

$$Q1a) \quad Z = (X - Y)^2$$

$$E(Z) = E((X - Y)^2)$$

$$= E(X^2 - 2XY + Y^2)$$

$$= E(X^2) - 2E(X)E(Y) + E(Y^2)$$

$$= \frac{1}{3} - 2E(X)E(Y) + \frac{1}{3}$$

$$= \frac{2}{3} - \frac{2}{4}$$

$$= \frac{1}{6}$$

$$E(X) = E(Y) = \frac{1-0}{2} = \frac{1}{2}$$

$$\text{Var}(X) = \text{Var}(Y) = \frac{(1-0)^2}{12} = \frac{1}{12}$$

$$E(X^2) = E(Y^2) = \text{Var}(X) + E(X)^2$$

$$= \frac{1}{12} + \frac{1}{4}$$

$$= \frac{1}{3}$$

integral formula

$$E(X^3) = \int_0^1 x^3 f(x) dx = \frac{1}{4}$$

$$E(X^4) = \int_0^1 x^4 f(x) dx = \int_0^1 x^4 dx = \frac{1}{5}$$

$$\text{Var}(Z) = E(Z^2) - E(Z)^2$$

$$= E((X - Y)^4) - \left(\frac{1}{6}\right)^2$$

$$= E(X^4) - 4E(X^3Y) + 6E(X^2Y^2) - 4E(Y^3X) + E(Y^4) - \frac{1}{36}$$

$$= 0.2 - 4 \times 0.25 \times \frac{1}{2} + 6 \times \left(\frac{1}{3}\right)^2 - 4 \times 0.25 \times \frac{1}{2} - \frac{1}{36}$$

$$= 0.038$$

$$Q1b) E(R) = E\left(\sum_{i=1}^d z_i\right)$$

$$= \sum_{i=1}^d E(z_i)$$

$$= \sum_{i=1}^d E(z)$$

$$= d E(z)$$

$$\text{Var}(R) = \text{Var}\left(\sum_{i=1}^d z_i\right)$$

$$= \sum_{i,j=1}^d \text{cov}(z_i, z_j)$$

$$= \sum_{i=1}^d \text{cov}(z_i, z_i) + \sum_{i \neq j}^d \text{cov}(z_i, z_j)$$

$$= \sum_{i=1}^d \text{Var}(z_i) + 0$$

z_i, z_j independent
if $i \neq j, \text{cov}(z_i, z_j) = 0$

$$= d \text{Var}(z)$$

```
/anaconda3/bin/python /Users/yufei/PycharmProjects/csc411/hw1/hw1_code.py
```

```
criterion=gini, max depth = 4, score is 0.7868852459016393
```

```
criterion=gini, max depth = 6, score is 0.819672131147541
```

```
criterion=gini, max depth = 8, score is 0.819672131147541
```

```
criterion=gini, max depth = 10, score is 0.8135245901639344
```

```
criterion=gini, max depth = 12, score is 0.8114754098360656
```

```
criterion=gini, max depth = 14, score is 0.8114754098360656
```

```
criterion=entropy, max depth = 4, score is 0.7991803278688525
```

```
criterion=entropy, max depth = 6, score is 0.7889344262295082
```

```
criterion=entropy, max depth = 8, score is 0.7930327868852459
```

```
criterion=entropy, max depth = 10, score is 0.7868852459016393
```

```
criterion=entropy, max depth = 12, score is 0.7848360655737705
```

```
criterion=entropy, max depth = 14, score is 0.7622950819672131
```

```
keyword: trump ;information gain: 0.0082
```

```
Process finished with exit code 0
```


Entropy , max_depth =4

