

Given any set U with $U \neq \emptyset$, the following are true.

The Distributive Lattice Axioms

Idempotence

A1: For all $X \subseteq U$, $X \cap X = X$.

O1: For all $X \subseteq U$, $X \cup X = X$.

Commutativity

A2: For all $X, Y \subseteq U$, $X \cap Y = Y \cap X$.

O2: For all $X, Y \subseteq U$, $X \cup Y = Y \cup X$.

Associativity

A3: For all $X, Y, Z \subseteq U$, $(X \cap Y) \cap Z = X \cap (Y \cap Z)$.

O3: For all $X, Y, Z \subseteq U$, $(X \cup Y) \cup Z = X \cup (Y \cup Z)$.

Absorption

A4: For all $X, Y \subseteq U$, $X \cup (X \cap Y) = X$.

O4: For all $X, Y \subseteq U$, $X \cap (X \cup Y) = X$.

Distributivity

D1: For all $X, Y, Z \subseteq U$, $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$ and $(Y \cup Z) \cap X = (Y \cap X) \cup (Z \cap X)$.

D2: For all $X, Y, Z \subseteq U$, $X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$ and $(Y \cap Z) \cup X = (Y \cup X) \cap (Z \cup X)$.

The Boundary Axioms

Universal Complement

U1: $U^c = \emptyset$.

U2: $\emptyset^c = U$.

Identity

A5: For all $X \subseteq U$, $X \cap U = X$ and $U \cap X = X$.

O5: For all $X \subseteq U$, $X \cup \emptyset = X$ and $\emptyset \cup X = X$.

Complementation

A6: For all $X \subseteq U$, $X \cap (X^c) = \emptyset$ and $X^c \cap X = \emptyset$.

O6: For all $X \subseteq U$, $X \cup (X^c) = U$ and $X^c \cup X = U$.

Annihilator

A7: For all $X \subseteq U$, $X \cap \emptyset = \emptyset$ and $\emptyset \cap X = \emptyset$.

O7: For all $X \subseteq U$, $X \cup U = U$ and $U \cup X = U$.

DeMorgan's Laws

DM1: For all $X, Y \subseteq U$, $(X \cap Y)^c = X^c \cup Y^c$.

DM2: For all $X, Y \subseteq U$, $(X \cup Y)^c = X^c \cap Y^c$.

The Order Axioms

Reflexivity

I1: For all $X \subseteq U$, $X \subseteq X$.

Antisymmetry

I2: For all $X, Y \subseteq U$, if $X \subseteq Y$ and $Y \subseteq X$, then $X = Y$.

Transitivity

I3: For all $X, Y, Z \subseteq U$, if $X \subseteq Y$ and $Y \subseteq Z$, then $X \subseteq Z$.

Consistency

I4: For all $X, Y, Z \subseteq U$, $X \subseteq X \cup Y$ and $X \cap Y \subseteq X$.

Order Preservation

I5: For all $X, Y, Z \subseteq U$, if $X \subseteq Y$, then $X \cap Z \subseteq Y \cap Z$ and $X \cup Z \subseteq Y \cup Z$.