

CSE 2331 Homework 8

1. Imagine contracting the edge above each red node in a red-black tree, so that it shrinks to nothing, merging it with its parent. All other edges are maintained. What is the most that this operation can change the height of the tree? What is the maximum number of children of any node?
2. Bound, as tightly as possible, the number of internal (non-nil-leaf) nodes in red-black trees of black-height, h_b .
3. Insert nodes 1 to 8 into a red-black tree, draw the tree before and after any fixup operations. How does the result compare to what happens with INSERT for regular BSTs?
4. Let G be a weighted graph containing a cycle with unique edge weights, and let T be a minimum spanning tree of G . Argue that T is also minimum spanning tree of $G' = (V(G), E(G) - \{e\})$, where e is a cycle edge with maximal weight among all edges on the cycle.
5. Argue that the minimum spanning tree is unique whenever the edge weights are unique. That is, whenever no two edges have the same weight. Hint: consider any cut of the graph.
6. Consider a graph with positive weight edges. Can the shortest path between any pair of vertices contain a cycle? Note we are talking about a cycle in a single path, not if the union of multiple shortest paths may form a cycle. (They can but this is irrelevant.)
7. Draw the shortest path's tree from H and label the nodes with their distances.

