A Boolean Algebra is a set $\mathbb{B}$ containing constants T and F with $T \neq F$.

On the set $\mathbb{B}$, the following are defined:

1. A binary operation $(x, y) \to x \wedge y$ (and).
2. A binary operation $(x, y) \to x \vee y$ (or).
3. A unary operation $x \to \neg x$ (negation).
4. A relation $\Rightarrow$.

These are subject to the following axioms:

The Distributive Lattice Axioms

Idempotence

(And) For all $x \in \mathbb{B}$, $x \wedge x = x$.

(Or) For all $x \in \mathbb{B}$, $x \vee x = x$.

Commutativity

(And) For all $x, y \in \mathbb{B}$, $x \wedge y = y \wedge x$.

(Or) For all $x,y \in \mathbb{B}$, $x \vee y = y \vee x$.

Associativity

(And) For all $x, y, z \in \mathbb{B}$, $(x \wedge y) \wedge z = x \wedge (y \wedge z)$.

(Or) For all $x, y, z \in \mathbb{B}$, $(x \vee y) \vee z = x \vee (y \vee z)$.

Absorption

(And over Or) For all $x, y \in \mathbb{B}$, $x \wedge (x \vee y) = x$.

(Or over And) For all $x, y \in \mathbb{B}$, $x \vee (x \wedge y) = x$.

Distributivity

(And over Or) For all $x, y, z \in \mathbb{B}$, $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$ and $(y \vee z) \wedge x = (y \wedge x) \vee (z \wedge x)$.

(Or over And) For all $x, y, z \in \mathbb{B}$, $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$ and $(y \wedge z) \vee x = (y \vee x) \wedge (z \vee x)$.

The Boundary Axioms

Boundedness

(Lower Bound) For all $x \in \mathbb{B}$, $F \Rightarrow x$.

(Upper Bound) For all $x \in \mathbb{B}$, $x \Rightarrow T$.

Identity

(And) For all $x \in \mathbb{B}$, $x \wedge T = x$ and $T \wedge x = x$.

(Or) For all $x \in \mathbb{B}$, $x \vee F = x$ and $F \vee x = x$.

Annihilation

(And) For all $x \in \mathbb{B}$, $x \wedge F = F$ and $F \wedge x = F$.

(Or) For all $x \in \mathbb{B}$, $x \vee T = T$ and $T \vee x = T$.

The Complement Axioms

Complementation

(And) For all $x \in \mathbb{B}$, $x \wedge (\neg x) = F$ and $\neg x \wedge x = F$.

(Or) For all $x \in \mathbb{B}$, $x \vee (\neg x) = T$ and $\neg x \vee x = T$.

DeMorgan's Laws

(Nand) For all $x, y \in \mathbb{B}$, $\neg (x \wedge y) = \neg x \vee \neg y$.

(Nor) For all $x, y \in \mathbb{B}$, $\neg (x \vee y) = \neg x \wedge \neg y$.

The Order Axioms

Reflexivity

For all $x \in \mathbb{B}$, $x \Rightarrow x$.

Antisymmetry

For all $x, y \in \mathbb{B}$, if $x \Rightarrow y$ and $y \Rightarrow x$, then $x = y$.

Transitivity

For all $x, y, z \in \mathbb{B}$, if $x \Rightarrow y$ and $y \Rightarrow z$, then $x \Rightarrow z$.

Consistency

(And) For all $x, y \in \mathbb{B}$, $x \wedge y \Rightarrow x$ and $x \wedge y \Rightarrow y$.

(Or) For all $x, y \in \mathbb{B}$, $x \Rightarrow x \vee y$ and $y \Rightarrow x \vee y$.

Monotonicity

(And) For all $x, y, z \in \mathbb{B}$, if $x \Rightarrow y$, then $x \wedge z \Rightarrow y \wedge z$.

(Or) For all $x, y, z \in \mathbb{B}$, if $x \Rightarrow y$, then $x \vee z \Rightarrow y \vee z$.