Radbound University

Recap of Introduction to Cryptography

Applied Cryptography – Spring 2024

Bart Mennink

January 29, 2024

Institute for Computing and Information Sciences
Radboud University

Outline

Course Organization

Security (NWI-IPC021)	$\longrightarrow$	Intro to Cryptography (NWI-IBC023)	$\longrightarrow$	**Applied Cryptography (NWI-IMC061)**	$\longrightarrow$	Cryptology (NWI-IMC063)

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Goal of the Course

- Learn what cryptography is used in applied settings
  - What is used in the real world
  - What is standardized
  - What will (?) be used in the future
- Prepare you for cryptographic aspects you might see later in your career

Feedback Welcome!

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- If you have feedback on the course, please contact the lecturers!

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- Course reasonably well-graded
- Some start-up problems identified by students and lecturers

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  - Further refinement with "Introduction to Cryptography" and "Cryptology"
  - More explanation on how cryptographic functions are used in practice
  - Further overall improvement of applications
- Tutorials/Assignments:
  - Make the assignments clearer
  - Less work-intensive assignments

Who?

Lecturers

- Bart Mennink, M1 3.05, `b.mennink@cs.ru.nl`
- Simona Samardjiska, M1 03.18, `simonas@cs.ru.nl`

Assignment Coordinators

- Mario Marhuenda Beltrán, M1 03.17, `mario.marhuendabeltran@ru.nl`

Tutorial Assistant

- Maximilian Pohl, `maximilian.pohl@ru.nl`

Lectures

- Weekly: Mon 13.30–15.15 in HG00.514
  - 5 lectures on symmetric cryptography (Bart Mennink)
  - 5 lectures on public-key/post-quantum cryptography (Simona Samardjiska)
  - 2–2.5 lectures on selected topics (guest lectures)
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- Weekly: Wed 10.30–12.15 in E1.17/EOSN01.560
  - 3 assignments on symmetric cryptography (after lectures 2, 3, 5)
  - 3 assignments on pk/pq cryptography (after lectures 7, 8, 11)
  - 1 assignment on selected topics (after lecture 12)

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  - New assignments on the web by **Monday evening**
  - Two tutorials for asking questions
  - Hand-in: **Sunday after second tutorial, before 23.59** via Brightspace
    - In LaTeX, as single pdf
    - Hint: you are allowed to hand in earlier!
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  - General rule: too late means score 0, no exceptions
- Assignment gives up to 1 point (out of 10) bonus on exam
- Assignments can be handed in in pairs (strongly encouraged)

Organization

Assessment

- Final mark is computed from:
  - Average of markings of assignments: A
  - Open-book on-campus exam: E
  - Final mark: $F = E + \frac{A}{10}$
- To pass: $E \geq 5$ and $F \geq 6$

Further Information

- All information on the course appears on Brightspace
- Read the course manual!

Keyed Symmetric Cryptography

General Setting

- Two parties, Alice and Bob, communicate over a public channel
  - They have agreed on a joint key and use it to transmit data

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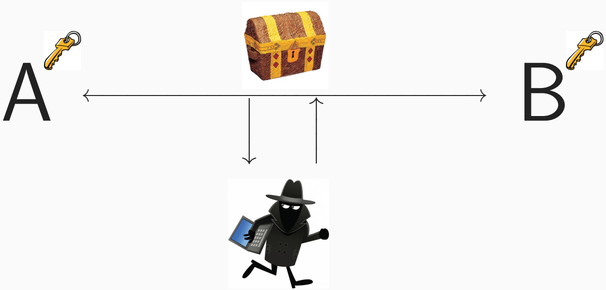

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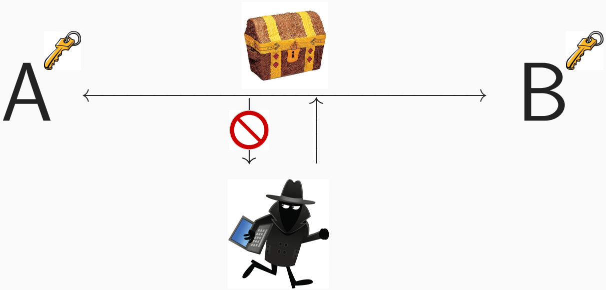

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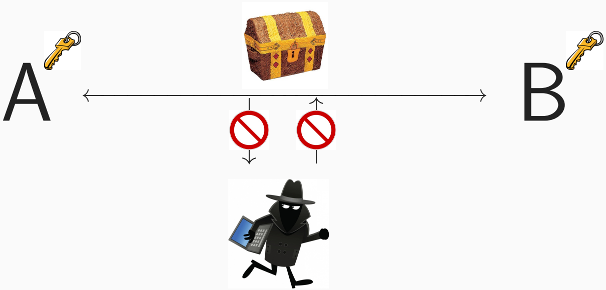

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- In symmetric cryptography, we are concerned with two main security properties:
  - **Confidentiality (or data privacy):** Eve cannot learn anything about data
  - **Authenticity:** Eve cannot manipulate the data

Core Functionalities

Encryption

- Uses key to transform data into ciphertext
- Only with the key, one can retrieve data back

Message authentication

- Uses key to complement data with a tag
- Only with the key, the tag can be verified

Authenticated encryption

- Combines encryption and authentication
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These (together with **hashing**) are the core functionalities in symmetric cryptography!

