APM466 Assignment 1

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1 2.1 Fundamental Questions (25 points)

1. (a)Governments issue bonds instead of printing money because printing money causes inflation, while bonds allow governments to borrow without destabilizing the currency.

   (b)The long-term part of the yield curve may flatten when investors expect slower economic growth and lower inflation in the future.

   (c)Quantitative easing is a policy where a central bank buys financial assets to lower interest rates, and since COVID-19 the U.S. Federal Reserve has used it by purchasing Treasury securities.

2. **(10 points)** To construct the 0–5 year yield and spot curves, we select 10 Government of Canada bonds with semi-annual coupons and maturities evenly spread across the short to medium term. These bonds are chosen to have similar credit quality, regular coupon structures, and staggered maturities, which ensures consistency across the curve and makes yields comparable at different maturities.

3. **(10 points)** The eigenvalues measure how much of the total variation in the stochastic curve is explained by each principal component, while the eigenvectors describe the shape of the movements across maturities associated with each component.

2.2 Empirical Questions

Question 4(a): Yield to Maturity Curves (10 points)

For each of the selected 10 Government of Canada bonds, we calculate the yield to maturity (YTM) for each trading day. All bonds pay semi-annual coupons, so we compute YTM on a semi-annual compounding basis. For each bond and date, the YTM is obtained numerically by solving the bond pricing equation using a bisection method. Figure 1 shows the 5-year yield curves for each trading day, with all curves plotted on the same graph.

Question 4(b): Spot Curve Construction (15 points)

1. Bootstrapping: Start with the shortest maturity bond.
2. For the first bond, solve directly for the spot rate using its price and cash flow.
3. Move to the next maturity bond.
4. Discount all earlier coupon payments using previously bootstrapped spot rates.
5. Solve for the new spot rate so that the present value equals the observed bond price.
6. Repeat until spot rates for maturities from 1 to 5 years are obtained.

This procedure is applied separately for each trading day.

Figure 2 shows the 5-year spot curves for all dates.

Question 4(c): Forward Rate Curves (15 points)

1. Choose two maturities t and $t + n$ with corresponding spot rates.
2. Use the continuous compounding forward rate formula:

$$F_{t,t+n} = \frac{S_{t+n}(t+n) - S_t t}{n}.$$

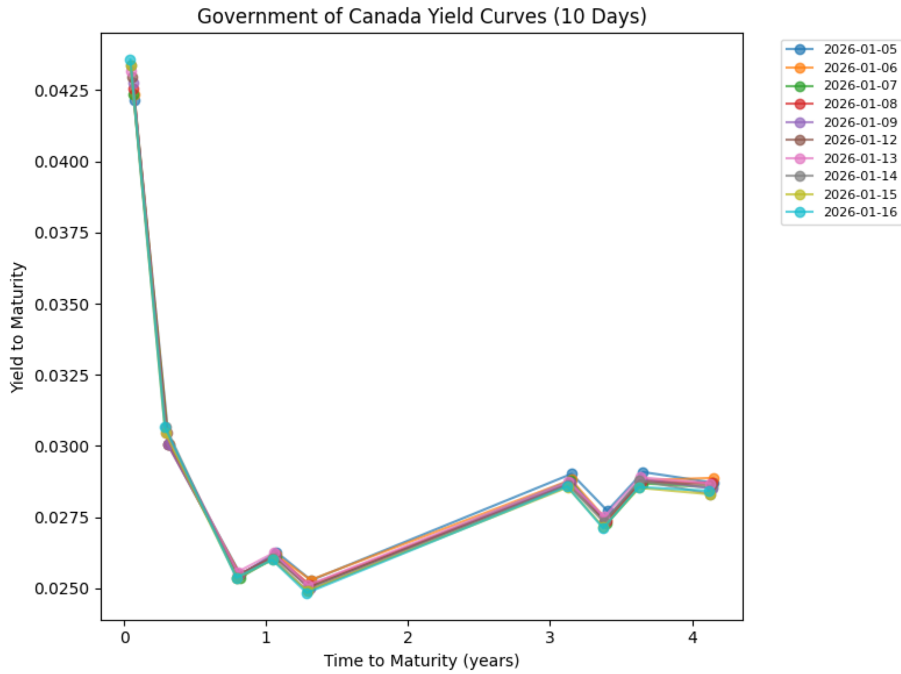

Figure 1: Yield to maturity curves constructed from Government of Canada bond prices over the 10-day sample period.

