Radbound University

Commitment schemes, Identification Protocols and Fiat-Shamir Signatures

Applied Cryptography – Spring 2024

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Summary

Last time:

- Public key encryption
- Key Encapsulation Mechanisms
- Digital Signatures from trapdoor permutations

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- Public key encryption
- Key Encapsulation Mechanisms
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Today:

- Commitment schemes
- Zero-Knowledge protocols
- Identification Protocols
- Fiat-Shamir Signatures
- DSA and ECDSA

Commitment schemes

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- **Second attempt**:

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- A deadlock! Any ideas?

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$c_A = commit(b_A)$	$\xrightarrow{c_A\text{(commit)}}$	
	$\xleftarrow{b_B}$	$b_B \xleftarrow{\$} \{heads, tails\}$
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- When does this protocol work?
  - If Alice can find another b'_A such that $commit(b'_A) = commit(b_A)$, then **Alice can cheat!** (*commit* **should be binding!**)
  - If Bob can find b_A given $commit(b_A)$, then **Bob can cheat!** (*commit* **should be hiding!**)

Commitment scheme – formally

Given security parameter $\lambda \in \mathbb{N}$ and two finite sets $\mathcal{M}, \mathcal{R} \subseteq \{0,1\}^*$, a commitment scheme $\mathsf{Comm} = (\mathsf{Setup}, \mathsf{Comm}, \mathsf{Open})$ consists of three algorithms:

- **Setup** $\mathsf{pk} \leftarrow \mathsf{Setup}(1^\lambda)$
- **Commitment algorithm**: Takes random $r \in \mathcal{R}$, the message $M \in \mathcal{M}$ and outputs $(C, D) \leftarrow \mathsf{Comm}(\mathsf{pk}, M, r)$. C is said to be the commitment of M, and D is the opening (decommitment).
- **Opening (verification) algorithm**: Takes as input the commitment of M, the opening D, and outputs $M' = \mathsf{Open}(\mathsf{pk}, C, D) \in \mathcal{M}$ or $\perp \notin \mathcal{M}$ to indicate an invalid commitment.

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Correctness: For all $M \in \mathcal{M}$

$$\mathsf{Open}(\mathsf{pk}, (\mathsf{Comm}(\mathsf{pk}, M, r))) = M$$

Note:

- Often the opening simply contains the message M and the random coins r used in the commitment generation,
- Now, the verification consists of running again the Comm algorithm and checking whether it matches

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- **Unconditional/statistical hiding:** for every $M_0, M_1 \in \mathcal{M}$ the two distribution ensembles

$$\{C_0 | (C_0, D_0) \leftarrow \mathrm{Comm}(\mathrm{pk}, M_0, R_0)\} \quad \text{and} \quad \{C_1 | (C_1, D_1) \leftarrow \mathrm{Comm}(\mathrm{pk}, M_1, R_1)\}$$

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Hash based commitment:

$$\mathrm{Comm}(\mathrm{pk}, M, R) = H(R||M) \text{ where } R \xleftarrow{\$} \{0,1\}^{t\lambda}$$

- Computationally binding if H- collision resistant
- Computationally hiding if H- preimage resistant

  In the ROM:
- If message length is bounded - statistical binding can be achieved
- Statistical hiding can be achieved for $t \geqslant 5$

Commitment scheme – examples

Pedersen bit-commitment:

$$\mathrm{Comm}(\mathrm{pk}, B, R) = g^R \cdot h^B \text{ where } R \xleftarrow{\$} \mathbb{Z}_n \text{ and } h \in \langle g \rangle, |\langle g \rangle| = n$$

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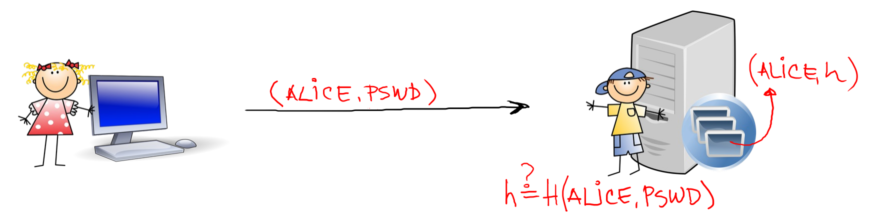
  - $\Rightarrow$ DL broken!
- Perfectly hiding - g^R and $g^R h$ are perfectly indistinguishable

Zero Knowledge Protocols

In practice ...

