = 2j -3 is satisfied?

7.2 Plane Trusses

A triangle is the building block of all plane trusses





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Is a truss always "stable" and "solvable" when m = 2j -3 is satisfied?

7.2 Plane Trusses

Rigidity and Solvability of A Truss

$$m = 2j - r$$

m: # of Members
j: # of Joints
r: # of Reactions

$$\boxed{m = 2j - 3} \qquad \boxed{m = 2j - r}$$

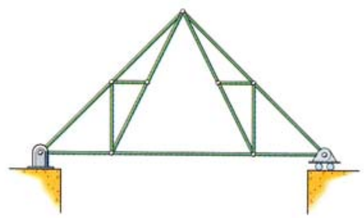


Figure 7-8 Plane truss with internal rigidity.

$j = 9 \quad m = 15$
$2j - 3 = 15 = m$

$j = 9 \quad m = 15 \quad r = 3$
$2j - r = 15 = m$

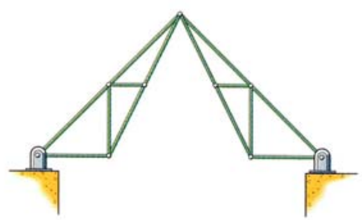


Figure 7-9 Plane truss that lacks internal rigidity.

$j = 9 \quad m = 14$
$2j - 3 = 15 \neq m$

$j = 9 \quad m = 14 \quad r = 4$
$2j - r = 14 = m$

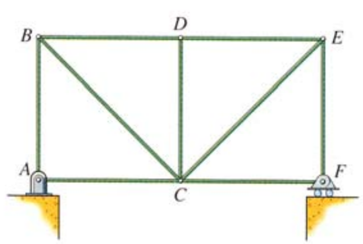


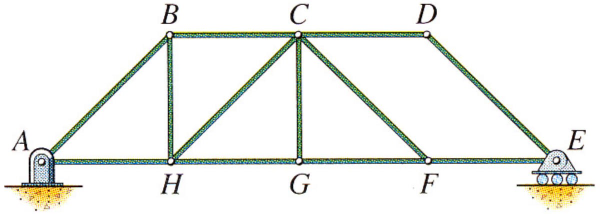
Figure 7-10 A simple planar truss constructed using triangular elements.

$j = 6 \quad m = 9$
$2j - 3 = 9 = m$

$j = 6 \quad m = 9 \quad r = 3$
$2j - r = 9 = m$

7.2 Plane Trusses

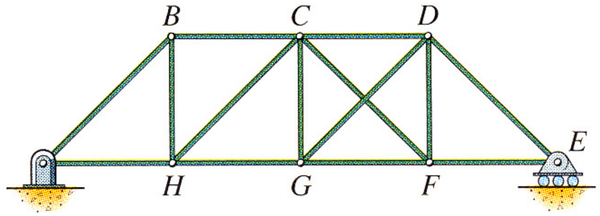
$$\boxed{m = 2j - r}$$



(b)

$$j = 8 \quad m = 12 \quad r = 3$$

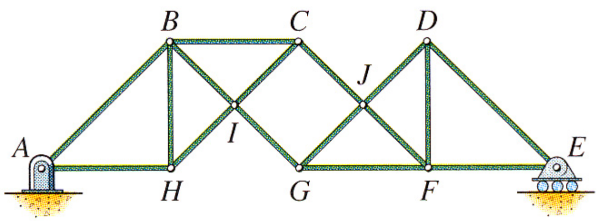
$$2j - r = 13 > m = 12$$



(c)

$$j = 8 \quad m = 14 \quad r = 3$$

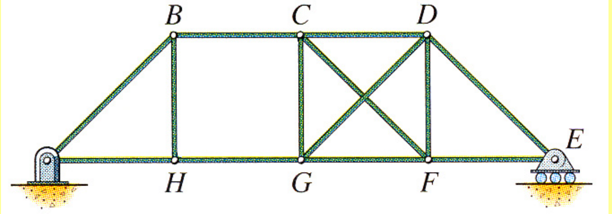
$$2j - r = 13 < m = 14$$



(d)

$$j = 10 \quad m = 16 \quad r = 3$$

$$2j - r = 17 > m = 16$$



$$j = 8 \quad m = 13 \quad r = 3$$

$$2j - r = 13 = m$$

Yet, it is unstable!