Core Functionalities

- Symmetric stands for:
  - same key for encryption and decryption
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  - This is a problem on its own!

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- Refinements often in:
  - data complexity: amount of observed data (limited by use case)
  - computation complexity: amount of computation (limited by budget)

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1

Length of the key

What Determines Security?

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2

$$K \longrightarrow \boxed{\text{primitive}} \longrightarrow Z$$

Security of the function
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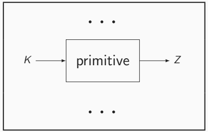

Method in which that
function is employed

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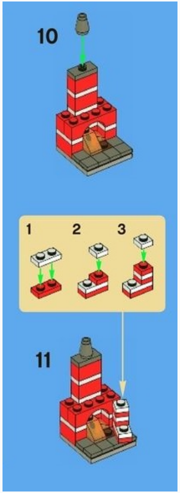

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- Primitives:
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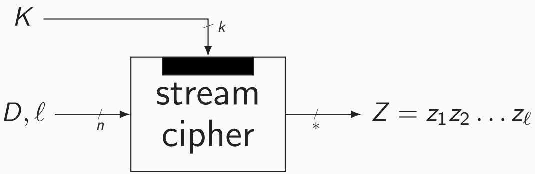
- Constructions:
  - Often come with a formal security proof
  - No unconditional security: based on assumption on the underlying primitive
  - Reductionist proof: breaking construction implies breaking primitive
  - Ideal model proof: assuming primitive is ideal, construction is secure

How to Model Security?

Modern Stream Ciphers



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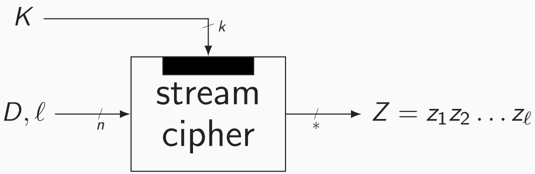

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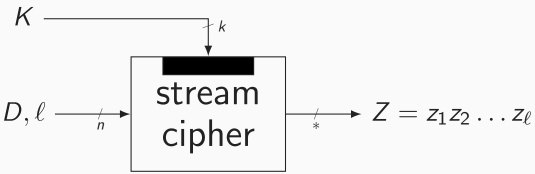

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- Example: data streams, e.g., pay TV and telephone, often split data in relatively short, numbered, frames. One can use frame number as diversifier and encrypt using stream:

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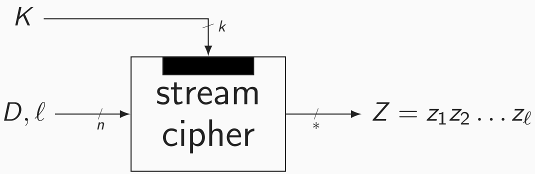

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When is a stream cipher strong enough?

Stream Cipher Security, Intuition (1/5)

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stream cipher

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  - $\ldots$

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  - $\ldots$
  - Intuitively, SC_K should not expose any irregularities
  - Its outputs should look completely random

Random Oracle

- A database of input-output tuples
- Initially empty

D	Z
$\ldots$	$\ldots$
$\ldots$	$\ldots$
$\ldots$	$\ldots$
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Intermezzo: Random Oracle

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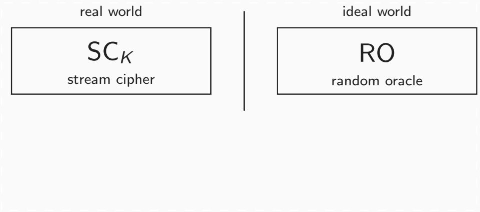
    - update (D, Z) in the list

Stream Cipher Security, Intuition (2/5)



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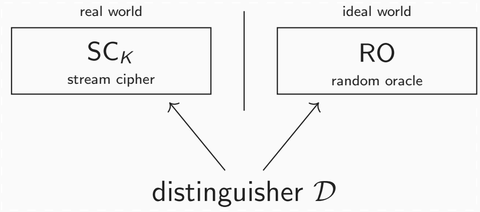