- Most common type of authentication - using passwords
- Typical flow:



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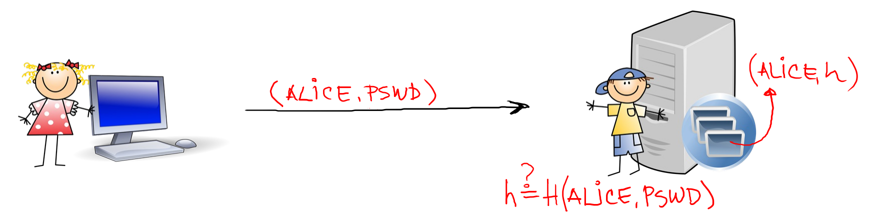

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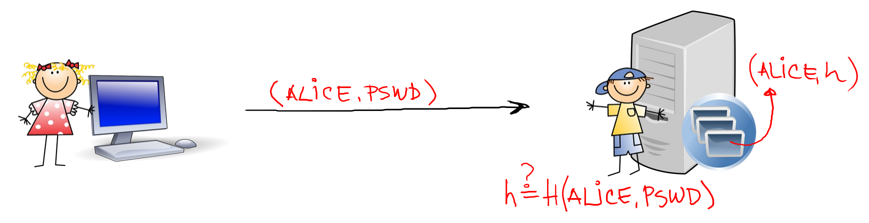

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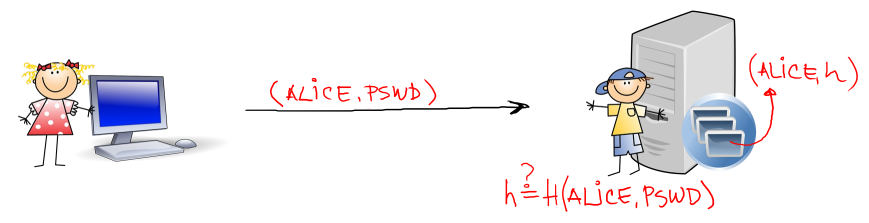

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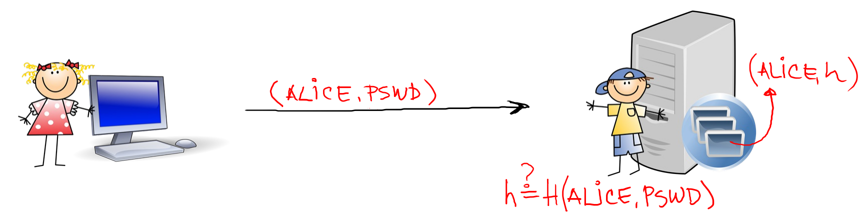

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- Solution?

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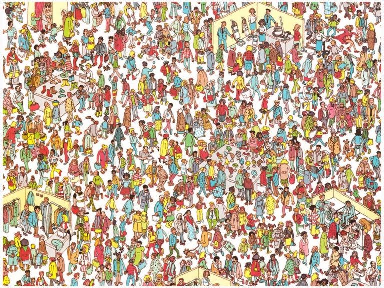
    - Ideally, not at all - **Zero-Knowledge (ZK) property**



Interactive proof/argument systems

Let L be a language. The pair $(\mathcal{P}, \mathcal{V})$ is an interactive proof system for L (proving membership of statement) if it satisfies the following two conditions:

- Completeness: If $x \in L$, then the probability that $(\mathcal{P}, \mathcal{V})$ rejects x is negligible in the length of x.
- Soundness: If $x \notin L$ then for any prover $\mathcal{P}^*$, the probability that $(\mathcal{P}^*, \mathcal{V})$ accepts x is negligible in the length of x. This probability is called **soundness error**.

