Characterizing Redundancy in Natural Images

Niru Maheswaranathan
Stanford University
nirum@stanford.edu

Abstract

The space of natural images is both high-dimensional and has striking non-Gaussian, higher-order statistical structure. It has been argued on information theoretic grounds that much of early visual processing consists of removing statistical regularities in visual input, in what is known as the redundancy reduction hypothesis. A key part of understanding the redundancy of natural scenes is understanding the entropy of image patches. Entropy allows us to quantify the information content of image patches, and bounds optimal compression schemes. This project aims to characterize the redundancy of natural images by quantifying the entropy of both the raw ensemble of images as well as other generative models of natural images.

1. Motivation

It has long been argued [1] that the human visual system has evolved to efficiently process the statistical structure of natural images. This suggests that computer vision systems can also be tuned to the structure of natural images. A useful metric for quantifying the redundancy in the distribution of image patches is the entropy of the distribution. Entropy can be thought of as a measure of the information content of a distribution, therefore, the entropy of the distribution of natural scenes bounds the amount of information available to vision systems (either man-made or in nature). Entropy also provides a lower bound on the number of bits needed for compression, thus knowledge of the entropy of natural images gives us an absolute threshold with which to compare compression schemes.

Another potential application of entropy is in comparing different generative models of images. In computer vision, people often reduce high-dimensional images into low-dimensional representations (in terms of projections onto a feature basis, for example) [2]. How can one compare these feature sets directly? One possibility would be that we wish to maximize the entropy of the reduced description. To do this, we need to also be able to estimate the entropy of natural images projected onto different basis sets.

2. Background

The difficulty in estimating the entropy of image patches arises due to the curse of dimensionality: for patches of size $n \times n$, we must estimate a probability distribution with dimension n^2, which quickly becomes infeasible as n grows. Despite these limitations, people have recently made estimates of the entropy using approximate methods.

Chandler and Field use an approximation that relies on nearest-neighbor (NN) distances. It has been shown [3] that the average log NN distance can be used to estimate the entropy without estimating the probability distribution $p(x)$. More specifically, the entropy estimate H_n given n samples from the distribution $p(x)$ can be written as follows:

$$H_n = \frac{1}{n} \sum_{i=1}^{n} \log(n\rho_{n,i}) + \log(2) + C_E \tag{1}$$

Where C_E is the Euler constant, $C_E = -\int_0^\infty e^{-t} \log(t) dt$, and $\rho_{n,i}$ is the Euclidean distance of the nearest-neighbor of the sample X_i:

$$\rho_{n,i} = \min_{j \neq i} ||X_i - X_j||$$

The advantage of this estimate is that it scales better with the dimension d of the data than say a kernel density estimate.

3. Methods

This section outlines the aims discussed in my proposal, and specifies which have been completed and which are still left to do. I have obtained a database of image patches by sampling from the van Hateren natural image database [4]. Figure 1 shows some examples of sampled 64×64 image patches, along with a probability distribution over single pixel intensities sampled from 100 such patches. Note the kurtotic structure of the pixel density.

1. I have generated samples of image patches from a multivariate gaussian distribution with fixed covariance

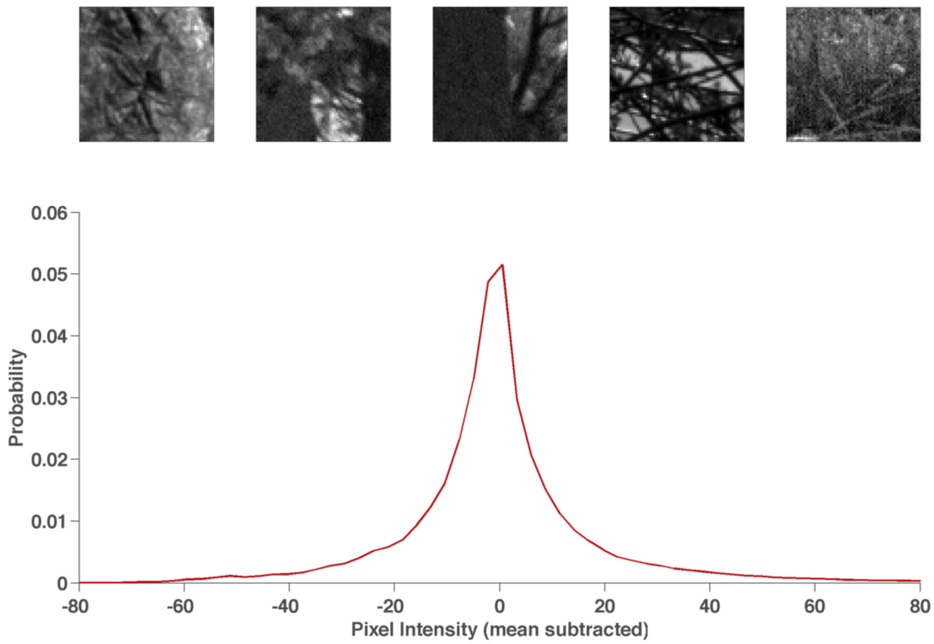

Figure 1. Sample natural image patches (top) and probability distribution over single pixel values (bottom)

