Radbound University

Cryptographic Hash Functions and Key Derivation

Applied Cryptography – Spring 2024

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Last Lectures

- We learned the basics of symmetric cryptography:
  - Encryption
  - Message authentication
  - Authenticated encryption
- These can be built from (a.o.):
  - Tweakable block ciphers
  - Block ciphers
  - Permutations

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  - Encryption
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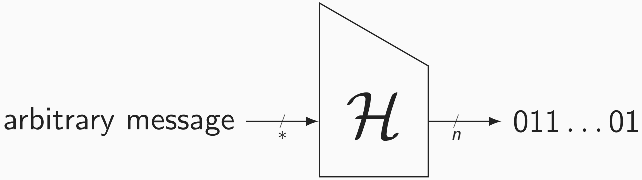
- These can be built from (a.o.):
  - Tweakable block ciphers
  - Block ciphers
  - Permutations
- There is one more core functionality of symmetric cryptography:

Cryptographic hashing

Outline

Hash Functions



- Function $\mathcal{H}$ from $\{0,1\}^*$ to $\{0,1\}^n$
  - No key input
  - Variable-length input
  - Classically fixed length output (but could be variable as well)

Example: Digital Signatures

- Suppose you want to sign a message M with a private key PrK:

$$\sigma = \mathrm{sign}(PrK, M)$$

- You can send (M, σ) to the receiver
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- This is called collision resistance of the hash function

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Many More Applications of Hash Functions

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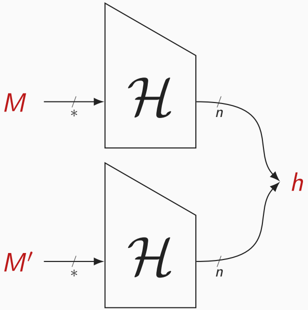
  - These are the classical security requirements
  - Focal point of first part of lecture
- Ideally, we want that a hash function behaves like a $\mathcal{RO}$
  - This is theoretically impossible
  - A security model that still solves this somewhat, is indifferentiability
  - Focal point of second part of lecture

Classical Security Requirements

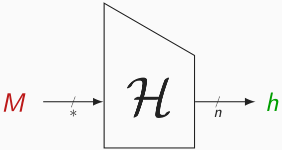
Collision



Find $M \neq M'$

Application:
2012 Flame virus

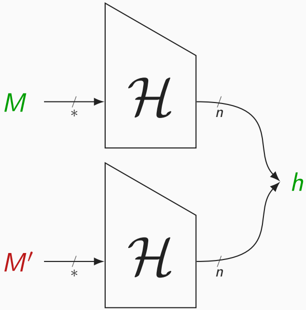
Preimage



Given h, find M

Application:
passphrase protection

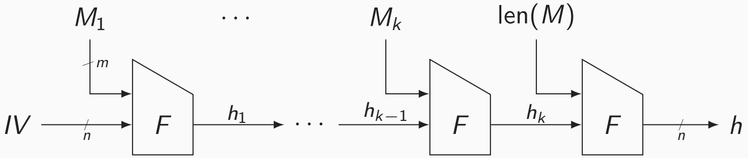
Second Preimage



Given M, find $M' \neq M$

Application:
data integrity

History



Merkle-Damgård with Strengthening

- Damgård [Dam89] and Merkle [Mer89]
- Consecutive evaluation of compression function F
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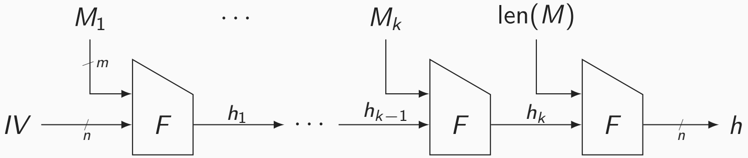

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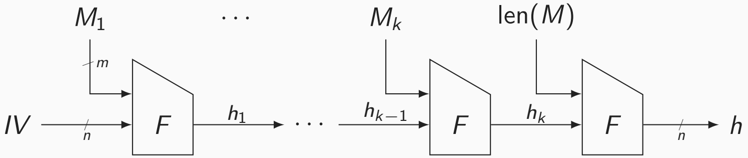