Some terminology:

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- **Transcript** - a record of the entire conversation between the parties
- The verifier is always polynomial time bounded
- The prover may be unbounded (proof systems) or polynomial time bounded (argument systems)
- In practice we will be always interested in **arguments**

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An interactive proof system or argument $(\mathcal{P}, \mathcal{V})$ for language L is **zero-knowledge** if for every PPT verifier $\mathcal{V}^*$, there is a simulator $\mathcal{S}_{\mathcal{V}^*}$ running in expected probabilistic polynomial time, such that:
For $x \in L$ the distribution of transcripts output by $\mathcal{S}_{\mathcal{V}^*}$ on input x is

- perfectly (perfect ZK)
- statistically (statistical ZK)
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- the simulator can generate messages of a transcript **in any order it wants**

An example walkthrough - Schnorr protocol

- Given p-prime, $q|p-1$, $g \in \mathbb{Z}_p^*$ of order q, $w \in \mathbb{Z}_q$ and $v = g^w \pmod{p}$.
- The prover $\mathcal{P}$ wants to prove to the verifier $\mathcal{V}$ knowledge of w without revealing any information about it

$$\overset{v}{\underset{w}{\dashrightarrow}}$$

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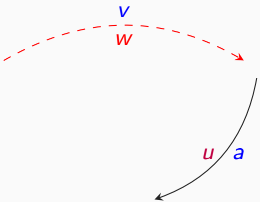

- Inside of circle - addition
- Outside of circle - multiplication

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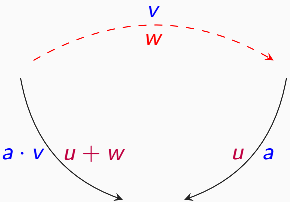

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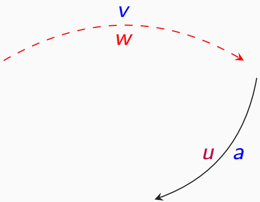

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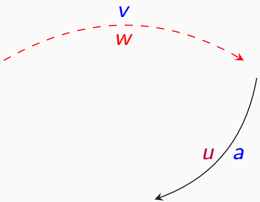

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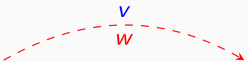

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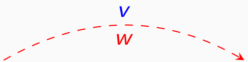

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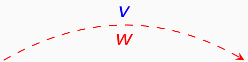

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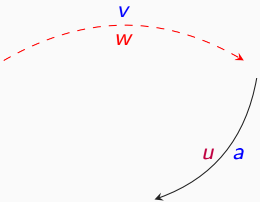

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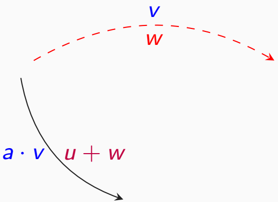

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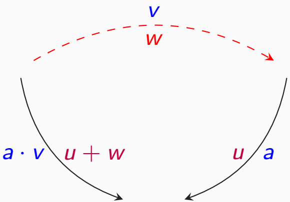

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- **Rewinding of adversaries/cheating parties**: We always consider these to be probabilistic Turing machines, so rewinding is possible, until a desired output is produced
- **$\mathcal{S}$ is expected probabilistic polynomial time:** One round is expected to be repeated 2 times, hence whole simulation takes expected 2λ time.
- **Distributions of simulated and real protocol are exactly the same**:

- **Zero-Knowledgeness:** We will show the protocol is zero-knowledge. We build a simulator $\mathcal{S}$ (no knowledge of the secret w, and is polynomially bounded) as follows:
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Homework - ZK for Graph Isomorphism (GI)

- Let ϕ be an isomorphism between two graphs $\mathcal{G}_0$ and $\mathcal{G}_1$ s.t. $\mathcal{G}_1 = \phi(\mathcal{G}_0)$.
- Given $\mathcal{G}_0, \mathcal{G}_1$, the prover $\mathcal{P}$ wants to prove to the verifier $\mathcal{V}$ knowledge of ϕ without revealing any information about it

$$\begin{array}{c} \mathcal{G}_0 \\ \phi \big\downarrow \\ \mathcal{G}_1 \end{array}$$

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Note:

- No probabilistic polynomial-time algorithms are known for the GI problem
- This generalizes to other isomorphisms on different objects
- **Homework: Show in detail that the above protocol is Zero-Knowledge!**