structure, as well as from the van Hateren database. The entropy of a multivariate gaussian is given as $H = \frac{1}{2}\log\left((2\pi e)^k|\Sigma|\right)$, where k is the dimension and $|\Sigma|$ is the determinant of the covariance matrix.

2. The simplest method of estimating entropy is via kernel density estimation (KDE), where the underlying probability distribution $p(x)$ is approximated given n samples from the distribution. This works well for one-dimensional distributions, and I use KDE as a baseline for comparison with the nearest-neighbor method for 1D distributions.

3. I implemented the nearest-neighbor algorithm to estimate the entropy from a sampled distribution, as described in Chandler and Field. First, I tested the algorithm on estimating the entropy of a 1D gaussian, which can be confirmed analytically. I also ran the algorithm to estimate the entropy of the distribution of single pixel values.

4. I am currently working on expanding the nearest-neighbor algorithm to estimate the entropy of high-dimensional distributions. There are some challenges with working in high-dimensions, which I outline in section 5.

5. I also hope to estimate the entropy of low-dimensional projections of natural images. For example, in class, we have discussed using PCA as a method to reduce the dimensionality of face images. Using this and other algorithms, such as ICA, I plan on generating low-dimensional representations of natural images and then computing entropy in the low-D space (computationally more feasable). This may also yield a method for comparing different reduced descriptions of natural images.

4. Preliminary Results

As described above, I have used a one-dimensional distributions as a baseline for comparing entropy estimation methods. Figure 2 shows the two methods' (kernel density and nearest-neighbor) estimates of the entropy of samples from a univariate normal distribution, along with the (analytically known) true entropy. We see that the estimates converge at similar rates and yield accurate estimates with roughly 10^3 samples. Estimating with 10^4 samples already takes a few seconds on my laptop, as computing the nearest-neighbors takes N^2 operations.

We can also estimate the entropy on the distribution of pixel intensities drawn from natural images, shown in Figure 3. Here, there is no analytical estimate, but we see that both the kernel density estimate and the nearest-neighbor estimate converge to around 6.5 bits per pixel.

5. Current Challenges and Future Work

The biggest challenge I am currently facing involve computational complexity. The previous work needed to go out to 10^8 or so samples to get accurate estimates of the entropy for large (8x8) pixel patches. To do this, I think I would have to offload the computation to corn or another cluster. There is also a trade-off between CPU and memory resources, as the distance computation can be done faster in parallel in matlab although it requires a lot of memory for large sample sizes.

While estimating the entropy of the raw distribution of images is a nice first step, I am also very interested in comparing estimates of the reduced descriptions of natural images. By reduced descriptions, I mean doing dimensionality reduction by projecting onto features learned from principal or independent components analysis, for example. I have implemented PCA and ICA on the natural image patches, and a future goal is to estimate the redundancy in these reduced descriptions of natural images.

References

[1] H. Barlow. Redundancy reduction revisited. *Network: Comput. Neural Syst.*, 12:241–253, 2001. 1

[2] M. Bethge. Factorial coding of natural images: how effective are linear models in removing higher-order dependencies? *J. Opt. Soc. Am.*, 23(6), 2006. 1

[3] J. B. et al. Nonparametric entropy estimation: An overview, 2001. 1

[4] J. van Hateren and A. van der Schaaf. Independent component filters of natural images compared with simple cells in primary visual cortex. *Proc.R.Soc.Lond. B*, 265:359–366, 1998. 1

6. Appendix

I certify that this project is my own original work. This project is not part of larger work from another class or lab.

