University of Windsor
Department of Mathematics and Statistics
Mathematical Foundations MATH 1020
Practice Questions for Midterm Test 1

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1. Using a proof by contraposition, prove $\forall x, y \in \mathbb{R}$, if $3y \leq 2x$, then $x \leq 0$ or $y \leq x$.
2. Using a proof by contraposition, prove $\forall x, y \in \mathbb{R}$, if $2y \leq x+1$, then $x \leq 1$ or $y \leq x$.
3. Using a proof by contradiction, prove $\forall a \in \mathbb{R}$, if $\forall x \in \mathbb{R}$, $ax \neq 1$, then $a = 0$.
4. Using a proof by contradiction, prove $\forall x, y \in \mathbb{R}$, if $xy < x + y$ and $x < y$, then $x < 2$.
5. Prove $\forall a \in \mathbb{R}$, $\exists b \in \mathbb{R}$, $b < a$.
6. Prove $\forall a \in \mathbb{R}$, if $a < 1$, then $\exists b \in (0, \infty)$, $a + 2b < 1$.
7. Prove $\forall a \in \mathbb{R}$, if $a < 1$, then $\exists b \in \mathbb{R}$, $2a < b < 2$.
8. Prove $\forall \varepsilon \in (0, \infty)$, $\exists \delta \in (0, \infty)$, $\forall x \in \mathbb{R}$, if $x - 2 < \delta$, then $3x - 6 < \varepsilon$.
9. Prove $\forall a \in \mathbb{R}$, if $\forall x \in \mathbb{R}$, $ax = x$, then $a = 1$.
10. Prove $\forall x \in \mathbb{R}$, if $\forall a \in \mathbb{R}$, $ax = a$, then $2x^2 + x = 2x + 1$.
11. Prove $\forall a \in \mathbb{R}$, if $\exists x \in (0, \infty)$, $x < a$, then $\forall x \in (-\infty, 0)$, $x < a$.
12. Prove $\forall x, y \in \mathbb{R}$, if $\forall a \in (-\infty, x]$, $a < y$, then $\exists b \in (0, \infty)$, $x + b \leq y$.
13. Prove $\forall a \in \mathbb{R}$, if $\exists x \in (0, \infty)$, $\forall y \in (0, \infty)$, $a + x \leq y$, then $\exists z \in (-\infty, 0)$, $a < z$.
14. Prove $\forall a \in \mathbb{R}$, if $\exists x \in (0, \infty)$, $\forall y \in (0, \infty)$, $a + x \leq y$, then $\forall z \in (0, \infty)$, $a < z$.
15. Prove $\forall x, y \in \mathbb{B}$, if $y \Rightarrow \neg x$, then $x \wedge \neg y = x$.
16. Prove $\forall a, x, y \in \mathbb{B}$, if $(\neg x) \wedge a \Rightarrow y$, then $(\neg y) \wedge a \Rightarrow x$.
17. Prove $\forall x, y, z \in \mathbb{B}$, if $x \Rightarrow y$ and $\neg y \Rightarrow z$, then $x \vee \neg z \Rightarrow y$.
18. Prove $\forall x, y \in \mathbb{R}$, if $|x - y| = 0$, then $x = y$.
19. Prove $\forall x, y \in \mathbb{R}$, $|x - y| = \max(x, y) - \min(x, y)$.
20. Prove $\forall x \in \mathbb{R}$, $|x| = \max(x, -x)$.
21. Prove $\forall x \in \mathbb{R}$, if $x < 1$, then $\exists n \in \mathbb{N}$, $x + \frac{2}{n} < 1$.
22. Prove $\forall x \in \mathbb{R}$, if $2 < x$, then $\exists n \in \mathbb{N}$, $2 < x - \frac{3}{n}$.
23. Prove $\forall x \in \mathbb{R}$, if $1 < x$, then $\exists n \in \mathbb{N}$, $\frac{n+1}{n} < x$.
24. Prove $\forall x \in \mathbb{R}$, if $3 < x$, then $\exists n \in \mathbb{N}$, $\frac{3n+4}{n} < x$.