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- Hence, for protecting confidentiality, privacy, anonymity
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- In Blockchain technologies
  - for privacy and anonymity

Sigma protocols

Σ protocols

$$
\begin{array}{lcl}
\mathcal{P}, \mathsf{sk} & & \mathcal{V}, \mathsf{pk} \\
\hline
\mathsf{com} \xleftarrow{\$} \mathcal{P}_0(\mathsf{sk}) & & \\
 & \xrightarrow{\quad \mathsf{com} \quad} & \\
 & & \mathsf{ch} \xleftarrow{\$} \mathsf{ChS}(1^k) \\
 & \xleftarrow{\quad \mathsf{ch} \quad} & \\
\mathsf{resp} \leftarrow \mathcal{P}_1(\mathsf{sk}, \mathsf{com}, \mathsf{ch}) & & \\
 & \xrightarrow{\quad \mathsf{resp} \quad} & \\
 & & b \leftarrow \mathsf{Vf}(\mathsf{pk}, \mathsf{com}, \mathsf{ch}, \mathsf{resp})
\end{array}
$$

Given a relation $R = V \times W$, where sk $\in W$ is called **witness** and language $L_R = \{v \in V | \exists w \in W, (v, w) \in R\}$ for the relation, a Σ protocol is a three move protocol as above satisfying:

- **Completeness**: (as before)
- **Special soundness**: There exists a PPT algorithm $\mathcal{K}$ - knowledge extractor, that given two valid transcripts $\mathsf{trans} = (\mathsf{com}, \mathsf{ch}, \mathsf{resp})$, $\mathsf{trans}' = (\mathsf{com}, \mathsf{ch}', \mathsf{resp}')$, $\mathsf{ch} \neq ch'$, extracts the witness sk with non-negligible probability
- **Special Honest Verifier ZK**: Same as before, except, the verifier is honest (follows the protocol), and the simulator $\mathcal{S}$ needs to output a valid transcript for a given challenge ch.

Recall Schnorr's protocol

p-prime, $q|p-1$, $g \in \mathbb{Z}_p^*$ of order q. $w \in \mathbb{Z}_q$ and $v = g^w \pmod p$

$\mathcal{P}, \mathsf{sk} = w$		$\mathcal{V}, \mathsf{pk} = v$
$u \xleftarrow{\$} \mathbb{Z}_q, a \leftarrow g^u$	$\xrightarrow{\quad a \quad}$	
	$\xleftarrow{\quad \mathsf{ch} \quad}$	$\mathsf{ch} \xleftarrow{\$} \{0,1\}$
$\mathsf{resp} \leftarrow u + \mathsf{ch} \cdot w$	$\xrightarrow{\quad \mathsf{resp} \quad}$	
		$g^{\mathsf{resp}} \stackrel{?}{=} a \cdot v^{\mathsf{ch}}$

- **Soundness error** - $1/2$, so needs many rounds to achieve negligible soundness error

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- Each round performs expensive exponentiations in a group of order q
- Can we do better?

Schnorr identification protocol

A more efficient **single round variant** - **Schnorr identification protocol**

$$\begin{array}{lcl} \mathcal{P}, \mathrm{sk} = w & & \mathcal{V}, \mathrm{pk} = v \\ \hline u \xleftarrow{\$} \mathbb{Z}_q, a \leftarrow g^u & & \\ & \xrightarrow{\quad a \quad} & \\ & & \mathrm{ch} \xleftarrow{\$} \mathbb{Z}_q \\ & \xleftarrow{\quad \mathrm{ch} \quad} & \\ \mathrm{resp} \leftarrow u + \mathrm{ch} \cdot w & & \\ & \xrightarrow{\quad \mathrm{resp} \quad} & \\ & & g^{\mathrm{resp}} \stackrel{?}{=} a \cdot v^{\mathrm{ch}} \end{array}$$

- We show Schnorr identification protocol is Σ-protocol
- **Completeness**: Prover that knows w always succeeds