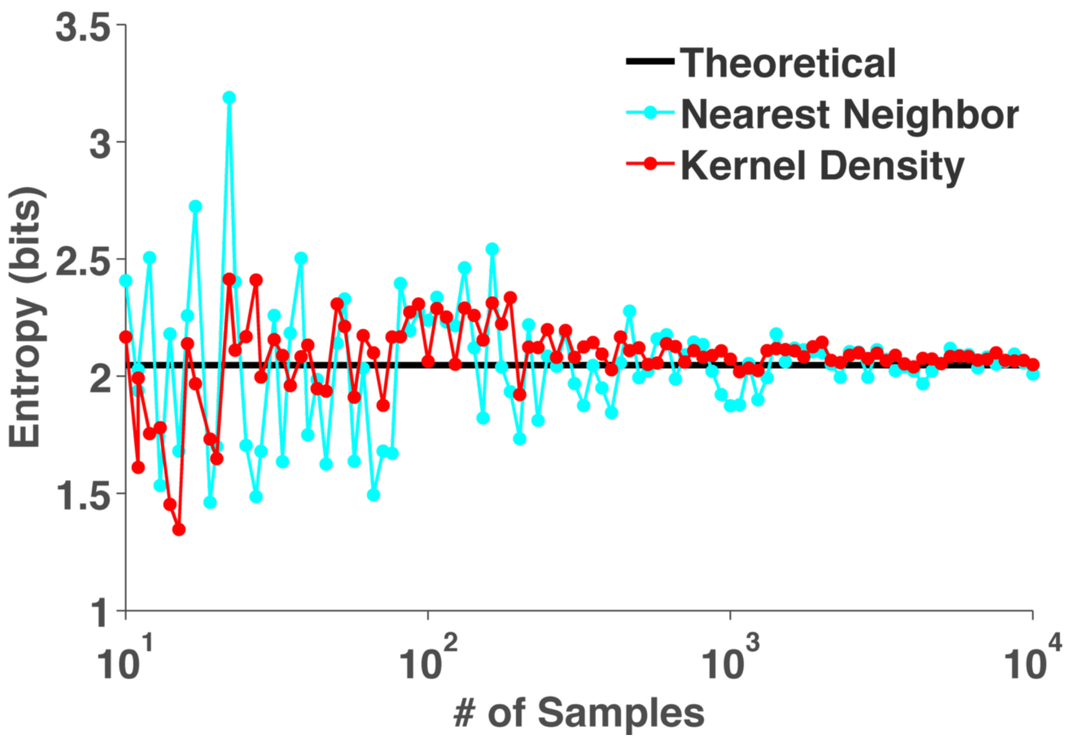

Figure 2. Entropy estimation of a normal distribution.

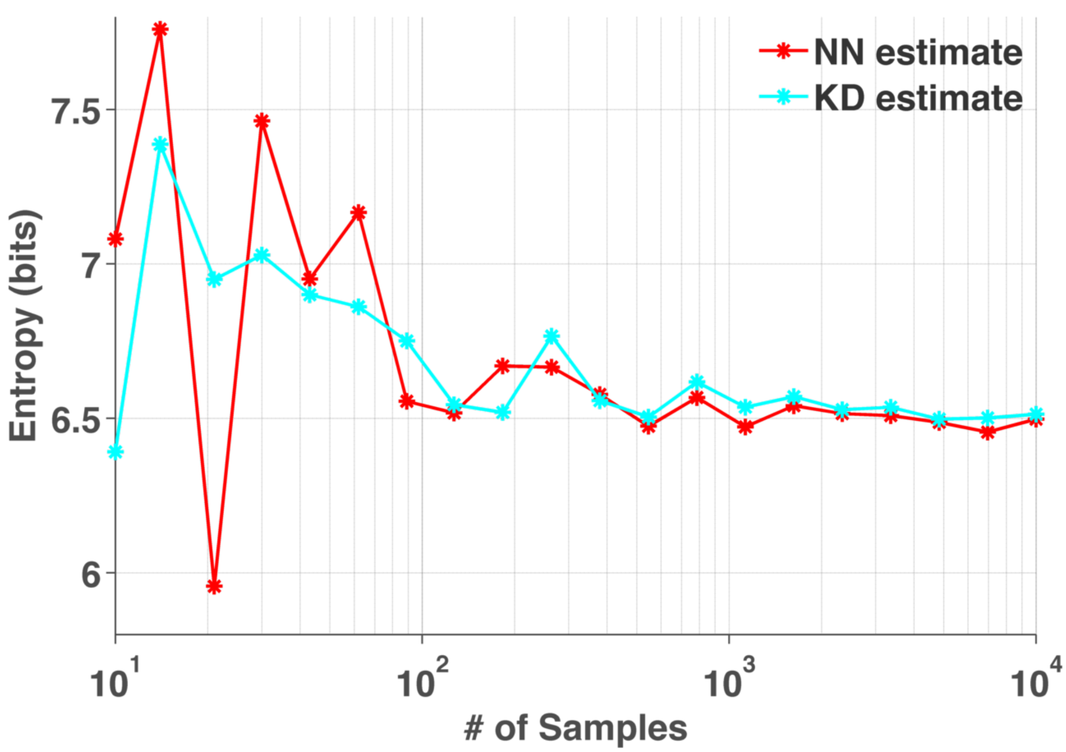

Figure 3. Entropy estimation of the distribution of pixel intensities in natural images.