# Lecture 15: Strong Form of Induction

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# Outline

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- Some burning questions from the mid-course feedback
- Lectures 13-14 review
- Strong form of mathematical induction
- Examples
  - Writing proper proofs

- A repeating note: **make sure you read the textbook**

# Burning Questions & Comments

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- Asynchronous sucks!
- I need the syllabus!
- I need more examples to understand the material!
- The course offered seems to lack structure!
- Why do I have to read the textbook? If I do that, why do I need to listen to the lectures!
- Quizzes are hard! Quiz time is too short!
- The instructor is mean!
- I want my points back!

- The curve will be generous if you are just looking for passing
  - A: ~35%, B+ & B: ~35%, C+, C: 30%
- We are here to help you learn – just ask

# L13-14: Mathematical Induction

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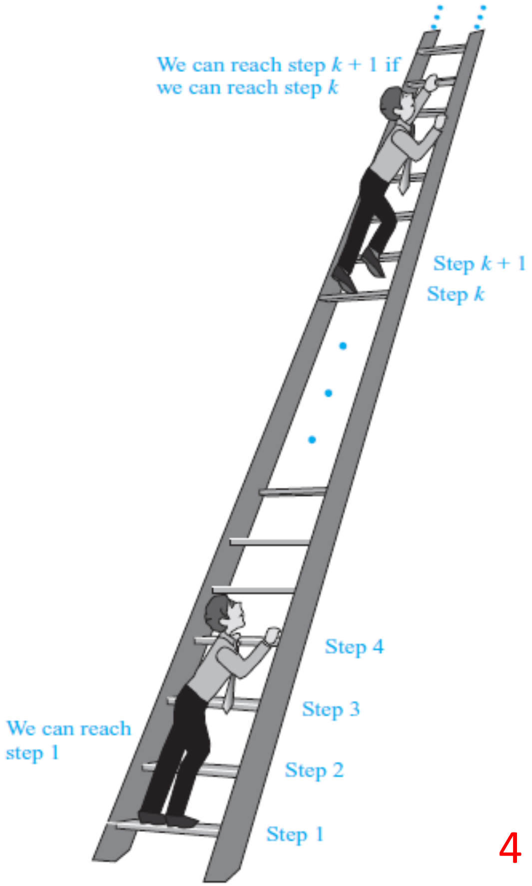
- Let $P(n)$ be a predicate depending on a positive integer $n \geq 1$. To show $P(n)$ is true for all $n$, two steps are needed:
  - (1) Basis step (or base case): verify that $P(1)$ is true.
  - (2) Inductive step: establish $P(k) \rightarrow P(k+1)$ for all $k \geq 1$.
- In (2), $P(k)$ is called the **inductive hypothesis**.



# Strong Induction

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- Let $P(n)$ be a predicate depending on a positive integer $n \geq 1$. To show $P(n)$ is true for all $n$, two steps are needed:
  - (1) Basis step (or base case): verify that $P(1)$ is true.
  - (2) Inductive step: assume $P(i)$ holds for all $1 \leq i \leq k$, show

We need to show

$$P(1) \wedge \cdots \wedge P(k) \rightarrow P(k+1)$$

- Note: we may use all of $P(1), \ldots, P(k)$ in the inductive step to prove that $P(k+1)$ hold. But we do not need to

# Example: Integers as Products of Primes

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$n=2$, $n=3$, $n=4=2\cdot 2$, $n=5$, $n=6=2\cdot 3$, ...

- Show that if $n > 1$ is an integer, then $n$ is a prime or can be written as a product of primes.

Proof: Base Case: $n=2$, 2 is a prime ✓

(2) Assume the statement holds $n=2,3,\dots,k$

To show the statement holds for $n=k+1$, there are two cases.

$1^o$ $n=k+1$ is a prime

$2^o$ $n=k+1$ is not a prime, $k+1=p\cdot q$, $p,q>1$

We have $2\le p\le k$, $2\le q\le k$

Apply IH to $p$ & $q$ $\begin{cases} p=p_1\cdot p_2\cdots p_r \\ q=q_1\cdot q_2\cdots q_s \end{cases}$ prime # prime #

$k+1=p\cdot q=p_1\cdots p_r\cdot q_1\cdots q_s$ ✓

# Example: Postage Composition

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- Show that every postage above 12 cents can be made using 4 and 5 cents stamps.

