9.1 Sequences

A sequence goes from $N \to R$

$$\lim_{n \to \infty} k = k$$

for $\varepsilon \geq 0$ there is N

$$n \geq N \Rightarrow |a_n - L| \leq \varepsilon$$

$$n \geq N \Rightarrow |k - k| \leq \varepsilon$$

$$\Rightarrow 0 \leq \varepsilon$$

$\varepsilon \geq 0 \Rightarrow$ it's true.

$$\frac{1}{n} = \frac{1}{n+1}$$

$$n = n+1$$

$$\Rightarrow n \to \infty$$

this uses trial and error thus it needs luck.

Theorm: $\{a_n\}$ and $\{b_n\}$ are sequences

if $\lim a_n = A$ and limit $b_n = B$

$$\lim(a_n \pm b_n) = A \pm B$$

$$\because \quad A \cdot B$$

$$\because \quad \frac{A}{B}$$

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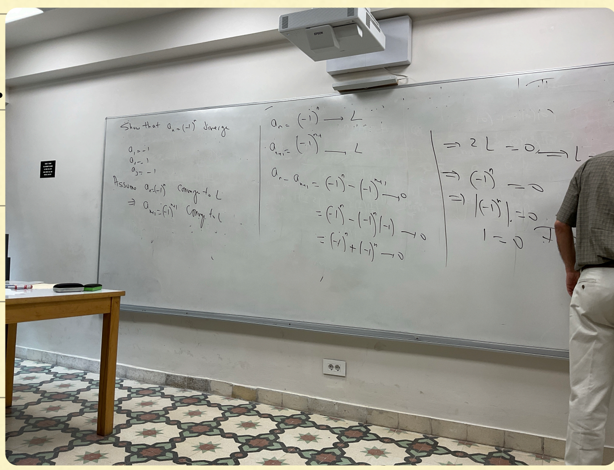
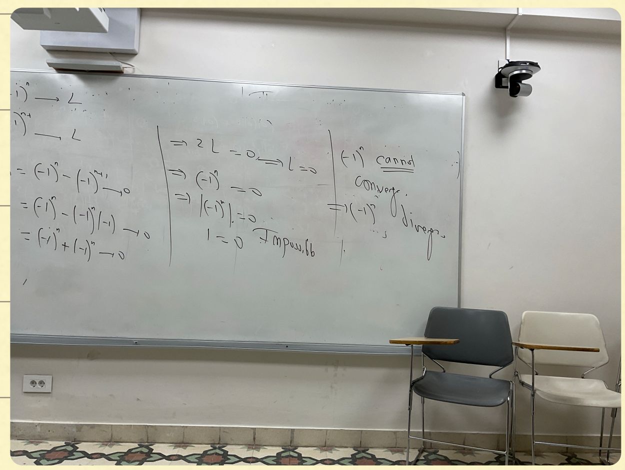
for $(-1)^n$ does it converge or div.?

Assume it converges

but...





we only care if a sequence increases or decrease <u>if</u> it converges (usually).

upper bounds and lower bounds.

Important Theorm (I.T)

$\{a_n\}$ as a sequence

(1) if $\{a_n\}$ is increasing

(2) bounded from above

$\Longrightarrow$ a_n converges.

opposite is the case too.

if it's unbounded then by def it diverges.

Sandwich Theorm:

let there be a_n, b_n, c_n

if $a_n \geq b_n \geq c_n$

and $\lim a_n = \lim c_n = L$

$\Longrightarrow \lim b_n = L$

Find $\lim \dfrac{\sin(n^3+5)}{n} = 0$ yd 3mmo

Fmn10

Proof of the Sandwich Theorm:

- we use def.

(1) $\lim\limits_{n \to +\infty} a_n = L \qquad n \geq N$

$$|a_n - L| < \varepsilon$$

$$-\varepsilon < a_n - L < \varepsilon$$

$$L - \varepsilon < a_n < L + \varepsilon$$

(2) $\lim\limits_{n \to \infty} c_n = L \qquad n \geq N$

$$|c_n - L| < \varepsilon$$

$$L - \varepsilon < c_n < L + \varepsilon$$

$$L - \varepsilon < a_n \leq b_n \leq c_n < L + \varepsilon$$

$$\underline{L - \varepsilon < b_n < L + \varepsilon}$$

therefore $-\varepsilon < b_n - L < \varepsilon$

$$|b_n - L| < \varepsilon$$

$$\Longrightarrow \lim_{n \to +\infty} b_n = L$$

Corollary to Sandwich theorm (CST)

if $|a_n| \leq b_n$

and $\lim_{n \to +\infty} b_n = 0 \Longrightarrow \lim_{n \to +\infty} a_n = 0$

Ex: $\lim_{n \to +\infty} \frac{(-1)^n}{n^2 + \cos^2 n}$

$$\left| \frac{(-1)^n}{n^2 + \cos^2 n} \right| = \frac{1}{n^2 + \cos^2 n} \leq \frac{1}{n^2}$$

$$\frac{(-1)^n}{n^2 + \cos^2 n} \leq \frac{1}{n^2}$$

goes to 0

$\Longrightarrow$ by CST goes to 0

Continuity Theorm

if $a_n \to L$, $f(a_n) \to f(L)$

$f(x)$ has to be continious at $x = L$

$\therefore$ we can't apply the theorm on the floor function.

So $a_n = \lfloor 2 - \frac{1}{n} \rfloor$

$a_1 = 1$

$a_2 = 1$

....

casually we take $a_n \to 2$ if it was continuous

but... it's actually $a_n \to 1$ due to the floor function.

Remark: if (a_n) converges
$\Rightarrow (a_{n+1})$ converges.

~ basic limits ~

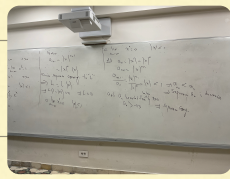
1. $\lim \frac{\ln n}{n^{c>0}} = 0$ ← l'hopital's rule

2. $\lim \sqrt[n]{n} = \lim n^{\frac{1}{n}} = 1$ ← we use $x = e^{\ln x}$

3. $\lim x^{\frac{1}{n}} = 1$ ↑ same.

   when we get to $e^{\frac{\ln x}{n}}$ ← when we plug the limit of $\frac{\ln x}{n} = 0$ we have to state that we used continiouity theorm.

4. $\lim x^{n} = 0$



5. $\lim \left(1 + \frac{x}{n}\right)^{n} = e^{x}$

Note: when we plug the limit of a function back $\{ e^{f(x)} \to e^{\lim f(x)}$ we **have** to use cont. theorm

6. $\lim \frac{x^{n}}{n!} = 0$