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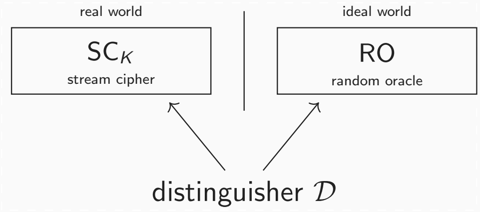
- We model a distinguisher $\mathcal{D}$ that is given oracle access to either of the worlds



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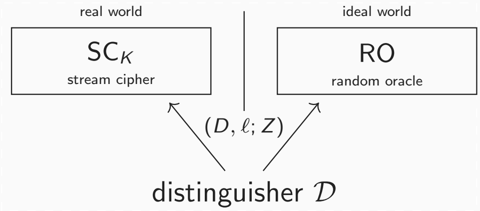
Stream Cipher Security, Intuition (2/5)



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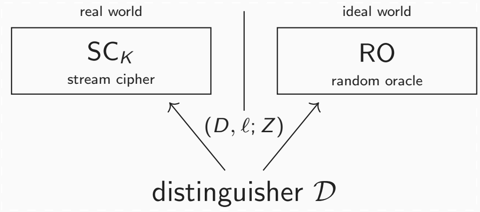

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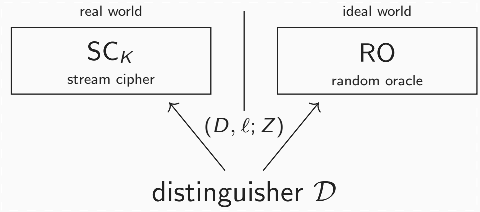
  - At the end, $\mathcal{D}$ has to guess the outcome of the coin toss (head/tail)

Stream Cipher Security, Intuition (3/5)



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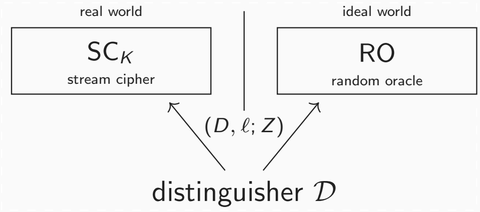

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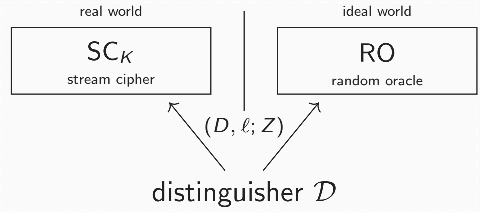

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- This turns out to be equal to (see Section 4.4 of "Intro2Crypto-symmetric.pdf")

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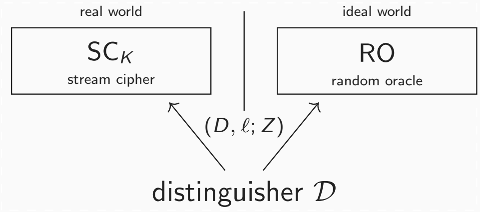
Stream Cipher Security, Intuition (4/5)



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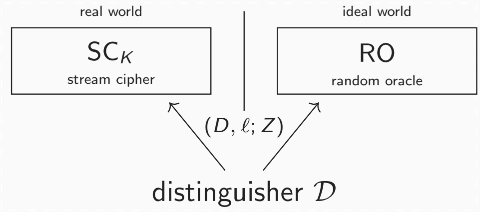

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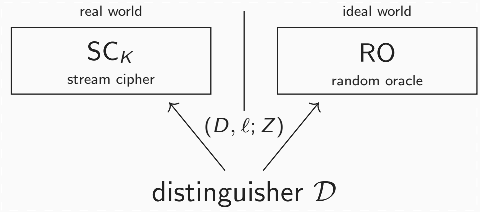

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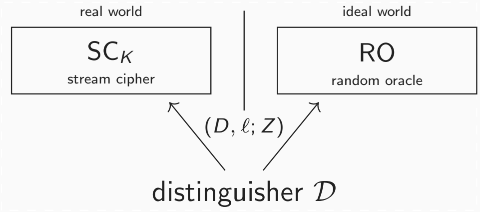

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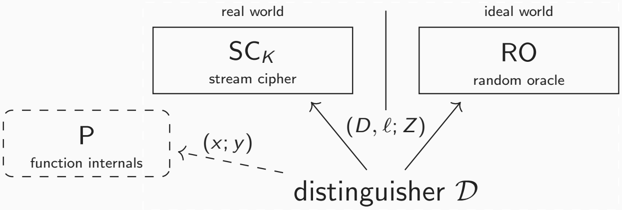

