

Probabilistic Analysis II  
Exercises

- (a) What is the asymptotic WORST case running time of functions func1-func3? Justify your solution.  
 (b) What is the asymptotic EXPECTED running time of functions func1-func3? Justify your solution.

Simplify and express your answer as  $\Theta(n^k)$  or  $\Theta(n^k(\log n))$  wherever possible. If the asymptotic running time is exponential, then just give exponential lower bounds.

**Random**( $n$ ) generates a random number between 1 and  $n$  with uniform distribution (every integer between 1 and  $n$  is equally likely.) **CoinFlip**() returns **heads** or **tails** with equal probability.

1.

```

func1(A, n)
  /* A is an array of integers */
1 if (n ≤ 10) then return (A[1]);
2 s ← A[1];
3 for i ← 1 to n do
4   for j ← 2 to n do
5     s ← s + A[i] * A[j];
6     c ← CoinFlip();
7     if (c = tails) then Return(s);
8   end
9 end
10 return (s);
  
```

2.

```

func2(A, n)
  /* A is an array of integers */
1 if (n ≤ 50) then return (A[1]);
2 k ← Random(n - 2);
3 s ← 0;
4 for i ← 1 to n - 8 do
5   for j ← 1 to n - 5 do
6     A[i] ← A[i] + A[i + 3] + A[j + 1];
7     s ← s + A[i];
8   end
9 end
10 s ← s + func2(A, k);
11 return (s);
  
```

3.

```

func3(A, n)
  /* A is an array of integers */
1 if (n ≤ 15) then return (A[1]);
2 s ← 0;
3 for i ← 1 to n - 5 do
4   B[i] ← A[i] + A[i + 3];
5   s ← s + A[i];
6 end
7 k ← Random(⌊n/7⌋);
8 s ← s + func3(A, k);
9 s ← s + func3(B, n-k);
10 return (s);
  
```

4. Consider the following function:

**Input** : Array  $A$  of at least  $j$  elements.

Integers  $i$  and  $j$ .

**Result** : A permutation of the  $i$  through  $j$  elements of  $A$  such that

$A[i] \leq A[i+1] \leq A[i+2] \leq \dots \leq A[j]$ .

```
Sort3(A[,i,j])
1 if (i < j) then
2   p ← NearCenter(A, i, j);
   /* Partition A[i, ..., j] using p s.t. A[s] = p and A[i'] ≤ p ≤ A[j'] for i ≤ i' ≤ s ≤ j' ≤ j
   */
3   s ← Partition(A, i, j, p);
4   Sort3(A[,i,s-1]);
5   Sort3(A[,s+1,j]);
6 end
```

Function **NearCenter** returns an element  $p$  of  $S = \{A[i], A[i+1], A[i+2], \dots, A[j]\}$  whose rank is between  $n/10$  and  $9n/10$ , i.e., at least  $n/10$  elements of  $S$  are less than or equal to  $p$  and at least  $9n/10$  elements of  $S$  are greater than or equal to  $p$ .

Let  $n = i - j + 1$  be the number of elements being sorted.

Assume that function **NearCenter** takes  $cn$  time where  $n = i - j + 1$  time.

Function **Partition** is the function defined in class and in the slides.

What is the asymptotic WORST CASE running time of **Sort3**. Justify your solution.

(You may assume that the more uneven the partition, the longer the combined running times of **Sort3**( $A[,i,s-1]$ ) and **Sort3**( $A[,s+1,j]$ ).)