$12 = 4+4+4$

Proof! (1) Base Case: will show $P(12), P(13), P(14), P(15)$ hold $12 \ne 4+4+4+4+5$

$12 = 4+4+4$, $13 = 5+4+4$, $14 = 5+5+4$, $15 = 5+5+5$

(2) Suppose $P(k), P(k-1), P(k-2), P(k-3), \ldots, P(13)$ hold for $k \geq 13$

For $n = k+1$, $\underline{k+1} = k-3+4$, by $P(k-3)$,
$k-3$ can be composed using 4s and 5s adding one more 4 yields $k+1$

$P(12), P(13), P(14), P(15)$ ✓ $P(k-3) \wedge \wedge P(k) \rightarrow P(k+1)$ $P(16) \Rightarrow P(17)$

# Example: Polygon Triangulation

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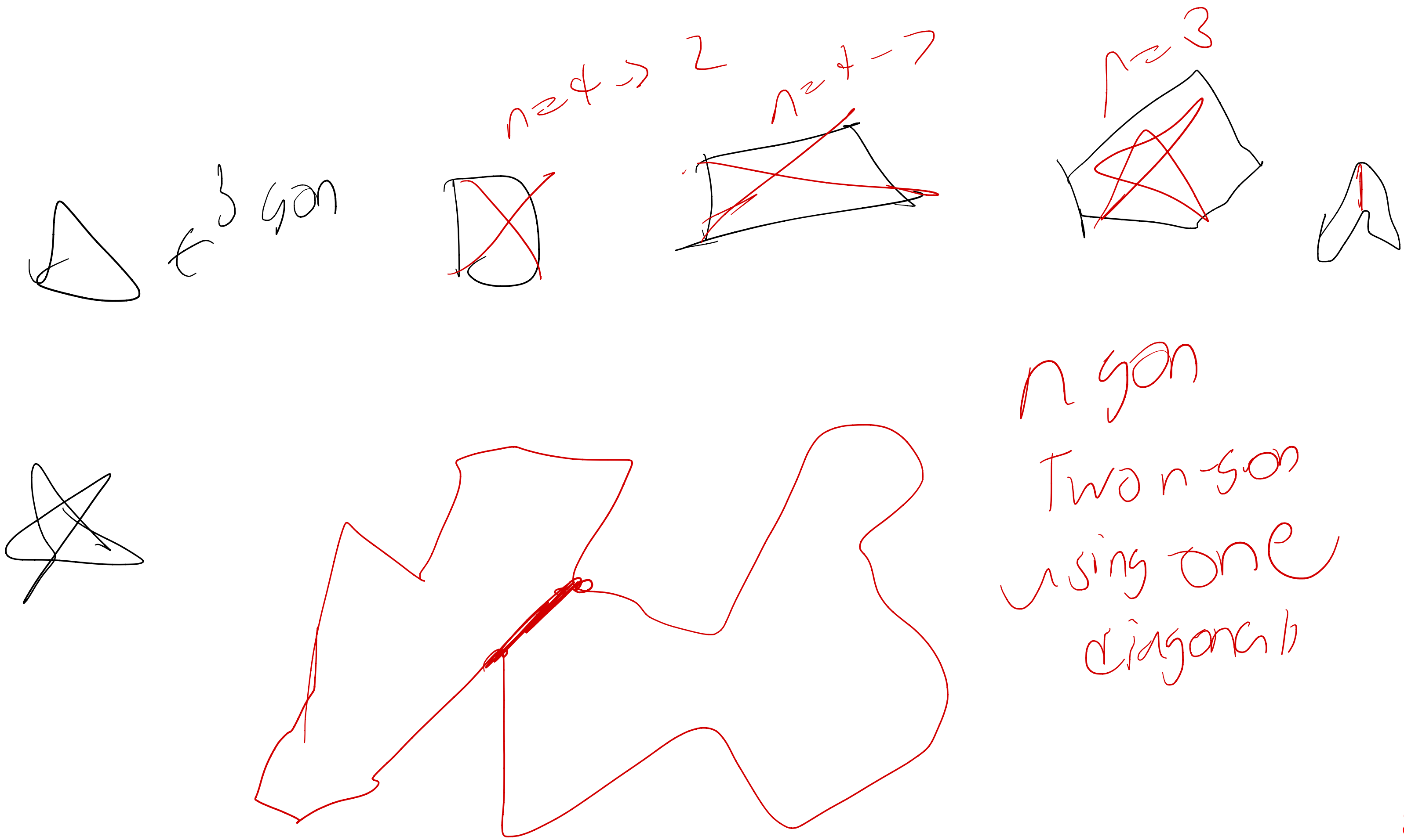
- Every simple polygon with $n \geq 3$ sides can be triangulated into $n - 2$ triangles.
  - Lemma: Every simple polygon with at least four sides has an internal diagonal.



# Example: Polygon Triangulation

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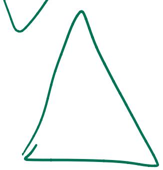
- Every simple polygon with $n \geq 3$ sides can be triangulated into $n-2$ triangles.
  - Lemma: Every simple polygon with at least four sides has an internal diagonal.

Proof. Base Case $n=3$, $3-2=1$ ✓



(2) Assume the statements for $3, 4, 5, \ldots k$

Given a $(k+1)$-gon that is simple, we apply lemma to get a $(p+1)$-gon or $(k+1-p+1)$-gon $2 \leq p$ edges,

$3 \leq p+1 \leq k$, $3 \leq k+1-p-1 \leq k$

Apply IH, $(p+1)$-gon $\Rightarrow p-1$ $\triangle$

$(k+2-p)$-gon $\Rightarrow k-p$ $\triangle$

There are $k-p+p-1 = k-1$ $\triangle$s ✓

$k+1-p \geq 2$