

Homework 8

Question 1

The height of the tree can be reduced by up to half.

The maximum number of children of any node can be 4.

Question 2

min nodes:

$$2^{(h_b-1)+1} - 1 = 2^{h_b} - 1$$

max nodes:

$$2^{(2h_b-1)+1} - 1 = 4^{h_b} - 1$$

Question 3

1.

1B

1.

1B

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2R

1.

1B

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2R \ CASE IV

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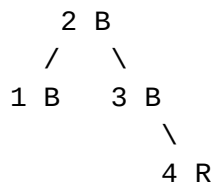
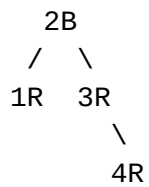
3R \

2B

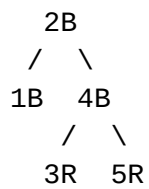
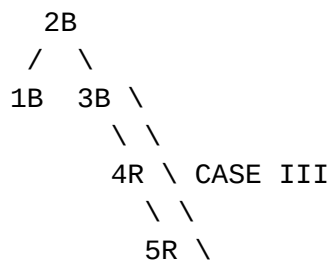
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1R 3R

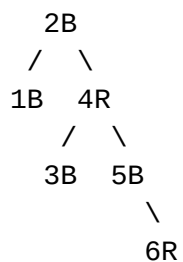
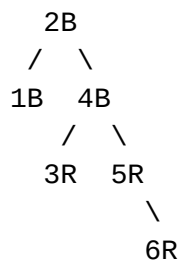
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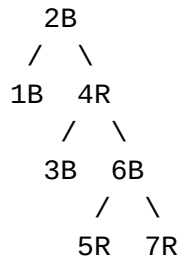
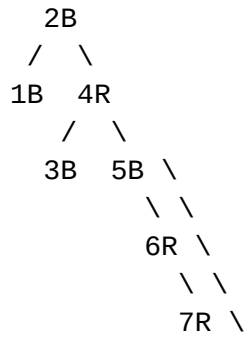
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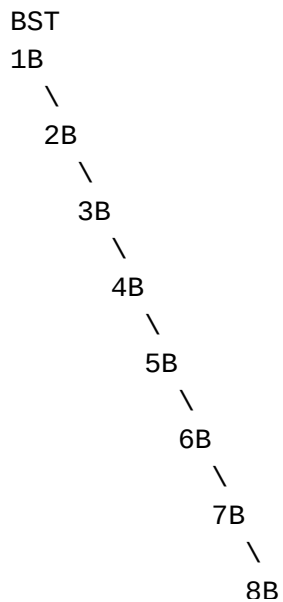
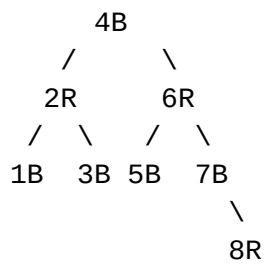
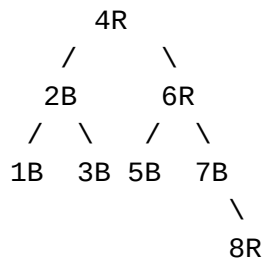
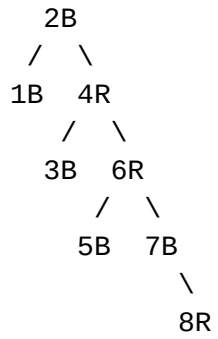
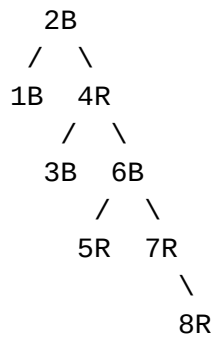
6.



7.



8.



Question 4

For any cycle in G let e denote its max weight edge. If T uses e_M then e can be swapped with any cycle-edge and the tree will remain connected. So e can be swapped for a cheaper tree so T does not use e . Thus T is a minimum spanning tree of G .

Question 5

Consider graph $U, V(G) - U$.

Assume many possible MSTs and fix one, T . An edge of T must cross a cut or the graph is disconnected. Since T spans $V(G)$ it spans U and $V(G) - U$. Every node on both sides of the cut is in the tree. Any edge crossing the cut results in a spanning tree. If any edge other than the minimum is selected the tree will not have minimal cost, since its cost could be reduced by using this edge. There is a unique choice of edge T across any cut.

Question 6

No.

Suppose the shortest path from s to t contains a cycle. Then some vertex v appears at least twice from s to t . Deleting a portion of the walk which loops back to v decreases the length since the graph has positive edge weights.

Question 7