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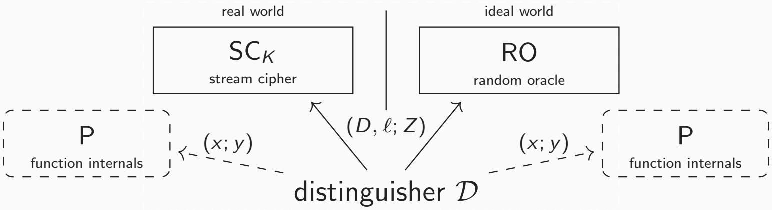

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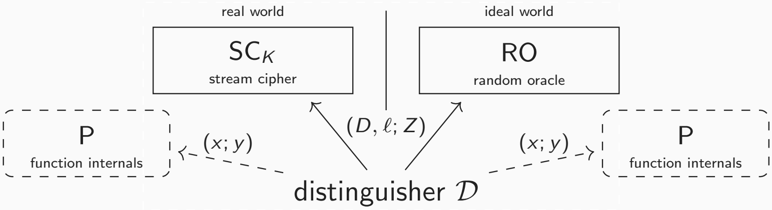

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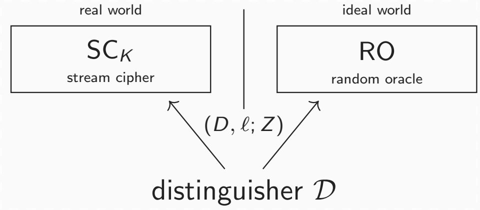

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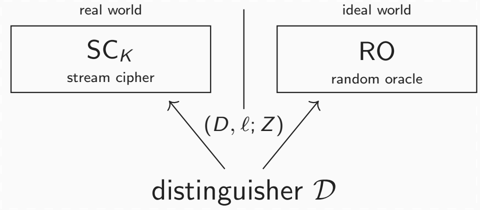

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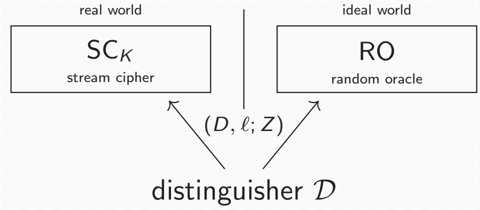

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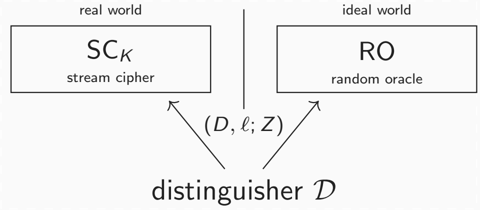

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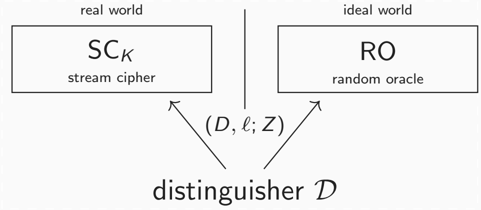

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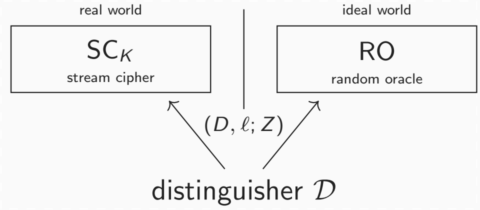

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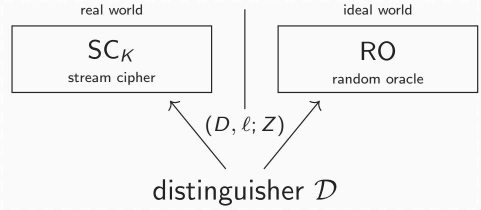

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- $\mathbf{Adv}_{\mathsf{SC}}^{\mathrm{prf}}(q, t)$: supremal advantage over any distinguisher with complexity q, t
  - More complexity parameters may apply, e.g., total length, different complexity bounds for different oracles, ...
  - In addition, t is sometimes left implicit if not needed for a security proof

Stream Cipher Security, Implication

- A bound $\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{SC}}(q, t)$ implies that
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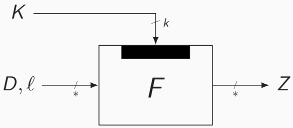
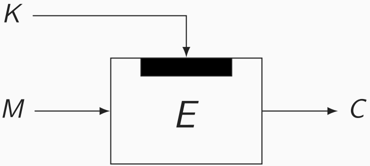
- Security definition of pseudorandom functions (PRF) is in fact more general: it applies to functions with possibly arbitrary length inputs and outputs



