CSE 2331 Homework 7

1. Recall the height of a tree is the maximum number of edges among all root-to-leaf paths. What are the minimum and maximum numbers of elements of a heap of height h?

2. Argue that for any subtree of a (max) heap, a maximum element occurs at its root.

3. Let H be an array with the (max) heap property. Characterize the range of elements where the minimum value *must* occur. In other words, what is the smallest range of indices where, if checked, one is *gauranteed* to find the minimum.

4. Assume the array $xs[0, \ldots, n-1]$ encodes a heap with at most one node in violation of the (max) heap property. Explain the purpose, function, and correctness of the procedure MAXHEAPIFY assuming node i's children are valid (max) heaps. Do the leaves ever need heapified? In lecture, MAXHEAPIFY was presented as part of HEAPEXTRACTMAX.

```
1   def max_heapify(xs, i, n):
2       while True:
3           li = left(i)
4           ri = right(i)
5
6           maxi = i
7           if li < n and xs[li] > xs[maxi]:
8               maxi = li
9
10          if ri < n and xs[ri] > xs[maxi]:
11              maxi = ri
12
13          if maxi == i:
14              break
15
16          swap(xs, i, maxi)
17          i = maxi
```

5. Let xs be an unsorted array. The procedure BUILDMAXHEAP converts an unsorted array to an a heap. Explain why it works.

```
1   def build_max_heap(xs):
2       n = len(xs)
3       for i in reversed(range(0,n//2)):
4           max_heapify(xs, i, n)
```

6. Explain HEAPSORT.

```
1  def heap_sort(xs):
2      build_max_heap(xs)
3      n = len(xs)
4      for i in reversed(range(0,n)):
5          swap(xs,0,i)
6          max_heapify(xs, 0, i)
```

7. Argue that if a binary search tree node has two children then its successor has no left child and its predecessor has no right child.

8. Argue that the in order walk on n-node binary search trees, performed by starting with TREEMIN followed by repeatedly calling TREESUCCESSOR runs in $\Theta(n)$-time.

9. How are the binary search tree property and max heap property different? Can the max heap property be used to print keys in sorted order in linear time? Why or why not?