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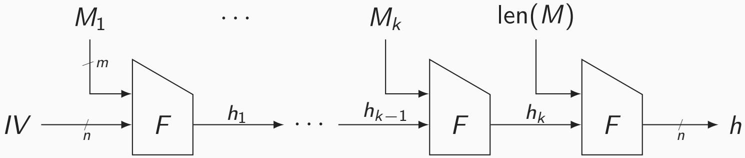
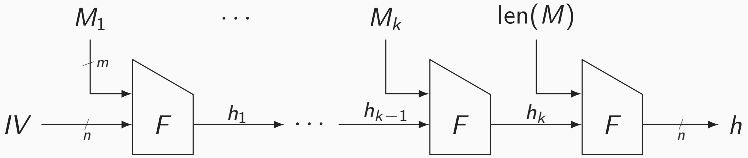
- Not a very good scheme, as we will see



Security of Merkle-Damgård

- $\mathcal{H}$ and F have same security models



Security of Merkle-Damgård

- $\mathcal{H}$ and F have same security models
- We happen to have (up to some degree):

$$F \text{ is col/sec/pre secure} \Longrightarrow \mathcal{H} \text{ is col/sec/pre secure}$$

MD5 and NIST Standards SHA-1/2

- MD5 [Rivest, 1991]
  - Based on MD4 that was an original design
  - 128-bit digest
- SHA-1 [NIST, 1995] (after SHA-0 [NIST, 1993])
  - Inspired by MD5, designed at NSA
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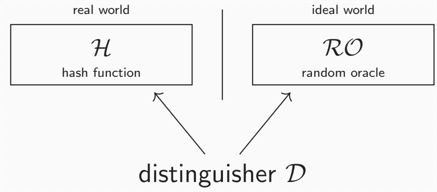
- Internally (for each of these):
  - Merkle-Damgård iteration mode
  - F based on a block cipher E in Davies-Meyer mode
  - Block cipher E: software oriented word-based design

Indifferentiability

Indistinguishability of Hash Functions (1/3)



- $\mathcal{H}$ should behave like random oracle $\mathcal{RO}$

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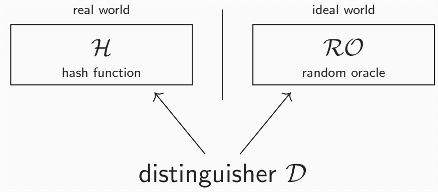

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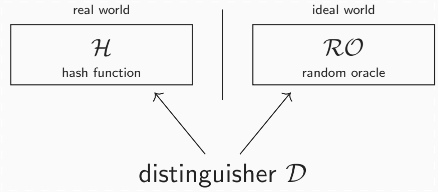

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- Solution: introduce randomness

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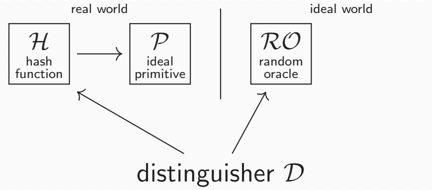

- $\mathcal{H}^{\mathcal{P}}$ for random primitive $\mathcal{P}$ should behave like random oracle $\mathcal{RO}$
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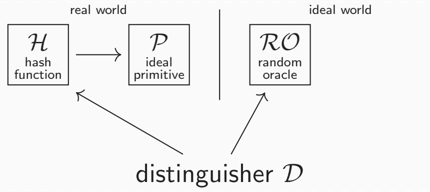

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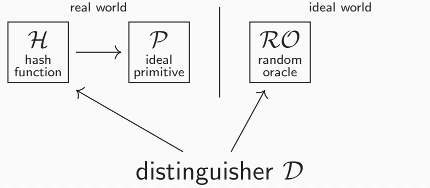

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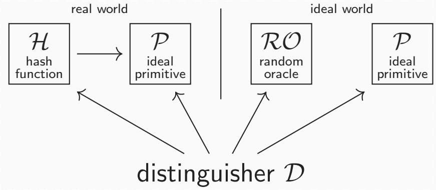
Indistinguishability of Hash Functions (3/3)