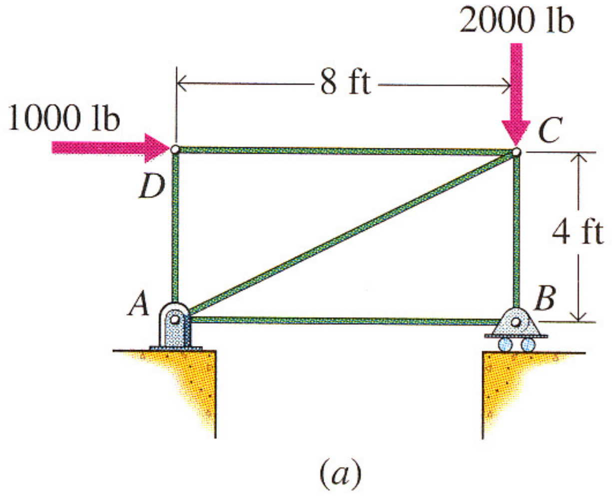
7.2 Plane Trusses

Method of Joints

1. *Draw a free-body diagram of the entire structure and determine the reactions (if $r = 3$).*
2. *Draw free-body diagrams for all members (assume tensile forces in all members) and all joints.*
3. *Set up the equilibrium equations for each joint and solve them one joint at a time, begin with those that have at most two unknowns.*
4. *Check the results at the last joint.*

