$$P(x^i, y^i|\theta) = p(x^i|y^i\theta)\, p(y).$$

$$= ((2\pi \prod_{j=1}^{D} \sigma_j^2)^{-\frac{1}{2}} \exp(-\sum (2\sigma_j^2)^{-1} (x_j^i - \mu_{kj})^2))(\alpha_{k^i}).$$

$$-\log P(y^{(1)}, x^{(1)}, \ldots, y^{(N)}, x^{(N)}|\theta) = \sum_{i=1}^{N} \Big(\log p(x^i|y^i, \theta) + \log p(y^i|\theta)\Big).$$

$$= -\Big(\log [(\prod_{j=1}^{D} 2\pi\sigma^2)^{\frac{-N}{2}}] + \sum_{i=1}^{N} (\log \alpha_{k^i} - \sum_{n=1}^{N}\sum_{m=1}^{D} (2\sigma_m^2)^{-1} (x_m - \mu_{km}^n)^2\Big)$$

$$= \frac{N}{2} \sum_{j=1}^{D} \log 2\pi\sigma^2 - \sum_{i=1}^{N} \log \alpha_{k^i} + \sum_{n=1}^{N}\sum_{m=1}^{D} \Big((2\sigma_m^2)^{-1} (x_j - \mu_{km}^n)^2\Big)$$

c). $$\frac{\partial(-\log L(\theta))}{\partial \mu_{ki}} = \frac{\partial \sum_{m=1}^{N} \left(\frac{1}{2}\log\left(\prod_{i=1}^{D} 2\pi\theta_i^2\right) + \sum_{i=1}^{D} \frac{1}{2\theta_i^2}(x_i^m - \mu_{y(m),i})^2 - \log \alpha_{y}\right)}{\partial \mu_{ki}}$$

$$= -\sum_{i=1}^{N} \mathbb{1}[y=k](x_{ij} - \mu_{kj})\frac{1}{\sigma^2}$$

$$\frac{\partial(-\log L(\theta))}{\partial \sigma_j^2} = \frac{\partial \sum_{m=1}^{N} \left(\frac{1}{2}\log\left(\prod_{i=1}^{D} 2\pi\theta_i^2\right) + \sum \frac{1}{2\theta_i^2}(x_i^m - \mu_{y(m),i})^2 - \log \alpha_{y}\right)}{\partial \sigma_j^2}$$

$$= \frac{1}{2}\sum_{m}^{N}\sum_{n=1}^{D} \mathbb{1}(n=i)\left[(\sigma_n^2)^{-1} - (\sigma_n^2)^{-2}(x_n^m - \mu_{y^m n})^2\right]$$

$$= \frac{N}{2\sigma_j^2} - \sum_{i=1}^{N}(x_{ij} - \mu_{kj})^2 \frac{1}{2\sigma_j^4}$$