The System of Boolean Values, denoted $\mathbb{B}$, is a set, containing constants T and F with $T \neq F$.

On the set $\mathbb{B}$, the following are defined:

1. A binary operation $(x, y) \rightarrow x \wedge y$ (and).
2. A binary operation $(x, y) \rightarrow x \vee y$ (or).
3. A unary operation $x \rightarrow \neg x$ (negation).
4. A relation $\Rightarrow$.

These are subject to the following axioms:

The Distributive Lattice Axioms

Idempotence

A1: For all $x \in \mathbb{B}$, $x \wedge x = x$.

O1: For all $x \in \mathbb{B}$, $x \vee x = x$.

Commutativity

A2: For all $x, y \in \mathbb{B}$, $x \wedge y = y \wedge x$.

O2: For all $x,y \in \mathbb{B}$, $x \vee y = y \vee x$.

Associativity

A3: For all $x, y, z \in \mathbb{B}$, $(x \wedge y) \wedge z = x \wedge (y \wedge z)$.

O3: For all $x, y, z \in \mathbb{B}$, $(x \vee y) \vee z = x \vee (y \vee z)$.

Absorption

A4: For all $x, y \in \mathbb{B}$, $x \vee (x \wedge y) = x$.

O4: For all $x, y \in \mathbb{B}$, $x \wedge (x \vee y) = x$.

Distributivity

D1: For all $x, y, z \in \mathbb{B}$, $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$ and $(y \vee z) \wedge x = (y \wedge x) \vee (z \wedge x)$.

D2: For all $x, y, z \in \mathbb{B}$, $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$ and $(y \wedge z) \vee x = (y \vee x) \wedge (z \vee x)$.

The Boundary Axioms

Universal Complement

U1: $\neg T = F$.

U2: $\neg F = T$.

Identity

A5: For all $x \in \mathbb{B}$, $x \wedge T = x$ and $T \wedge x = x$.

O5: For all $x \in \mathbb{B}$, $x \vee F = x$ and $F \vee x = x$.

Complementation

A6: For all $x \in \mathbb{B}$, $x \wedge (\neg x) = F$ and $\neg x \wedge x = F$.

O6: For all $x \in \mathbb{B}$, $x \vee (\neg x) = T$ and $\neg x \vee x = T$.

Annihilator

A7: For all $x \in \mathbb{B}$, $x \wedge F = F$ and $F \wedge x = F$.

O7: For all $x \in \mathbb{B}$, $x \vee T = T$ and $T \vee x = T$.

DeMorgan's Laws

DM1: For all $x, y \in \mathbb{B}$, $\neg (x \wedge y) = \neg x \vee \neg y$.

DM2: For all $x, y \in \mathbb{B}$, $\neg (x \vee y) = \neg x \wedge \neg y$.

The Order Axioms

Reflexivity

I1: For all $x \in \mathbb{B}$, $x \Rightarrow x$.

Antisymmetry

I2: For all $x, y \in \mathbb{B}$, if $x \Rightarrow y$ and $y \Rightarrow x$, then $x = y$.

Transitivity

I3: For all $x, y, z \in \mathbb{B}$, if $x \Rightarrow y$ and $y \Rightarrow z$, then $x \Rightarrow z$.

Consistency

I4: For all $x, y, z \in \mathbb{B}$, $x \Rightarrow x \vee y$ and $x \wedge y \Rightarrow x$.

Order Preservation

I5: For all $x, y, z \in \mathbb{B}$, if $x \Rightarrow y$, then $x \wedge z \Rightarrow y \wedge z$ and $x \vee z \Rightarrow y \vee z$.