An example walkthrough - Schnorr protocol

- **Special soundness**: Given two accepting transcripts for the same commitment $\mathrm{trans} = (\mathrm{com}, \mathrm{ch}, \mathrm{resp}) = (a, \mathrm{ch}, u + \mathrm{ch} \cdot w)$, and $\mathrm{trans}' = (\mathrm{com}, \mathrm{ch}', \mathrm{resp}') = (a, \mathrm{ch}', u + \mathrm{ch}' \cdot w)$, we have

$$w = \frac{\mathrm{resp} - \mathrm{resp}'}{(\mathrm{ch} - \mathrm{ch}')} \qquad \Rightarrow \text{ the witness can be extracted with probability 1.}$$

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  - The distributions of the real transcripts and the simulated transcripts are the same – in both a given valid transcript occurs with prob. $1/q$ (in one first u is chosen a random, in the other first resp is chosen at random.)

Fiat-Shamir signatures

The Fiat-Shamir transform

Σ-protocol

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$\downarrow \quad \downarrow \quad \downarrow$

FS signature

Signer

$\mathsf{com} \leftarrow \mathcal{P}_0{}^{r}(\mathsf{sk})$

$\mathsf{ch} \leftarrow H(m, \mathsf{com})$

$\mathsf{resp} \leftarrow \mathcal{P}_1{}^{r}(\mathsf{sk}, \mathsf{com}, \mathsf{ch})$

output : $\sigma = (\mathsf{com}, \mathsf{resp})$

Verifier

$\mathsf{ch} \leftarrow H(m, \mathsf{com})$

$b \leftarrow \mathsf{Vf}^{r}(\mathsf{pk}, \mathsf{com}, \mathsf{ch}, \mathsf{resp})$

output : b

The Fiat-Shamir transform

Let $\lambda \in \mathbb{N}$ the security parameter, $\mathsf{IDS} = (\mathsf{KGen}, \mathcal{P}, \mathcal{V})$ an identification scheme that is a Σ-protocol (special sound with knowledge error κ and HVZK).

Let $H : \{0,1\}^* \to \{0,1\}^r$ be modelled as a random oracle. The **Fiat-Shamir signature scheme** derived from IDS is the triplet of algorithms $(\mathsf{KGen}, \mathsf{Sign}, \mathsf{Vf})$ s.t.:

- $(\mathsf{sk}, \mathsf{pk}) \leftarrow \mathsf{KGen}(1^k)$,
- $\sigma = (\mathsf{com}, \mathsf{resp}) \leftarrow \mathsf{Sign}(\mathsf{sk}, m)$ where $\mathsf{com} \leftarrow \mathcal{P}_0^r(\mathsf{sk})$, $h = H(m, \mathsf{com})$, $\mathsf{resp} \leftarrow \mathcal{P}_1^r(\mathsf{sk}, \mathsf{com}, h)$.
- $\mathsf{Vf}(\mathsf{pk}, m, \sigma)$ parses $\sigma = (\mathsf{com}, \mathsf{resp})$, computes $h = H(m, \mathsf{com})$, and outputs $\mathcal{V}^r(\mathsf{pk}, \mathsf{com}, h, \mathsf{resp})$.

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- The number of rounds r is chosen such that $\kappa^r < \frac{1}{2^\lambda}$.
- Note that the "full" signature is $(\mathsf{com}, h, \mathsf{resp})$, but h can be omitted, since it can be recreated from m and com.

Structure of proof:

IDS		**FS signature**
Special soundness	$\longrightarrow$	Secure under key-only attack
+		
HVZK	$\longrightarrow$	**eu-cma secure**

Security of FS signatures [Pointcheval & Stern '96]

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Proof in ROM (see additional literature if interested!)

Schnorr signature

KeyGen:

1. Choose two primes p, q s.t. $q | p - 1$, and $g \in \mathbb{Z}_p^*$ of order q.
2. Choose a random $w \in \mathbb{Z}_q$ and compute $v = g^w \pmod p$
3. Output public key $\mathsf{pk} = v$ and private key $\mathsf{sk} = w$

Sign: Given message M,

1. $u \xleftarrow{\$} \mathbb{Z}_q, \mathsf{com} \leftarrow g^u$,
2. Set $\mathsf{ch} = H(M, \mathsf{com})$ and calculate $\mathsf{resp} \leftarrow u + \mathsf{ch} \cdot w$
3. Set $\sigma = (\mathsf{com}, \mathsf{resp})$
4. Output message - signature pair (M, σ)

