The System of Integers, denoted $\mathbb{Z}$, is a set, containing constants 0 and 1 with $0 \neq 1$.

On the set $\mathbb{Z}$, the following are defined:

1. A binary operation $(x, y) \to x + y$ (addition).
2. A binary operation $(x, y) \to xy$ (multiplication).
3. A unary operation $x \to -x$ (negation).
4. A relation $<$.

These are subject to the following axioms:

The Domain Axioms

Commutativity

(Addition) For all $x, y \in \mathbb{Z}$, $x + y = y + x$.

(Multiplication) For all $x, y \in \mathbb{Z}$, $xy = yx$.

Associativity

(Addition) For all $x, y, z \in \mathbb{Z}$, $(x + y) + z = x + (y + z)$.

(Multiplication) For all $x, y, z \in \mathbb{Z}$, $(xy)z = x(yz)$.

Identity

(Addition) For all $x \in \mathbb{Z}$, $x + 0 = x$ and $0 + x = x$.

(Multiplication) For all $x \in \mathbb{Z}$, $x1 = x$ and $1x = x$.

Invertibility

(Addition) For all $x \in \mathbb{Z}$, $x + (-x) = 0$ and $-x + x = 0$.

The Zero-Product Property

For all $x, y \in \mathbb{Z}$, if $xy = 0$, then $x = 0$ or $y = 0$.

Distributivity

For all $x, y, z \in \mathbb{Z}$, $x(y + z) = xy + xz$ and $(y + z)x = yx + zx$.

The Order Axioms

Trichotomy

For all $x, y \in \mathbb{Z}$, exactly one of $x = y$, $x < y$, or $y < x$ is true.

Transitivity

For all $x, y, z \in \mathbb{Z}$, if $x < y$ and $y < z$, then $x < z$.

Monotonicity

(Addition) For all $x, y, z \in \mathbb{Z}$, if $x < y$, then $x + z < y + z$.

(Positive Multiplication) For all $x, y, z \in \mathbb{Z}$, if $x < y$ and $0 < z$, then $xz < yz$.

The Well-Ordering Property

For any non-empty subset $A \subseteq \mathbb{Z}$, with the property that for all $x \in A$, $0 \leq x$, A has a smallest element. In other words, there is an element $a \in A$ with the property that for all $x \in A$, $a \leq x$.