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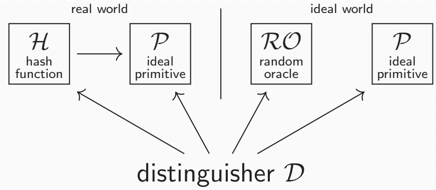

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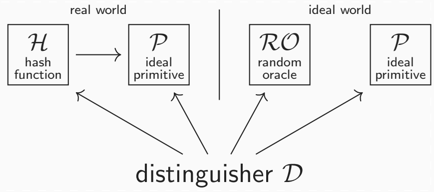

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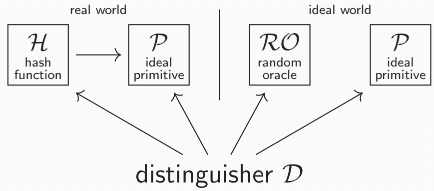

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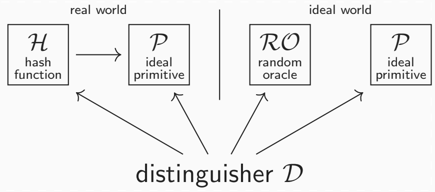

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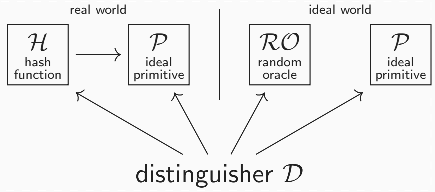

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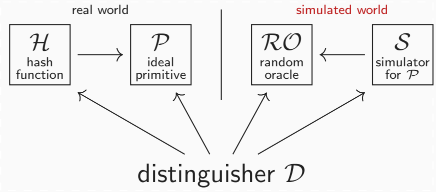
- Solution: indifferentiability

Indifferentiability (1/2)



- Maurer et al. [MRH04] and Coron et al. [CDMP05]
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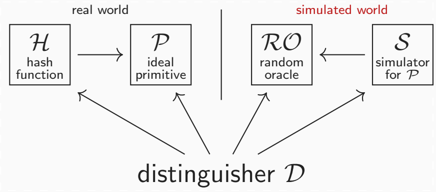

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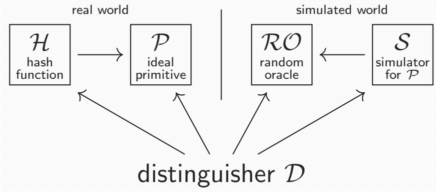
- Based on composition: distinguisher in one game is simulator in another one

Indifferentiability (2/2)



- $\mathcal{H}$ is indifferentiable from $\mathcal{RO}$ if for some simulator $\mathcal{S}$:

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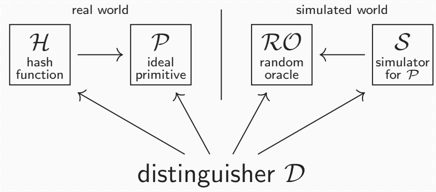

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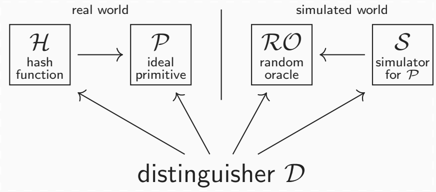

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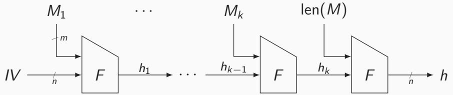
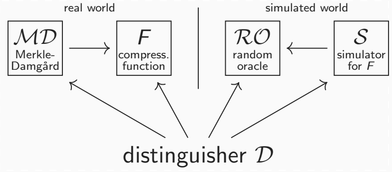
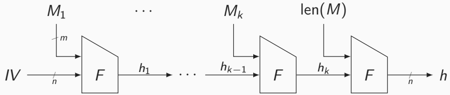
- Indifferentiability $\Longrightarrow$ coll/pre/sec security