Verify: To verify the message - signature pair (M, σ)

1. Parse $\sigma = (\mathsf{com}, \mathsf{resp})$ and calculate $\mathsf{ch} = H(M, \mathsf{com})$
2. Check $g^{\mathsf{resp}} \stackrel{?}{=} a \cdot v^{\mathsf{ch}}$ and output Accept if check succeeds, otherwise Fail

Schnorr signature and Digital Signature Algorithm (DSA)

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Digital Signature Algorithm (DSA)

KeyGen: Same as for Schnorr with private key $x \in \mathbb{Z}_q$ and public key $y = g^x \pmod{p}$

1. Choose two primes p, q s.t. $q | p - 1$, and $g \in \mathbb{Z}_p^*$ of order q.
2. Choose a random $x \in \mathbb{Z}_q$ and compute $y = g^w \pmod{p}$
3. Output public key $\mathsf{pk} = y$ and private key $\mathsf{sk} = x$

Sign: Given message M,

1. $k \xleftarrow{\$} \mathbb{Z}_q, r \leftarrow (g^k \pmod{p}) \pmod{q}$,
2. Set $h = H(M)$ and calculate $s \leftarrow (h + xr)k^{-1}$ (In Schnorr: $h = H(M, r)$ and $s \leftarrow k + h \cdot x$)
3. Set $\sigma = (r, s)$ and output message - signature pair (M, σ)

Verify: To verify the message - signature pair (M, σ)

1. Parse $\sigma = (r, s)$, verify $0 < r, s < q$ and calculate $h = H(M)$
2. Compute $u_1 = H(m)s^{-1} \pmod{q}$ and $u_2 = rs^{-1} \pmod{q}$
3. Check $(g^{u_1} g^{u_2} \pmod{p}) \pmod{q} \stackrel{?}{=} r$ and output Accept if check succeeds, otherwise Fail

Sensitive security of DSA and ECDSA

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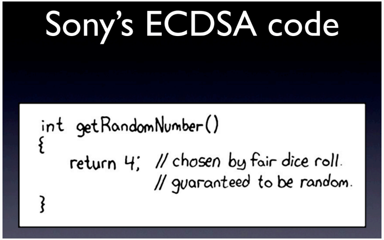
- In December 2010, a group calling itself fail0verflow announced recovery of ECDSA private key used by Sony to sign software for PlayStation 3
    - Sony's "epic fail": *k was static instead of random!*

- Suppose $s_1 = (H(M_1) + xr)k^{-1}$ and $s_2 \leftarrow (H(M_1) + xr)k^{-1}$ use same k for two messages M_1 and M_2
- Now: $s_1 - s_2 = (H(M_1) - H(M_2))k^{-1}$
- I.e., $k = (H(M_1) - H(M_2))(s_1 - s_2)^{-1}$
- Once k is known, the secret key can be easily derived: $x = (s_1 k - H(M_1))(r)^{-1}$



Recommendations for key sizes and usage by NIST, ECRYPT

Algorithm	Param.	Key length	Classical security	Usage
RSA-1024	N	1024	80 bits	disallowed for key transport
RSA-1024/DSA-1024	N/q	1024	80 bits	disallowed signature key gen./legacy signature verification
RSA-2048/DSA-2048	N/q	2048	112 bits	acceptable
RSA-3072/DSA-3072	N/q	3072	128 bits	recommended
RSA-7680/DSA-7680	N/q	7680	192 bits	long term
RSA-15360/DLP-15360	N/q	15360	256 bits	long term
ECDSA-160	p	80	80 bits	disallowed signature key gen./legacy signature verification
ECDSA-224	p	224	112 bits	acceptable
ECDSA-256	p	256	128 bits	recommended
ECDSA-384	p	384	192 bits	long term
ECDSA-512	p	512	256 bits	long term

Summary

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- Commitments
- Zero-Knowledge protocols
- Identification Protocols
- Fiat-Shamir Signatures
- DSA and ECDSA

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Next time:

- Post-Quantum Cryptography
- Hash-Based signatures