CSE 2331 Homework 6

Please do not write on the question sheet. You must show all work to receive any credit.

1. Let $xs = [11, 5, 24, 13, 6, 2, 9, 14, 4, 7, 8]$.

   (a) Draw the execution of PARTITION($xs, 0, 10, 9$) by showing the state of the array xs after each while-loop iteration.[1]

   (b) What is its return value?

2. We saw in class that deterministic and randomized QUICKSORT both have $O(n^2)$ worst-case time.

   (a) What causes the worst-case behavior? You may assume that the pivot is always the first element of the array.

   (b) Why does randomizing the pivot help? Hint: Think about the analysis of randomized QUICKSORT.

3. Suppose we are given a procedure, PICKPIVOT, that takes an array as input and is *guaranteed* to return an element that is a k-th order statistic for some $\frac{1}{4}n \leq k \leq \frac{3}{4}n$ on arrays of size n in worst-case linear-time. Here, suppose it returns the *value* of the pivot and not the index.

   (a) Give pseudocode demonstrating how PICKPIVOT can be used to improve QUICK-SORT and perform worst-case analysis of its running time, assuming distinct values in the input array.[2]

   (b) Give pseudocode demonstrating how PICKPIVOT can be used to find the (lower) median and perform worst-case analysis of its running time assuming distinct values in the input array.

   (c) What, if anything, changes in your analysis if PICKPIVOT's guarantee only holds asymptotically?

4. Suppose we are given a worst-case linear-time (lower) median-finding procedure, HAP-PYMEDIAN. In other words, perhaps your answer to 3b. Give a worst-case linear-time algorithm to find the k-th order statistic. Show your analysis that it runs in worst-case linear-time. Note that this is an improvement over randomized selection because its worst-case running time is linear.

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[1]Please use the pseudocode from the forum post "Partitioning Notes".

[2]I am not asking for you to give pseudocode for QUICKSORT. I ask you to give pseudocode for how PICKPIVOT. I am asking for a modified QUICKSORT that uses PICKPIVOT.