Invariant Set Lane Keeping Controller Explained

Petter Nilsson
pettni@umich.edu

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1 Model

The controller is based on the following model, which is prof. Grizzle's "Very Much Simplified Model" extended with an additional state to be able to constrain the derivative of the yaw rate:

$$\begin{bmatrix} \dot{y} \\ \dot{\psi} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} 0 & u_0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ \psi \\ r \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r_{dot} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} r_{road} \tag{1}$$

Therefore r_{dot} is the control input in this model, and r_{road} is an uncontrolled disturbance modeling the change in curvature of the road.

Parameter explanations:

- y [m] - distance from center of lane
- ψ [rad] - yaw angle
- r [rad/s] - yaw rate, time derivative of ψ
- u_0 [m/s] - speed of car which is constant

2 Specifications

We require the following safety specs to be satisfied at all times:

- $|y| \leq 0.9$ - to prevent lane departure
- $\left| r_{dot} + \frac{b^2 C_{ar}}{u_0 I_z} r \right| \leq \frac{a F_{yf,max}}{I_z}$ - Front axle force constraint from Yuxiao's notes.
- $|r| \leq \frac{a u_0 F_{yf,max}}{b^2 C_{ar}}$ - also from Yuxiao's notes.

The following parameter values are assumed, most are taken from prof. Grizzle's simulation:

- $u_0 = 30$ m/s
- $b = 1.59$ m
- $a = 1.11$ m
- $I_z = 2315.3$ kg m^2
- $C_{ar} = 9.882 \times 10^4$ N/rad
- $F_{yf,max} = 1500$ N - based on the simulation of prof. Grizzle's controller, might be too small.

0.1 s was used as the time discretization step.

3 Assumptions

The road does not force the car to turn sharper than a lateral acceleration corresponding to $\alpha_{road} g$. Equating the lateral acceleration to u_0^2/R, this gives a minimal turning radius of $R = u_0^2/\alpha_{road} g$. The derivative of the angle is equal to $r_{road} = u_0/R$, so we assume

$$|r_{road}| \leq \frac{\alpha_{road} g}{u_0}. \tag{2}$$

In the following $\alpha_{road} = 0.2$ has been used.

4 Invariant set

The red set in the Figure 3 below is found to be controlled-invariant for these dynamics and assumptions.

5 Simulation

A 15-step (corresponding to 1.5s) look ahead MPC controller was used. The road in the simulation has $\alpha = 0.15$. A simple way to implement the controller on the 4-state model, which takes steering angle as input, is to invert the mapping from δ_f to $\dot{r}$:

$$\delta_f(r_{dot}, v, , r) = \frac{I_z}{aC_{af}} \left(r_{dot} - \frac{bC_{ar} - aC_{af}}{I_z u_0} v + \frac{a^2 C_{af} + b^2 C_{ar}}{I_z u_0} r \right). \tag{3}$$

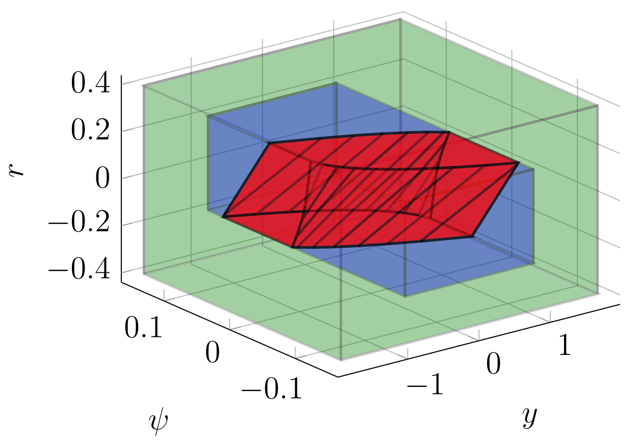

Figure 1: Invariant set (red) inside safe set (blue).