7.2 Plane Trusses

Method of Joints

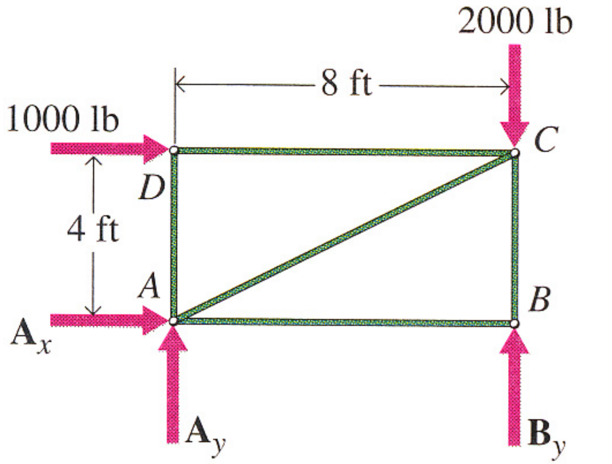


Reactions

$$\sum M_A = (8)B_y - (4)(1000) - (8)(2000) = 0$$

$$\sum M_B = -(8)A_y - (4)(1000) = 0$$

$$\sum F_x = A_x + 1000 = 0$$



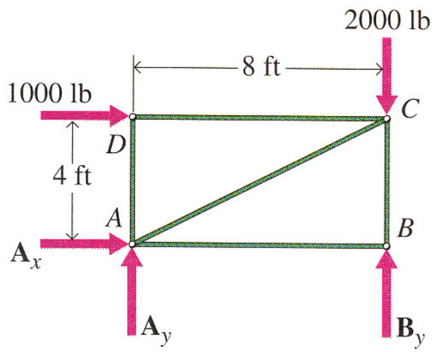
$$A_x = -1000\,\text{lb}$$

$$A_y = -500\,\text{lb}$$

$$B_y = 2500\,\text{lb}$$

7.2 Plane Trusses

Method of Joints

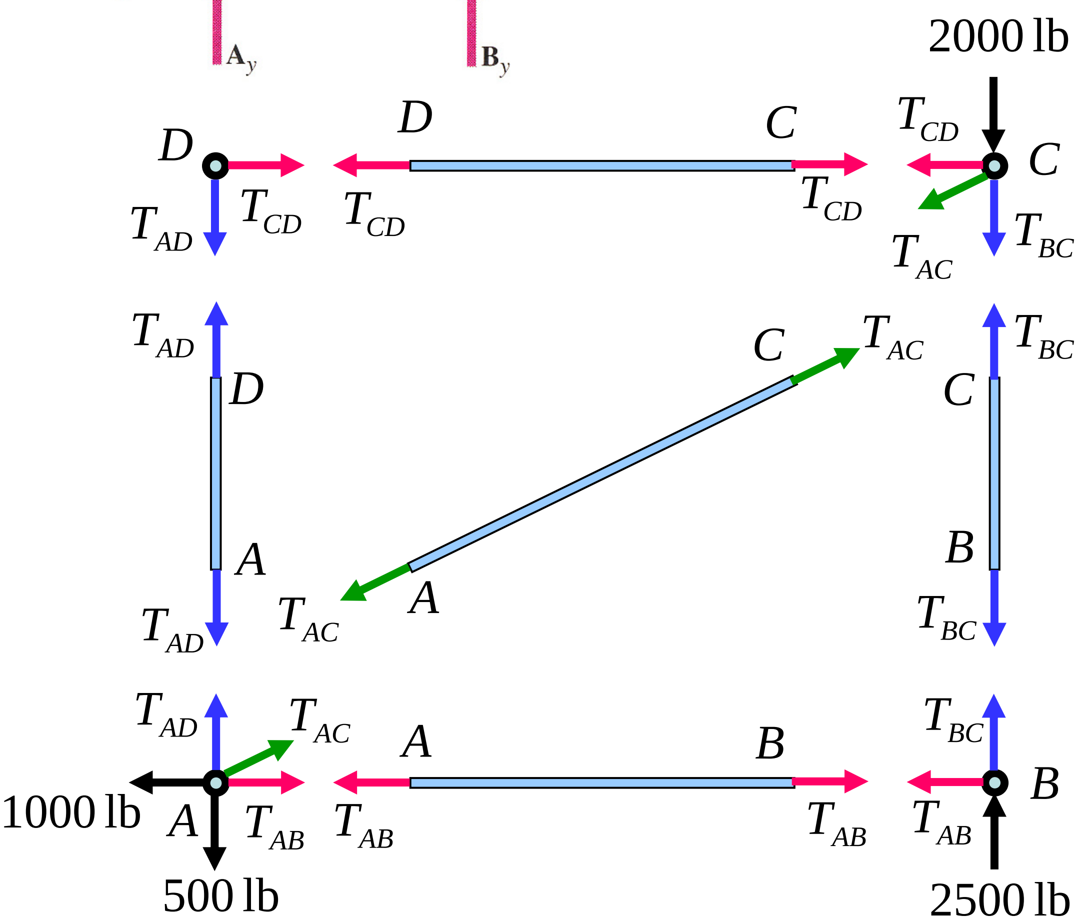


$$A:\begin{cases}\sum F_x = T_{AB} + T_{AC}\cos\theta - 1000 = 0\\ \sum F_y = T_{AD} + T_{AC}\sin\theta - 500 = 0\end{cases}$$

$$B:\begin{cases}\sum F_x = T_{AB} = 0\\ \sum F_y = T_{BC} + 2500 = 0\end{cases}$$

$$C:\begin{cases}\sum F_x = -T_{CD} - T_{AC}\cos\theta = 0\\ \sum F_y = -T_{BC} - T_{AC}\sin\theta - 2000 = 0\end{cases}$$

$$D:\begin{cases}\sum F_x = T_{CD} + 1000 = 0\\ \sum F_y = T_{AD} = 0\end{cases}$$

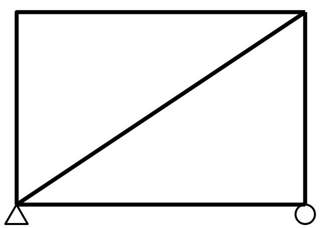


$$D(T_{AD}, T_{CD}) \Rightarrow C(T_{AC}, T_{BC})$$
$$\Rightarrow A(T_{AB}, T_{AC}) \Rightarrow B(T_{AB}, T_{BC})$$