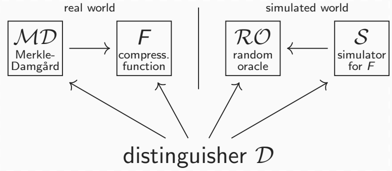




Merkle-Damgård is Easily Differentiable

- Goal is to prove that there exists a distinguisher that fools any simulator





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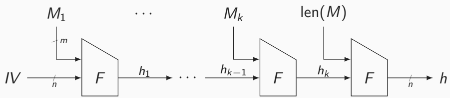

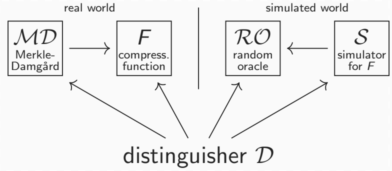

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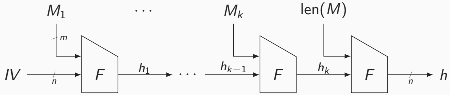
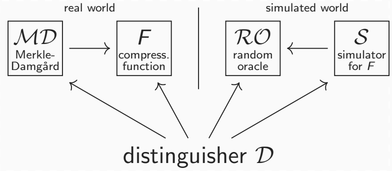
- Denote construction oracle by $\mathcal{H} \in \{\mathcal{MD}, \mathcal{RO}\}$ and primitive by $\mathcal{P} \in \{F, \mathcal{S}\}$





Merkle-Damgård is Easily Differentiable

- Distinguisher $\mathcal{D}$ operates as follows:
  - Pick arbitrary M_1
  - Query $\mathcal{H}(M_1) = h$, $\mathcal{H}(M_1\|\text{len}(M_1)) = h'$, and $\mathcal{P}(h, \text{len}(M_1\|\text{len}(M_1))) = y$
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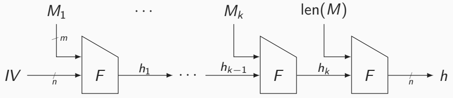
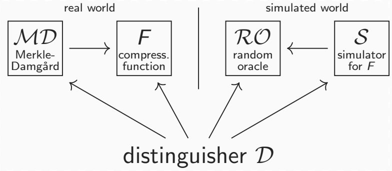
Differentiability of Merkle-Damgård (2/2)





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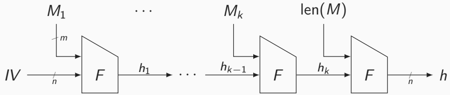

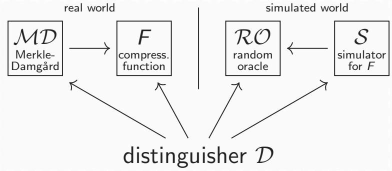

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- Simulated world: $\mathcal{S}$ must choose output y only based on knowledge of h and $\text{len}(M_1 \| \text{len}(M_1))$, but it cannot deduce M_1 from these values and it will likely fail

Sponges

A Bit of History (1/3)

- 2005-2006: MD5 and SHA-1 crisis
  - Actual collisions for MD5
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- 2008: NIST publishes SHA-3 requirements
  - *More efficient than SHA-2*
  - Output lengths: 224, 256, 384, 512 bits
  - Security: collision and (second) preimage resistance

A Bit of History (2/3)

- Competition started in 2008
- Three-round public process
  - round 1: 64 submissions, 51 accepted
  - round 2: 14 semi-finalists
  - round 3: 5 finalists
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- August 2015: NIST finally publishes the SHA-3 standard: FIPS 202