Block Ciphers



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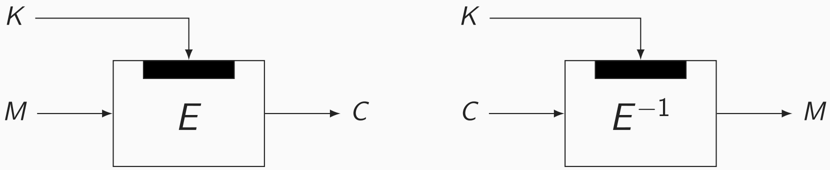

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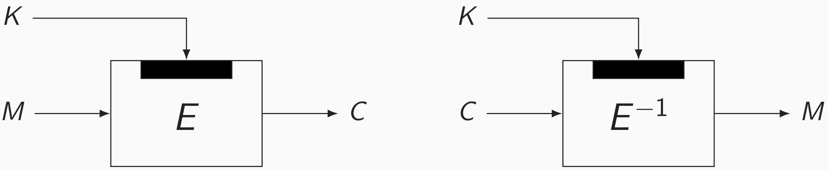

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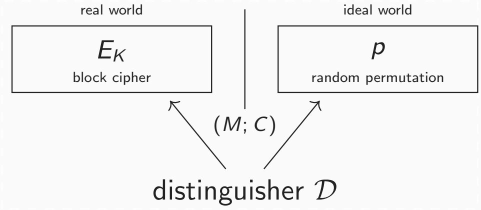

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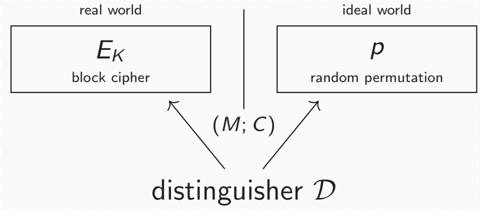

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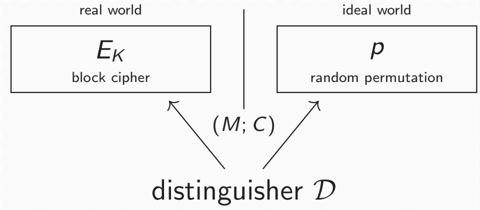

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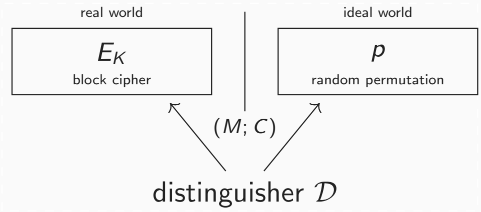

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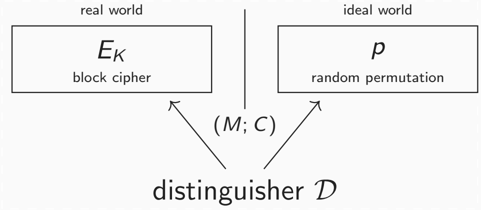

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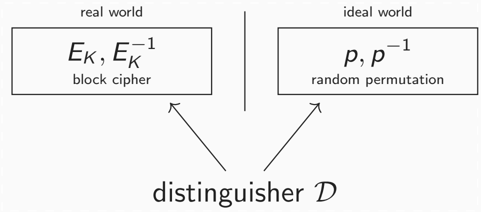

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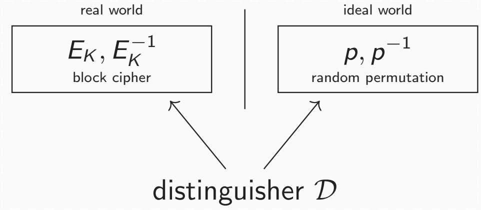
Strong Block Cipher Security



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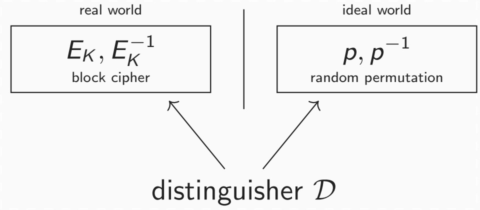

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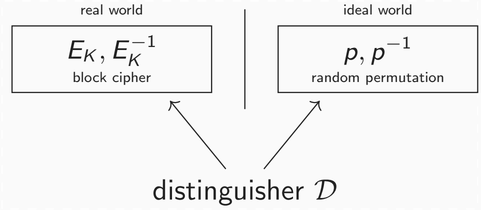

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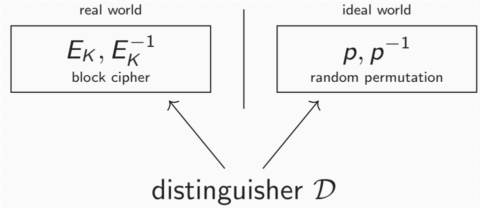

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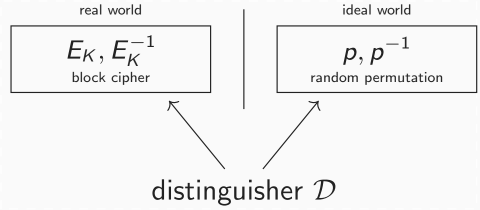
Block Cipher Security: How to Model Key Recovery?