7.2 Plane Trusses

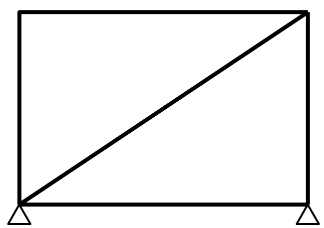
Method of Joints

$$\boxed{?\ \ m = 2j - r\ \ ?}$$



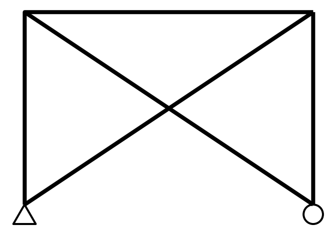
$j = 4 \quad m = 5 \quad r = 3$

$2j - r = 5 = m$



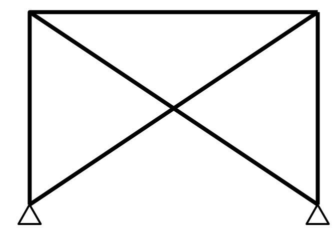
$j = 4 \quad m = 5 \quad r = 4$

$2j - r = 4 < m = 5$



$j = 4 \quad m = 5 \quad r = 3$

$2j - r = 5 = m$

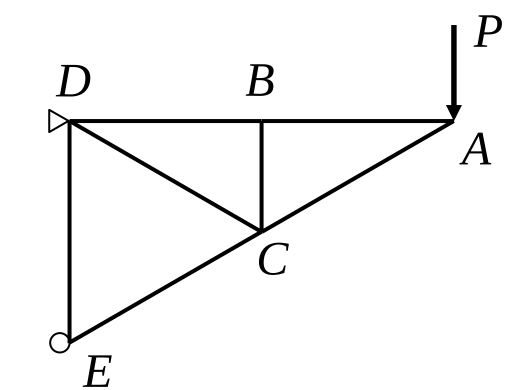


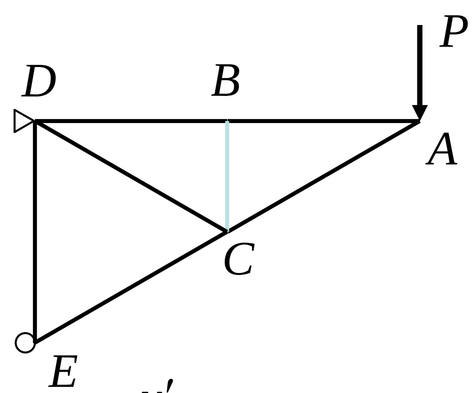
$j = 4 \quad m = 5 \quad r = 4$

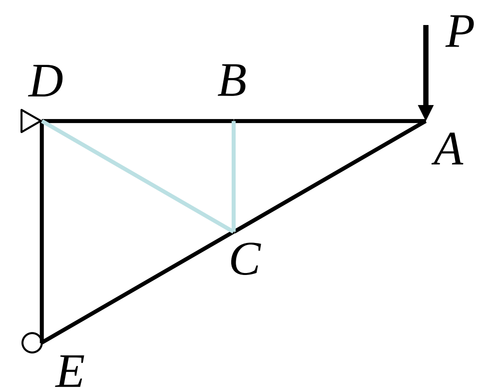
$2j - r = 4 < m = 5$

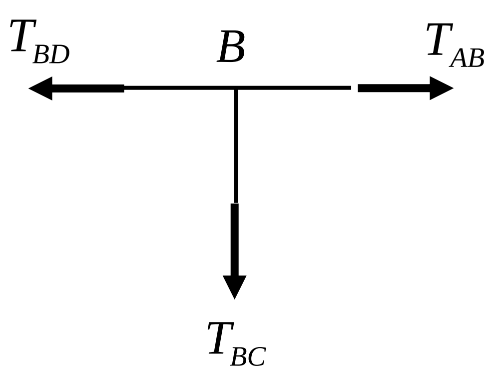
7.2 Plane Trusses

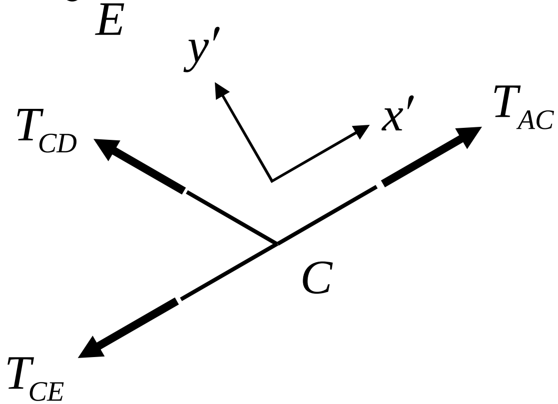
