Analysis of Loops
Exercises

Give the asymptotic running time of each the following functions in Θ notation. Justify your answer. (Show your work.)

1.
```
Func1(n)
1  s ← 0;
2  for i ← 3 to n² do
3      for j ← 7 to 2i⌊log₅(i)⌋ do
4          s ← s + i − j;
5      end
6  end
7  return (s);
```

2.
```
Func2(n)
1  s ← 0;
2  for i ← 3 to ⌊√n⌋ do
3      j ← i³;
4      while (j ≥ i) do
5          s ← s + i − j;
6          j ← j − 4 ;              /* Note: Subtraction */
7      end
8  end
9  return (s);
```

3.
```
Func3(n)
1  s ← 0;
2  for i ← 2n to n² do
3      for j ← i to n² do
4          for k ← 6i to 6i + 21 do
5              s ← s + i − j + k;
6          end
7      end
8  end
9  return (s);
```

4.
```
Func4(n)
1  s ← 0;
2  for i ← ⌊n/2⌋ to ⌊4n√n⌋ do
3      for j ← 3 to i do
4          for k ← j to i do
5              s ← s + i − j + k;
6          end
7      end
8  end
9  return (s);
```

5.

```
Func5(n)
1  s ← 0;
2  i ← 5;
3  while i ≤ n² do
4  |    j ← n;
5  |    while j ≥ 9 do
6  |    |    j ← j − ⌈log₂(n)⌉;
7  |    |    s ← s + i − j;
8  |    end
9  |    i ← i + ⌈√n⌉;
10 end
11 return (s);
```

$$\textbf{Func5}(n)$$
$$1\quad s \leftarrow 0;$$
$$2\quad i \leftarrow 5;$$
$$3\quad \textbf{while } i \leq n^2 \textbf{ do}$$
$$4\quad\quad j \leftarrow n;$$
$$5\quad\quad \textbf{while } j \geq 9 \textbf{ do}$$
$$6\quad\quad\quad j \leftarrow j - \lceil \log_2(n) \rceil;$$
$$7\quad\quad\quad s \leftarrow s + i - j;$$
$$8\quad\quad \textbf{end}$$
$$9\quad\quad i \leftarrow i + \lceil \sqrt{n} \rceil;$$
$$10\quad \textbf{end}$$
$$11\quad \textbf{return } (s);$$

6.

$$\textbf{Func6}(n)$$
$$1\quad s \leftarrow 0;$$
$$2\quad i \leftarrow 5;$$
$$3\quad \textbf{while } i \leq n^{3/2} \textbf{ do}$$
$$4\quad\quad j \leftarrow 7;$$
$$5\quad\quad \textbf{while } j \leq i^3 \textbf{ do}$$
$$6\quad\quad\quad j \leftarrow j + i;$$
$$7\quad\quad\quad s \leftarrow s + i - j;$$
$$8\quad\quad \textbf{end}$$
$$9\quad\quad i \leftarrow i + n;$$
$$10\quad \textbf{end}$$
$$11\quad \textbf{return } (s);$$

7.

$$\textbf{Func7}(n)$$
$$1\quad s \leftarrow 0;$$
$$2\quad i \leftarrow 1;$$
$$3\quad \textbf{while } (i < \lfloor 6n^{3/2} \rfloor) \textbf{ do}$$
$$4\quad\quad \textbf{for } j \leftarrow 1 \textbf{ to } i \textbf{ do}$$
$$5\quad\quad\quad s \leftarrow s + i - j;$$
$$6\quad\quad \textbf{end}$$
$$7\quad\quad i \leftarrow 7 * i ; \qquad\qquad \textit{/* Note: Multiplication */}$$
$$8\quad \textbf{end}$$
$$9\quad \textbf{return } (s);$$

8.

$$\textbf{Func8}(n)$$
$$1\quad s \leftarrow 0;$$
$$2\quad \textbf{for } i \leftarrow n \textbf{ to } 2n^2 \textbf{ do}$$
$$3\quad\quad j \leftarrow 7;$$
$$4\quad\quad \textbf{while } (j < 3i) \textbf{ do}$$
$$5\quad\quad\quad s \leftarrow s + i - j;$$
$$6\quad\quad\quad j \leftarrow 3 * j ; \qquad\qquad \textit{/* Note: Multiplication */}$$
$$7\quad\quad \textbf{end}$$
$$8\quad \textbf{end}$$
$$9\quad \textbf{return } (s);$$

9.

```
Func9(n)
1  s ← 0;
2  i ← n;
3  while (i < 5n³) do
4  │   j ← 3n³;
5  │   while (j > 18) do
6  │   │   s ← s + i − j;
7  │   │   j ← ⌊j/4⌋ ;                    /* Note: Division */
8  │   end
9  │   i ← 4 * i ;                        /* Note: Multiplication */
10 end
11 return (s);
```

10.

```
Func10(n)
1  s ← 0;
2  i ← n;
3  while (i ≤ ⌊n log₅(n)⌋) do
4  │   j ← 9;
5  │   while (j < i²) do
6  │   │   s ← s + i − j;
7  │   │   j ← 3 * j ;                     /* Note: Multiplication */
8  │   end
9  │   i ← i + 4 ;                         /* Note: Addition */
10 end
11 return (s);
```

11.

```
Func11(n)
1  s ← 0;
2  i ← 1;
3  while (i < 3n) do
4  │   j ← 5;
5  │   while (j < n²) do
6  │   │   s ← s + i − j;
7  │   │   j ← j + i ;                     /* Note: Addition */
8  │   end
9  │   i ← (2.5) * i ;                     /* Note: Multiplication */
10 end
11 return (s);
```

12.

```
1  s ← 0;
2  i ← 1;
3  while (i < n²) do
4  │   j ← 2n³;
5  │   while (j > n) do
6  │   │   s ← s + i − j;
7  │   │   j ← j − 3 ;                     /* Note: Subtraction */
8  │   end
9  │   i ← 4 * i ;                         /* Note: Multiplication */
10 end
11 return (s);
```

13.

```
1  for i ← 3 to n² do
2  │   j ← 5;
3  │   while j ≤ i do
4  │   │   for k ← 1 to j do
5  │   │   │   s ← s + i * (j² − k²);
6  │   │   end
7  │   │   j ← 3 * j ;                    /* Note: Multiplication */
8  │   end
9  end
10 return (s);
```

14.

```
1  i ← n³;
2  while i ≥ n do
3  │   j ← 6;
4  │   while j ≤ ⌊i log₂(i)⌋ do
5  │   │   k ← 9;
6  │   │   while k ≤ 2j do
7  │   │   │   s ← s + i * (j² − k²);
8  │   │   │   k ← k + 3 ;               /* Note: Addition */
9  │   │   end
10 │   │   j ← 5 * j ;                   /* Note: Multiplication */
11 │   end
12 │   i ← i − 5 ;                       /* Note: Subtraction */
13 end
14 return (s);
```