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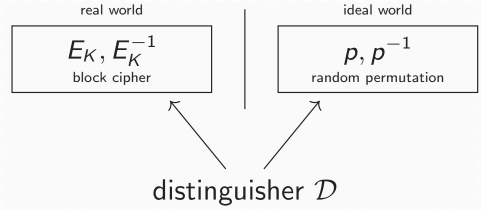

- Suppose $\mathcal{D}$ has $q \geq 1$ query and t time
- It can mount the following attack:
  - Make 1 construction query $(0; \mathcal{O}(0))$
  - Make t offline key attempts $E_{L_i}(0)$
  - If $E_{L_i}(0) = \mathcal{O}(0)$ for some i, key recovery very likely
- For this distinguisher (simplified, ignoring false positives): $\mathbf{Adv}_E^{\mathrm{sprp}}(\mathcal{D}) \approx t/2^k$
- Supremized: $\mathbf{Adv}_E^{\mathrm{sprp}}(q, t) \geq t/2^k$

AES

- Block cipher with block and key lengths $\in \{128, 160, 192, 224, 256\}$
  - Set of 25 block ciphers
  - AES limits block length to 128 and key length to multiples of 64
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- Iteration of a round function with following properties:
  - 4 layers: nonlinear, shuffling, mixing and round key addition
  - All rounds are identical ...
  - ... except for the round keys
  - ... and omission of mixing layer in last round
  - Parallel and symmetric

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  - 4 layers: nonlinear, shuffling, mixing and round key addition
  - All rounds are identical ...
  - ... except for the round keys
  - ... and omission of mixing layer in last round
  - Parallel and symmetric
- Key schedule
  - Expansion of cipher key to round key sequence
  - Recursive procedure that can be done in-place

- Block cipher with block and key lengths $\in \{128, 160, 192, 224, 256\}$
  - Set of 25 block ciphers
  - AES limits block length to 128 and key length to multiples of 64
- We only consider AES in this course
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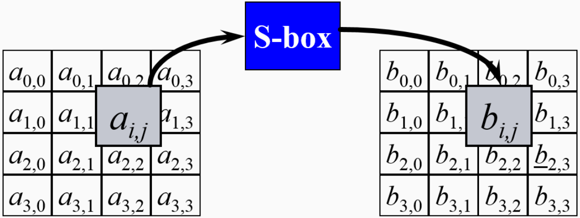
- Manipulates bytes rather than bits

The Non-Linear Layer: SubBytes



- The same invertible S-box applied to all bytes of the state

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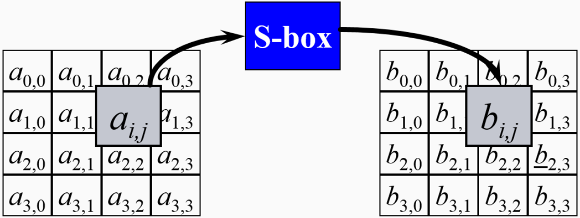

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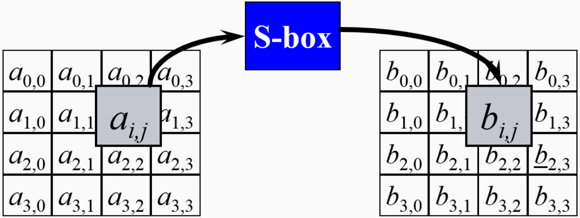
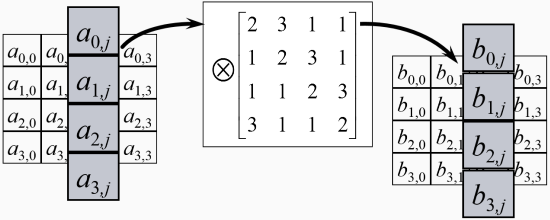


- The same invertible S-box applied to all bytes of the state
- Assembled from building blocks that were proposed and analyzed in cryptographic literature
- Criteria:
  - Offer resistance against DC, LC and *algebraic* attacks . . .
  - . . . when combined with the other layers

The Mixing Layer: MixColumns



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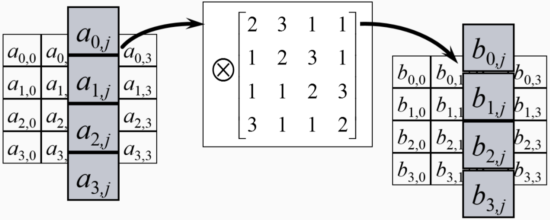