Zero-Force Members









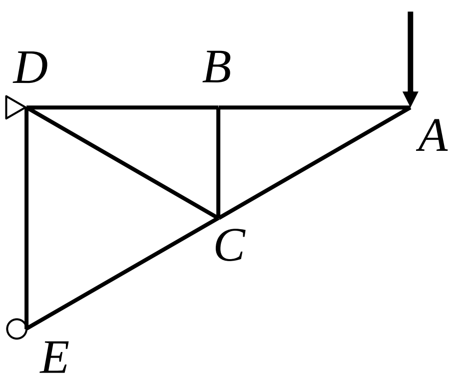


$$\sum F_y = 0 \Rightarrow T_{BC} = 0$$

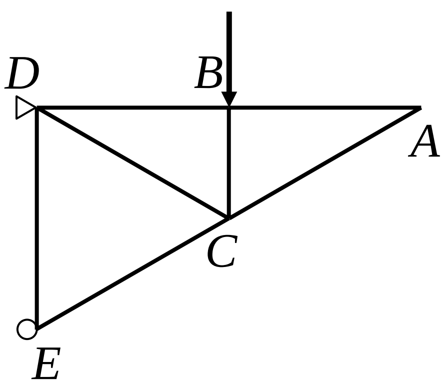
$$\sum F_{y'} = 0 \Rightarrow T_{CD} = 0$$

7.2 Plane Trusses

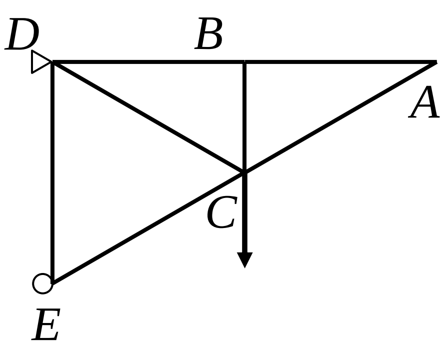
Zero-Force Members



Zero-force member:
BC, *CD*



Zero-force member:
AB, *AC*

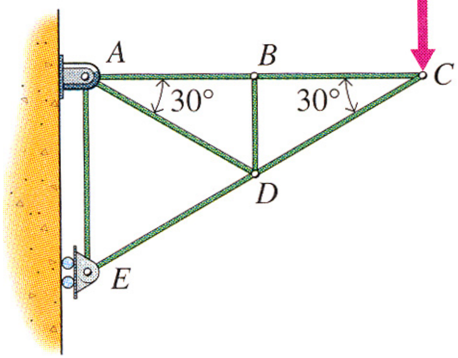


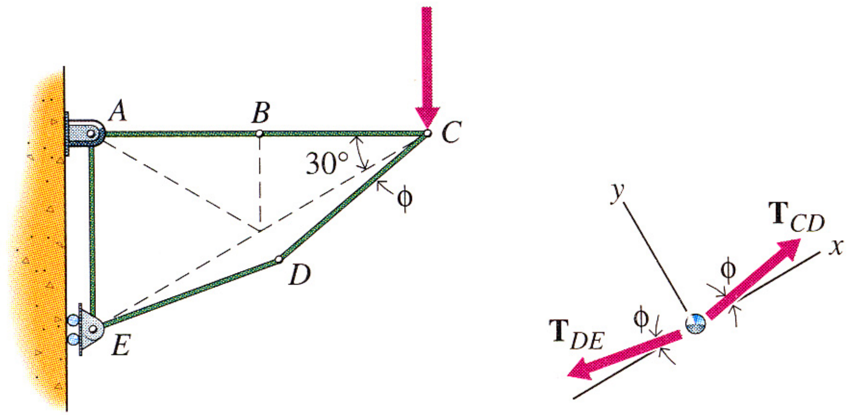
Zero-force member:
AB, *AC*, *BC*

7.2 Plane Trusses

More About Zero-Force Members

Zero force members cannot simply be removed from the truss and discarded, as they are needed to guarantee stability of the truss.