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- F must be collision resistant for $\mathcal{H}$ to be collision resistant
- But this means: we require F to be cryptographically strong
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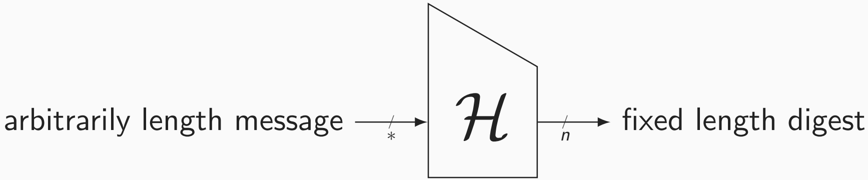
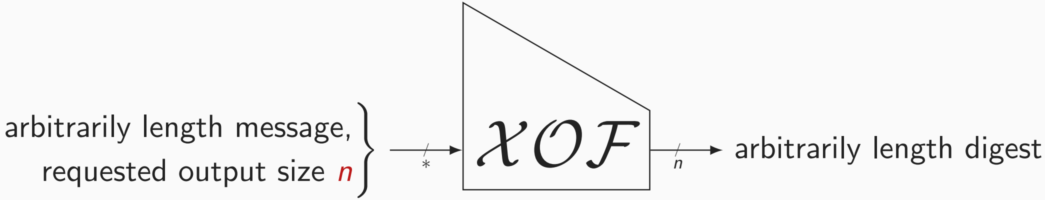
2. **Block Ciphers Have a Key Schedule and Data Path**
    - F is in turn often built from a block cipher (like Davies-Meyer)
    - While data paths are reasonably well-understood, key schedules not so much
    - In addition, final state of key schedule is discarded
    - Block cipher is weirdly compressing function from $n + k$ to n bits
    - Solution in sponge: use (iterative) permutation from b to b bits

Ancient Definition of Hashing



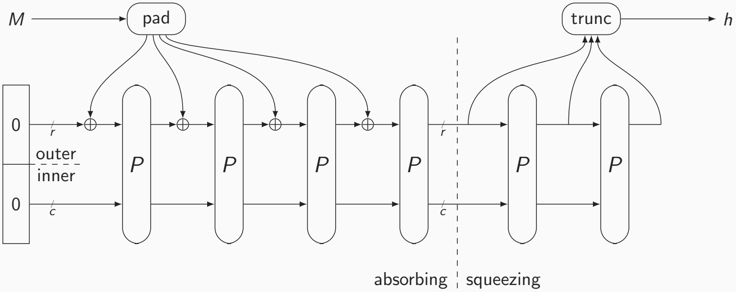
- Function $\mathcal{H}$ from $\{0,1\}^*$ to $\{0,1\}^n$
  - Variable-length input
  - Fixed-length output
  - Mode *on top of* $\mathcal{H}$ might give variable-length output

Modern Definition of Hashing



- Function $\mathcal{XOF}$ from $\{0,1\}^*$ to $\{0,1\}^\infty$
  - Variable-length input
  - Variable-length output
  - User specifies output length n when calling the function

Sponges [BDPV07]



- P is a b-bit permutation, with $b = r + c$
  - r is the rate
  - c is the capacity (security parameter)

Indifferentiability of the Sponge [BDPV08]

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Collision resistance:	$N^2/2^{c+1} + N^2/2^{n+1}$
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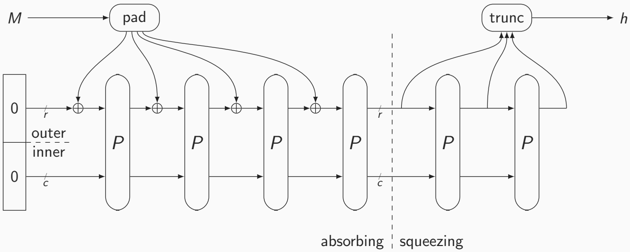
distance from sponge to RO
(N is # primitive evaluations)

classical attacks against RO
(N is # oracle evaluations)

Sponge Recap



- Relevant parameters:

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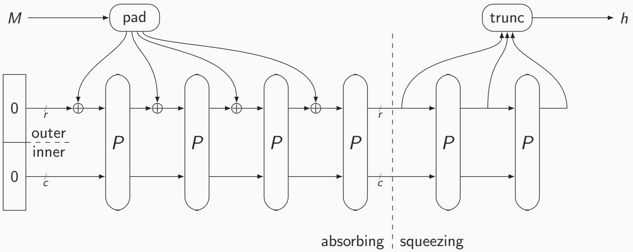