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    - Difference in 1 input byte propagates to 4 output bytes

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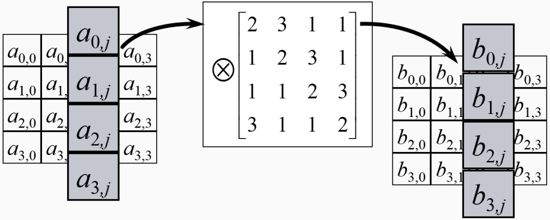

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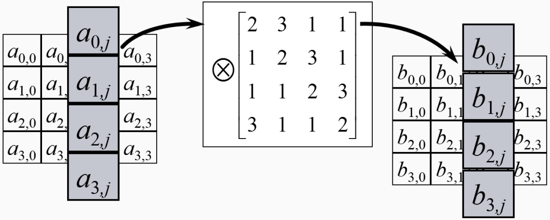

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Slide credit: Joan Daemen

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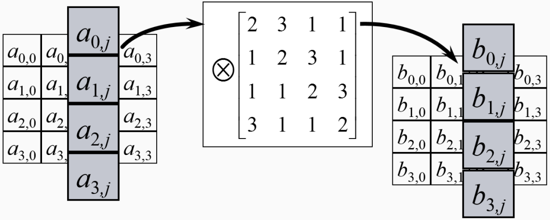

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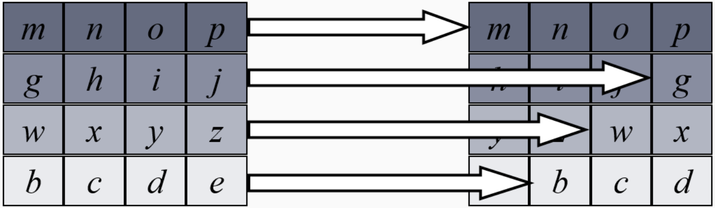
  - Difference in 4 input bytes propagates to 1 output byte

The Shuffling Layer: ShiftRows



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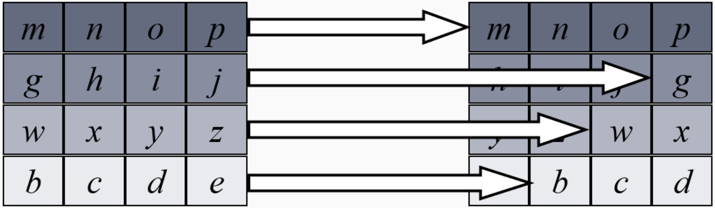

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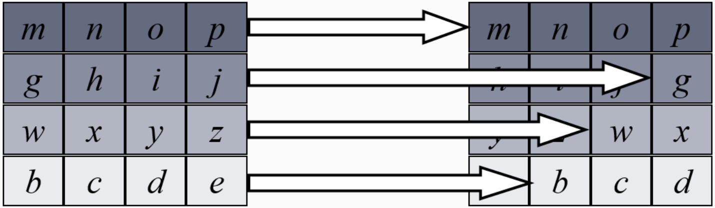

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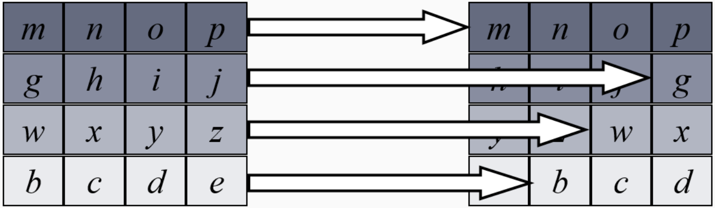


- Each row is shifted by a different amount
- Different shift offsets for higher block lengths
- Moves bytes in a given column to 4 different columns
- Combined with MixColumns and SubBytes this gives fast diffusion

Round Key Addition: AddRoundKey

$$\begin{array}{|c|c|c|c|}\hline a_{0,0} & a_{0,1} & a_{0,2} & a_{0,3} \\ \hline a_{1,0} & a_{1,1} & a_{1,2} & a_{1,3} \\ \hline a_{2,0} & a_{2,1} & a_{2,2} & a_{2,3} \\ \hline a_{3,0} & a_{3,1} & a_{3,2} & a_{3,3} \\ \hline \end{array} + \begin{array}{|c|c|c|c|}\hline k_{0,0} & k_{0,1} & k_{0,2} & k_{0,3} \\ \hline k_{1,0} & k_{1,1} & k_{1,2} & k_{1,3} \\ \hline k_{2,0} & k_{2,1} & k_{2,2} & k_{2,3} \\ \hline k_{3,0} & k_{3,1} & k_{3,2} & k_{3,3} \\ \hline \end{array} = \begin{array}{|c|c|c|c|}\hline b_{0,0} & b_{0,1} & b_{0,2} & b_{0,3} \\ \hline b_{1,0} & b_{1,1} & b_{1,2} & b_{1,3} \\ \hline b_{2,0} & b_{2,1} & b_{2,2} & b_{2,3} \\ \hline b_{3,0} & b_{3,1} & b_{3,2} & b_{3,3} \\ \hline \end{array}$$