If members *BD* and *AD* are removed, then a slight disturbance would cause joint *D* to buckle outward. The equilibrium at joint *D* (see the free-body diagram) requires that

$$\left.\begin{array}{l}\sum F_x = -T_{DE}\cos\phi + T_{CD}\cos\phi = 0 \Rightarrow T_{DE} = T_{CD} \\ \sum F_y = T_{DE}\sin\phi + T_{CD}\sin\phi = 0 \Rightarrow T_{DE} = -T_{CD}\end{array}\right\} \Rightarrow T_{DE} = T_{CD} = 0$$

Yet equilibrium at joint *C* requires that $T_{CD} \neq 0$. Thus, joint *D* will continue to buckle outward and the truss is no longer stable.

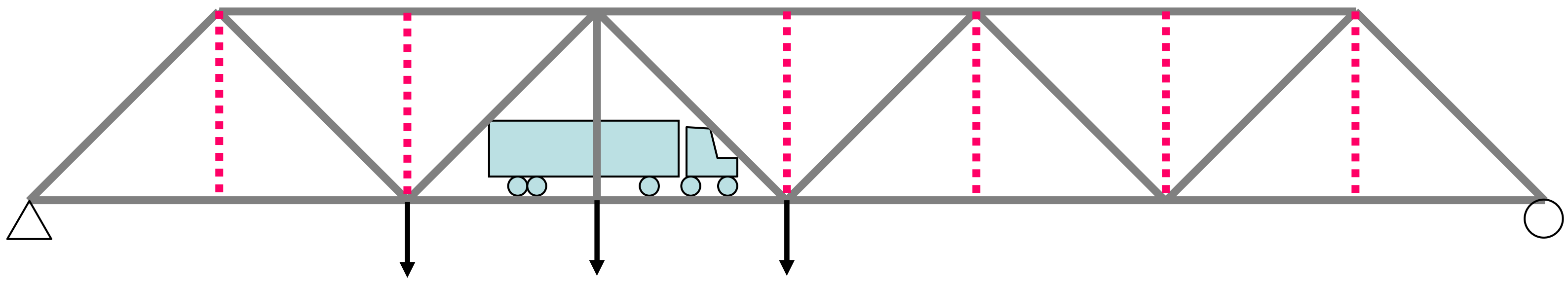
7.2 Plane Trusses

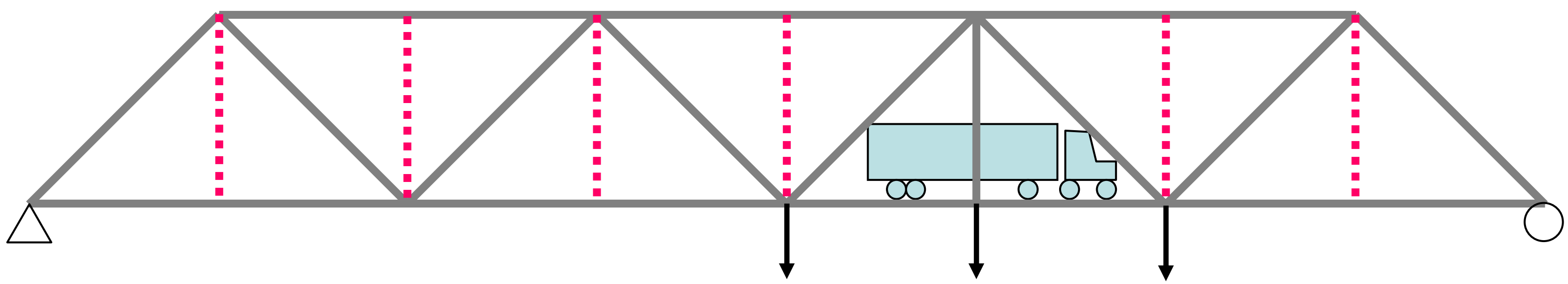
More About Zero-Force Members

Zero-force members may become non-zero-force members, or vice versa, as the load moves from one location to another.

