M1F Summary Notes (JMC Year 1, 2017/2018 syllabus)

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UNDER CONSTRUCTION

This document contains a list of definitions and a list of theorems.

Note that the exam will probably require you to PROVE some of these theorems, so you should refer back to the original notes for the proofs.

Boxes cover content in more detail.

Contents

1 Definitions

2 Theorems

2.1 Sets

2.2 Complex numbers

2.3 Number theory

Completeness Axiom
A set S has a least upper bound (sup) iff:

- S is non-empty
- S is bounded above

Corresponding statement for greatest lower bound (inf).

Bezout's Theorem

$$\gcd(a, b) = \lambda a + \mu b \quad \text{(for some } \lambda, \mu \in \mathbb{R}) \tag{1}$$

Let:

- $a = \alpha \gcd(a, b)$
- $b = \beta \gcd(a, b)$

In general the solution to equation 1 is given by:

$$\gcd(a, b) = (\lambda + \beta n)a + (\mu - \alpha n)b \tag{2}$$

noting that the extra terms will always cancel out. So we have a set of solutions (λ_n, μ_n), where:

$$\lambda_n = \lambda + \beta n, \quad \mu_n = \mu - \alpha n \tag{3}$$

Every integer larger than 1 can be written as a product of primes. (use strong induction to prove)

Fundamental Theorem of Arithmetic
Every integer larger than 1 can be written UNIQUELY as a product

of primes.
(proof not needed)

2.4 Equivalence relations and functions

2.5 Combinatorics