M1F Summary Notes (JMC Year 1, 2017/2018 syllabus)

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This document contains a list of definitions and a list of theorems.

Note that the exam will probably require you to PROVE some of these theorems, so you should refer back to the original notes for the proofs.

Boxes cover content in more detail.

Contents

1 Definitions

2 Theorems

2.1 Sets

2.2 Complex numbers

2.3 Number theory

Bezout's Theorem

$$\gcd(a, b) = \lambda a + \mu b \quad (\text{for some } \lambda, \mu \in \mathbb{R}) \tag{1}$$

Let:

- $a = \alpha \gcd(a, b)$
- $b = \beta \gcd(a, b)$

In general the solution to equation 1 is given by:

$$\gcd(a, b) = (\lambda + \beta n)a + (\mu - \alpha n)b \tag{2}$$

noting that the extra terms will always cancel out. So we have a set of solutions (λ_n, μ_n), where:

$$\lambda_n = \lambda + \beta n, \quad \mu_n = \mu - \alpha n \tag{3}$$

Every integer > 1 can be written as a product of primes.
(use strong induction to prove)

Fundamental Theorem of Arithmetic
Every integer > 1 can be written UNIQUELY as a product of primes.
(proof not needed)

2.4 Equivalence relations and functions

2.5 Combinatorics