Consider a "Warren" truss with vertical supports.

(See http://www.geocities.com/Baja/8205/truss.htm for other types of bridge truss)

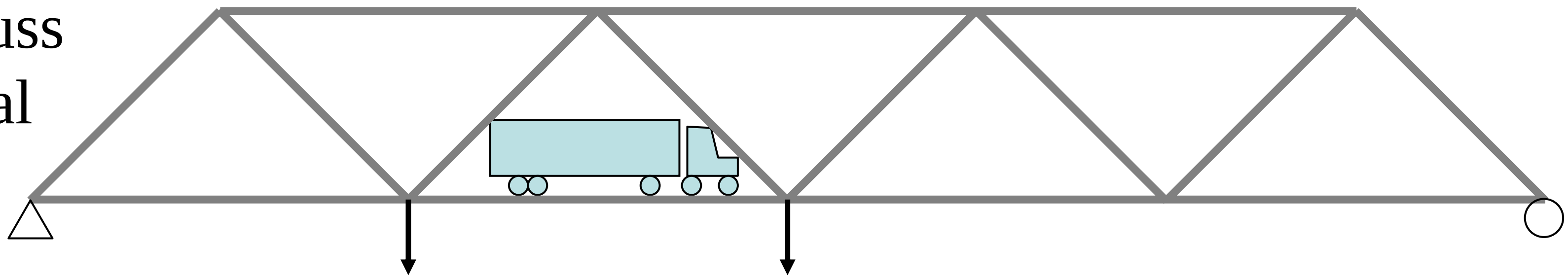




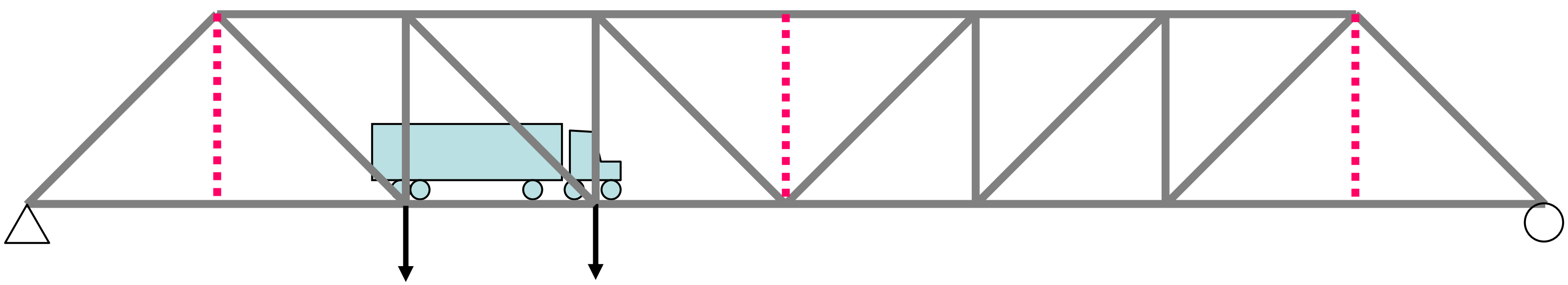
7.2 Plane Trusses

Other Bridge Trusses

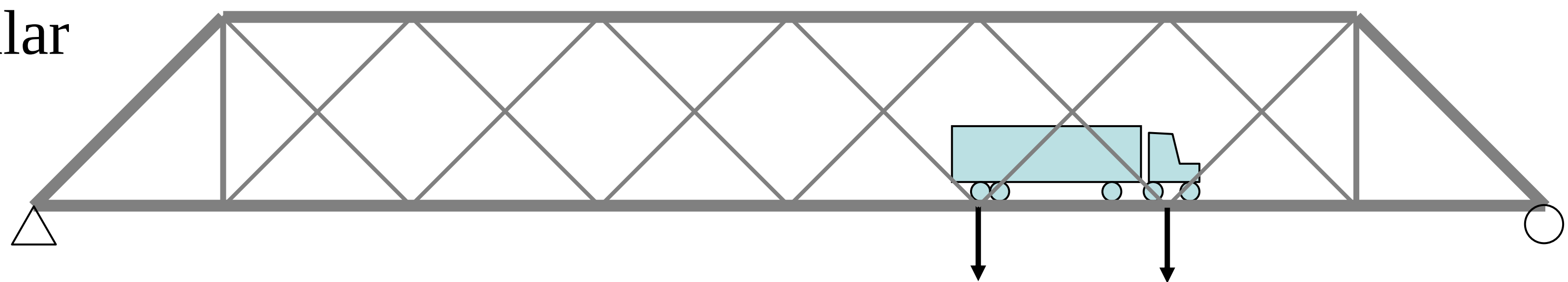
Warren Truss w/o Vertical Supports



Pratt Truss