- Round key is computed from the cipher key K

Key Schedule: Example with 192-bit Key K

k_0	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}	k_{13}	k_{14}	k_{15}	$\cdots$

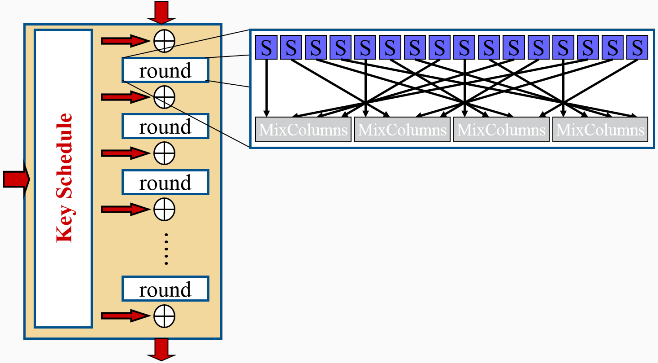
Round key 0	Round key 1	Round key 2	$\cdots$

- Expansion: put K in 1st columns and compute others recursively:

$$k_{6n} = k_{6n-6} \oplus f(k_{6n-1})$$
$$k_i = k_{i-6} \oplus k_{i-1}, \quad i \neq 6n$$

  with f: 4 parallel AES S-boxes followed by 1-byte cyclic shift
- Selection: round key i is columns $4i$ to $4i+3$

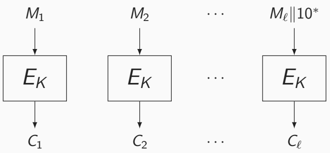
AES: Summary



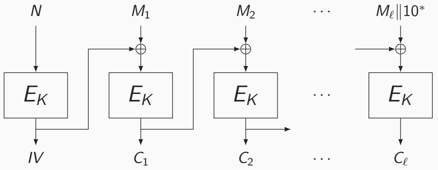
- 10 rounds for 128-bit key, 12 for 192-bit key and 14 for 256-bit key
- Last round has no MixColumns so that inverse is similar to cipher

Block Cipher Based Encryption Modes

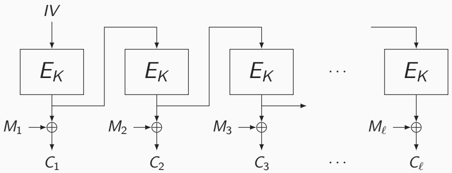
Block Cipher Encryption Modes



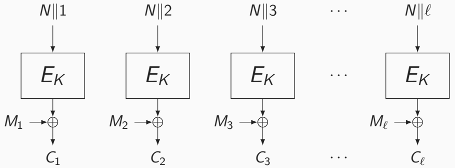
ECB



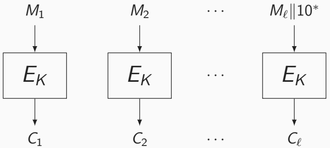
CBC



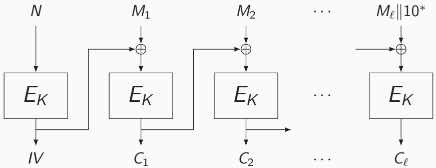
OFB



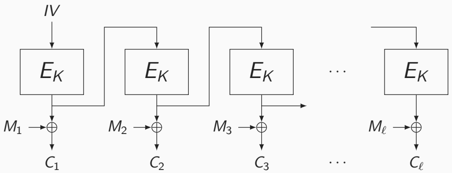
CTR



ECB



CBC



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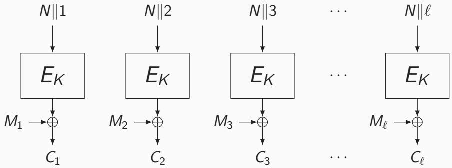

CTR

Open question: advantages/disadvantages?

Overview

	ECB	CBC	OFB	CTR
parallel encryption	✓	—	—	✓
parallel decryption	✓	✓	—	✓
inverse free	—	—	✓	✓
absence of message expansion	—	—	✓	✓
tolerant to bit flips in $C \to P$	—	—	✓	✓
graceful degradation if nonce violation	n/a	✓	—	—

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Next Lectures

- Advanced techniques on how to argue security
- More involved functions such as authenticated encryption
- Standardization efforts (NIST, ISO, CFRG, PKCS)