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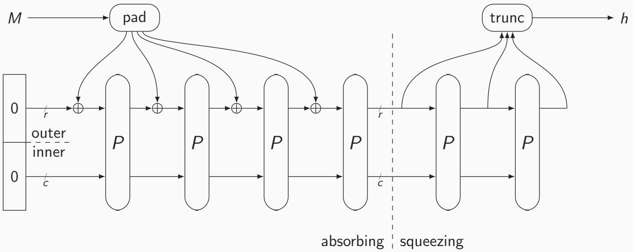

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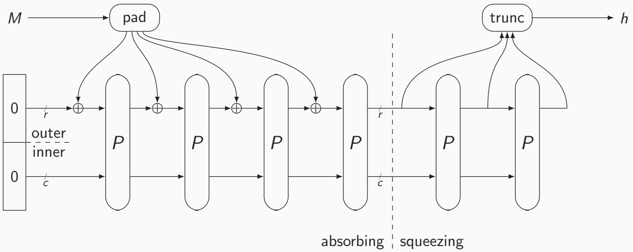

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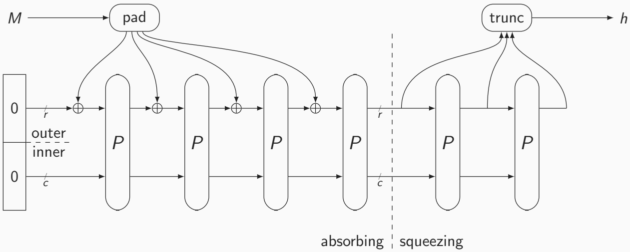

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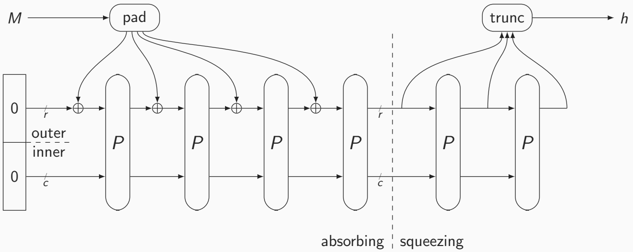

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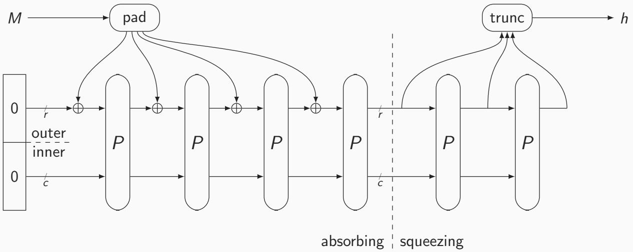

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Keccak and SHA-3

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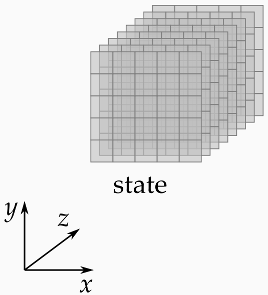
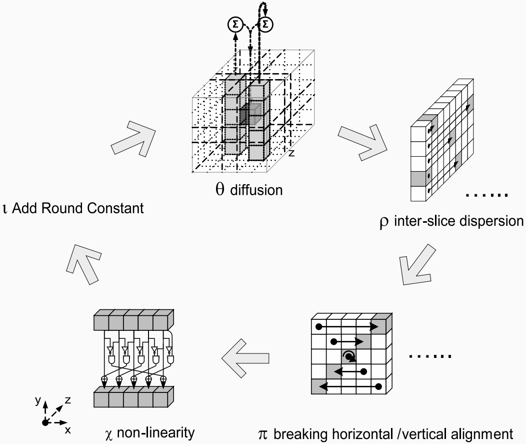
- Keccak-f operates on 3-dimensional state:
  - 5×5 *lanes*, each containing 2^ℓ bits (1, 2, 4, 8, 16, 32 or 64)
  - (5×5)-bit *slices*, 2^ℓ of them



Keccak-f: Steps of the Round Function



bit-oriented highly-symmetric *wide-trail* design

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- Security status:
  - Best attack on hash function covers 6-round version
  - $\#$ rounds ranges from 18 for $b = 200$ to 24 for $b = 1600$