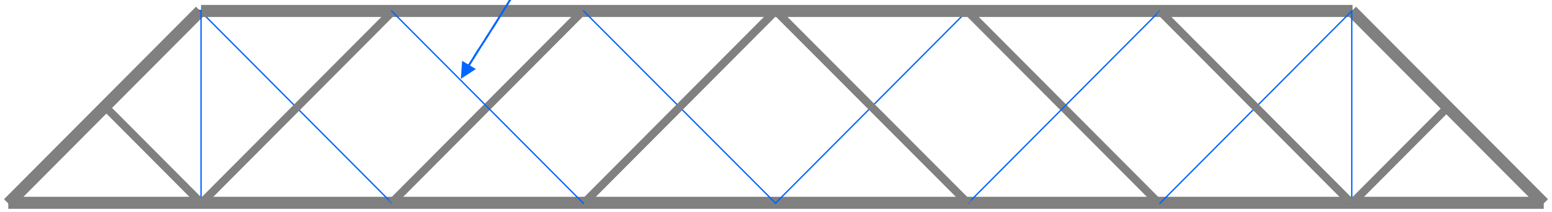
Quadrangular Warren Trusses



7.2 Plane Trusses

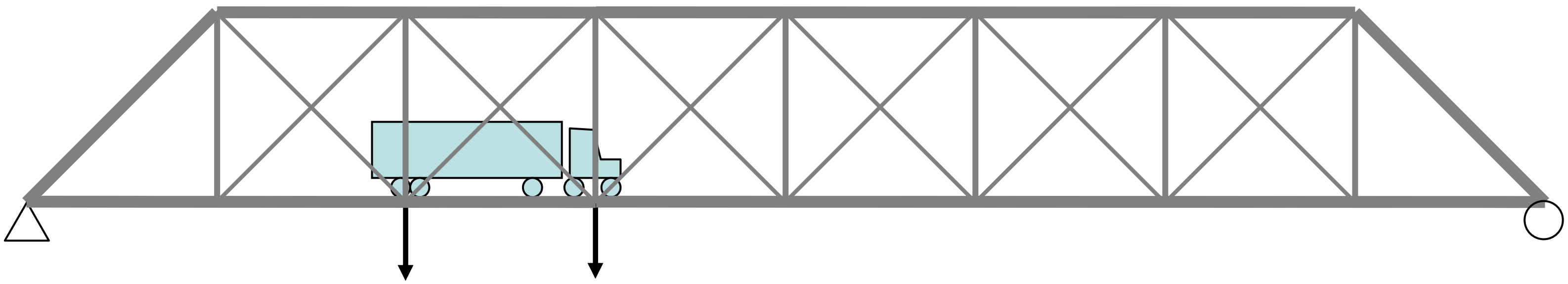
More About Zero-Force Members
Washington Crossing Bridge

Tension-only members



7.2 Plane Trusses

More About Zero-Force Members



Cross members are slender tension-only members. Any compressive force will buckle the member, render it useless.