M1J2 Summary Notes (JMC Year 1, 2017/2018 syllabus)

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STILL UNDER CONSTRUCTION

Dr Lawn refers to propositions, theorems, corollaries and lemmas. In this document I will refer to them all as 'theorems'.

This document only contains a list of definitions and a list of theorems.

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Sequence   A sequence is simply a map $f : \mathbb{N} \to \mathbb{R}$, denoted by a_n

Convergence   A sequence a_n converges to a limit L if for all real numbers $\epsilon > 0$, there exists an $N \in \mathbb{N}$ such that for all $n \geq N$ we have $|a_n - L| < \epsilon$.

$$\forall \epsilon > 0 \quad \exists N \in \mathbb{N} \quad s.t \quad \forall n \geq N \quad |a_n - L| < \epsilon \tag{1}$$

Shift   The shift of a sequence by say, k, is the sequence $b_n = a_{n+k}$

Tending to infinity   We say a sequence tends to infinity if for all $R \in \mathbb{R}$, the sequence a_n is eventually bigger than R.

$$\forall R \in \mathbb{R} \quad \exists N \in \mathbb{N} \quad s.t. \quad \forall n \geq N \quad a_n > R \tag{2}$$

Triangle inequality   The general triangle inequality is:

$$|x - y| < |x - z| + |z - y| \tag{3}$$

Setting $z = 0$ gives us:

$$|x - y| > |x| - |y| \tag{4}$$

Then setting $y = -y$ gives us the familiar case:

$$|x + y| < |x| + |y| \tag{5}$$

Bounded above   A sequence a_n is bounded above if there's a real number A such that $a_n < A$ for all n.

Bounded below   A sequence a_n is bounded below if there's a real number A such that $a_n > A$ for all n.

Bounded   A sequence a_n is bounded if there's a real number A such that $|a_n| < A$ for all n.

Increasing   A sequence is increasing if $a_{n+1} \geq a_n$ for all n.

Strictly increasing   A sequence is strictly increasing if $a_{n+1} > a_n$ for all n.

Decreasing   A sequence is decreasing if $a_{n+1} \leq a_n$ for all n.

Strictly decreasing   A sequence is strictly decreasing if $a_{n+1} < a_n$ for all n.

Monotonic   A sequence is monotonic if it is increasing or decreasing.

Supremum   The supremum A of a set S is the least upper bound of that set i.e. the smallest number such that $\forall s \in S \quad s \leq A$

Infimum   The infimum B of a set S is the greatest lower bound of that set i.e. the largest number such that $\forall s \in S \quad s \geq B$

Subsequence   A subsequence of a_n is a sequence $a_{f(n)}$, where $f(n)$ is a strictly increasing function.

Cauchy sequence   A sequence is Cauchy if the terms get arbitrarily close to one another. To put it mathematically:

$$\forall \epsilon > 0 \quad \exists N \in \mathbb{N} \quad s.t \quad \forall m, n \geq N \quad |a_n - a_m| < \epsilon \qquad (6)$$

Partial sum   The nth partial sum S_n of a sequence a_n is the sum of terms up to that point:

$$S_n = \sum_{i=1}^{n} a_n \qquad (7)$$

Summable   A sequence is summable if the sequence of its partial sums converges. The limit of the sequence of partial sums will be:

$$L = \sum_{i=1}^{\infty} a_n \qquad (8)$$

Absolutely summable   A sequence a_n is absolutely summable if $|a_n|$ is summable.

Conditionally summable   A sequence is conditionally summable if it is summable but not absolutely summable.

Power series   The power series associated with a sequence a_n is the sequence of partial sums:

$$\sum_{i=1}^{n} a_i x^i \tag{9}$$

Radius of convergence   The radius of convergence R of a power series $P(x)$ is defined as the largest x for which $P(x)$ is convergent.

6   Theorems