Key Derivation Functions

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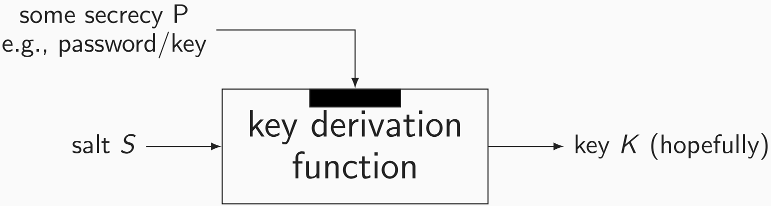

- Derive secret key from a password, passphrase, ...
- Key stretching, strengthening, ...
- Key diversification
- ...

How to Build Hash-Based PRF?

- Ideally, one does $\mathrm{PRF}(K, M) = \mathcal{H}(K \parallel M)$
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Intermezzo: HMAC Message Authentication Code

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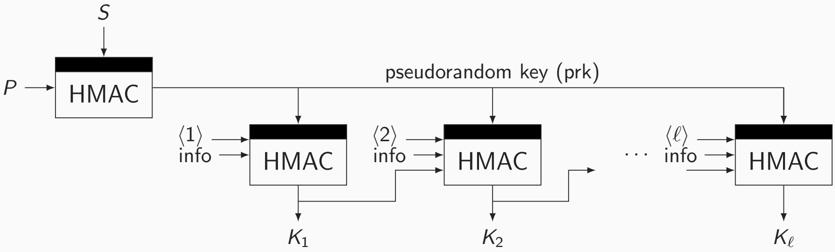
HMAC (Bellare et al. [BCK96])

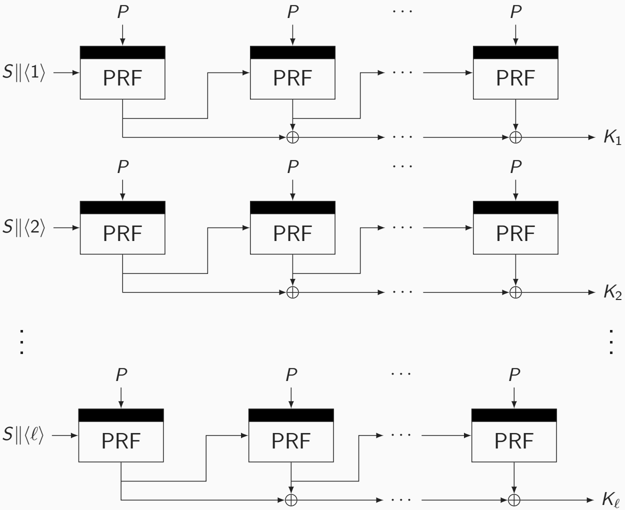
- Let opad be a constant string consisting of repetition of 0x5c
  ipad be a constant string consisting of repetition of 0x36
- $\mathrm{HMAC}(K \| M) = \mathcal{H}\Big(K \oplus \mathrm{opad} \parallel \mathcal{H}(K \oplus \mathrm{ipad} \parallel M)\Big)$
- Band-aid cryptography, not the most beautiful construction, but very popular!

HKDF Key Derivation Function



- RFC 5869 (2010)
- "info" is optional material, e.g., to bind application to use case

PBKDF2 Key Derivation Function



- RFC 2898 (2000)
- Standardized in PKCS #5 v2.0
- Popular PRF choices:
  - HMAC-SHA-1 (in WPA2)
  - HMAC-SHA-256, HMAC-SHA-512, HMAC-RIPEMD-160 (in VeraCrypt)

Conclusion

Next Week

- Sponge construction solved the problems that were present in Merkle-Damgård
- No band-aid-type cryptography (like HMAC) needed
  - $\mathrm{PRF}(K, M) = \mathrm{sponge}(K \parallel M)$ would have done the job
- Sponges can also be used for
  - Message authentication
  - Keystream generation
  - Authenticated encryption
  - ...
- This will be the topic of next week!