Asymptotic Analysis
Exercises

1. Give the asymptotic complexity of each of the following functions in simplest terms. Your solution should have the form $\Theta(n^\alpha)$ or $\Theta((\log_\mu(n))^\beta)$ or $\Theta(n^\alpha(\log_\mu(n))^\beta)$ or $\Theta(\gamma^{\delta n})$ or $\Theta(1)$ where $\alpha, \beta, \gamma, \delta, \mu$ are constants. (No need to give any justification or proof.)

   (a) $f_a(n) = 6\log_5(n^5 + 3n^3) + 3n^{0.2}$;
   (b) $f_b(n) = \log_3(6n+7) \times \log_2(5n^{0.3}+21) + \log_4((3n+2)(2n+1)(5n+6))$;
   (c) $f_c(n) = 5n^{0.6} + 3n^{0.7}$;
   (d) $f_d(n) = \sqrt{3n^2 + 2n + 74}$;
   (e) $f_e(n) = 2\log_4(4n + 17) + 9\log_5(6n + 8)$;
   (f) $f_f(n) = 6^{13} + 2^6 \times 7\log_4(62)$;
   (g) $f_g(n) = 5\log_2(3n^2 + n + 8) + 7\log_6(3n^3 + 4n^2 + 75)$;
   (h) $f_h(n) = \sqrt{2\log_2(n) + 3 + 7n}$;
   (i) $f_i(n) = 2n\log_3(2n^3 + 17n + 1) + 5n$;
   (j) $f_j(n) = 2\log_3(n) + \sqrt{2n} + 3n$;
   (k) $f_k(n) = 5\log_2(3^n + n^3 + 1)$;
   (l) $f_l(n) = 3(n + 17)\log_5(2n^2 + 17n + 1) + 4n + 64$;
   (m) $f_m(n) = 5^n + 10^n + 15^n$;
   (n) $f_n(n) = 3^{2n} + 2 \times 3^n$;
   (o) $f_o(n) = 7n^2 + 2^{n+5} + 2^{n+9}$;
   (p) $f_p(n) = 3 \times 5^{n+9} + 6 \times 3^{n+9}$;
   (q) $f_q(n) = \sqrt{2n^3 + 3n^2}$;
   (r) $f_r(n) = 3n^3 + 2^{3n} + 7 \times 5^n$;
   (s) $f_s(n) = 9 \times 2^{\log_2(n^2+2n)}$;
   (t) $f_t(n) = (3\log_4(n^2 + 8) + 6\sqrt{n}) \times (7\log_5(2n + 9) + 4\log_3(6n + 7))$;

2. Give an example of a function $f(n)$ such that:

   - $f(n) \in O(n^2)$ and $f(n) \in \Omega(n)$ but $f(n) \notin \Theta(n^2)$ and $f(n) \notin \Theta(n)$.

3. Give an example of a function $f(n)$ such that:

   - $f(n) \in O(n)$ and $f(n) \in \Omega(\log_2(n))$ but $f(n) \notin \Theta(n)$ and $f(n) \notin \Theta(\log_2(n))$.

4. Prove that $7\sqrt{2n^4 + 6n^3 + 5n^2 + 9} \in \Theta(n^2)$ using the definition of $\Theta(n^2)$ as functions $f(n)$ such that $c_1 n^2 \leq f(n) \leq c_2 n^2$ for constants $c_1, c_2 \geq 0$ for all large n.

5. Let $f(n) = 8\log_5(n^3 + 6) \times 7\log_3(n^2 + n)$ and $g(n) = 5(\log_6(5n + 8))^3$. Prove that $f(n) \in O(g(n))$ using $\lim_{n\to\infty} f(n)/g(n)$.