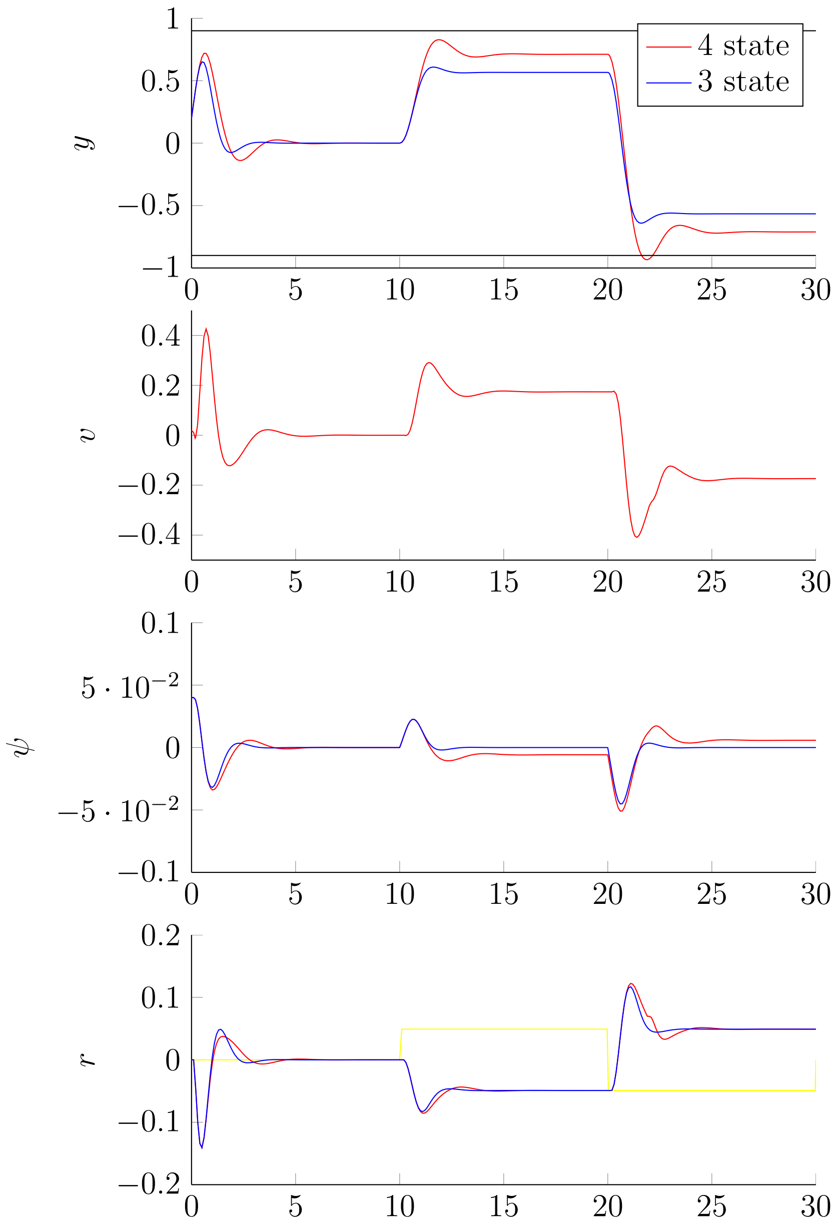

Figure 2: Car states in simulation. Yellow line is r_{road}.

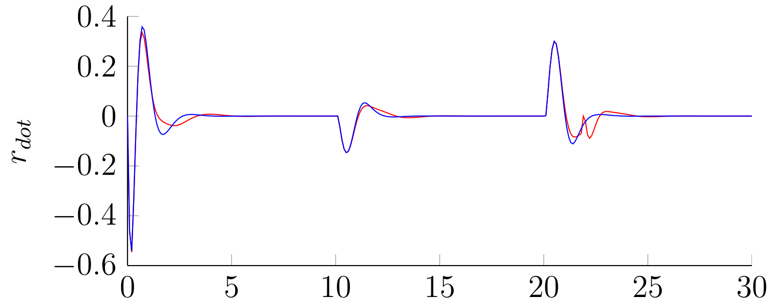

Figure 3: Input: derivative of car yaw rate.