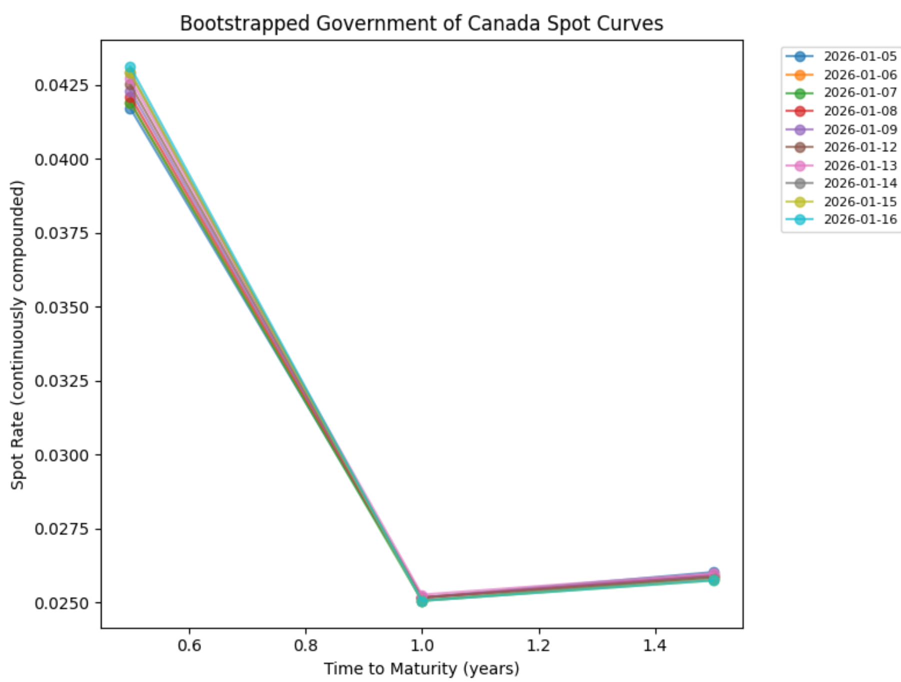

Figure 2: Bootstrapped spot rate curves implied by Government of Canada bond prices.

3. Compute the implied forward rate for the period from t to $t + n$.
4. Repeat for all required maturities.
5. Apply the same steps for each trading day.

In our implementation, we compute 3-month forward rates derived from the spot curves. Figure 3 shows the

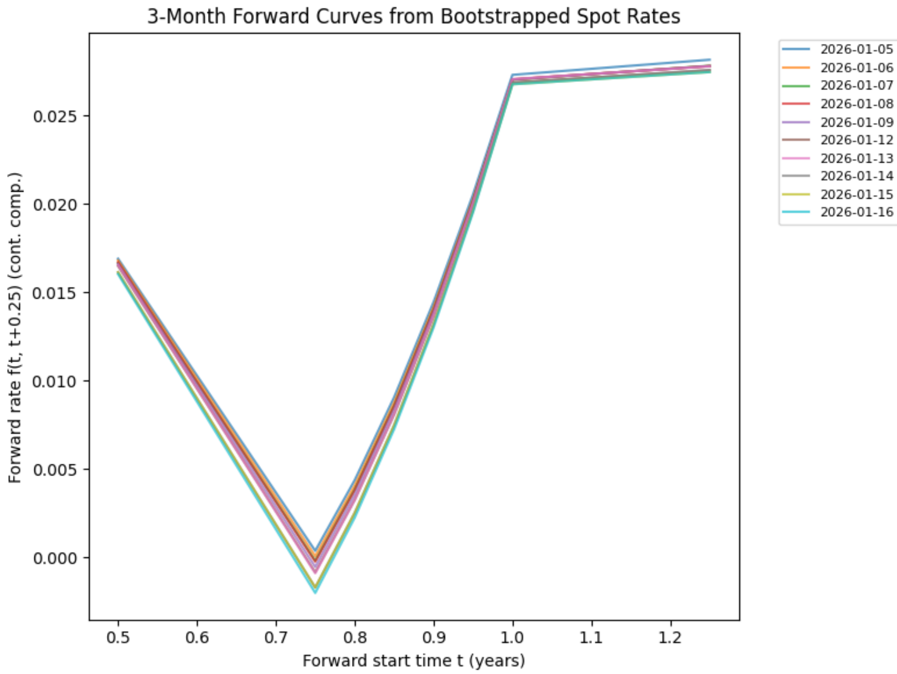

Figure 3: Three-month forward rate curves derived from bootstrapped spot rates.

forward curves for all dates.

Question 5: Covariance Matrices (15 points)

We construct two covariance matrices using the 10-day sample of yields:

(i) the covariance matrix of yield levels, and

(ii) the covariance matrix of daily yield changes.

Daily yield changes are computed as first differences across consecutive trading days. All calculations are performed in Python using the yield data from Question 4.

(i) Covariance matrix of yield levels. The covariance matrix of yield levels shows strong positive covariances across maturities. This reflects the fact that yields at different maturities tend to move together over time. Longer maturities generally exhibit larger covariances, indicating higher variability in yield levels.

(ii) Covariance matrix of daily yield changes. Compared to yield levels, the covariance matrix of daily yield changes has smaller magnitudes. This is expected since differencing removes common level effects and reduces non-stationarity.Nevertheless, positive covariances across maturities remain, suggesting common underlying factors driving yield movements.

Question 6: Eigenvalues and Eigenvectors (5 points)

We compute the eigenvalues and eigenvectors of the covariance matrix of daily yield changes. The eigenvalues measure how much variation is explained by each principal component, while the eigenvectors represent factor loadings across maturities. The results show that the first principal component explains approximately 59% of the total variance. The second and third components explain about 22% and 14%, respectively. Together, the first three components account for over 95% of the total variation in daily yield changes. The loadings of the first component have the same sign across maturities, which corresponds to a parallel shift of the yield curve. The second component reflects changes in the slope, and the third component captures curvature effects. These findings are consistent with standard empirical results in term structure analysis.

References

- Bond price and yield data retrieved from Business Insider Markets: `https://markets.businessinsider.com/bonds/finder?borrower=71&maturity=midterm&yield=&bondtype=2,3,4,16&coupon=&currency=184&rating=&country=19`

Code availability. All Python code and data used in this assignment are available at: `https://github.com/fayeeewang9/APM466-A1`